A Tutorial for G@RCH 2.3, a Complete Ox Package for Estimating and Forecasting ARCH Models

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Abstract

This tutorial documents G@RCH 2.3, an Ox package dedicated to the estimation and forecast of various univariate ARCH-type models in the conditional variance and an AR(FI)MA specification in the conditional mean. These ARCH processes include ARCH, GARCH, EGARCH, GJR, APARCH, IGARCH, FIGARCH, FIEGARCH, FIAPARCH and HYGARCH.

These models can be estimated by Approximate (Quasi-) Maximum Likelihood under four assumptions: normal, Student-t, GED or skewed Student-t errors. Explanatory variables can enter both the conditional mean and the conditional variance equations. One-step-ahead (density) forecasts of both the conditional mean and variance are available as well as some miss-specification tests and several graphical features.

After a brief introduction of the package, we present the G@RCH class member functions. We then propose a theoretical overview of all the specifications both in the conditional mean and the conditional variance where the usefulness of asymmetric and fractionally integrated models is highlighted. Next, further explanations are given about the estimation methods. A concrete application of G@RCH 2.3 is then provided with the daily CAC40 French index. Numerous print-screens are given with this application so that this document can be considered as an user guide. That way, the interface of the program is illustrated and its ease of use is highlighted. Finally, a note on the history of the releases of the G@RCH package as well as a brief list of possible future improvements of the software is given.

Contents

1 Introduction

Well known statistical packages such as Eviews 4.0, Rats 5.0, Microfit 4.0, Matlab 12 or S-Plus 6.0 provide various options to estimate sophisticated econometric models, in completely different areas such as cointegration, panel data, etc.

This paper documents G@RCH 2.3, an Ox package dedicated to the estimation and forecast of various univariate models. Contrary to the softwares mentioned above, G@RCH 2.3 is only concerned with ARCH-type models (Engle, 1982), including some of the most recent contributions in this field: GARCH (Bollerslev, 1986), EGARCH (Nelson, 1991), GJR (Glosten, Jagannathan, and Runkle, 1993), APARCH (Ding, Granger, and Engle, 1993), IGARCH (Engle and Bollerslev, 1986), FIGARCH (Baillie, Bollerslev, and Mikkelsen, 1996a and Chung, 1999), FIEGARCH (Bollerslev and Mikkelsen, 1996), FIAPARCH (Tse, 1998) and HYGARCH (Davidson ,2001) specifications of the conditional variance. Moreover an AR(FI)MA process can be specified in the conditional mean (see Baillie, Chung, and Tieslau, 1996b, Tschernig, 1995, Teyssière, 1997, or Lecourt, 2000, for further details about ARFIMA models)

Our package has been developed with the Ox 3.10 matrix programming language of Doornik (2001)[1]. G@RCH 2.3 should be compatible with a lot of platforms, including Windows, Linux, Unix and Solaris. For most of the specifications, it is generally very fast and one of its main characteristic is its ease of use.

This tutorial is structured as follows: the structure and the functions of the package are detailed in Section 2 while we propose an overview of the package features in Section 3, with the presentation of the models (in the mean and in the variance). Comments over estimation procedures (parameters constraints, distributions, standard deviation estimation methods, tests, forecasting procedures and accuracy of the package) are introduced in Section 4. Then a user guide is provided for both versions of G@RCH 2.3 in Section 5 with an application using the CAC40 French index. Section 6 proposes an history of the releases of G@RCH and future improvements of the package.

1.1 The G@RCH Package

1.1.1 Definition

G@RCH 2.3 is an Ox package dedicated to the estimation and the forecasting of GARCH models and many of its extensions. It can be used via OxPack (with a dialog-oriented interface) or via the classic programming way.

The available models are ARCH (Engle, 1982), GARCH (Bollerslev, 1986), EGARCH (Nelson, 1991), GJR (Glosten, Jagannathan, and Runkle, 1993), APARCH (Ding, Granger, and Engle,

[1]For a comprehensive review of this language, see Cribari-Neto and Zarkos (2001)

1993), IGARCH (Engle and Bollerslev, 1986), FIGARCH (Baillie, Bollerslev, and Mikkelsen, 1996 and Chung, 1999), FIEGARCH (Bollerslev and Mikkelsen, 1996), FIAPARCH (Tse, 1998) and HYGRACH (Davidson, 2001). This package provides a lot of features unavailable in most traditional econometric softwares, including various model specifications, two standard errors estimation methods (Approximate Maximum Likelihood and Approximate Quasi-Maximum Likelihood) and four distributions (normal, Student-t, GED or skewed Student-t). Moreover, explanatory variables can enter the mean and/or the variance equations. Finally, one-step-ahead (density) forecasts of both the conditional mean and variance are available as well as many miss-specification tests (Nyblom, SBT, Pearson goodness-of-fit, Box-Pierce,...).

1.1.2 Program Versions

Two versions of our program are available and called the "Light Version" and the "Full Version".

The "Light Version" can be launched from the Ox file `GarchEstim.ox`. This version requires some experience with the program and its structure. The "Light Version" is therefore dedicated to advanced users. This is also dedicated to users who do not possess the GiveWin software. Indeed, since the OxPack module is only available for registered Ox users, users who have a free (console) Ox version cannot use our dialogs-oriented (or "Full") version. Hence, the "Light Version" is the solution, since its use just requires an Ox executable and a text editor.

The "Full Version" provides the same features as the other version, but also offers a friendly interface and some graphical features. This version needs to be launched from OxPack.

1.2 Disclaimer

This package is functional but no warranty is given whatsoever. The most appropriate way to discuss about problems and issues of the G@RCH package is the Ox-users forum (go to `http://www.mailbase.ac.uk/lists/ox-users` for registration and archives). Suggestions, mistakes, typos and possible improvements can be reported to the authors via e-mail: `S.Laurent@ulg.ac.be` for Sébastien and `jp.peters@ulg.ac.be` for Jean-Philippe.

1.3 Availability and Citation

The G@RCH package is available for downloading at the following address:

`http://www.egss.ulg.ac.be/garch`

This package is free of charge for academic and research purposes. For a commercial use of the program, please contact the authors. Moreover, for easier validation and replication of empirical findings, please cite this documentation (or Laurent and Peters, 2002) in all reports and publications involving the use of G@RCH 2.3.

1.4 Installing and Running G@RCH 2.3

To run G@RCH 2.3, unzip the `garch23.zip` (or `garch23_tut.zip`) file in the *ox/packages* directory (using folders names). A new *Garch23* folder should be automatically created. The files contained in this `garch23.zip` file are listed in `readme.txt`.

If you want to use the "Light Version", you need `garch.oxo` and `garch.h`. The example of section 5 is based on `GarchEstim.ox`. To run it, type

```
oxl GarchEstim
```

at the command prompt. Alternatively, you may use `oxrun`. OxRun can easily be launched from OxEdit, a free text editor[2]. GiveWin is needed to display the graphics but is not mandatory for running the estimation.

To run the "Full Version" of the program, `garch.oxo` and `garch.h` must also be installed. Moreover, you have to possess a registered version of Ox 3.10 Professional, and you need to be sure that GiveWin and OxPack are correctly installed on your computer. See Section 5.2 for more details.

1.5 What's new in this version ?

- Ox 3.10 compatible.
- Improved OxPack interface.
- Allows to run tests on the raw series from OxPack.
- The Hyperbolic GARCH (HYGARCH) of Davidson (2001) is available.
- Improved forecasts, with new graphical options.
- Stationarity and positivity constraints are checked for most the models.
- All the GARCH models and all the tests can be called from external code. See Section 2.3 for further details.
- Possibility to fix some parameters at any defined value. It allows thus to estimate, for instance, an AR(2) model with only lag 2 estimated (by fixing the AR(1) parameter to 0).
- Possibility to save graphics when using the "Light Version" without GiveWin.
- Several minor bugs corrected.

[2]You can download OxEdit at `http://www.oxedit.com`

2 Structure of the Program

2.1 Classes and Functions

Ox provides support for object-oriented programming. An interesting concept is therefore the "Classes". Indeed, one can create new classes based on other existing parent-classes and use the functions of these parents, therefore avoiding to rewrite these procedures for derived classes. In our case, the Garch class is defined as a Modelbase type of class. This Modelbase class derives itself from the Database class.

The Database class is dedicated to the handling of the database, the sample, the names of the variables, the selection of the variables... The Modelbase class implements model estimation features. It is not intended to be used directly but as a base for a more specialized class, such as our Garch class or already available classes such as ARFIMA, DPD (Panel Data estimation), SVPack (Stochastic Volatility models) or SsfPack (State space forms).

See Doornik (2001) for more details about the notion of "Classes".

2.2 G@RCH Member Functions List

Here is the list of the Garch member functions and a brief description for each of them. Our program also uses functions from other classes (Modelbase and Database). New and significantly modified functions for this version are indicated with the † symbol.

Constructor

Garch	Constructor

Model Formulation (used in the "Light Version")

ARCHLAGS	Specifies the desired lags for the Engle's LM test for ARCH
ARFIMA	Specifies if ARFIMA is wanted in the mean
ARMA_ORDERS	Specifies the AR and MA orders in the mean
BOUNDS	Specifies if estimated parameters are bounded with the low and up bounds entered in `startingvalues.txt`
BOXPIERCE	Specifies the desired lags for the Box-Pierce test
COVAR	Specifies if the Variance-Covariance matrix of the estimated parameters is printed in the output
CSTS	Specifies if constants are wanted in the mean and in the variance
DISTRI	Specifies the desired distribution
FIXPARAM(†)	Allows to fix some parameters to their starting values.
FOREGRAPHS	Plots and saves various forecast-related graphs in GiveWin
FORECASTS	Specifies if forecasts are wanted and the number of forecasts

GARCH_ORDERS	Specifies the p and q orders of the GARCH(p, q)
GRAPHS	Plots and saves various estimation related graphs in GiveWin
ITER	Specifies the number of iterations between prints of intermediary results
MLE	Specifies the estimation method of the standard errors
MODEL(†)	Specifies the GARCH-type of models in the conditional variance.
NYBLOM	Specifies if the Nyblom stability test is wanted
PEARSON	Specifies the desired lags for the adjusted Pearson goodness-of-fit test
SAVEPAR	Saves the parameters estimates and their standard errors in an Excel spreadsheet
STORE	Allows storing estimated ε_t, ε_t^2 and σ_t^2 series
TESTS(†)	Allows to run tests either on raw data (prior to estimation) or on the estimated series.
TRUNC(†)	Truncation order for the F.I. models using the method of BBM (96).

Initialization

CheckPara	Check initial values
FixBounds	Fixes the values of the lower and upper bounds of the estimated parameters
InitData	Initializes the characteristics of the model (sample, regressors...)
InitStartValues	Initializes the starting values of the parameters to estimate

Parameters related functions

Dialogs	Parameter starting values (only "Full Version")
GetPara	Constructs the parameters vector
PAR(†)	Creates a matrix with the parameters estimates, their standard errors and their robust s.e.
SplitPara	Allocates the value of each element of the parameters vector to the correct variable
Transform	Computes the transformation of Equation 28

Filters

AParch	APARCH filter
EGarch	EGARCH filter
Garch_Filter	GARCH filter
GJR_Filter	GJR filter
FIEGarch	FIEGARCH filter (the Fractional Integration process uses Chung's method)
Figarch_BBM	FIGARCH filter with the Baillie *et al.* (1996) method (BBM)
Figarch_Chung	FIGARCH filter with the Chung (1999) method

Distributions Related

CD	Computes the Cumulative Distribution Function of the Gaussian distribution

CDFGED	Computes the Cumulative Distribution Function of the GED
CDFTA	Computes the Cumulative Distribution Function of the (skewed) Student-t distribution
GaussLik	Computes the log-likelihood for the Gaussian distribution
GEDLik	Computes the log-likelihood for the GED
INVCDFGED	Computes the Inverse CDF of the GED
INVCDFTA	Computes the Inverse CDF of the (skewed) Student-t distribution
KiAparch	Computes the stationary condition of Equation 18
SkStudentLik	Computes the log-likelihood for the skewed Student distribution
StudentLik	Computes the log-likelihood for the Student distribution

Forecasting

Fig_FOR	Allocates the right forecasts filter depending on the specification
FOR_APARCH(†)	Forecasts filter of the APARCH process
FOR_ARFIMA	Forecasts filter of the ARFIMA process
FOR_ARMA	Forecasts filter of the ARMA process
FOR_EGARCH(†)	Forecasts filter of the EGARCH process
FOR_FIAPARCH_BBM(†)	Forecasts filter of the FIAPARCH process (BBM method)
FOR_FIAPARCH_Chung(†)	Forecasts filter of the FIAPARCH process (Chung method)
FOR_FIEGARCH(†)	Forecasts filter of the FIEGARCH process.
FOR_FIGARCH_BBM(†)	Forecasts filter of the FIGARCH process (BBM method)
FOR_FIGARCH_Chung(†)	Forecasts filter of the FIGARCH process (Chung method)
FOR_GARCH	Forecasts filter of the GARCH process
FOR_GJR	Forecasts filter of the GJR process
FOR_GRAPHS(†)	Draws the forecasts graphics
FORECASTING(†)	Launches the one-step ahead forecast process

General

GetCovar	Gets the covariance matrix of the estimated parameters
GetcT(†)	Gets the number of observations
GetForcData	Gets all the post-estimation data (if any)
GetForErrors	Gets the forecasts errors
GetINames	Gets the name of the realized volatility series (if it exists)
GetNbPar(†)	Gets the number of parameters
GetParNames	Gets the names of the parameters
GetSeries	Returns a matrix with the Y series, the mean residuals and the conditional variance
GetXB	Gets μ_t of Equation 2

GetXBetaForc	Gets the post-estimation values of the regressor(s) in the mean (if any)
GetXNames	Gets the names of the regressors in the mean equation
GetYNames(†)	Gets the name of the dependent variable
GetZB	Gets α_{0t} of Section 3.2.1
GetZBetaForc	Gets the post-estimation values of the regressor(s) in the variance (if any)
GetZNames	Gets the names of the regressors in the variance equation

Model Estimation

DoEstimation	Estimates the model ("Light version")
Estimate	Estimates the model ("Full version")
FigLL	This is the function optimized by BFGS
Filter	Allocates the filter number depending on the model specification
GetRes	Gets the mean residuals
ResVar(†)	Gets the variance residuals
ScoreContributions	Computes the numerical derivatives

Post Estimation

absha	Computes frequencies in an interval defined by upper and lower bounds
APGT	Computes adjusted Pearson Chi-square Goodness-of-fit test (Vlaar and Palm, 1993)
ArchTest	Computes and prints the Engle's LM ARCH test (Engle, 1982)
AUTO	Computes and plots the autocorrelation functions
BoxPQ	Computes and prints the modified Box-Pierce Q-statistics and the associated p-values
confidence_limits	Computes the confidence bounds of a confidence interval from the vector of assumed uniform 0-1 "z series"
FEM(†)	Computes and prints 10 forecasts errors measures
ICriterion	Computes the four Information Criteria (Akaike, Hannan-Quinn, Schwarz and Shibata)
Normality	Computes the skewness, kurtosis and Jarque and Bera (1987) test, with associated t-test and p-values
MLEMeth	Prints the estimated parameters, their standard deviations, t-tests and p-values
MZ(†)	Computes and prints the Mincer and Zarnowitz (1969) regression for the conditional variance
Nyblom	Computes and prints the Nyblom (1989) stability test
Output	Prints the model specification and launches other post-estimation procedures
Positivity(†)	Checks the positivity constraints with the estimated parameters
QuantileGraphics	Plots the quantiles of a distribution
SBT	Computes the sign bias test, the negative size bias test, the positive size bias test

	and a joint test of the three
Stationarity(†)	Checks the stationarity constraints with the estimated parameters
Tests	Launches the selected tests and prints their results
TestGraphicAnalysis	Draws the graphics of the estimation

2.3 G@RCH Members Functions

Garch::AParch, Garch::EGarch,Garch::Garch_Filter, Garch::GJR_Filter, Garch::Figarch_BBM, Garch::Figarch_Chung, Garch::FIEGarch, Garch::FIAParch,

AParch (const e, const level, const p, const q, const par) ;

EGarch (const e, const level, const p, const q, const par, const dist) ;

Garch_Filter (const e, const level, const p, const q, const par) ;

GJR_Filter (const e, const level, const p, const q, const par) ;

Figarch_BBM (const e, const level, const p, const q, const laglamb, const par) ;

Figarch_Chung (const e, const level, const p, const q, const par);

FIEGarch (const e, const level, const p, const q, const laglamb, const par, const dist) ;

FIAParch (const e, const level, const p, const q);

e	in: (m_cT x 1) matrix, residuals series (from the mean).
level	in: (m_cT x 1) or (m_cT x m_cX) matrix, constant + independent variables.
p	in: integer, GARCH order
q	in: integer, ARCH order
laglamb	in: integer, truncation order (BBM method)
par	in: (*# param* x 1) matrix, parameters values (size of the vector depends on the model).
dist	in: integer, distribution used (0: Normal, 1:Student-t, 2: GED, 3: Skewed Student-t)

Return value

A (m_cT x 1) matrix with the estimated conditional variance (σ^2).

Description

These are the filters of the different models. Recall that two methods are available for the FIGARCH models: the Baillie, Bollerslev, and Mikkelsen (1996) method (BBM) that includes a truncation order or the Tse (1998) method that does not. Moreover, the HYGARCH model is launched with the FIGARCH function: the last element of *par* is $ln(\alpha)$. If it is equal to 0, then we have the traditional FIGARCH model. Note also that *level* is a (m_cT x 1) vector of ones if there is no independent variable and a (m_cT x m_cXV) matrix equals to $b'X$ if there are explanatory variables in the variance equation. Finally, the arguments of the functions are sufficient to call them from an external Ox code. Indeed no class member is used in these procedures anymore.

Here is an example of an Ox code that creates randomly a series with 1000 obseravtions, runs a GARCH(1,1), a GJR(1,1) and an APARCH(1,1) models on it (all with a normal distribution) and then displays the graphs of the estimated conditional variance series (in GiveWin):

```
decl garchobj; decl p, q, alpha, beta, apa, gjr ;

decl parameters,res, level ;

decl garch, gjr, aparch;

garchobj = new Garch();
res = rann(1000,1) ;
level = ones(1000,1);
p = 1 ;
q = 1 ;
alpha = 0.05 ;
beta = 0.90 ;
gjr = -0.15 ;

apa = <0.15;1.5> ;
parameters = alpha|beta ;

garch = garchobj.Garch_Filter(res, level, p, q, parameters) ;
gjr = garchobj.GJR_Filter(res, level, p, q, parameters|gjr) ;
aparch = garchobj.APArch(res, level, p, q, parameters|apa) ;

Draw(0, garch');
DrawTitle(0, "Garch");
Draw(1, gjr');
DrawTitle(1, "GJR");
Draw(2, figarch');
DrawTitle(2, "Aparch");
ShowDrawWindow() ;

delete garchobj ;
```

The elements of the *par* argument have to be entered in a specific order. See the *GetPara* function for more details on the order.

Garch::APGT
APGT(const cd, const ng, const np) ;

cd	in: (m_cT x 1) matrix, values of the cumulative distribution function.
ng	in: integer, number of classification groups
np	in: integer, number of parameters

Return value
1 if the test is successfully run, 0 otherwise.
Description
Computes and prints adjusted Pearson χ^2 goodness-of-fit test (Vlaar and Palm, 1993). See Section 4.3 for more detail about this test.

Garch::ARCHLAGS
ARCHLAGS(const lags) ;
No return value
Description
Fix the lags wanted when computing Engle's LM test for ARCH processes. By default, *lags* is $< 2; 5; 10 >$. This means that three Engle's LM tests for ARCH are computed, with lags 2, 5 and 10. If *lags* is <>, the test will not be reported.

Garch::ARMA_ORDERS, Garch::GARCH_ORDERS
ARMA_ORDERS(const cAR, const cMA) ;
GARCH_ORDERS(const cP, const cQ) ;

cAR	in: integer, AR order, p
cMA	in: integer, MA order, q
cP	in: integer, GARCH order, p
cQ	in: integer, ARCH order, q

No return value
Description
Fixes the ARMA and GARCH orders.

Garch::AUTO
AUTO(const z, const ncor, const min, const max, const plot) ;

z	in: (n x 1) matrix, series to be tested
ncor	in: integer, maximum lag of the autocorrelation
min, max	in: integer, coordinates of the Y-axis (for the 4 graphs)
plot	in: integer, area wherein the first graph is plotted

No return value
Description
Computes and plots the autocorrelations (with maximum lag = *ncor*).

Garch::BOUNDS
BOUNDS(const method) ;

method	in: 1 if bounded parameters wanted, 0 otherwise.

No return value
Description
If *method* is 1, the estimated parameters are bounded between (editable) lower and upper values defined in `startingvalues.txt`. If it is equal to 0, the parameters are not bounded.

Garch::BOXPIERCE
BOXPIERCE(const lags) ;
No return value
Description
Fixes the lags wanted when computing Box-Pierce statistics. By default, *lags* is $< 5; 10; 20 >$. This means that BP(5), BP(10) and BP(20) are computed for the standardized residuals and squared standardized residuals. If *lags* is <>, the test will not be reported.

Garch::BoxPQ
BoxPQ(const eh, const e2h, const ncor, const adjM, const adjV, const sq) ;

eh	in: (m_cT x 1) matrix, series to be tested.
e2h	in: (m_cT x 1) matrix, squared series.
ncor	in: (m_cT x 1) matrix, vector of lags.
adjM	in: integer, number of degrees of freedom to be subtracted for eh
adjV	in: integer, number of degrees of freedom to be subtracted for e2h
sq	in: if 0, test on the residuals, if 1, test on the squared residuals

Return value

1 if the test is successfully run, 0 otherwise.

Description

Computes and prints Box-Pierce Q-statistics on standardized residuals and/or squared standardized residuals. See Section 4.3 for more details about this test.

Garch::CD

CD (const e, const var, const dis, const par) ;

e	in: (m_cT x 1) matrix, the residuals series.
var	in: (m_cT x 1) matrix, the variance series.
dis	in: integer, the distribution (0:normal, 1:Student, 2:GED or 3:skewed Student).
par	in: (2 x 1) matrix, $< log(\xi); \upsilon >$, i.e. the asymmetry coefficient and the degree of freedom.
	par is <> if dist = 0 and $log(\xi) = 0$ if dist = 1 or 2.

Return value

A (m_cT x 1) vector with the CD.

Description

Computes the CDF of ϵ/σ for the selected distribution.

Garch::CDFGED, Garch::CDFTA

CDFGED (const ee, const nu) ;

CDFTA (const ee, const logxi, const nu) ;

ee	in: (m_cT x 1) matrix, the series.
logxi	in: double, logarithm of the skewness parameter.
nu	in: double,degree of freedom (kurtosis parameter).

Return value

A ($rows(ee)$ x 1) matrix with the cdf of the selected distribution.

Description

Computes the cdf of the GED (CDFGED) and the skewed Student (CDFTA).

Garch::COVAR

COVAR(const p) ;

p	in: 1 or 0, to print the variance-covariance matrix of the estimated parameters.

No return value

Description

If *p* is 1, the variance-covariance matrix of the estimated parameters is printed in the output.

Garch::Dialogs

Dialogs() ;

No return value

Description

Only used with OxPack. Launches dialogs related to the starting values. There are two possibilities: the *"Parameter-by-parameter"* dialog or the *"Vector"* dialog. The former is launched if the user has previously selected the *"Manually (Individual Form)"* option in OxPack while the latter is available when having chosen *"Manually (Vector Form)"*.

Garch::DISTRI

DISTRI(const dist) ;

dist	in: integer (0, 1, 2 or 3), distribution selection

No return value

Description

Selects the distribution. If *dist* is 0, the Gaussian (Normal) distribution is selected, if it is 1, it is the Student-t distribution, if it is 2, it is the Generalized Error distribution (GED) and if it is 3, it is the skewed Student-t.

Garch::DoEstimation, Garch::Estimate

DoEstimation(const vStart) ;

Estimate () ;

vStart	in: (*m_cPar* x 1) matrix, starting values of the parameters to be estimated.

Return value

1 if model successfully estimated, 0 if it failed.

Description

$DoEstimation$ is launched from the "Light version" of the program while $Estimate$ is launched from the "Full version". These procedures are the core procedures of the program. They successively launch others procedures to initialize parameters, to estimate the formulated model, to print the results, to run the tests or to display the graphics.

Garch::FEM

FEM(const forc, const obs) ;

forc	in: (*m_cTforc* x 2) matrix, mean forecast $\sim$ variance forecast
trunc	in: (*m_cTforc* x 2) matrix, observed series (*m_Yfor*) $\sim$ observed variance (*m_Hfor*)

Return value

1 if the tests are successfully run, 0 otherwise.

Description

Computes and prints 10 forecast error measures: Mean Squared Error (MSE), Median Squared Error (MedSE), Mean Error (ME), Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), Heteroskedastic Mean Squared Error (HMSE), Mean Absolute Percentage Error (MAPE), Adjusted Mean Absolute Percentage Error (AMAPE), Percentage Correct Sign (PCS), Theil Inequality Coefficient (THEIL) and Logarithmic Loss Function (LL). See Brooks, Burke, and Persand (1997) for more details about these measures.

Garch::FigLL

FigLL(const vP, const adFunc, const avScore, const amHessian);

vP	in: (m_cP x 1) matrix, parameters to be estimated
	out: estimated parameters
adFunc	in: addresse
	out: double, log-likelihood function value at vP
avScore	in: 0, or an address
	out: if not 0 on input, (m_cP x 1) matrix with first derivatives at vP
amHessian	in: 0, as MaxBFGS does not require the Hessian.

Return value

1 if successful, 0 if evaluation failed.

Description

This procedure is optimized by Ox with the MaxBFGS function. This function uses the Broyden, Fletcher, Goldfarb and Shanno (BFGS) quasi-Newton method (see Doornik, p.240, for further details).

Garch::Filter

Filter() ;

Return value

1 if successful, 0 if failure.

Description

Allocates the correct filter to the specified model.

Garch::FixBounds

FixBounds(const m);

m	in: (18 x 2) matrix, bounds values in `startingvalues.txt`.

Return value

1 if successful, 0 if allocation failed.

Description

This allocates the upper and lower bounds for the parameters to the class member m_mBound variable. Note the these bounds are editable by changing default values in `startingvalues.txt`. See Section 4.1 for more details.

Garch::FixParam

FixParam(const cfix, const fix);

cfix	in: integer, 1 to enable this option, 0 to disable
fix	in: (k x 1) vector of 1's (fixed) or 0's (estimated)

No return value

Description

When cfix=1, this procedure fixes several of the k parameters to their starting values. For instance, in an ARCH(1) model their are by default 3 parameters to be estimated. When using $DoEstimation(<0.01;0.01;0.5>)$, to launch the estimation, the ARCH parameter is initialized at 0.5. However, when setting $FixParam(1,<0;0;1>)$, the ARCH parameter is not estimated and fixed at 0.5.

This option thus allows to estimate an AR(2)-ARCH(1) model with the estimation of only the lag 2 of the AR. 5 parameters are considered here (two constants, two AR parameters and the ARCH parameter). To do so, one has to select $FixParam(1,<0;1;0;0;0>)$ for the AR(1) parameter to be fixed and $DoEstimation(<0.01;0;0.1;0.01;0.5>)$ to fix it to 0.

Garch::FOR_APARCH, Garch::FOR_EGARCH, Garch::FOR_FIGARCH_BBM, Garch::FOR_FIGARCH_Chung, Garch::FOR_GARCH, Garch::FOR_GJR

FOR_APARCH(const e, const h, const p, const q, const alpha, const beta, const gamma, const delta, const level_forc, const Ki);

FOR_EGARCH(const e, const h, const p, const q, const alpha, const beta, const theta1, const theta2, const level_forc, const dist);

FOR_GARCH(const e, const h, const p, const q, const alpha, const beta, const level_forc);

FOR_GJR(const e, const h, const p, const q, const alpha, const beta, const leverage, const level_forc, const prob_neg);

FOR_FIAPARCH_BBM(const e, const hh, const p, const q, const d, const alpha, const beta, const gamma, const delta, const level_forc, const laglamb, const Ki);

FOR_FIAPARCH_Chung(const e, const hh, const p, const q, const d, const alpha, const beta, const gamma, const delta, const level_forc, const Ki);

FOR_FIEGARCH(const e, const h, const p, const q, const d, const alpha, const beta, const theta1, const theta2, const level_forc, const dist, const laglamb);

FOR_FIGARCH_BBM(const e, const h, const p, const q, const d, const alpha, const beta, const level_forc, const laglamb);

FOR_FIGARCH_Chung(const e, const h, const p, const q, const d, const alpha, const beta, const level_forc);

e	in: (m_cT x 1) matrix, pre-forecasts residual values
h	in: (m_cT x 1) matrix, pre-forecasts conditional variance
p	in: integer, GARCH order.
q	in: integer, ARCH order.
alpha	in: (q x 1) matrix, ARCH coefficients.
beta	in: (p x 1) matrix, GARCH coefficients.
leverage	in: (q x 1) matrix, asymmetry coefficients of the GJR (ω_i in Equation 16).
gamma	in: (q x 1) matrix, asymmetry coefficients of the APARCH (γ_i in Equation 17).
delta	in: double, standard deviation exponent (δ in Equation 17).
theta1	in: (q x 1) matrix, sign effect of the EGARCH (θ_1 in Equation 12).
theta2	in: (q x 1) matrix, magnitude effect of the EGARCH (θ_2 in Equation 12).
Ki	in: (q x 1) matrix, output of KiAparch(dist, q, par, delta, gamma)
prob_neg	in: double, probability that $\epsilon < 0$ (it equals 0.5 for the symmetric distributions and $1/(1+\xi^2)$ for the skewed Student).
dist	in: integer, selected distribution (0: Normal, 1: Student, 2: GED, 3: Skewed Student)
level_forc	in: (m_cTforc x 1) or (m_cTforc x m_cXM) matrix, it is the level, computed as $b * X_i (i = 1, ..., m\_cTforc)$
laglamb	in: integer, truncation order (BBM method).

Return value

A (m_cTforc x 1) matrix with the forecasts of the variance.

Description

These are the forecasting procedures. *level_forc* is a (*m_cTforc* x 1) vector of ones if there is no independent variable and a (m_cTforc x m_cXV) matrix equals to $b'X$ if there are explanatory variables in the variance.

Garch::FOR_ARMA, Garch::FOR_ARFIMA

FOR_ARMA (const y_l, const p, const q, const arma, const level_forc, const e) ;
FOR_ARFIMA (const y_l, const p, const q, const d, const arma, const level_forc, const e) ;

y_l	in: (m_cT x 1) matrix, with $y\_l = y - b * X$ (X is a matrix of explanatory variables and b the estimated parameters).
p	in: integer, AR order.
q	in: integer, MA order.
arma	in: (1 x (p+q)) matrix, AR coefficients followed by MA coefficients.
level_forc	in: (*m_cTforc* x 1) matrix, independent variables.
e	in: (m_cT x 1) matrix, residuals series.
d	in: double, long memory coefficient.

Return value

A (*m_cTForc* x 1) matrix with the forecasts of the mean.

Description

These are the filters for the mean equation. They compute forecasts for ARMA and ARFIMA specifications (without explanatory variables).

Garch::FOR_GRAPHS

FOR_GRAPHS(const plot, const pre, const type, const valcrit) ;

plot	in: integer, area wherein the first graph is plotted
pre	in: integer, number of pre-observations
type	in: integer, type of confidence intervals (0: none, 1: bands, 2: bars, 3: fans).
valcrit	in: double, critical value for the confidence interval (forecasts $\pm$ *valcrit* x std.err.).

No return value

Description

Displays graphics of the mean forecasts (with or without confidence intervals) and/or the variance forecasts and/or the forecasted series and/or the observed series in the GiveWin front-end.

Garch::FORECAST

FORECAST(const i, const nbForc, const iprint) ;

i	in: 1 to compute forecasts, 0 otherwise.
nbForc	in: integer, number of forecasts.
iprint	in: 1 to print the forecasts, 0 otherwise.

No return value

Description

If *i* is 1, one-step ahead forecasting will be executed. The number of forecasts is given by the value of $nbForc$. These forecasts will be printed in the output if *iprint* is 1.

Garch::FORCASTING

FOR_STAT_1() ;

Return value

m_mForc, a (m_cTForc x 2) matrix containing the forecasts for the mean and for the variance.

Description

This procedure launches the forecasts for the mean, then for the variance and allocates different filters depending on the specification of the model.

Garch::FOREGRAPHS

FOREGRAPHS(const d, const s, const file) ;

d	in: 1 to draw forecasts graphics, 0 otherwise.
s	in: 1 to save forecasts graphics, 0 otherwise.
file	in: string, name of the EPS file containing the saved graphics.

No return value

Description

This function calls FOR_GRAPHS to draw forecasts graphics in GiveWin when using the "Light" version. It also allows to save these graphs in a EPS (Encapsulated PostScript) file.

Garch::Garch

Garch() ;

No return value

Description

Constructor.

Garch::GaussLik, Garch:: GEDLik, Garch::StudentLik, Garch::SkStudentLik

GaussLik(const vE, const vSigma2) ;
GEDLik(const vE, const vSigma2, const a) ;
StudentLik(const vE, const vSigma2, const v) ;
SkStudentLik(const vE, const vSigma2, const s, const v) ;

vE	in: (m_cT x 1) matrix, residuals.
vSigma2	in: (m_cT x 1) matrix, conditional variance.
a	in: double, asymmetric coefficient.
s	in: double, skewness parameter.
v	in: double, degree of freedom (kurtosis parameter).

Return value

The log-likelihood function value associated with vE and vSigma2.

Description

Computes the log-likelihood functions of the four available distributions.

Garch::GetForcData, Garch::GetXBetaForc, Garch::GetZBetaForc;

GetForcData(const iGroup, const cTforc);
GetXBetaForc(const cTforc);
GetZBetaForc(const cTforc);

iGroup	in: name of the variable group.
cTforc	in: number of forecasts.

Return value

(m_cTforc x 1) matrix containing the realized values of the regressors (in the mean or in the variance).

Description

GetForcData collects the realized values matrix of all the regressor(s) for the forecasting period $[t+1\ ;\ t+m\_cForc]$. GetXBetaForc and GetZBetaForc do the same for the regressor(s) in the mean and the regressor(s) in the variance, respectively.

Garch::GetINames, Garch::GetParNames, Garch::GetXNames, Garch::GetYNames, Garch::GetZNames;

GetINames();
GetParNames();
GetXNames();
GetYNames();
GetZNames();

Return value

Array of strings with the names of the variables or parameters.

Description

These procedures collect the names of the series (Y), the realized volatility (I), the regressors in the mean equation (X) or in the variance equation (Z) or the estimated parameters (Par).

Garch:: GetPara;

GetPara();

Return value

1 if successful, 0 if failure.

Description

It constructs the parameters vector and allocates it to *m_vPar*. The order of the parameters is the following: constant in mean (1 item), regressors in mean coefficients (m_cXM items), d coefficient (1 item), AR coefficients (p items), MA coefficients (q items), constant in var. (1 item), regressors in variance coefficients (m_cXV items), F.I.(d) coefficient (1 item), GARCH coefficients (p items), ARCH coefficients (q items), GJR coefficients (q items), EGARCH coefficients (q x 2 items), APARCH coefficients (q + 1 items), GED degrees of freedom (1 item) and Student degrees of freedom (1 item).

Garch::GetRes

GetRes(const y, const x);

y	in: (m_cT x 1) matrix, dependent variable
x	in: (m_cT x m_cX) matrix, regressors

Return value

(m_cT x 1) matrix containing the residuals.

Description

Computes the residuals of the mean equation without taking into account the AR(FI)MA process.

Garch::GRAPHS
GRAPHS(const d, const s, const file) ;

d	in: 1 to draw graphics of the estimation, 0 otherwise.
s	in: 1 to save graphics of the estimation, 0 otherwise.
file	in: string, name of the EPS file with the saved graphics.

No return value
Description

This function calls *Test_Graphic_Analysis* to draw various graphics resulting from the estimation. These graphs are displayed in GiveWin. *GRAPHS* also allows to save these graphs in a EPS (Encapsulated PostScript) file.

Garch::ICriterion
ICriterion(const LogL, const n, const q) ;

LogL	in: double, the value of the log-likelihood function.
n	in: integer, number of observations.
q	in: integer, number of parameters.

Return value

1 if the test is successfully run, 0 otherwise.

Description

Computes and prints four information criteria : the Akaike, Schwarz, Shibata and Hannan-Quinn tests. See Section 4.3 for more details about this test.

Garch::InitData
InitData() ;
Return value

1 if successful, 0 if failure.

Description

Initializes the model by allocating the Y series and the regressors to class members, computing the number of observations of the sample and the number parameters to be estimated.

Garch::InitStartValues
InitStartValues(const init_par, const init_bounds) ;

init_par	in: 1 if default starting values used, 0 otherwise.
init_bounds	in: 1 if bounded parameters used, 0 otherwise.

No return value
Description
Initializes the starting values when the user do not enter any specific starting values. These values are:

- Constant in the mean: 0.05
- Regressors in the mean: 0.01
- ARFIMA(p,d,q): $p_1 = 0.2$, $p_{>1} = 0.05$, $d = 0.1$, $q_1 = 0.15$, $q_{>1} = 0.02$
- Constant term in the variance equation: 0.01
- GARCH: $\beta_1 = 0.7$ (if GARCH) or 0.45 (if FIGARCH), $\beta_{>1} = 0.1$.
- ARCH: $\alpha_i = 0.1$
- FIGARCH: $d = 0.5$
- GJR: $\omega_i = 0.01$
- EGARCH: $\phi_1 =$ -0.1 and $\phi_2 = 0.2$
- APARCH: $\delta = 1.2$, $\gamma_1 = 0.15$, $\gamma_{>1} = 0.05$
- skewed Student distribution: $\xi = 0.01$ (asymmetry coefficient).
- Student distribution: $\upsilon = 6$ (degrees of freedom).
- GED distribution: $\upsilon = 2$.

All these values can easily be modified by the user just by editing the `startingvalues.txt` file. This file is automatically installed in the *.../ox/packages/garch* directory when unzipping `garch22.zip`. In this `startingvalues.txt` file, one may also change lower and upper bounds of all the parameters.

Garch::INVCDFGED, Garch::INVCDFTA
INVCDFGED (const p, const nu) ;
INVCDFTA (const p, const logxi, const nu);

p	in: double, probability.
logxi	in: double, logarithm of the skewness parameter.
nu	in: double,degree of freedom (kurtosis parameter).

Return value
The solution of the integral equation $p = F(x|logxi, nu)$.
Description
Computes the the inverse cdf of the GED (INVCDFGED) and the skewed Student (INVCDFTA).

Garch::ITER
Integrate(const i) ;

i	in: integer, number of iterations between intermediary prints.

No return value

Description

With G@RCH, it is possible to print intermediary results of the estimation. This function allows the user to easily select the number of iteration between printing results. For instance, if i is 10, intermediary values of the parameters and the log-likelihood function will be printed every 10 iterations. When i is 0, no intermediary result is printed.

Garch::MLE

MLE(const method);

method	in: integer, method selection.

No return value

Description

Selection of the estimation method. If method = 0, both (Approximate) Maximum Likelihood and Quasi-Maximum Likelihood Estimates will be computed. If method = 1, only MLE is selected and if it is equal to 2, only QMLE are computed.

Garch::MLEMeth

MLEMeth(const par, const parnames, const title, const nbpar);

par	in: (m_cPar x 1) matrix, estimated parameters
parnames	in: array of m_cPar strings, estimated parameters names
title	in: string, selected method name
nbpar	in: integer, number of parameters (m_cPar)

No return value

Description

Prints the estimated parameters, their standard deviations, t-tests and p-values with their names. Depending on the user's choice, ML estimates, QML estimates or both will be printed.

Garch::MODEL

MODEL(const mod) ;

mod	in: integer, used to select the Garch model.

No return value

Description

The argument takes a value between 1 and 11 to select the ARCH model to be used in the conditional variance:

1:	GARCH	6:	FIGARCH (BBM)
2:	EGARCH	7:	FIGARCH (Chung)
3:	GJR	8:	FIEGARCH

4:	APARCH	9:	FIAPARCH (BBM)
5:	IGARCH	10:	FIAPARCH (Chung)
		11:	HYGARCH

Garch::MZ
MZ(const HFor, const MatFor, const nbFor) ;

HFor	in: blabla
MatFor	in: blabla
nbFor	in: integer, number of forecasts

Return value
1 if the tests are successfully run, 0 otherwise.
Description
Computes and prints the Mincer-Zarnowitz regression on the forecasted volatility. See Section 4.3 for more details about this regression.

Garch::Normality
Normality(const e) ;

e	in: (m_cT x 1) matrix, series to be tested.

Return value
1 if the test is successfully run, 0 otherwise.
Description
Computes and prints skewness, excess kurtosis and Jarque-Bera normality test with the associated adjusted t-statistics and p-values. See Section 4.3 for more details about this test.

Garch::NYBLOM
NYBLOM(const i) ;

i	in: 1 or 0, to compute the Nyblom test.

No return value
Description
If i is 1, the parameters stability test of Nyblom (1989) is computed.

Garch::Nyblom
Nyblom(const eh, const grad) ;

eh	in: (m_cTx1) matrix, parameters to be tested.
grad	in: (m_cParx1) matrix, gradients.

Return value

1 if the test is successfully run, 0 otherwise.

Description

Computes and prints the Nyblom test. This checks the constancy of parameters over time. See Nyblom (1989) and Lee and Hansen (1994) for more details.

Garch::Output

Output() ;

No return value

Description

Prints the specification of the formulated model and launches the standard errors computations.

Garch::PAR

PAR() ;

Return value

A (m_cPar x 3) matrix structured as ($m\_vPar \sim m\_vStdErrors \sim m\_vRobStdErrors$)

Description

Returns a (m_cPar x 3) matrix with the parameters estimates, their standard errors and their robust standard errors. It is used together with $SAVEPAR$ to store estimation results of a model in an external file (Microsoft Excel spreadsheet).

Garch::PEARSON

PEARSON(const lags) ;

lags	in: (l x 1) matrix, vector containing the l wanted lags for the test. It must have the following form:$< lag_1; lag_2; ...; lag_l >$

No return value

Description

Fixes the lags wanted when computing the adjusted Pearson goodness-of-fit test (see Section 4.3 for more details). By default, *lags* is $< 40; 50; 60 >$.

Garch::SAVEPAR

SAVEPAR(const i, const file);

i	in: 0: store nothing, 1: stores parameters estimates, 2: stores estimates and std.errors, 3: stores estimates, std.errors and robust std. errors.
file	in: string, name of the Excel file wherein the values will be stored.

Return value

1 if successful, 0 otherwise.

Description

Allows to store optimized parameters estimates, their standard errors and their robust standard errors in a *.xls* file (Excel spreadsheet) for further analysis.

Garch::SBT
SBT(const res, const cvar) ;

res	in: (m_cT x 1) matrix, residuals.
cvar	in: (m_cT x 1) matrix, conditional variance.

Return value
1 if the test is successfully run, 0 otherwise.
Description
Computes and prints Sign Bias Test, negative Size Bias Test, positive Size Bias Test and joint Test for the three effects described in Engle and Ng (1993). See Section 4.3 for more details about this test.

Garch::SplitPara
SplitPara(vP) ;

vP	in: (m_cPar x 1) matrix, parameters vector.

Return value
1 if successful, 0 if failure.
Description
Splits the parameters vector and allocates each one to the corresponding class member.

Garch::STORE
STORE(const res, const res2, const condv, const mfor, const vfor, const name, const file);

res	in: 1 or 0, to store the residuals.
res2	in: 1 or 0, to store the squared residuals.
condv	in: 1 or 0, to store the conditional variance.
mfor	in: 1 or 0, to store the mean forecasts.
vfor	in: 1 or 0, to store the variance forecasts
name	in: string, suffix added to "Res_", "SqRes_", "CondV_", "MeanFor_" or "VarFor_" to name the saved series.
file	in: if 0, saves as a new *.xls* file. If 1, saves as a new *.in*7 file.

No return value
Description
Stores the residuals, the squared residuals and the conditional variance of the estimated models, but also the forecasted mean and variance. Argument 6 provides a default suffix ("01") that can be modified. If argument 7 equals 0 (default value), the series will be stored in a new *.xls* file

(Microsoft Excel spreadsheet). If it is equal to 1, the series will be stored in a new *.in*7 file (GiveWin database).

Garch::Tests
Tests() ;
No return value
Description
Runs the selected tests.

Garch::TESTS
TESTSONLY(const p, const a) ;

p	in: 0 or 1.
a	in: 0 or 1.

No return value
Description
Allows to run the test either for the raw series, prior to any estimation ($p = 1$) or for the estimated series, after the optimization ($a = 1$).

Garch::TestGraphicAnalysis
TestGraphicAnalysis(const ser, const res, const sqres, const h, const plot) ;

ser	in: 1 or 0; 1 if raw series graph wanted.
res	in: 1 or 0; 1 if residuals graph wanted.
sqres	in: 1 or 0; 1 if squared residuals graph wanted.
h	in: 1 or 0; 1 if cond.variance graph wanted.
plot	in: integer, area wherein the first graph is plotted

No return value
Description
Displays graphics of the series and/or the residuals and/or the squared residuals and/or the conditional variance in the GiveWin front-end.

Garch::Transform
Transform(const cpar, const bornes, const meth);

cpar	in: (m_cPar x 1) matrix, estimated parameters
bornes	in: (m_cPar x 2) matrix, upper and lower bounds
meth	in: 1, 2 or 3.

Return value

(m_cPar x 1) matrix containing the transformed parameters.

Description

Transforms the values of the estimated parameters to take account of the lower and upper bounds of these parameters (see Section 4.1 for details). When *meth* is 0, the constrained parameters are transformed into unconstrained parameters. The opposite is obtained by equalling *meth* to 1. If *meth* is 2, unconstrained standard errors of the estimated parameters are transformed into constrained ones.

Garch::TRUNC

TRUNC(const t) ;

trunc	in: integer, truncation order (this argument is only used with BBM's approach)

No return value

Description

It is related to the fractionally integrated (FI) model selection. If the estimation method follows BBM's (1996) specification, the value of t will be used as the truncation order. If the estimation method follows Chung's (1999) specification, t will equal all the previous observations. One notable difference (among others) between these two methods is indeed that BBM uses a fix number of lags to compute the binomial expansion (Taylor's theorem) while Chung proposes an increasing number of lags (it includes thus all previous observations). The FIAPARCH model is estimated with the FIGARCH procedure. Here is an example for a FIAPARCH(1,d,1) with the method of BBM:

```
Figarch_BBM(sqrt(G),level,p,q,laglamb,alpha|beta|d|hy).^(2/delta)
```

with `G = |residuals| - gamma * (residuals.^delta)`

OxPack Functions

OxPack related functions are described in Doornik (2001).

3 Features of the package

Our attention will be first devoted to review the specifications of the conditional mean equation. Then, some recent contributions in the ARCH modelling framework will be presented.

3.1 Mean equation

Let us consider an univariate time series y_t. If Ω_{t-1} is the information set at time $t-1$, we can define its functional form as:

$$y_t = E(y_t|\Omega_{t-1}) + \varepsilon_t, \tag{1}$$

where $E(.|.)$ denotes the conditional expectation operator and ε_t is the disturbance term (or unpredictable part), with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t \varepsilon_s) = 0, \forall\ t \neq s$.

This is the mean equation which has been studied and modelled in many ways. Two of the most famous specifications are the Autoregressive (AR) and Moving Average (MA) models. Mixing these two processes and introducing n_1 explanatory variables in the equation, we obtain this ARMAX(n, s) process,

$$\begin{aligned} \Psi(L)(y_t - \mu_t) &= \Theta(L)\varepsilon_t \\ \mu_t &= \mu + \sum_{i=1}^{n_1} \delta_i x_{i,t}, \end{aligned} \tag{2}$$

where L is the lag operator[3], $\Psi(L) = 1 - \sum_{i=1}^{n} \psi_i L^i$ and $\Theta(L) = 1 + \sum_{j=1}^{s} \theta_j L^j$. To start the recursion, it is convenient to set the initial conditions as $\varepsilon_t = 0$ for all $t \leq \max\{p, q\}$.

Several studies have shown that the dependent variable (interest rate returns, exchange rate returns, etc.) may exhibit significant autocorrelation between observations widely separated in time. In such a case, we can say that y_t displays long memory, or long-term dependence and is best modelled by a fractionally integrated ARMA process (so called ARFIMA process) initially developed in Granger (1980) and Granger and Joyeux (1980) among others.[4] The ARFIMA(n, ζ, s) is given by:

$$\Psi(L)(1-L)^{\zeta}(y_t - \mu_t) = \Theta(L)\varepsilon_t, \tag{3}$$

where the operator $(1-L)^{\zeta}$ accounts for the long memory of the process and is defined as:

$$\begin{aligned} (1-L)^{\zeta} &= \sum_{k=0}^{\infty} \frac{\Gamma(\zeta+1)}{\Gamma(k+1)\,\Gamma(\zeta-k+1)} L^k \\ &= 1 - \zeta L - \frac{1}{2}\zeta(1-\zeta)L^2 - \frac{1}{6}\zeta(1-\zeta)(2-\zeta)L^3 - \ldots \\ &= 1 - \sum_{k=1}^{\infty} c_k(\zeta) L^k, \end{aligned} \tag{4}$$

[3]Recall that $L^k y_t = y_{t-k}$.

[4]ARFIMA models have been combined with an ARCH-type specification by Baillie, Chung, and Tieslau (1996), Tschernig (1995), Teyssière (1997), Lecourt (2000) and Beine, Laurent, and Lecourt (2000).

with $0 < \zeta < 1$, $c_1(\zeta) = \zeta, c_2(\zeta) = \frac{1}{2}\zeta(1-\zeta)$, ... and $\Gamma(.)$ denoting the Gamma function (see Baillie, 1996, for a survey on this topic). The truncation order of the infinite summation is set to $t-1$.

It is worth noting that Doornik and Ooms (1999) recently provided an Ox package for estimating, forecasting and simulating ARFIMA models. However, in opposition to our package, they assume that the conditional variance is constant over time.

3.2 Variance equation

The ε_t term in Eq. (1)-(3) is the innovation of the process. Two decades ago, Engle (1982) defined as an Autoregressive Conditional Heteroscedastic (ARCH) process, all ε_t of the form:

$$\varepsilon_t = z_t \sigma_t, \tag{5}$$

where z_t is an independently and identically distributed (*i.i.d.*) process with $E(z_t) = 0$ and $Var(z_t) = 1$. By definition, ε_t is serially uncorrelated with a mean equal to zero, but its conditional variance equals σ_t^2 and, therefore, may change over time, contrary to what is assumed in the standard regression model.

The models provided by our program are all ARCH-type.[5] They differ on the functional form of σ_t^2 but the basic principles are the same. Besides the traditional ARCH and GARCH models, we focus mainly on two kinds of models: the asymmetric models and the fractionally integrated models. The former are defined to take account of the so-called "leverage effect" observed in many stock returns, while the latter allows for long-memory in the variance. Early evidence of the "leverage effect" can be found in Black (1976), while persistence in volatility is a common finding of many empirical studies; see for instance Bera and Higgins (1993), Bollerslev, Chou, and Kroner (1992) or Palm (1996) for excellent surveys on ARCH models.

3.2.1 ARCH Model

The ARCH (q) model can be expressed as:

$$\begin{aligned} \varepsilon_t &= z_t \sigma_t \\ z_t &\sim i.i.d.\ D(0,1) \\ \sigma_t^2 &= \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2, \end{aligned} \tag{6}$$

where $D(.)$ is a probability density function with mean 0 and unit variance (it will be defined in Section 4.2).

The ARCH model can describe volatility clustering. The conditional variance of ε_t is indeed an increasing function of the square of the shock that occurred in $t-1$. Consequently, if ε_{t-1} was large in absolute value, σ_t^2 and thus ε_t is expected to be large (in absolute value) as well. Notice

[5] For stochastic volatility models, see Koopman, Shepard, and Doornik (1998).

that even if the conditional variance of an ARCH model is time-varying ($\sigma_t^2 = E(\varepsilon_t^2|\psi_{t-1})$), the unconditional variance of ε_t is constant and, provided that $\omega > 0$ and $\sum\limits_{i=1}^{q} \alpha_i < 1$, we have:

$$\sigma^2 \equiv E(\varepsilon_t^2) = \frac{\omega}{1 - \sum\limits_{i=1}^{q} \alpha_i}. \tag{7}$$

Note also that the ARCH model can explain part of the excess kurtosis that we observe in financial time series. As shown by Engle (1982) for the ARCH(1) case under the normality assumption, the kurtosis of ε_t is equal to $\frac{3(1-\alpha_1^2)}{1-3\alpha_1^2}$. The kurtosis is thus finite if $\alpha_1 < \frac{1}{3}$ and larger than 3 (the kurtosis of a standard normal distribution) if $\alpha_1 > 0$.

The computation of σ_t^2 in Eq. (6) depends on past (squared) residuals (ε_t^2), that are not observed for $t = 0, -1, \ldots, -q+1$. To initialize the process, the unobserved squared residuals have been set to their sample mean.

In the rest of the paper, ω is assumed fixed. If n_2 explanatory variables are introduced in the model, $\omega_t = \omega + \sum\limits_{i=1}^{n_2} \omega_i x_{i,t}$ with an exception for the exponential models (EGARCH and FIEGARCH) where $\omega_t = \omega + \ln\left(1 + \sum\limits_{i=1}^{n_2} \omega_i x_{i,t}\right)$.

Finally, σ_t^2 has obviously to be positive for all t. Sufficient conditions to ensure that the conditional variance in Eq. (6) is positive are given by $\omega > 0$ and $\alpha_i \geq 0$. However, these conditions are not necessary as shown by Nelson and Cao (1992). Furthermore, when explanatory variables enter the ARCH equation, these positivity constraints are not valid anymore (even if the conditional variance still has to be non-negative).

3.2.2 GARCH Model

Early empirical evidence has shown that a high ARCH order has to be selected to catch the dynamics of the conditional variance (thus involving the estimation of numerous parameters). The Generalized ARCH (GARCH) model of Bollerslev (1986) is an answer to this issue. It is based on an infinite ARCH specification and it allows to reduce the number of estimated parameters by imposing non-linear restrictions on them. The GARCH (p, q) model can be expressed as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2. \tag{8}$$

Using the lag or backshift operator L, the GARCH (p, q) model is:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2, \tag{9}$$

with $\alpha(L) = \alpha_1 L + \alpha_2 L^2 + \ldots + \alpha_q L^q$ and $\beta(L) = \beta_1 L + \beta_2 L^2 + \ldots + \beta_p L^p$.

If all the roots of the polynomial $|1 - \beta(L)| = 0$ lie outside the unit circle, we have:

$$\sigma_t^2 = \omega \left[1 - \beta(L)\right]^{-1} + \alpha(L) \left[1 - \beta(L)\right]^{-1} \varepsilon_t^2, \tag{10}$$

which may be seen as an ARCH(∞) process since the conditional variance linearly depends on all previous squared residuals. In this case, the conditional variance of y_t can become larger than the unconditional variance given by:

$$\sigma^2 \equiv E(\varepsilon_t^2) = \frac{\omega}{1 - \sum\limits_{i=1}^{q} \alpha_i - \sum\limits_{j=1}^{p} \beta_j},$$

if past realizations of ε_t^2 are larger than σ^2 (Palm, 1996).

As in the ARCH case, some restrictions are needed to ensure σ_t^2 to be positive for all t. Bollerslev (1986) shows that imposing $\omega > 0$, $\alpha_i \geq 0$ (for $i = 1, \ldots, q$) and $\beta_j \geq 0$ (for $j = 1, \ldots, p$) is sufficient for the conditional variance to be positive. In practice, the GARCH parameters are often estimated without the positivity restrictions. Nelson and Cao (1992) argued that imposing all coefficients to be nonnegative is too restrictive and that some of these coefficients are found to be negative in practice while the conditional variance remains positive (by checking on a case-by-case basis). Consequently, they relaxed this constraint and gave sufficient conditions for the GARCH$(1, q)$ and GARCH$(2, q)$ cases based on the infinite representation given in Eq. (10). Indeed, the conditional variance is strictly positive provided $\omega \left[1 - \beta(1)\right]^{-1} > 0$ is positive and all the coefficients of the infinite polynomial $\alpha(L) \left[1 - \beta(L)\right]^{-1}$ in Eq. (10) are nonnegative. The positivity constraints proposed by Bollerslev (1986) can be imposed during the estimation (see 4.1). If not, these constraints, as well as the ones implied by the ARCH(∞) representation, will be tested a posteriori and reported in the output.

3.2.3 EGARCH Model

The Exponential GARCH (EGARCH) model is introduced by Nelson (1991). Bollerslev and Mikkelsen (1996) propose to re-express the EGARCH model has follows:

$$\ln \sigma_t^2 = \omega + \left[1 - \beta(L)\right]^{-1} \left[1 + \alpha(L)\right] g(z_{t-1}). \tag{11}$$

The value of $g(z_t)$ depends on several elements. Nelson (1991) notes that, *"to accommodate the asymmetric relation between stock returns and volatility changes (...) the value of $g(z_t)$ must be a function of both the magnitude and the sign of z_t".*[6] That is why he suggests to express the function $g(.)$ as

$$g(z_t) \equiv \underbrace{\gamma_1 z_t}_{sign\ effect} + \underbrace{\gamma_2 [|z_t| - E|z_t|]}_{magnitude\ effect}. \tag{12}$$

$E|z_t|$ depends on the assumption made on the unconditional density of z_t. Indeed, for the normal distribution,

$$E\left(|z_t|\right) = \sqrt{\frac{2}{\pi}}. \tag{13}$$

[6]Note that with the EGARCH parameterization of Bollerslev and Mikkelsen (1996), it is possible to estimate an EGARCH $(p, 0)$ since $\ln \sigma_t^2$ depends on $g(z_{t-1})$, even when $q = 0$.

For the skewed Student distribution,

$$E\left(|z_t|\right) = \frac{4\xi^2}{\xi + \frac{1}{\xi}} \frac{\Gamma\left(\frac{1+\upsilon}{2}\right)\sqrt{\upsilon - 2}}{\sqrt{\pi}(\upsilon - 1)\Gamma\left(\frac{\upsilon}{2}\right)} \tag{14}$$

where $\xi = 1$ for the symmetric Student.

For the GED, we have

$$E\left(|z_t|\right) = \lambda_{upsilon} 2^{\frac{1}{\upsilon}} \frac{\Gamma\left(\frac{2}{\upsilon}\right)}{\Gamma\left(\frac{1}{\upsilon}\right)}. \tag{15}$$

ξ, υ and $\lambda_{upsilon}$ concern the shape of the non-normal densities and will be defined in Section 4.2.

Note that the use of a ln transformation of the conditional variance ensures that σ_t^2 is always positive.

3.2.4 GJR Model

This popular model is proposed by Glosten, Jagannathan, and Runkle (1993). Its generalized version is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} (\alpha_i \varepsilon_{t-i}^2 + \gamma_i S_{t-i}^- \varepsilon_{t-i}^2) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{16}$$

where S_t^- is a dummy variable.

In this model, it is assumed that the impact of ε_t^2 on the conditional variance σ_t^2 is different when ε_t is positive or negative. The TGARCH model of Zakoian (1994) is very similar to the GJR but models the conditional standard deviation instead of the conditional variance. Finally, Ling and McAleer (2002) has proposed, among other stationarity conditions for GARCH models, the conditions of existence of the second and fourth moment of the GJR.

3.2.5 APARCH Model

This model has been introduced by Ding, Granger, and Engle (1993). The APARCH (p, q) model can be expressed as:

$$\sigma_t^\delta = \omega + \sum_{i=1}^{q} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}\right)^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta, \tag{17}$$

where $\delta > 0$ and $-1 < \gamma_i < 1$ $(i = 1, ..., q)$.

This model couples the flexibility of a varying exponent with the asymmetry coefficient (to take the "leverage effect" into account). The APARCH includes seven other ARCH extensions as special cases:[7]

- The ARCH of Engle (1982) when $\delta = 2,\ \gamma_i = 0$ $(i = 1, \ldots, p)$ and $\beta_j = 0$ $(j = 1, \ldots, p)$.
- The GARCH of Bollerslev (1986) when $\delta = 2$ and $\gamma_i = 0$ $(i = 1, \ldots, p)$.
- Taylor (1986)/Schwert (1990)'s GARCH when $\delta = 1$, and $\gamma_i = 0$ $(i = 1, \ldots, p)$.

[7]Complete developments leading to these conclusions are available in Ding, Granger, and Engle (1993).

- The GJR of Glosten, Jagannathan, and Runkle (1993) when $\delta = 2$.
- The TARCH of Zakoian (1994) when $\delta = 1$.
- The NARCH of Higgins and Bera (1992) when $\gamma_i = 0$ $(i = 1, \ldots, p)$ and $\beta_j = 0$ $(j = 1, \ldots, p)$.
- The Log-ARCH of Geweke (1986) and Pentula (1986), when $\delta \to 0$.

Following Ding, Granger, and Engle (1993), if $\omega > 0$ and $\sum\limits_{i=1}^{q} \alpha_i \, E(|z| - \gamma_i z)^\delta + \sum\limits_{j=1}^{p} \beta_j < 1$, a stationary solution for Eq. (17) exists and is:

$$E\left(\sigma_t^\delta\right) = \frac{\alpha_0}{1 - \sum\limits_{i=1}^{q} \alpha_i \left(|z| - \gamma_i z\right)^\delta - \sum\limits_{j=1}^{p} \beta_j}. \tag{18}$$

Notice that if we set $\gamma = 0$, $\delta = 2$ and z_t has zero mean and unit variance, we have the usual stationarity condition of the GARCH(1,1) model $(\alpha_1 + \beta_1 < 1)$. However, if $\gamma \neq 0$ and/or $\delta \neq 2$, this condition depends on the assumption made on the innovation process.

Ding, Granger, and Engle (1993) derived a closed form solution to $\kappa_i = E\left(|z| - \gamma_i z\right)^\delta$ in the Gaussian case. Lambert and Laurent (2001) show that for the standardized skewed Student:[8]

$$\kappa_i = \left\{ \xi^{-(1+\delta)} \left(1 + \gamma_i\right)^\delta + \xi^{1+\delta} \left(1 - \gamma_i\right)^\delta \right\} \frac{\Gamma\left(\frac{\delta+1}{2}\right) \Gamma\left(\frac{\upsilon - \delta}{2}\right) (\upsilon - 2)^{\frac{1+\delta}{2}}}{\left(\xi + \frac{1}{\xi}\right) \sqrt{(\upsilon - 2)\pi}\, \Gamma\left(\frac{\upsilon}{2}\right)}.$$

For the GED, we can show that:

$$\kappa_i = \frac{\left[(1+\gamma_i)^\delta + (1-\gamma_i)^\delta\right] 2^{\frac{\delta - \upsilon}{\upsilon}} \Gamma\left(\frac{\delta+1}{\upsilon}\right) \lambda_{upsilon}^\delta}{\Gamma\left(\frac{1}{\upsilon}\right)}.$$

Note that ξ, υ and $\lambda_{upsilon}$ concern the shape of the non-normal densities and will be defined in Section 4.2.

3.2.6 IGARCH Model

In many high-frequency time-series applications, the conditional variance estimated using a GARCH(p, q) process exhibits a strong persistence, that is:

$$\sum_{j=1}^{p} \beta_j + \sum_{i=1}^{q} \alpha_i \approx 1.$$

If $\sum\limits_{j=1}^{p} \beta_j + \sum\limits_{i=1}^{q} \alpha_i < 1$, the process (ε_t) is second order stationary, and a shock to the conditional variance σ_t^2 has a decaying impact on σ_{t+h}^2, when h increases, and is asymptotically negligible.

However, if $\sum\limits_{j=1}^{p} \beta_j + \sum\limits_{i=1}^{q} \alpha_i \geq 1$, the effect on σ_{t+h}^2 does not die out asymptotically. This property is called persistence in the literature.

[8]For the symmetric Student density, $\xi = 1$.

When this sum is equal to one, we are confronted to an Integrated GARCH (IGARCH) model, meaning that current information remains of importance when forecasting the volatility for all horizons. A similar concept is present in the mean equation: when the sum of all AR coefficients and MA coefficients is equal to one, the ARMA process in integrated (ARIMA).

Recall that the GARCH(p,q) model can be expressed as an ARMA process. Using the lag operator L, we can rearrange Eq. (8) as:

$$[1-\alpha(L)-\beta(L)]\varepsilon_t^2 = \omega + [1-\beta(L)]\left(\varepsilon_t^2-\sigma_t^2\right).$$

When the $[1-\alpha(L)-\beta(L)]$ polynomial contains a unit root, i.e. the sum of all the α_i and the β_j is one, we have the IGARCH(p,q) model of Engle and Bollerslev (1986). It can then be written as:

$$\phi(L)(1-L)\varepsilon_t^2 = \omega + [1-\beta(L)](\varepsilon_t^2-\sigma_t^2), \tag{19}$$

where $\phi(L)=[1-\alpha(L)-\beta(L)](1-L)^{-1}$ is of order [max{p,q}-1].

We can rearrange Eq. (19) to express the conditional variance as a function of the squared residuals. After some manipulations, we have:

$$\sigma_t^2 = \frac{\omega}{[1-\beta(L)]} + \left\{1-\phi(L)(1-L)[1-\beta(L)]^{-1}\right\}\varepsilon_t^2. \tag{20}$$

3.2.7 Fractionally Integrated Models

Volatility tends to change quite slowly over time, and, as shown in Ding, Granger, and Engle (1993) among others, the effects of a shock can take a considerable time to decay.[9] Therefore, the distinction between I(0) and I(1) processes seems to be far too restrictive. Indeed, the propagation of shocks in an I(0) process occurs at an exponential rate of decay (so that it only captures the short-memory), while for an I(1) process the persistence of shocks is infinite. In the conditional mean, the ARFIMA specification has been proposed to fill the gap between short and complete persistence, so that the short-run behavior of the time-series is captured by the ARMA parameters, while the fractional differencing parameter allows for modelling the long-run dependence.[10]

To mimic the behavior of the correlogram of the observed volatility, Baillie, Bollerslev, and Mikkelsen (1996) (hereafter denoted BBM) introduce the Fractionally Integrated GARCH (FIGARCH) model by replacing the first difference operator of Eq. (20) by $(1-L)^d$.

The conditional variance of the FIGARCH (p,d,q) is given by:

$$\sigma_t^2 = \underbrace{\omega[1-\beta(L)]^{-1}}_{\omega^*} + \underbrace{\left\{1-[1-\beta(L)]^{-1}\phi(L)(1-L)^d\right\}}_{\lambda(L)}\varepsilon_t^2, \tag{21}$$

or $\sigma_t^2 = \omega^* + \sum_{i=1}^{\infty}\lambda_i L^i \varepsilon_t^2 = \omega^* + \lambda(L)\varepsilon_t^2$, with $0 \le d \le 1$. It is fairly easy to show that $\omega > 0, \beta_1 - d \le \phi_1 \le \frac{2-d}{2}$ and $d\left(\phi_1 - \frac{1-d}{2}\right) \le \beta_1(\phi_1 - \beta_1 + d)$ are sufficient to ensure that the

[9]In their study of the daily S&P500 index, they find that the squared returns series has positive autocorrelations over more than 2,500 lags (or more than 10 years !).

[10]See Bollerslev and Mikkelsen (1996, p.158) for a discussion on the importance of non-integer values of integration when modelling long-run dependencies in the conditional mean of economic time series.

conditional variance of the FIGARCH $(1, d, 1)$ is positive almost surely for all t. Setting $\phi_1 = 0$ gives the condition for the FIGARCH $(1, d, 0)$.

Davidson (2001) notes the interesting and counterintuitive fact that the memory parameter of this process is $-d$, and is increasing as d approaches zero, while in the ARFIMA model the memory increases when ζ increases. According to Davidson (2001), the unexpected behavior of the FIGARCH model may be due less to any inherent paradoxes than to the fact that, embodying restrictions appropriate to a model in levels, it has been transplanted into a model of volatility. The main characteristic of this model is that it is not stationary when $d > 0$. Indeed,

$$\begin{aligned}(1-L)^d &= \sum_{k=0}^{\infty} \frac{\Gamma(d+1)}{\Gamma(k+1)\,\Gamma(d-k+1)} L^k \\ &= 1 - dL - \frac{1}{2}d(1-d)L^2 - \frac{1}{6}d(1-d)(2-d)L^3 - \ldots \\ &= 1 - \sum_{k=1}^{\infty} c_k(d) L^k, \end{aligned} \tag{22}$$

where $c_1(d) = d, c_2(d) = \frac{1}{2}d(1-d)$, etc. By construction, $\sum_{k=1}^{\infty} c_k(d) = 1$ for any value of d, and consequently, the FIGARCH belongs to the same "knife-edge-nonstationary" class represented by the IGARCH. To test whether this nonstationarity feature holds, Davidson (2001) proposes a generalized version of the FIGARCH and calls it the HYperbolic GARCH. The HYGARCH is given by Eq. (21), when $\lambda(L)$ is replaced by $1 - [1-\beta(L)]^{-1}\phi(L)\left\{1 + \alpha\left[(1-L)^d\right]\right\}$. Note that we report $\ln(\alpha)$ and not α. The $c_k(d)$ coefficients are thus weighted by α. Interestingly, the HYGARCH nests the FIGARCH when $\alpha = 1$ (or equivalently when $\ln(\alpha) = 0$) and if the GARCH component observes the usual covariance stationarity restrictions, then this process is stationary with $\alpha < 1$ (or equivalently when $\ln(\alpha) < 0$) (see Davidson, 2001 for more details).

Chung (1999) underscores some little drawbacks in the BBM model: there is a structural problem in the BBM specification since the parallel with the ARFIMA framework of the conditional mean equation is not perfect, leading to difficult interpretations of the estimated parameters. Indeed the fractional differencing operator applies to the constant term in the mean equation (ARFIMA) while it does not in the variance equation (FIGARCH). Chung (1999) proposes a slightly different process:

$$\phi(L)(1-L)^d\left(\varepsilon_t^2 - \sigma^2\right) = [1-\beta(L)](\varepsilon_t^2 - \sigma_t^2), \tag{23}$$

where σ^2 is the $\underline{\text{un}}$conditional variance of ε_t .

If we keep the same definition of $\lambda(L)$ as in Eq. (21), we can formulate the conditional variance as:

$$\sigma_t^2 = \sigma^2 + \left\{1 - [1-\beta(L)]^{-1}\phi(L)(1-L)^d\right\}\left(\varepsilon_t^2 - \sigma^2\right)$$

or

$$\sigma_t^2 = \sigma^2 + \lambda(L)\left(\varepsilon_t^2 - \sigma^2\right). \tag{24}$$

$\lambda(L)$ is an infinite summation which, in practice, has to be truncated. BBM propose to truncate $\lambda(L)$ at 1000 lags (this truncation order has been implemented as the default value in our package, but it may be changed by the user) and initialize the unobserved ε_t^2 at their unconditional moment. Contrary to BBM, Chung (1999) proposes to truncate $\lambda(L)$ at the size of the information set $(t-1)$ and to initialize the unobserved $\left(\varepsilon_t^2 - \sigma^2\right)$ at 0 (this quantity is small in absolute values and has a zero mean).[11]

The idea of fractional integration has been extended to other GARCH types of models, including the Fractionally Integrated EGARCH (FIEGARCH) of Bollerslev and Mikkelsen (1996) and the Fractionally Integrated APARCH (FIAPARCH) of Tse (1998).[12]

Similarly to the GARCH(p,q) process, the EGARCH(p,q) of Eq. (11) can be extended to account for long memory by factorizing the autoregressive polynomial $[1-\beta(L)] = \phi(L)(1-L)^d$ where all the roots of $\phi(z) = 0$ lie outside the unit circle. The FIEGARCH (p,d,q) is specified as follows:

$$\ln\left(\sigma_t^2\right) = \omega + \phi(L)^{-1}(1-L)^{-d}[1+\alpha(L)]g(z_{t-1}). \tag{25}$$

Finally, the FIAPARCH (p,d,q) model can be written as:[13]

$$\sigma_t^\delta = \omega + \left\{1 - [1-\beta(L)]^{-1}\phi(L)(1-L)^d\right\}(|\varepsilon_t| - \gamma\varepsilon_t)^\delta . \tag{26}$$

[11]See Chung (1999) for more details.

[12]Notice that the GJR has not been extended to the long-memory framework. It is however nested in the FIAPARCH class of models.

[13]When using the BBM option in G@RCH for the FIEGARCH and FIAPARCH, $(1-L)^d$ and $(1-L)^{-d}$ are truncated at some predefined value (see above). It is also possible to truncate this polynomial at the information size at time t, i.e. $t-1$.

4 Estimation Methods

4.1 Parameters Constraints

When numerical optimization is used to maximize the log-likelihood function with respect to the vector of parameters Ψ, the inspected range of the parameter space is $]-\infty;\infty[$. The problem is that some parameters might have to be constrained in a smaller interval. For instance, the leverage effect parameter γ of the APARCH model must lie between -1 and 1. To impose these constraints one could estimate Ψ^* (which ranges from $-\infty$ to $+\infty$) instead of Ψ where Ψ is recovered using the non-linear function: $\Psi = x\left(\Psi^*\right)$. In our package, $x(.)$ is defined as:

$$x(\Psi^*) = Low + \frac{Up - Low}{1 + e^{-\Psi^*}}, \tag{27}$$

where *Low* is the lower bound and *Up* the upper bound (i.e. in our example, $Low = -1$ and $Up = 1$).

So, applying unconstrained optimization of the log-likelihood function with respect to Ψ is equivalent to applying constrained optimization with respect to Ψ^*. Therefore, the optimization process of the program results in $\hat{\Psi}^*$ with the covariance matrix being noted $Cov\left(\hat{\Psi}^*\right)$. The estimated covariance of the parameters of interest $\hat{\Psi}$ is:

$$Cov\left(\hat{\Psi}\right) = \left(\frac{\partial x\left(\hat{\Psi}^*\right)}{\partial \Psi^*}\right) Cov\left(\hat{\Psi}^*\right) \left(\frac{\partial x\left(\hat{\Psi}^*\right)}{\partial \Psi^*}\right)'. \tag{28}$$

In our case, we have $Cov\left(\hat{\Psi}\right) = Cov\left(\hat{\Psi}^*\right) \frac{\exp\left(-\hat{\Psi}^*\right)(Up-Low)}{\left[1+\exp\left(-\hat{\Psi}^*\right)\right]^2}$. Note that, in G@RCH 2.3, lower and upper bounds of the parameters can be easily modified by the user in the file *startingvalues.txt*.

4.2 Distributions

Weiss (1986) and Bollerslev and Wooldridge (1992) show that under the normality assumption, the quasi maximum likelihood estimator is consistent if the conditional mean and the conditional variance are correctly specified. This estimator is, however, inefficient with the degree of inefficiency increasing with the degree of departure from normality (Engle and González-Rivera, 1991). Since the issue of fat-tails is an important one in empirical finance, it may be expected that using a more appropriate distribution would reduce the excess kurtosis displayed by the residuals of conditional heteroscedasticity models. As reported by Palm (1996), Pagan (1996) and Bollerslev, Chou, and Kroner (1992), the use of a fat-tailed distributions is widespread in the literature. In particular, Bollerslev (1987), Hsieh (1989), Baillie and Bollerslev (1989) and Palm and Vlaar (1997) among others show that these distributions perform better in order to capture the higher observed kurtosis.

Four distributions are available in our program: the usual Gaussian (normal) distribution, the Student-*t* distribution, the Generalized Error Distribution (GED) and the skewed Student-*t* distribution.

The GARCH models are estimated using a maximum likelihood (ML) approach. The logic of ML is to interpret the density as a function of the parameters set, conditional on a set of sample outcomes. This function is called the *likelihood function*. It is quite evident from equation (6) (and all the following equations of Section 3) that the recursive evaluation of this function is conditional on unobserved values. The ML estimation is therefore not perfectly exact. To solve the problem of unobserved values, we have set these quantities to their unconditional expected values.

If we express the mean equation as in Eq. (1) and $\varepsilon_t = z_t \sigma_t$, the log-likelihood function of the standard normal distribution is given by:

$$L_{norm} = -\frac{1}{2}\sum_{t=1}^{T}\left[\ln(2\pi) + \ln\left(\sigma_t^2\right) + z_t^2\right], \tag{29}$$

where T is the number of observations.

For a Student-t distribution, the log-likelihood is:

$$\begin{aligned} L_{Stud} &= T\left\{\ln\Gamma\left(\frac{\upsilon+1}{2}\right) - \ln\Gamma\left(\frac{\upsilon}{2}\right) - \frac{1}{2}\ln\left[\pi(\upsilon-2)\right]\right\} \\ &- \frac{1}{2}\sum_{t=1}^{T}\left[\ln(\sigma_t^2) + (1+\upsilon)\ln\left(1+\frac{z_t^2}{\upsilon-2}\right)\right], \end{aligned} \tag{30}$$

where υ is the degrees of freedom, $2 < \upsilon \leq \infty$ and $\Gamma(.)$ is the gamma function.

The GED log-likelihood function of a normalized random variable is given by:

$$L_{GED} = \sum_{t=1}^{T}\left[\ln\left(\upsilon/\lambda_\upsilon\right) - 0.5\left|\frac{z_t}{\lambda_\upsilon}\right|^{\upsilon} - \left(1+\upsilon^{-1}\right)\ln(2) - \ln\Gamma\left(1/\upsilon\right) - 0.5\ln\left(\sigma_t^2\right)\right], \tag{31}$$

where $0 < \upsilon < \infty$ and

$$\lambda_\upsilon \equiv \sqrt{\frac{\Gamma\left(\frac{1}{\upsilon}\right)2^{-\frac{2}{\upsilon}}}{\Gamma\left(\frac{3}{\upsilon}\right)}}. \tag{32}$$

The main drawback of the last two densities is that even if they may account for fat tails, they are symmetric. Skewness and kurtosis are important in financial applications in many respects (in asset pricing models, portfolio selection, option pricing theory or Value-at-Risk among others). Quite recently, Lambert and Laurent (2000, 2001) applied and extended the skewed Student density proposed by Fernández and Steel (1998) to the GARCH framework.

If $\Gamma(.)$ denotes the gamma function, the log-likelihood of a standardized (zero mean and unit variance) skewed Student is:

$$\begin{aligned} L_{SkSt} &= T\left\{\ln\Gamma\left(\frac{\upsilon+1}{2}\right) - \ln\Gamma\left(\frac{\upsilon}{2}\right) - 0.5\ln\left[\pi\left(\upsilon-2\right)\right] + \ln\left(\frac{2}{\xi+\frac{1}{\xi}}\right) + \ln\left(s\right)\right\} \\ &- 0.5\sum_{t=1}^{T}\left\{\ln\sigma_t^2 + (1+\upsilon)\ln\left[1+\frac{\left(sz_t+m\right)^2}{\upsilon-2}\xi^{-2I_t}\right]\right\}. \end{aligned} \tag{33}$$

where $I_t = \begin{cases} 1 \text{ if } z_t \geq -\frac{m}{s} \\ -1 \text{ if } z_t < -\frac{m}{s} \end{cases}$, ξ is the asymmetry parameter, υ is the degree of freedom of the distribution, $m = \frac{\Gamma\left(\frac{\upsilon+1}{2}\right)\sqrt{\upsilon-2}}{\sqrt{\pi}\Gamma\left(\frac{\upsilon}{2}\right)}\left(\xi - \frac{1}{\xi}\right)$ and $s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2}$. See Lambert and Laurent (2001) for more details.

In principal, the gradient vector and the hessian matrix can be obtained numerically or by evaluating its analytic expressions. Due to the high number of possible models and distributions, we use numerical techniques to approximate the derivatives of the log-likelihood function with respect to the parameter vector.

4.3 Tests

In addition to the possibilities offered by GiveWin (ACF, PACF, QQ-plots...), several tests are provided in the G@RCH package:

- Four Information Criteria (divided by the number of observations): [14]
  - Akaike $= -2\frac{LogL}{n} + 2\frac{k}{n}$
  - Hannan-Quinn $= -2\frac{LogL}{n} + 2\frac{k\log[\log(n)]}{n}$
  - Schwartz $= -2\frac{LogL}{n} + 2\frac{\log(k)}{n}$
  - Shibata $=-2\frac{LogL}{n} + \log\left(\frac{n+2k}{n}\right)$

- The value of the skewness and the kurtosis of the standardized residuals ($\hat{z}_t$) of the estimated model, their t-tests and p-values. Moreover, the Jarque-Bera normality test (Jarque and Bera, 1987) is also reported.

- The Box-Pierce statistics at lag l^* for both standardized, i.e. $BP\left(l^*\right)$, and squared standardized, i.e. $BP^2\left(l^*\right)$, residuals. Under the null hypothesis of no autocorrelation, the statistics $BP\left(l^*\right)$ and $BP^2\left(l^*\right)$ should be evaluated against the $\chi^2\left(l^* - m - l\right)$ and $\chi^2\left(l^* - p - q\right)$, respectively (see McLeod and Li, 1983).

- The Engle LM ARCH test (Engle, 1982) to test the presence of ARCH effects in a series.

- The diagnostic test of Engle and Ng (1993) that investigate possible misspecification of the conditional variance equation. The Sign Bias Test (SBT) examines the impact of positive and negative return shocks on volatility not predicted by the model under construction. The negative Size Bias Test (resp. positive Size Bias Test) focuses on the different effects that large and small negative (resp. positive) return shocks have on volatility, which is not predicted by the volatility model. Finally, a joint test for these three tests is also provided.

- The adjusted Pearson goodness-of-fit test that compares the empirical distribution of the innovations with the theoretical one. In order to carry out this testing procedure, it is necessary to first classify the residuals in cells according to their magnitude.[15] Let n be the number of observations, r the number of categories we consider, p_i ($i = 1$,...,r) the observed

[14] $LogL$ = log likelihood value, n = # observations and k the number of estimated parameters.
[15] See Palm and Vlaar (1997) for more details.

proportion of observations being in the i^{th} category and p_i^{t} (i = 1,...,r) the theoretical probability for an observation to be in the i^{th} category. The Pearson goodness-of-fit test has the null H$_0$: $p_1 = p_1^{\text{t}}$, $p_2 = p_2^{\text{t}}, \ldots, p_r = p_r^t$. The statistic is computed as

$$P(g) = \sum_{i=1}^{r} \frac{(n_i - En_i)^2}{En_i}, \tag{34}$$

where n_i is the observed number in the sample that fall into the i$^{\text{th}}$ category and En_i is the number of observations expected to be in this i^{th} category when Ho is true. The Pearson statistic is therefore "small" when all of the observed counts (proportions) are close to the expected counts (proportions) and it is "large" when one or more observed counts (proportions) differs noticeably from what is expected when H_0 is true.[16] For *i.i.d.* observations, Palm and Vlaar (1997) show that under the null of a correct distribution the asymptotic distribution of $P(g)$ is bounded between a $\chi^2(r-1)$ and a $\chi^2(r-k-1)$ where k is the number of estimated parameters. As explained by Palm and Vlaar (1997), the choice of r is far from being obvious. For $T = 2252$, these authors set r equal to 50. According to König and Gaab (1982), the number of cells must increase at a rate equal to $T^{0.4}$.

- The Nyblom test (Nyblom, 1989 and Lee and Hansen, 1994) to check the constancy of parameters over time. See also Hansen (1994) for an overview of this test.

4.4 Forecasts

Estimating a model can be useful to try to understand the mechanism that produces the series of interest. It can also suggest a solution to an economic problem. Is it the only game in town ? Certainly not. Indeed, the main purpose of building and estimating a model with financial data is to produce a forecast. G@RCH 2.3 also provides forecasting tools. Indeed, forecasts of both the conditional mean and the conditional variance are available as well as several forecast error measures.

4.4.1 Forecasting the conditional mean

Our first goal is to give the optimal h-step-ahead predictor of y_{t+h} given the information we have up to time t.

For instance, for the following AR(1) process,

$$y_t = \mu + \psi_1(y_{t-1} - \mu) + \varepsilon_t. \tag{35}$$

The optimal[17] h-step-ahead predictor of y_{t+h}, i.e. $\hat{y}_{t+h|t}$, is its conditional expectation at time t (given the estimated parameters $\hat{\mu}$ and $\hat{\psi}_1$):

$$\hat{y}_{t+h|t} = \hat{\mu} + \hat{\psi}_1(\hat{y}_{t+h-1|t} - \hat{\mu}), \tag{36}$$

[16]Large values of GoF suggest therefore that H_0 is false.

[17]By optimal, we mean optimal under expected quadratic loss, or in a mean square error sense.

where $\hat{y}_{t+i|t} = y_{t+i}$ for $i \leq 0$.

For the AR(1), the optimal 1-step-ahead forecast equals $\hat{\mu} + \hat{\psi}_1(\hat{y}_t - \hat{\mu})$. For $h > 1$, the optimal forecast can be obtained recursively or directly as $\hat{y}_{t+h|t} = \hat{\mu} + \hat{\psi}_1^h(\hat{y}_t - \hat{\mu})$.

In the general case of an ARFIMA(n, ζ, s) as given in Eq. (3), the optimal h-step-ahead predictor of y_{t+h} is:

$$\begin{aligned}\hat{y}_{t+h|t} &= \left[\hat{\mu}_{t+h|t} + \sum_{k=1}^{\infty} \hat{c}_k(\hat{y}_{t+h-k} - \hat{\mu}_{t+h|t})\right] \\ &+ \sum_{i=1}^{n} \hat{\psi}_i \left\{\hat{y}_{t+h-i} - \left[\hat{\mu}_{t+h|t} + \sum_{k=1}^{\infty} \hat{c}_k(\hat{y}_{t+h-i-k} - \hat{\mu}_{t+h|t})\right]\right\} \\ &+ \sum_{j=1}^{s} \hat{\theta}_j(\hat{y}_{t+h-j} - \hat{y}_{t+h-j|t}). \qquad (37)\end{aligned}$$

Recall that when exogenous variables enter the conditional mean equation, μ becomes $\mu_t = \mu + \sum_{i=1}^{n_1} \delta_i x_{i,t}$ and consequently, provided that the information $x_{i,t+h}$ is available at time t (which is the case for instance if $x_{i,t}$ is a “day-of-the-week” dummy variable), $\hat{\mu}_{t+h|t}$ is also available at time t. When there is no exogenous variable in the ARFIMA model and $n = 1, s = 0$ and $\zeta = 0$ ($c_k = 0$), the forecast of the AR(1) process given in Eq. (36) can be recovered.

4.4.2 Forecasting the conditional variance

Independently from the conditional mean, one can forecast the conditional variance. In the simple GARCH(p, q) case, the optimal h-step-ahead forecast of the conditional variance, i.e. $\hat{\sigma}^2_{t+h|t}$ is given by:

$$\sigma^2_{t+h|t} = \hat{\omega} + \sum_{i=1}^{q} \hat{\alpha}_i \varepsilon^2_{t+h-i|t} + \sum_{j=1}^{p} \hat{\beta}_j \sigma^2_{t+h-j|t}, \qquad (38)$$

where $\varepsilon^2_{t+i|t} = \sigma^2_{t+i|t}$ for $i > 0$ while $\varepsilon^2_{t+i|t} = \varepsilon^2_{t+i}$ and $\sigma^2_{t+i|t} = \sigma^2_{t+i}$ for $i \leq 0$. Eq. (38) is usually computed recursively, even if a closed form solution of $\sigma^2_{t+h|t}$ can be obtained by recursive substitution in Eq. (38).

Similarly, one can easily obtain the h-step-ahead forecast of the conditional variance of an ARCH, IGARCH and FIGARCH model. By contrast, for thresholds models, the computation of the out-of-sample forecasts is more complicated. Indeed, for the EGARCH, GJR and APARCH models (as well as for their long-memory counterparts), the assumption made on the innovation process may have an effect on the forecast (especially for $h > 1$).

For instance, for the GJR (p, q) model,

$$\hat{\sigma}^2_{t+h|t} = \hat{\omega} + \sum_{i=1}^{q} (\hat{\alpha}_i \varepsilon^2_{t-i+h|t} + \hat{\gamma}_i S^-_{t-i+h|t} \varepsilon^2_{t-i+h|t}) + \sum_{j=1}^{p} \hat{\beta}_j \sigma^2_{t-j+h|t}. \qquad (39)$$

When all the γ_i parameters equal 0, one recovers the forecast of the GARCH model. Otherwise, one has to compute $S^-_{t-i+h|t}$. Note first that $S^-_{t+i|t} = S^-_{t+i}$ for $i \leq 0$. However, when $i > 1$, $S^-_{t+i|t}$

depends on the choice of the distribution of z_t. When the distribution of z_t is symmetric around 0 (for the Gaussian, Student and GED density), the probability that ε_{t+i} will be negative is $S^-_{t+i|t} = 0.5$. If z_t is (standardized) skewed Student distributed with asymmetry parameter ξ and degree of freedom υ, $S^-_{t+i|t} = \frac{1}{1+\xi^2}$ since ξ^2 is the ratio of probability masses above and below the mode.

For the APARCH (p,q) model,

$$\begin{aligned}
\hat{\sigma}^{\delta}_{t+h|t} &= E\left(\sigma^{\delta}_{t+h}|\Omega_t\right) \\
&= E\left(\hat{\omega} + \sum_{i=1}^{q} \hat{\alpha}_i \left(|\varepsilon_{t+h-i}| - \hat{\gamma}_i \varepsilon_{t+h-i}\right)^{\hat{\delta}} + \sum_{j=1}^{p} \hat{\beta}_j \sigma^{\hat{\delta}}_{t+h-j} \mid \Omega_t\right) \\
&= \hat{\omega} + \sum_{i=1}^{q} \hat{\alpha}_i E\left[\left(\varepsilon_{t+h-i} - \hat{\gamma}_i \varepsilon_{t+h-i}\right)^{\hat{\delta}} |\Omega_t\right] + \sum_{j=1}^{p} \hat{\beta}_j \sigma^{\hat{\delta}}_{t+h-j|t}, \qquad (40)
\end{aligned}$$

where $E\left[\left(\varepsilon_{t+k} - \hat{\gamma}_i \varepsilon_{t+k}\right)^{\hat{\delta}} |\Omega_t\right] = \kappa_i \sigma^{\hat{\delta}}_{t+k|t}$, for $k > 1$ and $\kappa_i = E\left(|z| - \gamma_i z\right)^{\hat{\delta}}$ (see Section 4.2).

For the EGARCH (p,q) model,

$$\begin{aligned}
\ln \hat{\sigma}^2_{t+h|t} &= E\left(\ln \sigma^2_{t+h}|\Omega_t\right) \\
&= E\left\{\hat{\omega} + \left[1 - \hat{\beta}(L)\right]^{-1} [1 + \hat{\alpha}(L)] \hat{g}(z_{t+h-1}) \mid \Omega_t\right\} \\
&= \left[1 - \hat{\beta}(L)\right] \hat{\omega} + \hat{\beta}(L) \ln \hat{\sigma}^2_{t+h|t} + [1 + \hat{\alpha}(L)] \hat{g}(z_{t+h-1|t}), \qquad (41)
\end{aligned}$$

where $\hat{g}(z_{t+k|t}) = \hat{g}(z_{t+k})$ for $k \leq 0$ and 0 for $k > 0$.

Finally, the h-step-ahead forecast of the FIAPARCH and FIEGARCH models are obtained in a similar way.

One of the most popular measures to check the forecasting performance of the ARCH-type models is the Mincer-Zarnowitz regression, i.e. ex-post volatility regression:

$$\check{\sigma}^2_t = a_0 + a_1 \hat{\sigma}^2_t + u_t, \qquad (42)$$

where $\check{\sigma}^2_t$ is the ex-post volatility, $\hat{\sigma}^2_t$ is the forecasted volatility and a_0, a_1 are parameters to be estimated. If the model for the conditional variance is correctly specified (and the parameters are known) and $E(\check{\sigma}^2_t) = \hat{\sigma}^2_t$, it follows that $a_0 = 0$ and $a_1 = 1$. The R^2 of this regression is often used as a simple measure of the degree of predictability of the ARCH-type model.

However, $\check{\sigma}^2_t$ is never observed. By default, G@RCH 2.3 uses $\check{\sigma}^2_t = (y_t - \overline{y})^2$, where $\overline{y}$ is the sample mean of y_t. The R^2 of this regression is often lower than 5% and this could lead to the conclusion that GARCH models produce poor forecasts of the volatility (see, among others, Schwert, 1990, or Jorion, 1996). But, as described in Andersen and Bollerslev (1998), the reason of these poor results is the choice of what is considered as the "true" volatility. G@RCH 2.3 allows to select any series as the "observed" volatility (Obs.-Var., see Figure 1). The user may then compute the daily realized volatility as the sum of squared intraday returns and use it as the "true" volatility. Actually, Andersen and Bollerslev (1998) show that this measure is a more proper one than squared daily returns. Therefore, using 5-minute returns for instance, the realized volatility can be expressed as:

$$\sigma_t^2 = \sum_{k=1}^{K} y_{k,t}^2, \tag{43}$$

where $y_{k,t}$ is the return of the k^{th} 5-minutes interval of the t^{th} day and K is the number of 5-minutes intervals per day.

Finally, to compare the adequacy of the different distributions, G@RCH 2.3 also allows the computation of density forecasts tests developed in Diebold, Gunther, and Tay (1998). The idea of density forecasts is quite simple.[18] Let $f_i(y_i|\Omega_i)_{i=1}^m$ be a sequence of m one-step-ahead density forecasts produced by a given model, where Ω_i is the conditioning information set, and $p_i(y_i|\Omega_i)_{i=1}^m$ the sequence of densities defining the Data Generating Process y_i (which is never observed). Testing whether this density is a good approximation of the true density $p(.)$ is equivalent to testing:

$$H_0 : f_i(y_i|\Omega_i)_{i=1}^m = p_i(y_i|\Omega_i)_{i=1}^m \tag{44}$$

Diebold, Gunther, and Tay (1998) use the fact that, under Eq. (44), the probability integral transform $\hat{\zeta}_i = \int_{-\infty}^{y_i} f_i(t)dt$ is *i.i.d.* $U(0,1)$, i.e. independent and identically distributed uniform. To check H_0, they propose to use goodness-of-fit test and independence test for *i.i.d.* $U(0,1)$. The *i.i.d.*-ness property of $\hat{\zeta}_i$ can be evaluated by plotting the correlograms of $\left(\zeta - \overline{\hat{\zeta}}\right)^j$, for $j = 1, 2, 3, 4, ...$, to detect potential dependence in the conditional mean, variance, skewness, kurtosis, etc. Departure from uniformity can also be evaluated by plotting an histogram of $\hat{\zeta}_i$. According to Bauwens, Giot, Grammig, and Veredas (2000), *a humped shape of the $\hat{\zeta}$-histogram would indicate that the issued forecasts are too narrow and that the tails of the true density are not accounted for. On the other hand, a U-shape of the histogram would suggest that the model issues forecasts that either under- or overestimate too frequently.* Moreover, Lambert and Laurent (2001) show that an *inverted S* shape of the histogram would indicate that the errors are skewed, i.e. the true density is probably not symmetric.[19] An illustration is provided in Section 5 with some formal tests and graphical tools.

4.5 Accuracy

McCullough and Vinod (1999) and Brooks, Burke, and Persand (2001) use the daily German mark/British pound exchange rate data of Bollerslev and Ghysels (1996) to compare the accuracy of GARCH model estimation among several econometric softwares. They choose the GARCH(1,1) model described in Fiorentini, Calzolari, and Panattoni (1996) (hereafter denoted FCP) as the benchmark. In this section, we use the same methodology with the same dataset to check the

[18]For more details about density forecasts and applications in finance, see the special issue of *Journal of Forecasting* (Timmermann, 2000).

[19]Confidence intervals for the $\hat{\zeta}$-histogram can be obtained by using the properties of the histogram under the null hypothesis of uniformity.

accuracy of our procedures. Coefficients and standard errors estimates of G@RCH 2.3 are reported in Table 1 together with the results of McCullough and Vinod (1999) (FCP in the table).

	Coefficient		Hessian		QMLE	
	G@RCH	FCP	G@RCH	FCP	G@RCH	FCP
μ	-0.006184	-0.006190	0.008462	0.008462	0.009187	0.009189
ω	0.010760	0.010761	0.002851	0.002852	0.006484	0.006493
α_1	0.153407	0.153134	0.026569	0.026523	0.053595	0.053532
β_1	0.805879	0.805974	0.033542	0.033553	0.072386	0.072461

Table 1: Accuracy of the GARCH procedure

G@RCH gives very satisfactory results since the first four digits (at least) are the same as those of the benchmark for all but two estimations. In additionn, it competes well compared to other well known econometric softwares : Table 2 gives indeed the coefficient estimates and the error percentage associated for 5 softwares. G@RCH, PcGive and TSP (which uses the analytical gradients for the GARCH(1,1) model) clearly outperform Eviews and S-Plus on this specification.

	FCP	G@RCH	Eviews	PcGive	TSP	S-Plus
μ	-0.00619	-0.00618	-0.00541	-0.00625	-0.00619	-0.00919
ω	0.010761	0.010760	0.009581	0.010760	0.010761	0.011696
α_1	0.153134	0.153407	0.142284	0.153397	0.153134	0.154295
β_1	0.805974	0.805879	0.821336	0.805886	0.805974	0.800276
μ	-	0.10%	12.58%	0.91%	0.00%	48.41%
ω	-	0.01%	10.96%	0.01%	0.00%	8.69%
α_1	-	0.18%	7.08%	0.17%	0.00%	0.76%
β_1	-	0.01%	1.91%	0.01%	0.00%	0.71%

Table 2: GARCH Accuracy Comparison

Moreover, to investigate the accuracy of our forecasting procedures, we have run a 8-step ahead forecasts of the model, similar to Brooks, Burke, and Persand (2001). Table 4 in Brooks, Burke, and Persand (2001) reports the conditional variance forecasts given by six well-known softwares and the correct values. Contrary to E-Views, Matlab and SAS, G@RCH hits the benchmarks for all steps to the third decimal (note that GAUSS, Microfit and Rats also do).

Finally, Lombardi and Gallo (2001) extends the work of Fiorentini, Calzolari, and Panattoni (1996) to the FIGARCH model of Baillie, Bollerslev, and Mikkelsen (1996) and develops the analytic Hessian matrices of this long memory process. For the same DEM/UKP database as in the previous example, Table 3 reports the coefficients estimates and their standard errors for our package (using numerical gradients and the BFGS optimization method) and for Lombardi and Gallo (2001) (using analytical gradients and the Newton-Raphson algorithm).

	Coefficient		Hessian	
	G@RCH	LG	G@RCH	LG
μ	0.003606	0.003621	0.009985	0.009985
ω	0.015772	0.015764	0.003578	0.003581
α_1	0.198134	0.198448	0.042508	0.042444
β_1	0.675652	0.675251	0.051800	0.051693
d	0.570702	0.569951	0.075039	0.074762

Table 3: Accuracy of the FIGARCH procedure

Results show that G@RCH provides excellent numerical estimates that are quite close to the analytical ones, even for an advanced model such as the FIGARCH. As expected, it is however more time-consuming than the C code of Lombardi and Gallo (2001)[20] (163 sec. vs 43 sec. using a Dell PC with a PIII processor).

4.6 Features Comparison

The goal of this section is to compare objectively, in a GARCH perspective, the features offered by G@RCH 2.3 and nine other econometric softwares, namely PcGive 10 (also programmed in Ox), GAUSS and its Fanpac extension, Eviews 4, S-Plus 6 and its GARCH module, Rats and its *garch.src* example[21], TSP 4.5, Microfit 4, SAS 8.2 and Stata 7. It is thus not our intention to evaluate a program against another, but we will rather show an overview of what you can or cannot do with these softwares.

[20]This C code is available at `http://www.ds.unifi.it/~mjl/` in the "software" section.

[21]This file is available at `http://www.estima.com/procindx.htm` for download.

Table 4: GARCH Features Comparison

	G@RCH	PcGive	Fanpac	Eviews	S-Plus	Rats	TSP	Microfit	SAS	Stata
Version	2.3	10	-	4.0	6	5.0	4.5	4	8.2	7
Conditional mean										
Explanatory variables	+	+	+	+	+	+	+	+	+	+
ARMA	+	+	+	+	+	+	+	+	+	+
ARFIMA	+	-	-	-	-	-	-	-	-	-
ARCH-in-Mean	-	+	+	+	+	+	+	-	+	+
Conditional variance										
Explanatory variables	+	+	+	+	+	+	+	+	+	+
GARCH	+	+	+	+	+	+	+	+	+	+
IGARCH	+	-	+	-	-	+	-	-	+	-
EGARCH	+	+	+	+	+	+	-	+	+	+
GJR	+	+	-	+	+	+	-	-	-	+
APARCH	+	-	-	-	+	-	-	-	-	+
C-GARCH	-	-	-	+	+	-	-	-	-	-
FIGARCH	+	-	+	-	+	-	-	-	-	-
FIEGARCH	+	-	-	-	+	-	-	-	-	-
FIAPARCH	+	-	-	-	-	-	-	-	-	-
HYGARCH	+	-	-	-	-	-	-	-	-	-
Distributions										
Normal	+	+	+	+	+	+	+	+	+	+
Student-t	+	+	+	-	+	+	-	+	+	-
GED	+	+	-	-	+	+	-	-	-	-
Skewed-t	+	-	-	-	-	-	-	-	-	-
Double Exponential	-	-	-	-	+	-	-	-	-	-
Estimation										
MLE	+	+	+	+	+	+	+	+	+	+
QMLE	+	+	+	+	-	-	-	-	-	+

A “+” (resp.“-”) means that the corresponding option is (resp. is not) available for this software. C-GARCH corresponds to the Component GARCH of Engle and Lee (1999).

The proposed models and options differ widely from one program to the other as can be seen in Table 4. Regarding the range of different univariate models[22] , if many programs propose asymmetric models, very few (G@RCH, S-Plus with the FIGARCH BBM and the FIEGARCH and Fanpac with only the FIGARCH BBM) offer long memory models in the variance equation and none (except G@RCH) offers fractionally integrated specification in the mean. As for the distribution, G@RCH and S-Plus are the only softwares that permit the use of four densities. Finally, robust standard errors are proposed in 5 programs out of the 10 we have compared (G@RCH, PcGive, GAUSS Fanpac, Eviews and Stata).

In summary, on a GARCH point of view, G@RCH 2.3 offers many more possibilities than most of the reviewed programs. S-Plus has certainly the most complete range of options among G@RCH competitors, while TSP and Microfit have quite poor GARCH features.

[22]For the multivariate models, only FANPAC and S-Plus currently provide such models. The inclusion of multivariate models in G@RCH is currently under way.

5 Application

5.1 Data and Methodology

To illustrate our G@RCH 2.3 package with a concrete application, we analyze the French CAC40 stock index for the years 1995-1999 (1249 daily observations). It is computed by the exchange as a weighted measure of the prices of its components and is available in the database on an intraday basis with the price index being computed every 15 minutes. For the time period under review, the opening hours of the French stock market were 10.00 am to 5.00 pm, thus 7 hours of trading per day. This translates into 28 intraday returns used to compute the daily realized volatility. Intraday prices are the outcomes of a linear interpolation between the closest recorded prices below and above the time set in the grid. Correspondingly, all returns are computed as the first difference in the regularly time-spaced log prices of the index. Because the exchange is closed from 5.00 pm to 10.00 am the next day, the first intraday return is the first difference between the log price at 10.15 am and the log price at 5.00 pm the day before. Then, the intraday data are used to compute the daily realized volatility using Eq. (43). Finally, daily returns in percentage are defined as 100 times the first difference of the log of the closing prices.[23]

The estimation of the parameters is carried out for the 800 observations while forecasting is computed for the last observations.

5.2 Using the "Full Version"

First, open the database you want to use in GiveWin and select the OxPack module. Then select *Add/Remove Package...* in the *Package* menu. Click on the *Browse* button, then find and select *garch.oxo*. Click on the *Add* button. The *Garch* class is now active.

Then, select *Package/Garch*. There are two model classes in G@RCH 2.3 : `Garch Models` and `Dataset Statistics`. This second option allows to run several tests (general statistics, ARCH test and Box-Pierce test) on the raw series of the dataset. It is thus similar to the `TESTSONLY` function of the "Light Version".

For this application, select `Garch Models` and *Model/Estimate* is automatically launched. The list of all the variables of the database appears in the *Database* section (see figure 1). There are four possible statuses for each variable: dependent variable (Y variable), regressor in the mean (Mean), regressor in the variance (Variance) or observed volatility (Obs. Var.). Note that, in the example, we include the observed volatility of the series. Our program provides estimations for univariate models[24], so only one Y variable per model is accepted. However one can include several regressors in the mean and the variance equations and the same variable can be a regressor in both equations.

[INSERT FIGURE 1]

[23]By definition and using the properties of the log distribution, the sum of the intraday returns is equal to the observed daily return based on the closing prices.

[24]The extension of this package to multivariate GARCH models is currently under development.

Once the *OK* button is pressed, the *Model Settings* box automatically appears. This box allows to select the specification of the model: AR(FI)MA orders for the mean equation, GARCH orders, type of GARCH model for the variance equation and the distribution (figure 2). The default specification is an ARMA(0,0)-GARCH(1,1) with normal errors. In our application, we run a ARMA(1,0)-APARCH(1,1).

[INSERT FIGURE 2]

As explained in Section 4.1, it is possible to constrain the parameters to range between a lower and an upper bound by selecting the `Bounded Parameters` option. The defaults bounds can be changed in the *startingvalues.txt* file.

In the next window, we are asked to make a choice regarding the starting values (figure 3): we might (1) let the program choose the starting values[25], (2) enter them manually, element by element, or (3) enter the starting values in a vector form (the required form is "value1;value2;value3").

The first method is obviously the easiest, and may be indicated for beginning users, since it prevents from entering aberrant values. If we want particular starting values for the estimation and if we do not know the sequence of the parameters in the parameter vector used in our program, the second method should be a solution. An advanced user knowing the program quite well may use the third option as it is faster to do than the previous one. Note that, in the output, the estimated parameters are notably given in a vector form, so that we can just copy the vector and paste it in this box for a subsequent estimation.

[INSERT FIGURE 3]

Then, the estimation method for standard deviations is selected: Maximum Likelihood (ML) or Quasi-Maximum Likelihood (QML) or both. In this box (see figure 4), one may also select the sample and some maximization options (such has the number of iterations between intermediary results prints) when clicking on the *Options* button.

[INSERT FIGURE 4]

When we click on the *OK* button, the estimation procedure is then launched and the program comes back to GiveWin. A new dialog box is launched if the starting values are entered manually. Let us assume that the element-by-element method has been selected. A new window appears (see figure 5) with all the possible parameters to be estimated. Depending on the specification, some parameters have a value, others have not. The user should replace only the former since they correspond to the parameters to be estimated for the specified model.

[INSERT FIGURE 5]

Once this step is completed, the program starts the iteration process. Depending on the options selected earlier, it prints intermediary iteration results or not. The final output is divided

[25]Note that these default values can be modified by the user. Indeed they are stored in the `startingvalues.txt` file installed with the package.

by default in two main parts: first, the model specification reminder; second, the estimated values and other useful statistics of the parameters.[26] The output is given in the box "Output 1".

```
                                                                                    Output 1

********************
* SPECIFICATIONS **
*******************
Mean Equation: ARMA (1, 0) model.
No regressor in the mean.
Variance Equation : APARCH (1, 1) model.
No regressor in the variance.
The distribution is a Skewed Student distribution, with a tail coefficient of 15.72 and an asymmetry coefficient of -0.08751.
Strong convergence using numerical derivatives

Maximum Likelihood Estimation

               Coefficient     Std.Error      t-value        t-prob
Cst(M)         0.065337        0.037157       1.758          0.0791
AR(1)          0.004704        0.037117       0.1267         0.8992
Cst(V)         0.017498        0.013488       1.297          0.1949
Beta1          0.947590        0.020193       46.93          0.0000
Alpha1         0.038464        0.017776       2.164          0.0308
Gamma1         0.676364        0.348702       1.940          0.0528
Delta          1.462837        0.533581       2.742          0.0063
Asymmetry      -0.087512       0.054314       -1.611         0.1075
Tail           15.718323       8.087414       1.944          0.0523

No. Observations: 800          No. Parameters: 9
Mean (Y): 0.08103              Variance (Y): 1.27405
Log Likelihood: -1190.521

The sample mean of squared residuals was used to start recursion.
The condition for existence of E(σ^δ) and E(|e^δ|) is observed.
The constraint equals 0.9926 and should be < 1.
Vector of estimated parameters:
0.065337; 0.004704; 0.017498; 0.947590; 0.038464; 0.676364; 1.462837;-0.087512; 15.718323
```

After the estimation of the model, new options are available in OxPack: `Menu/Tests, Menu/Graphic Analysis, Menu/Forecasts, Menu/Exclusion Restrictions, Menu/Linear Restrictions` and `Menu/Store`.

The `Menu/Graphic Analysis` option allows to plot different graphics (see Figure 6 for details). Just as any other graphs in the GiveWin environment, they can be easily edited (color, size,...) and exported in many formats (.eps, .ps, .wmf, .emf and .gwg). Figure 7 provides the graphs of the squared residuals and the conditional mean with a 95% confidence interval.

[INSERT FIGURES 6 and 7]

The `Menu/Tests` option allows to run different tests (see Section 4.2 for further explanations). It also allows to print the variance-covariance matrix of the estimated parameters (Figure 8). The results of these tests are printed in GiveWin. An example of output is reported in the next box ("Output 2").

[INSERT FIGURE 8]

[26] Recall that the estimations are based on the numerical evaluation of the gradients.

```
                                                                                Output 2

TESTS:
----------

                         Statistic              t-value               t-prob
Skewness                 -0.2135                2.47                  0.0135
Excess Kurtosis          0.4684                 2.713                 0.006674
Jarque-Bera              13.39                  13.39                 0.001235
----------
Information Criterium (minimize)
Akaike                   2.998802               Shibata               2.998553
Schwarz                  3.051504               Hannan-Quinn          3.019048
----------
BOX-PIERCE:
H0: No serial correlation ⇒ Accept H0 when prob. is High [Q < Chisq(lag)]
Box-Pierce Q-statistics on residuals
→ P-values adjusted by 1 degree(s) of freedom
Q(10) = 14.47 [0.1064]
Q(20) = 21.67 [0.3012]

Box-Pierce Q-statistics on squared residuals
→ P-values adjusted by 2 degree(s) of freedom
Q(10) = 9.887 [0.2731]
Q(20) = 16.13 [0.5838]
----------
Diagnostic test based on the news impact curve (EGARCH vs.GARCH)
                                                Test                  Prob
Sign Bias t-Test                                0.98838               0.32297
Negative Size Bias t-Test                       0.14581               0.88407
Positive Size Bias t-Test                       0.62400               0.53263
Joint Test for the Three Effects                5.13914               0.16189
----------
Joint Statistic of the Nyblom test of stability: 2.727
Individual Nyblom Statistics:
Cst(M)                   0.72438
AR(1)                    0.68524
Cst(V)                   0.51505
Beta1                    0.42785
Alpha1                   0.46229
Gamma1                   0.43489
Delta                    0.54130
Asymmetry                0.21342
Tail                     0.08950
Rem: Asymptotic 1% critical value for individual statistics = 0.75.
Asymptotic 5% critical value for individual statistics = 0.47.
----------
Adjusted Pearson Chi-square Goodness-of-fit test
Lags                     Statistic              P-Value(lag-1)        P-Value(lag-k-1)
40                       24.9000                0.961261              0.729877
50                       26.7500                0.995994              0.946240
60                       32.6500                0.997893              0.972622


Rem.: k = # estimated parameters
```

We do not intend to comment this application in details. However, looking at these results, one can briefly argue that the model seems to capture the dynamics of the first and second moments of the CAC40 (see the Box-Pierce statistics). Moreover, the Sign Bias tests show that there is no remaining leverage component in the innovations while the Nyblom stability test suggests that the estimated parameters are quite stable during the investigated period. Finally, our model specification is not rejected by the goodness-of-fit tests for various lag lengths.

To obtain the h-step-ahead forecasts, access the menu `Test/Forecast` and set the number of forecasts, pre-sample observations (to be plotted) as well as some other graphical options (Figure

9).

Figure 10 shows 10 pre-sample observations and the forecasts up to horizon 10 of the conditional mean. The forecasted bands are $\pm 2\hat{\sigma}_{t+h|t}$ (note that the critical value 2 can be changed).

[INSERT FIGURE 9]

[INSERT FIGURE 10]

```
Forecast Evaluation Measures

                                                   Mean            Variance
Mean Squared Error(MSE)                            2.2253          11.3109
Median Squared Error(MedSE)                        0.6754          2.0989
Mean Error(ME)                                     0.0476          -0.0803
Mean Absolute Error(MAE)                           1.1022          1.9847
Root Mean Squared Error(RMSE)                      1.4918          3.3632
Mean Absolute Percentage Error(MAPE)               .NaN            1.4964
Adjusted Mean Absolute Percentage Error(AMAPE)     .NaN            0.3726
Percentage Correct Sign(PCS)                       0.5434          .NaN
Theil Inequality Coefficient(TIC)                  0.9559          0.5126
Logarithmic Loss Function(LL)                      .NaN            1.0699
```

Finally, the residuals, the squared residuals, the conditional variance, the (forecasted) probability integral transform and the forecasted conditional mean and conditional variance series can be stored in the database as a new variable. When selecting this option, a first window appears and the user selects the series to be stored (figure 11). A default name is then proposed for this series.

[INSERT FIGURE 11]

5.3 Using the "Light Version"

First, to specify the model you want to estimate, you have to edit *GarchEstim.ox* with any text editor. Yet we recommend OxEdit. It is a shareware that highlights Ox syntax in color (see http://www.oxedit.com for more details). An example of the *GarchEstim.ox* file is displayed here below.

```
                                                                                GarchEstim.ox
#import <packages/garch23/garch>

main()
{
    decl i,j,k,l,garchobj;

    garchobj = new Garch();

//*** DATA ***//
    garchobj.Load("/data/cac40.xls");
    garchobj.Info();

    garchobj.Select(Y_VAR, {"CAC40",0,0} );
//  garchobj.Select(X_VAR, {"NAME",0,0});           // REGRESSOR IN THE MEAN
//  garchobj.Select(Z_VAR, {"NAME",0,0});           // REGRESSOR IN THE VARIANCE
//  garchobj.Select(O_VAR, {"REALVOLA",0,0} );      // REALIZED VOLATILITY

    garchobj.SetSelSample(-1, 1, -1, 1);

//*** SPECIFICATIONS ***//
    garchobj.CSTS(1,1);             // cst in Mean (1 or 0), cst in Variance (1 or 0)
    garchobj.DISTRI(0);             // 0 for Gauss, 1 for Student, 2 for GED, 3 for Skewed-Student
    garchobj.ARMA_ORDERS(0,0);      // AR order (p), MA order (q).
    garchobj.ARFIMA(0);             // 1 if Arfima wanted, 0 otherwise
    garchobj.GARCH_ORDERS(1,1);     // p order, q order
    garchobj.MODEL(1);              //  1:GARCH     2:EGARCH    3:GJR   4:APARCH    5:IGARCH
                                    //  6:FIGARCH(BBM)  7:FIGARCH(Chung)    8:FIEGARCH(BBM only)
                                    //  9:FIAPARCH(BBM) 10: FIAPARCH(Chung) 11: HYGARCH(BBM)

    garchobj.TRUNC(1000);           // Truncation order (only F.I. models with BBM method)

//*** PARAMETERS ***//
    garchobj.BOUNDS(0);             // 1 if bounded parameters wanted, 0 otherwise
    garchobj.FixParam(0,<0;0;1>);   // Arg.1 : 1 to fix some parameters to their starting values, 0 otherwize
                                    // Arg.2 : 1 to fix (see garchobj.DoEstimation(<>))
                                    //and 0 to estimate the corresponding parameter

//*** ESTIMATION OPTIONS ***//
    garchobj.MLE(1);                // 0 : both, 1 : MLE, 2 : QMLE
    garchobj.COVAR(0);              // if 1, prints variance-covariance matrix of the parameters.
    garchobj.ITER(0);               // Interval of iterations between printed intermediary results
                                    // (if no intermediary results wanted, enter '0')
    garchobj.GRAPHS(0,0,"");        // Arg.1 : if 1, displays graphics of the estimations (only when using GiveWin).
                                    // Arg.2 : if 1, saves these graphics in a EPS file (OK with all Ox versions)
                                    // Arg.3 : Name of the saved file.
    garchobj.FOREGRAPHS(1,0,"");    // Same as GRAPHS(p,s,n) but for the graphics of the forecasts.

//*** TESTS & FORECASTS ***//
    garchobj.BOXPIERCE(<5;10;20>);  // Lags for the Box-Pierce Q-statistics, <> otherwise
    garchobj.ARCHLAGS(<2;5;10>);    // Lags for Engle's LM ARCH test, <> otherwise
    garchobj.NYBLOM(1);             // 1 to compute the Nyblom stability test, 0 otherwise
    garchobj.PEARSON(<40;50;60>);   // Cells (<40;50;60>) for the adjusted Pearson Chi-square Goodness-of-fit test,
                                    // <> otherwise
    garchobj.FORECAST(0,10,0);      // Arg.1 : 1 to launch the forecasting procedure, 0 otherwize
                                    // Arg.2 : Number of forecasts
                                    // Arg.3 : 1 to Print the forecasts, 0 otherwise
    garchobj.TESTS(0,1);            // Arg.1 : if 1, runs tests for the raw Y series, prior to any estimation.
                                    // Arg.2 : if 1, runs tests after the estimation.
    garchobj.DoEstimation(<>);

// m_vPar = m_clevel | m_vbetam |  m_dARFI | m_vAR | m_vMA | m_calpha0 | m_vgammav | m_dD |  m_vbetav |
//          m_valphav | m_vleverage | m_vtheta1 | m_vtheta2 | m_vpsy | m_ddelta | m_cA | m_cV | m_vHY
//garchobj.DoEstimation(<0.02;0.05;0.45;0.22;0.01;0.025;0.8;0.1;-0.15;0.2;6>);

    garchobj.STORE(0,0,0,0,0,"01",0);  //  Arg.1,2,3,4,5 : if 1 -> stored. (Res-SqRes-CondV-MeanFor-VarFor)
                                       //  Arg.6 : Suffix. The name of the saved series will be "Res_ARG6"
                                       //  Arg.7 : if 0, saves as an Excel spreadsheet (.xls).
                                       //  If 1, saves  as a GiveWin dataset (.in7)
    delete garchobj;
}
```

Let us study this file more in details. The `#import` statement indicates that this file is linked with the *Garch.oxo* and *Garch.h* files. In the body of the file (after the `main()` instruction), a new Garch object is first created and a database is loaded. The user has to enter the correct path of the database, but also has to pay attention to the structure of the database he will use. For instance, to use a Microsoft Excel file, the format of the spreadsheet is of crucial importance. The following convention has to be adopted when loading an Excel spreadsheet: variables are in columns, columns with variables are labelled, there is an unlabelled column containing the dates (with the form Year-Period) and the data form a contiguous sample. Here is an example: [27]

	A	**B**	**C**	**D**
1		RET	MON	HOL
2	1990-1	0.0439	1	0
3	1990-2	-0.0302	0	0
4	1990-3	0.0845	0	1

We note then that the dependent variable (Y), the regressor(s) in the mean equation (X), the regressor(s) in the variance equation (Z) and the "realized" volatility (0) are selected with the `Select` function. Ox being case- sensitive, the exact name of the variable has to be entered. The second and third arguments denote the starting and ending observations to be considered. By default, "0" and "0" mean that all the observations are selected. From this selection, a sample can be extracted with the `SetSelSample` function. The arguments are ordered as (StartYear, StartPeriod, EndYear, EndPeriod2) and the default (-1, 1, -1, 1) means all the selected observations.

The ***GarchEstim.ox*** file consists in six parts: [28]

- the **"Data"** part deals with the database, the sample and the variables selection;
- the **"Specification"** part is related to the choice of the model, the lag orders and the shape of the distribution;
- the **"Parameters"** part consists in two procedures. `BOUNDS` to constraint or not several parameters to range between a lower and an upper bound (see Section 4.1), `FixParam` to fix some parameters to their starting values;
- the **"Output"** part includes several options including `MLE` that refer to the computation method of the standard deviations of the estimated parameters, `TESTONLY`, useful when

[27]See Doornik (2001) for the supported formats, the Load functions and other related information. Interested reader can also take a look at the *DJIA.xls* file included in the package for an example of Excel file ready to be loaded by Ox. Please be very careful when editing numbers in an Excel file, especially regarding the decimal separator ("," or "." depending on the language used).

[28]All the functions cited here are described in details in section 2.3

you want to run some tests on the raw series, prior to any estimation and `GRAPHS` and `FOREGRAPHS`, to print graphs for the estimation and the forecasting, respectively;[29]

- the **"Tests & Forecasts"** part allows to compute different tests and to parameterize the forecasting part. Note that `BOXPIERCE`, `ARCHLAGS` and `PEARSON` all require a vector of integers corresponding to the lags used in the computation of the statistics;

- `DoEstimation` launches the estimation of the model and the `STORE` function allowing to store some series. The argument of the `DoEstimation` procedure is a vector containing starting values of the parameters in a specified order (but the user can also let the program take defaults values).

Note that the "Light Version" is more than just a replication of the "Full Version" without the graphical interface. Indeed, G@RCH uses the object-oriented programming features of Ox and provides a new class called `Garch`. All the functions of this class can thus be used within an Ox program. To illustrate the potentiality of our package, we also provide *Forecast.ox*, an example that computes 448 one-step-ahead forecasts of the conditional mean and conditional variance (using the estimated parameters presented in the previous section), computes the Mincer-Zarnowitz regression and performs some out-of-sample density forecast tests as suggested by Diebold, Gunther, and Tay (1998).

The interesting part of *Forecast.ox* is printed in the next box. This code has been used to produce Figure 12 and the outputs associated with this forecasting experiment (see below).

[29]Graphics will only be displayed when using GiveWin as front-end.

```
                                                                                    Forecast.ox
#import <packages/garch/garch> main() {
    decl garchobj;
    garchobj = new Garch();

    ...

    garchobj.DoEstimation(<>);
    decl number_of_forecasts=448; // number of h_step_ahead forecasts
    decl step=1;                  // specify h (h-step-ahead forecasts)
    decl T=garchobj.GetcT();
    decl par=garchobj.PAR()[][0];
    println("!!! Please Wait while computing the forecasts !!!");
    decl forc=<>,h,yfor=<>,Hfor=<>;
    decl RV=columns(garchobj.GetGroup(O_VAR));
    decl shape=<>;
    if (garchobj.GetDistri()==1 || garchobj.GetDistri()==2)  // Except for the HYGARCH
        shape=par[rows(par)-1];
    else if (garchobj.GetDistri()==3)
        shape=par[rows(par)-2:rows(par)-1];
    for (h=0; h<number_of_forecasts; ++h)
    {
        garchobj.FORECAST(1,step,0);
        garchobj.SetSelSample(-1, 1, T+h, 1);
        garchobj.InitData();
        yfor|=garchobj.GetForcData(Y_VAR, step);
        forc|=garchobj.FORECASTING();
        if (RV==1)
            Hfor|=garchobj.GetForcData(O_VAR, step);        // If you use the realized volatility
    }
    decl cd=garchobj.CD(yfor-forc[][0],forc[][1],garchobj.GetDistri(),shape);
    println("Density Forecast Test on Standardized Forecast Errors");
    garchobj.APGT(cd,20|30,rows(par));
    garchobj.AUTO(cd, number_of_forecasts, -0.1, 0.1, 0);
    garchobj.confidence_limits_uniform(cd,30,0.95,1,4);
    if (RV==0)
    {
        DrawTitle(5, "Conditional variance forecast and absolute returns");
        Hfor = (yfor - meanc(yfor)).^2;
    }
    else
        DrawTitle(5, "Conditional variance forecast and realized volatility");
    Draw(5, (Hfor~forc[][1])');
    ShowDrawWindow();
    garchobj.MZ(Hfor, forc, number_of_forecasts);
    garchobj.FEM(forc, yfor~Hfor);

    garchobj.STORE(0,0,0,0,0,"01",0);  //  Arg.1,2,3,4,5 ...
                                       //  Arg.6 : Suffix. ...
                                       //  Arg.7 : if 0, ...
    delete garchobj;
}
```

In the first four panels of Figure 12, we show the correlograms of $\left(\hat{\zeta} - \overline{\hat{\zeta}}\right)^j$, for $j = 1, 2, 3, 4$. This graphical tool has been proposed by Diebold, Gunther, and Tay (1998) to detect potential remaining dependence in the conditional mean, variance, skewness, kurtosis. In our example, it seems that the probability integral transform is independently distributed.

[INSERT FIGURE 12]

Panel 5 of Figure 12 also shows the histogram (with 30 cells) of $\hat{\zeta}$ with the 95 % confidence bands. From this figure, it is clear that the AR(1)-APARCH(1,1) model coupled with a skewed Student distribution for the innovations performs very well with the dataset we have investigated. This conclusion is reinforced by the Pearson Chi-square goodness-of-fit test printed hereafter that provides a statistical version of the graphical test presented in Figure 12. Finally, the program performs the Mincer-Zarnowitz regression given in Eq. (42) that regresses the observed volatility (in our case the realized volatility) on a constant and a vector of 448 one-step-ahead forecasts of the conditional variance (produced by the APARCH model).[30] The results (reported in the next box) suggest that the APARCH model gives good forecasts of the conditional variance. Indeed, looking at the estimated parameters of this regression, one can hardly conclude that the APARCH model provides biases forecasts. Moreover, the R^2 of this regression is higher than 40 % (See Andersen and Bollerslev (1998) for more details).

```
Density Forecast Test on Standardized Forecast Errors Adjusted Pearson Chi-square Goodness-of-fit test

Lags           Statistic      P-Value(lag-1)    P-Value(lag-k-1)
20             21.0179        0.335815          0.020969
30             26.5089        0.598181          0.149654
Rem.: k = number of estimated parameters

Mincer-Zarnowitz regression on the forecasted volatility
               Coefficient    Std.Error         t-value           t-prob
a_0            -0.225818      0.264837          -0.8527           0.3940
a_1            1.370648       0.176086          7.784             0.0000
R^2: 0.402914
Note: S.E. are Heteroskedastic Consistent (White, 80)
```

[30]The realized and one-step-ahead forecasts are plotted in the last panel of Figure 12.

6 Versions and Future Improvements

6.1 Releases History

Here is the history of the G@RCH package releases:

v.2.30 : April, 22nd
v.2.20 : February, 5th, 2001.
v.2.10 : September, 4th, 2001.
v.2.00 : April, 23th, 2001.
v.1.11 : November, 18th, 2000.
v.1.10 : October, 30th, 2000.
v.1.00 : September, 4th, 2000.

6.2 Future Improvements

- Analytical gradients
- Multivariate GARCH models (under development)
- New "add-ons" with specific applications such as Value-at-Risk...
- ...

We Wish You A Productive Use Of G@RCH 2.3 !

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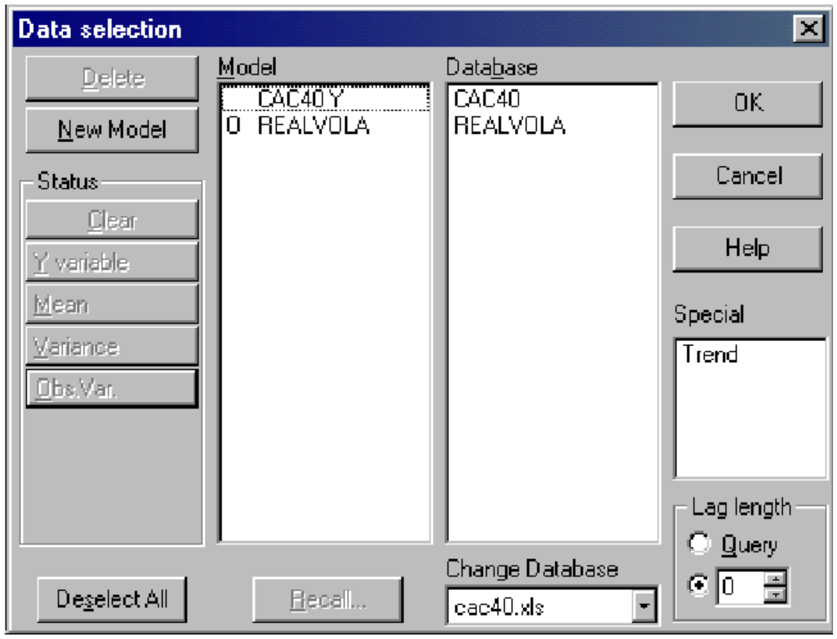


Model: CAC40 Y; 0 REALVOLA

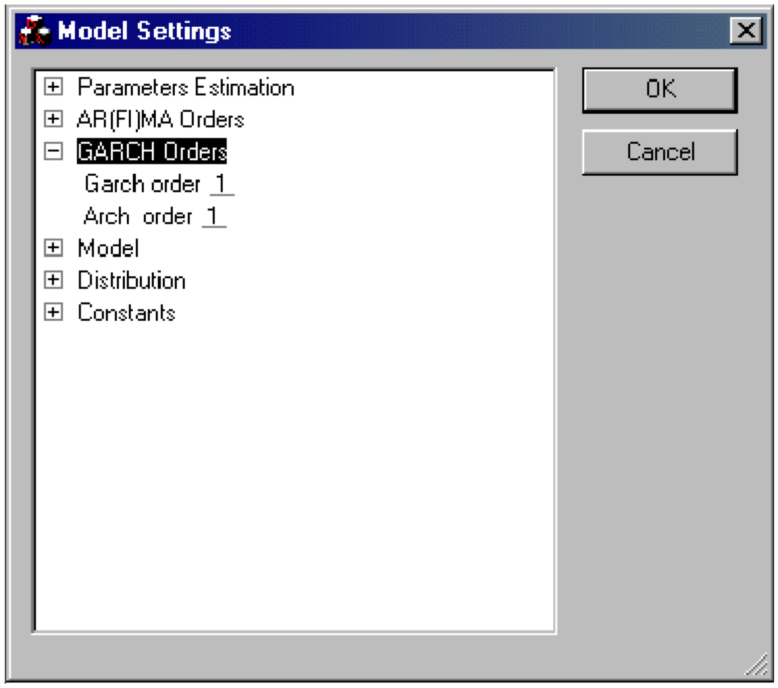
Database: CAC40; REALVOLA

Special: Trend

Lag length: 0

Change Database: cac40.xls

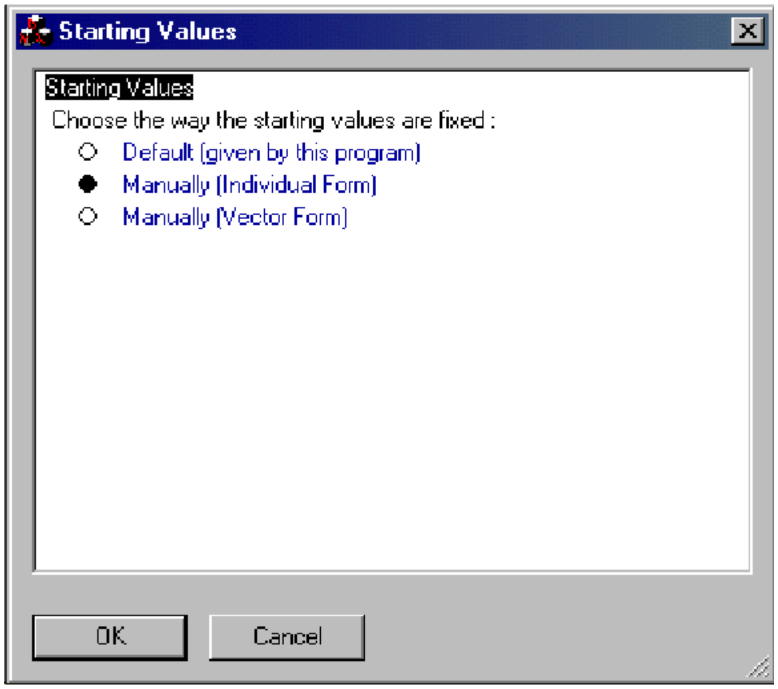
Figure 1: Selecting the variables



Garch order 1

Arch order 1

Figure 2: Model Settings



Starting Values

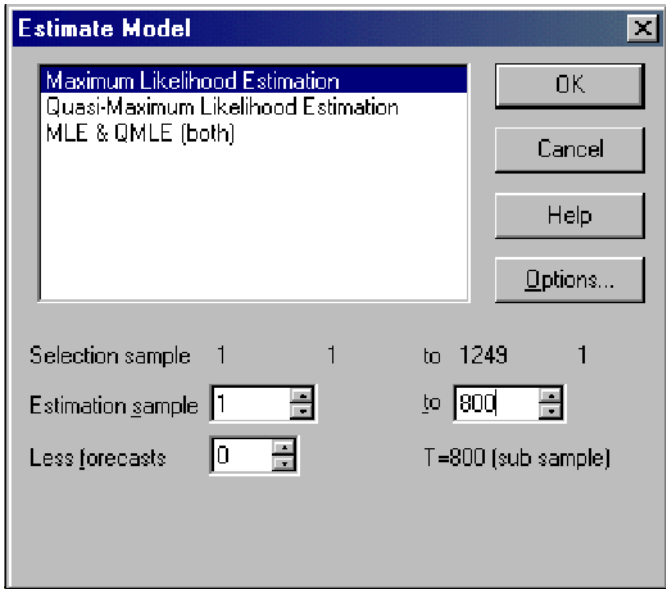
Choose the way the starting values are fixed :

○ Default (given by this program)

● Manually (Individual Form)

○ Manually (Vector Form)

Figure 3: Selecting the Starting Values Method



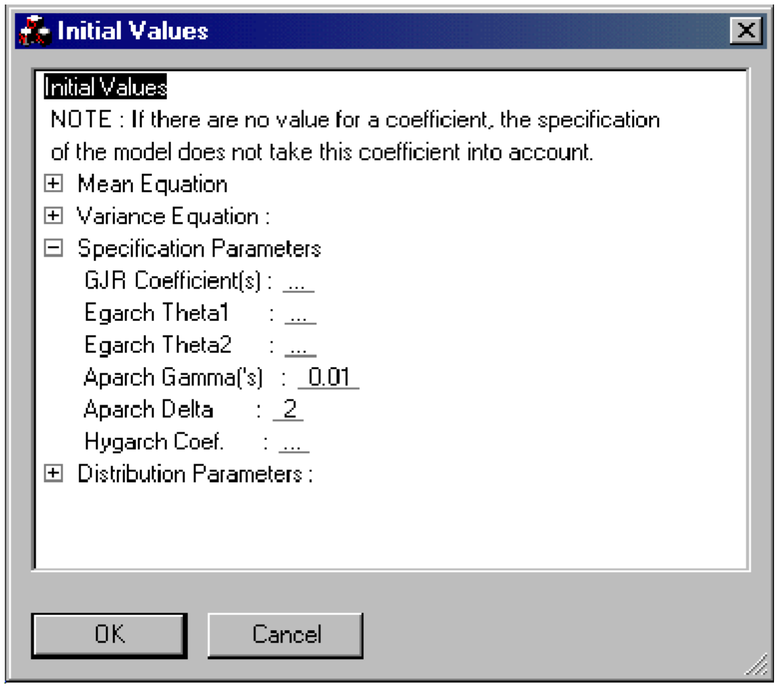
Maximum Likelihood Estimation

Quasi-Maximum Likelihood Estimation

MLE & QMLE (both)

Selection sample	1	1	to 1249	1
Estimation sample	1		to 800	
Less forecasts	0		T=800 (sub sample)	

Figure 4: Standard Errors Estimation Methods



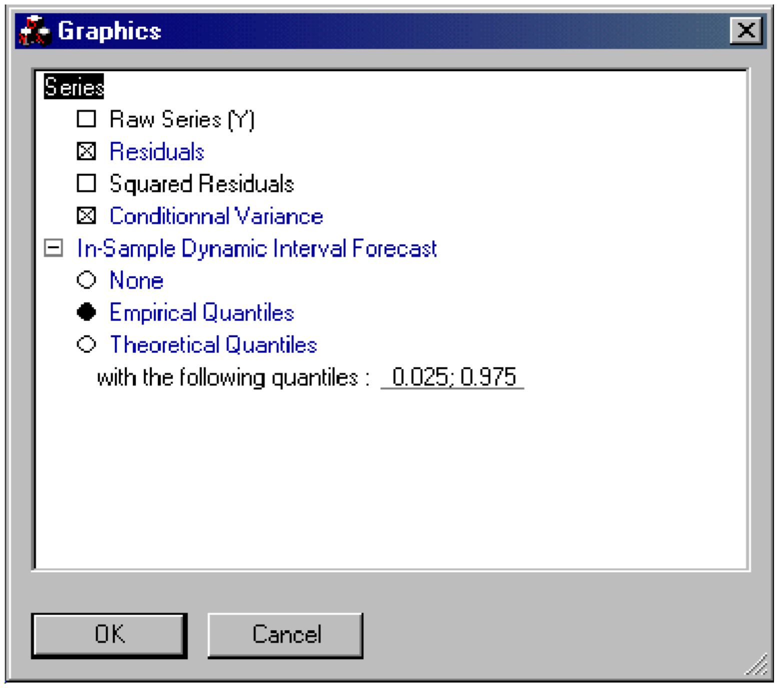
Initial Values

NOTE : If there are no value for a coefficient, the specification of the model does not take this coefficient into account.

⊞ Mean Equation
⊞ Variance Equation :
⊟ Specification Parameters
- GJR Coefficient(s) : ...
- Egarch Theta1 : ...
- Egarch Theta2 : ...
- Aparch Gamma('s) : 0.01
- Aparch Delta : 2
- Hygarch Coef. : ...

⊞ Distribution Parameters :

Figure 5: Entering the Starting Values



Series
- ☐ Raw Series (Y)
- ☒ Residuals
- ☐ Squared Residuals
- ☒ Conditionnal Variance

⊟ In-Sample Dynamic Interval Forecast
- ○ None
- ● Empirical Quantiles
- ○ Theoretical Quantiles

with the following quantiles : 0.025; 0.975

Figure 6: Graphics Menu

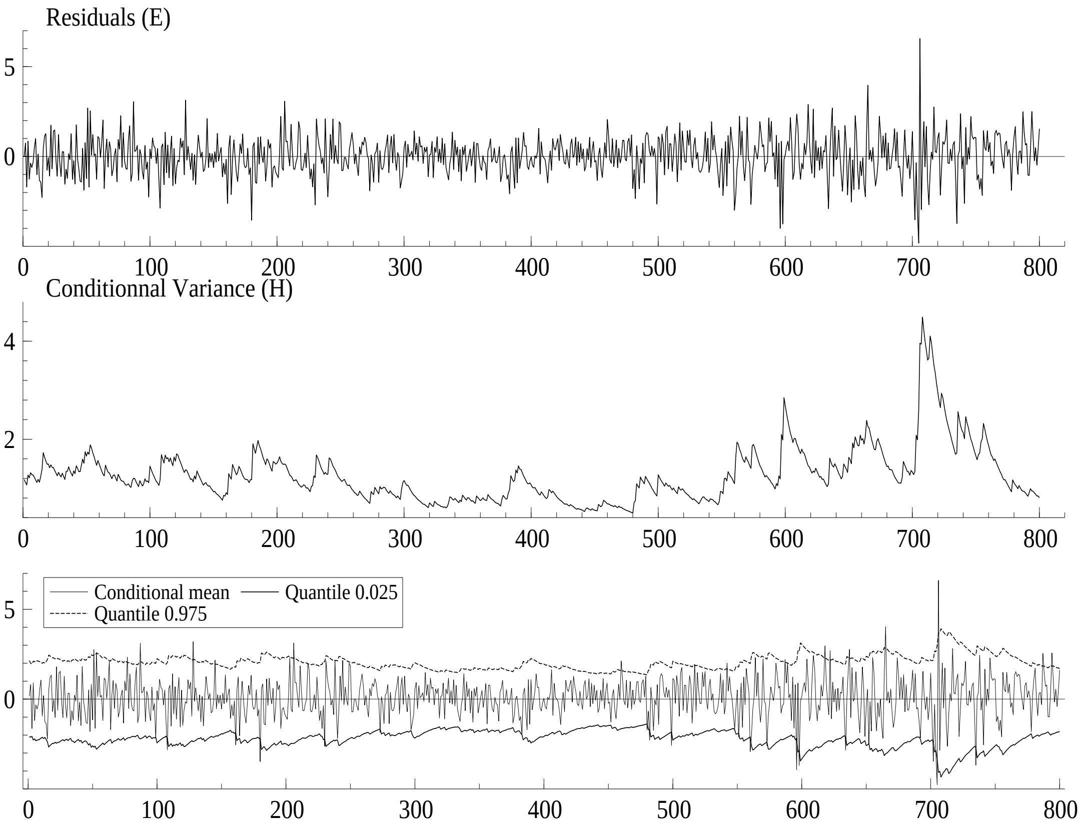
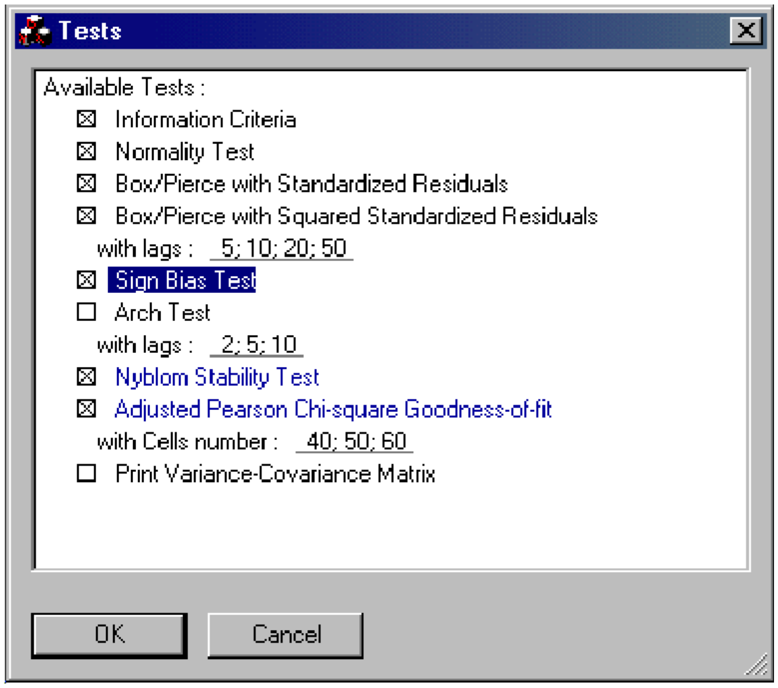
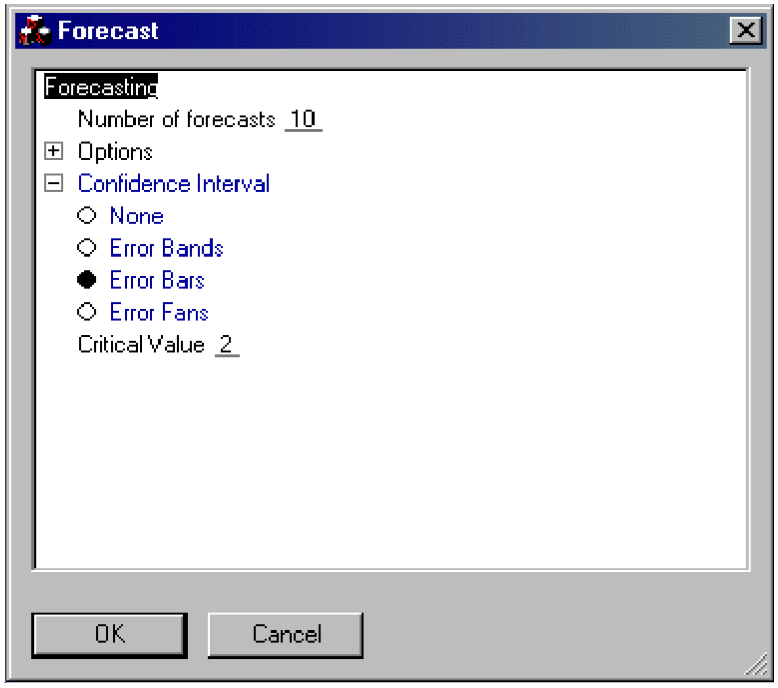


Figure 7: Graphical Analysis



Available Tests :
- ☒ Information Criteria
- ☒ Normality Test
- ☒ Box/Pierce with Standardized Residuals
- ☒ Box/Pierce with Squared Standardized Residuals
  with lags : 5; 10; 20; 50
- ☒ Sign Bias Test
- ☐ Arch Test
  with lags : 2; 5; 10
- ☒ Nyblom Stability Test
- ☒ Adjusted Pearson Chi-square Goodness-of-fit
  with Cells number : 40; 50; 60
- ☐ Print Variance-Covariance Matrix

Figure 8: Tests Dialog Box



Forecasting
Number of forecasts 10
⊞ Options
⊟ Confidence Interval
- ○ None
- ○ Error Bands
- ● Error Bars
- ○ Error Fans

Critical Value 2

Figure 9: Forecasting Menu

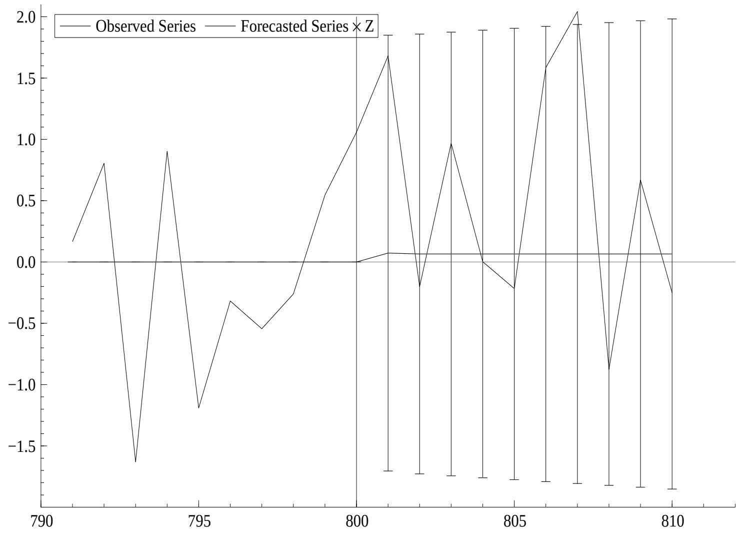
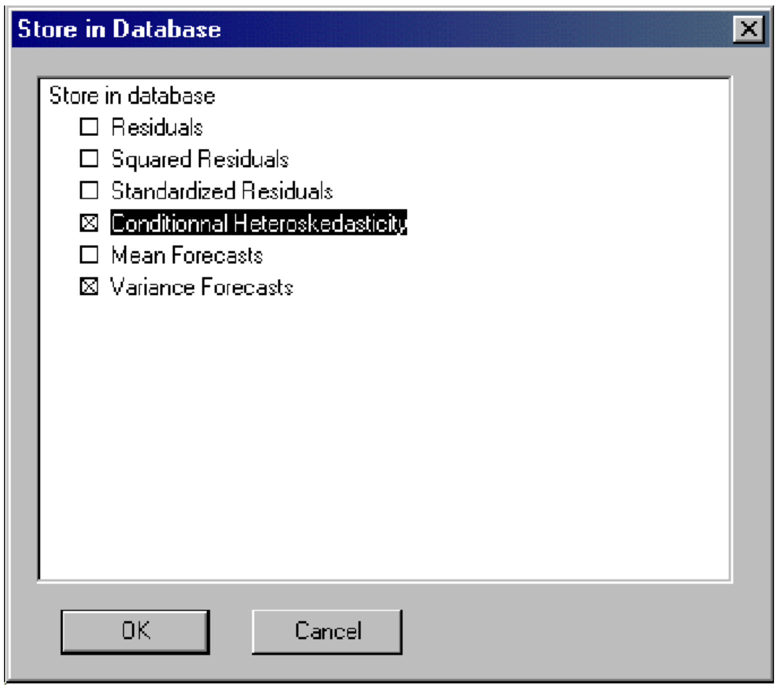


Figure 10: Forecasts from an AR(1)-APARCH(1,1).



Store in database

- ☐ Residuals
- ☐ Squared Residuals
- ☐ Standardized Residuals
- ☒ Conditionnal Heteroskedasticity
- ☐ Mean Forecasts
- ☒ Variance Forecasts

Figure 11: Storing in the Database

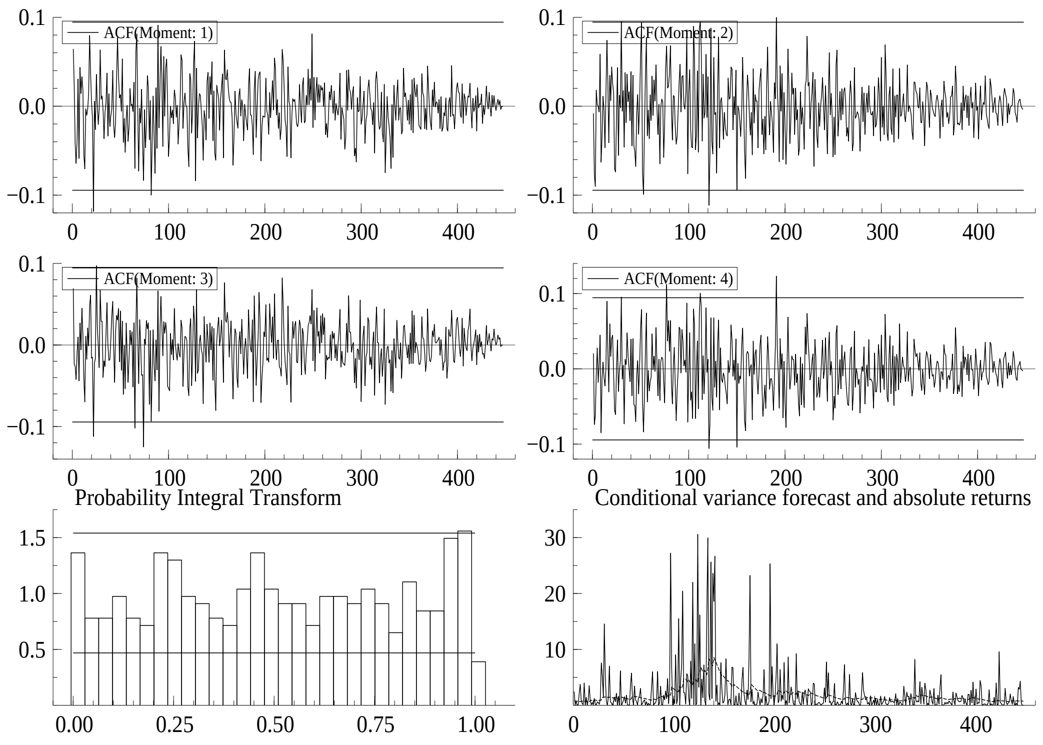


Figure 12: Density Forecast Analysis