Contract Mix and Ownership

Chong-en Bai *and Zhigang Tao†

August 1996

Abstract

This paper analyzes a model with many homogeneous agents, whose effort can be allocated to two tasks. One task produces a public good that is an important input for the production of the final output. The other task only affects the agent's own output. We show that, when the public input and the private input are complementary, the principal should offer a fixed-wage contract to some agents and a revenue-sharing contract to the remaining agents. Furthermore, we show that, when the ex ante contracts are subject to ex post renegotiation, agents with the fixed-wage contract should not own any asset, whereas agents with the revenue-sharing contract should own the physical asset in which the private input is embedded. Meanwhile, the principal should retain residual rights of control over the public good. This paper offers an explanation of the co-existence of company-owned units and franchised units in a franchise company. It adopts and extends important features from both the multi-task theory of the firm and the incomplete-contract theory of the firm.

Key words: Contract Mix, Ownership Structure, Contractual Incompleteness, Multitask, Franchise

JEL classification: L22, D23

*Department of Economics, Boston College, Chestnut Hill, MA 02167.
†Department of Economics, Hong Kong University of Science and Technology, Hong Kong.

1. Introduction

As an increasingly prominent economic organization,[1] franchise has both motivated and challenged recent developments in the theory of the firm. A franchise company typically has both company-owned units and franchised units which differ in at least two important aspects:[2] (1) Managers in the company-owned units have low-powered incentive contracts; they receive fixed wages. Managers in the franchised units, however, have high-powered incentive contracts; they pay a portion of their revenue as royalty to the company and keep the remainder. (2) Managers in the company-owned units do not own any assets of the units, whereas managers in the franchised units own part or all of the units' physical assets.

The multi-task theory of the firm, pioneered by Holmstrom and Milgrom (1991), emphasizes how incentive pay affects the allocation of an agent's attention among his various tasks, and rationalizes different-powered contracts based on ex ante heterogeneity in task importance and task measurability. The incomplete-contract theory of the firm (Grossman and Hart (1986), Hart and Moore (1990), Hart (1995)) elucidates how ownership structure determines payoffs to all parties involved, and justifies various ownership structures according to the relative importance of the parties' investment. For *ex ante homogeneous* units of the franchise company, however, neither contract mix nor multiple ownership structure is an obvious implication of the two theories, let alone the correspondence between contract mix and multiple ownership structure.

This paper attempts to develop a theory of contract mix and multiple ownership structure. It adopts and extends important features from both the multi-task theory and the incomplete-contract theory of the firm.[3]

---

[1] Business format franchise involves not only product, service and trademark, but also the entire business format. According to Lafontaine (1992), both the number of the outlets of business format franchisors and total nominal sales through them had phenomenal growth between 1972 and 1986.

[2] In 1986, the percentage of franchised units ranged from 76% in McDonald's to 82% in Burger King, see Lafontaine (1992).

[3] Hart, Shleifer, and Vishny (1996) discuss incomplete contracts with multiple tasks in a different context. In particular, they exclude the possibility of revenue-sharing contracts, and focus on how ownership structure affects the allocation of an agent's attention among his various

The analysis begins with an observation that it is essential for a franchise company to have reputable goods and services. For example, it was inspired by the success of McDonald brothers' California drive-in that Ray Kroc (the founder of McDonald's) proposed to sell the winning business format together with its brand name to would-be entrepreneurs. Crucial to the franchise business are thus the effort to develop and maintain brand name products and services shared by all units of the company (henceforth, goodwill), as well as the effort in production and distribution (henceforth, sales).[4]

If a manager's payoff is tied to the revenue of her unit with a high-powered incentive contract, she will allocate too little of her effort on goodwill and too much of it on sales activities. This is because goodwill is company wide and thus the manager can free ride on goodwill provided by other units, whereas sales effort is unit specific and she can not rely on other units for its provision. When there are many small units in the company, as is the case of many franchise companies, such misallocation of effort between the two tasks is the most severe. In that case, if given a high-powered contract, the manager will not expend any goodwill effort. In contrast, if given a low-powered contract, then she is indifferent to the allocation of her effort between the two tasks and thus is willing to expend some goodwill effort to the extent that it does not bring about disutility.[5] Therefore, in order to induce goodwill effort, the company offers some of its managers low-powered contracts, despite their adverse implications on overall effort level. With the goodwill provided by these units, sales effort becomes more important in the remaining units of the company, and high-powered incentive contracts are thus optimal for managers of the latter units. Contract mix is optimal in our model, as it allows the company to induce public good (goodwill) production on the one hand and capture the positive externality of goodwill on the other hand.

---

tasks.

[4]The importance of the brand name products and services is well recognized. For example, Kaufmann and Lafontaine (1994) document incidents where franchised units suffered significant loss of revenue once their brand names were revoked.

[5]Following Holmstrom and Milgrom (1991), we assume that the manager may take pleasure in working up to some limit, and incentives are only required to encourage work beyond that limit.

Our theory of contract mix is consistent with stylized facts about franchise.[6] It is well documented that company-owned units are much better than franchised units in terms of quality of services and adherence to uniform standards.[7] This directly supports our explanation of why there are company-owned units. Our model predicts that franchised units are more profitable than company-owned units, because the latter are to provide goodwill effort for the whole company at the expense of sales effort that would enhance their own profitability. This is borne out by Shelton's (1967) finding that the profit margin was 9.5 percent under franchisee ownership and was 1.8 percent under company ownership.

Compared with Holmstrom and Milgrom's multi-task model (1991) which emphasizes the difference in the measurability of various tasks, ours focuses on the difference in the scope of influence of the tasks. Moreover, heterogeneity in task importance is endogenously determined in our model, which explains contract mix for ex ante homogeneous units of the company.[8]

With contract mix understood, it remains to be explained why managers with the high-powered contracts own part or all of the units' physical assets whereas these with the low-powered contracts do not. Our analysis on this issue is inspired by a stylized fact of franchise, namely incompleteness of franchise contracts. While brand name goods and services are of paramount importance to a franchisor, their retail markets are significantly uncertain. It is difficult or even impossible for the company to specify ex ante (in its contracts with the managers) how to develop and maintain the brand name products and

[6] An alternative theory of contract mix in franchise was provided by Gallini and Lutz (1992). In their asymmetric-information model, a franchisor owns some units in order to signal the profitability of his business. However, empirical studies by Lafontaine (1993) and Lafontaine and Shaw (1996) have rejected two major predictions of the signalling theory, namely, higher company ownership is more likely for higher-quality franchisors, and company ownership declines as a franchise company matures over time.

[7] In a detailed study of McDonald's, Love (1986) documents that there were problems of quality and cleanliness in franchised units (Chapter 4) and that company-owned units were set up to "encourage wayward McDonald's franchisees to clean up their act" (Chapter 9).

[8] Optimality of contract mix depends crucially on ex ante heterogeneity when all tasks are of private good nature.

services ex post in response to market changes.[9] Instead, the company keeps the residual rights of control and makes necessary business decisions ex post.[10] A recent example is the outbreak of mad cow disease and McDonald's subsequent decision of not using UK beef. However, the residual rights of control also gives the company an opportunity to hold up the managers ex post. In particular, the company may abuse its power "in order to transfer the franchisees to more profitable franchisees or to convert the outlets to company ownership (Hadfield (1990))."

Note that, when ex ante contracts are incomplete about development and maintenance of brand name products and services, disputes between the HQ and the managers are inevitable. Furthermore, the presence of contractual incompleteness makes it difficult for courts to verify ex post which party is at fault in the event of the disputes. Thus, to examine implications of contractual incompleteness in franchise, we assume that the ex ante contracts could be renegotiated ex post. From MacLeod and Malcomson (1993), however, we know that an ex ante contract may or may not be renegotiated depending on the outside options of the HQ and the manager.

To probe how to make the ex ante contracts renegotiation-proof, we first assume away any contractual remedies, in which case the outside options of the company and the manager are determined solely by the ownership arrangements of the unit's physical asset. When the manager owns the asset, he can deny the company access to the asset. Then, the manager's outside option is to provide generic products and services, while the company's is to conduct business with only the goodwill. When the company owns the asset, it can deny the manager access to both the goodwill and the asset. The manager thus does not have any outside option, while the company has the outside option of

[9]Through examining McDonald's franchise contracts, Hadfield (1990) found that "many of the standards with which a franchisee must comply will not even be articulated until well after the contract has been signed."

[10]The franchise company's residual rights of control are well protected by the business judgment approach that is currently adopted in the courts. For example, from *Amerada Hess*, 142 N.J. Super. at 251, 362 A.2d at 1266 (adapted from Hadfield (1990)), "(t)he substantiality of a franchisee's noncompliance, as a legal concept, must be gauged in light of its effect upon or potential to affect the franchisor's trade name, trademark, good will and image which, after all, is in the heart and substance of the franchising method of doing business."

capturing all the revenue. Under reasonable conditions, we show that, to make their contracts renegotiation-proof, managers with the high-powered contracts need to own their units' physical assets whereas those with the low-powered contracts should not. To conclude our analysis, we then show that contractual remedies, even optimally chosen, cannot mimic what the ownership arrangements have achieved. Thus, given there is severe contractual incompleteness in franchise, the different-powered contracts can only be made renegotiation-proof by the corresponding ownership arrangements.

It is generally held that, because the value of franchisees' assets depends crucially on their access to the brand name, franchisees are extremely vulnerable to franchisors' residual rights of control in general and the power to terminate the franchise contracts in particular. However, empirical studies by Kostecka (1987) found that franchisors terminated 2,651 units in 1985, which equals only .87 percent of the estimated 301,689 units of business-format franchise companies existing then. It is interesting to note that our theory of multiple ownership structure can explain this long-standing puzzle in the franchise literature.

Our model extends the existing incomplete-contracts approach by considering contracts in settings of contractual incompleteness and exploring other roles of ownership. structures (is this controversial?) In Grossman and Hart (1986), ex ante contracts are not possible. Consequently, the residual rights of control solely determines the payoffs of concerned parties, and optimal ownership structure is chosen to elicit ex ante incentive. In our model, ex ante contracts are possible, but they are subject to ex post renegotiation. Optimal ownership structure is chosen to make the ex ante contracts renegotiation-proof. More importantly, when ex ante contracts are not possible, ownership of complementary assets should be rested in a single party to avoid ex post bargaining and ex ante inefficiency (Hart and Moore (1990)). However, when contracts are possible as in franchise, the optimality of contract mix in the presence of a multi-task framework implies *multiple* ownership arrangements of complementary assets.

Our paper is related to Holmstrom and Milgrom (1994) as both papers address contractual and ownership arrangements. Holmstrom and Milgrom (1994) provides a general theory to explain many features of an employment relationship as opposed to an independent contracting relationship, including the power of incentive contracts, ownership of assets, and freedom to act. However, their theory can only explain contract mix and multiple ownership structure based on ex ante heterogeneity of units (say, in monitoring costs). Furthermore, it predicts franchised units over company units when monitoring cost is low, whereas the empirical studies found the opposite (Brickley and Dark (1987)).

The plan of the paper is as follows. In Section 2, we introduce a multi-task model where there is no contractual incompleteness. In Section 3, the company's contract design problem is studied, and the optimality of contract mix is established. In Section 4, we introduce contractual incompleteness into our model and consider contract renegotiation. We show that different-powered incentive contracts can only be implemented by various ownership arrangements of the units' physical assets as observed in the case of franchise. The paper concludes with Section 5.

2. The Model

2.1. Production technology

Consider a company (or chain) that consists of the headquarters (HQ) and many units. The units are indexed by $i \in \mathcal{I}$, where $\mathcal{I}$ is a measurable space with its probability measure denoted by d. To highlight the free-rider problem, we assume that there are infinite units, and,

Assumption 1 *The measure, d, on space $\mathcal{I}$ is atomless. That is, there does not exists an $i \in \mathcal{I}$ such that $d(i) > 0$.*

The manager of each unit performs two tasks: s and g. s is a unit-specific effort that affects only the revenue of the unit, and g is a general effort that increases the revenue of all units of the company. For a fast food chain, for example, s is the sales effort, and g is the effort to develop customer goodwill towards the brand name of the chain, or to learn about customer tastes, or to develop new products. From now on, we will call s the sales effort and g goodwill. The level of the two efforts are not verifiable and hence cannot be contracted on.

We assume, however, that the revenue of each unit is verifiable, as is the case for fast food chains. Furthermore, it is given by

$$x(i) = y(s(i), G) + \epsilon(i),$$

where $\epsilon(i)$ is a normally distributed random variable with mean 0 and variance σ^2, ϵ is independent across units, and $G = \int_{\mathcal{I}} g(i)di$ is the total stock of goodwill possessed by the company. This revenue function, y, assumes that g is a pure public good. The results would not change if the revenue function included $g(i)$ as an additional argument to incorporate the idea that $g(i)$ contributes more to unit i than to other units. (is this true?) We assume that,

Assumption 2 *y is increasing and concave in (s, G).*

The manager incurs a private cost of $c(s, g)$ to provide these efforts. We assume that these efforts are perfectly substitutable in the manager's cost function, i.e., $c = c(s + g)$. In addition, there exists some positive number T such that $c'(t) = 0$ for $t \leq T$, $c(T) = 0$, and $c'(t) > 0$, $c''(t) > 0$ for $t > T$.[11]

It is clear that we adopt a multi-task model pioneered by Holmstrom and Milgrom (1991). However, we emphasize the difference in the scope of influence of the two tasks, whereas they focus on the difference in the measurability of the tasks.

2.2. Incentive contracts

We assume that the manager has constant absolute risk aversion. That is, the manager's utility function is $u(z) = -e^{-rz}$, where r is the coefficient of absolute risk aversion and z is the manager's net (but risky) payoff. The HQ is assumed to be a risk-neutral profit maximizer.

[11]We follow Holmstrom and Milgrom (1991) and (1994) in using this cost function.

The HQ chooses a compensation scheme to induce the manager's efforts. The HQ can offer different contracts to managers, although all managers are identical ex ante. Without losing generality, we consider two types of contracts: (1) a fixed-wage contract; (2) a high-powered contract that rewards the manager according to the revenue of the manager's unit.[12]

Let $w_i(x(i))$ be the incentive contract for the manager of unit i. Then the manager's expected utility is assumed to take the form

$$u(CE) \equiv E\{u[w_i(x(i)) - c(s(i) + g(i))]\},$$

where CE is the manager's certainty equivalent money payoff, and E is the expectation operator. Given contract $w_i(x(i))$, the manager chooses $s(i)$ and $g(i)$ to maximize $E(CE)$.

To summarize, the timing of events is as follows.

(1) At t=0, the HQ chooses $w_i(x(i))$ for all $i \in \mathcal{I}$.

(2) At t=1, the manager of unit i chooses $s(i)$ and $g(i)$.

3. Contract Mix

3.1. Contract design problem

In this analysis, we constrain the HQ to the choice of linear contracts. There is no loss of generality though. Holmstrom and Milgrom (1987) show that the optimal incentive contract in suitably stationary dynamic environments in which the agent can continuously monitor his own performance is equivalent to the optimum of a reduced-form static model in which the principal is constrained to linear contracts.

[12]One may argue that the HQ could also write: (a) a contract that bases a manager's reward only on the revenue of other units, or (b) a contract that bases a manager's reward on the revenue of other units as well as on that of the manager's own unit. Under Assumption 1, each unit is so small that its level of goodwill does not affect the total stock of goodwill of the whole company, G. Furthermore, revenue is stochastically independent across units. Therefore, the revenue of other units does not contain any information about a unit's efforts and thus should not affect the unit's compensation; including the revenue of other units in the reward of the unit only add unnecessary uncertainty to the income of the risk averse manager. Refer to Holmstrom (1982).

For $w_i = \alpha(i)x(i) + \beta(i)$ where $\alpha(i)$ and $\beta(i)$ are constants unrestricted in sign, we have:

$$CE = \alpha(i)y(s(i), G) + \beta(i) - c(s(i) + g(i)) - \frac{1}{2}r\sigma^2\alpha(i)^2.$$

At $t = 1$, given the contracts offered by the HQ, $\{(\alpha(i), \beta(i)) : i \in \mathcal{I}\}$, the managers choose $(s(i), g(i))$. Specifically, taking G as given, the manager of unit i solves

$$\max_{s(i)\geq 0, g(i)\geq 0} CE = \alpha(i)y(s(i), G) + \beta(i) - c(s(i) + g(i)) - \frac{1}{2}r\sigma^2\alpha(i)^2.$$

If $\alpha(i) > 0$, the manager of unit i chooses $g(i) = 0$. This is because, in this case, a manager has an incentive to increase the revenue of his unit. The optimal way to do so is to allocate no effort to goodwill and all his effort to sales, as goodwill is a pure public good and has only an infinitesimal effect on the revenue of the unit while sales effort has a non-trivial positive effect. $s(i)$ is uniquely determined by

$$s(i) = \arg\max_{s(i)\geq 0} \alpha(i)y(s(i), G) - c(s(i)). \qquad (OP - s)$$

Such an incentive contract is called a high-powered contract. A manager who receives a high-powered contract is called manager H, and his unit H unit.

If the manager of unit i receives a fixed-wage contract ($\alpha(i) = 0$), then he puts a combined effort level of T (i.e., $s(i) + g(i) = T$) and is indifferent between the sales and goodwill efforts (or the optimal choice of $(s(i), g(i))$ is not unique). This is because, in contrast to the case of $\alpha(i) > 0$, the manager's utility does not depend on the revenue of his unit and consequently not on how his effort is allocated between the two tasks. Therefore, when $\alpha(i) = 0$, the manager is assumed to do what is requested by the HQ among the set of $\mathcal{O}_i = \{(s(i), g(i)) : s(i) + g(i) = T\}$. Such an incentive contract is called a low-powered contract. A manager who receives a low-powered contract is call manager L, and his unit L unit.[13]

[13] If $\alpha(i) < 0$, the manager will choose $(s(i), g(i)) = (0, T) \in \mathcal{O}_i$. For the HQ, a contract with a negative $\alpha(i)$ is strictly dominated by a fixed-wage contract because, under the former contract, the HQ has less flexibility in choosing $(s(i), g(i))$ than under the latter contract and furthermore has to pay a risk premium, $\frac{1}{2}r\sigma^2\alpha(i)^2$, to the manager. Therefore, the HQ will never offer a contract with $\alpha(i) < 0$.

The above discussion shows that the HQ can only induce goodwill effort by offering extreme (fixed-wage) incentive contracts to managers. The intuition for this result should be emphasized. For any manager, his goodwill effort has negligible effect on his unit's revenue, whereas the sales effort has non-trivial effect on the revenue. It follows that, once a manager's payoff is slightly related to his unit's revenue, the manager would shift all of his effort to sales. (would risk aversion matters?)

Let $\mathcal{L} \subseteq \mathcal{I}$ be the set of L units and p be the measure of this set, then $G = \int_{\mathcal{L}} g(j)dj$ and the HQ's expected total profit is

$$\Pi = \int_{\mathcal{L}} [y(T - g(j), G) - w(j)]dj + \int_{\mathcal{I}-\mathcal{L}} [(1 - \alpha(i))y(s(i), G) - \beta(i)]di,$$

where $g(j)$ is the level of goodwill effort chosen by the HQ for the j^{th} L-unit and $s(i)$ is the level of sales effort chosen by the manager of the i^{th} H unit. At $t = 1$, given p, $\{w(j)\}_{j\in\mathcal{L}}$ and $\{\alpha(i), \beta(i)\}_{i\in\mathcal{I}-\mathcal{L}}$, and also taking $\{s(i)\}_{i\in\mathcal{I}-\mathcal{L}}$ as given, the HQ chooses $\{g(j)\}_{j\in\mathcal{L}}$ to maximize Π, i.e.,

$$\{g(j)\} = \arg\max_{\{g(j)\}} \Pi. \qquad (OP - G)$$

In summary, at $t = 1$, $\{s(i)\}_{i\in\mathcal{I}-\mathcal{L}}$ and $\{g(j)\}_{j\in\mathcal{L}}$ are jointly determined by $(OP - s)$ and $(OP - G)$.

At $t = 0$, the HQ chooses p, $\{w(j)\}_{j\in\mathcal{L}}$, and $\{\alpha(i), \beta(i)\}_{i\in\mathcal{I}-\mathcal{L}}$ to maximize the expected total profit, subject to the incentive compatibility constraints, $(OP-s)$ and $(OP-G)$, and the individual rationality constraints that managers are willing to accept the contracts. We normalize the reservation utility of the managers to be 0. Then, the HQ's optimization problem at $t = 0$ is

$$\max_{\{\alpha(i),\beta(i)\}_i,\{w(j)\}_j,p} \int_{\mathcal{L}} [y(T-g(j), G)-w(j)]dj+\int_{\mathcal{I}-\mathcal{L}} [(1-\alpha(i))y(s(i), G)-\beta(i)]di \qquad (OP-HQ)$$

$$\begin{aligned} s.t. \quad & (OP - s), (OP - G) & (IC) \\ & w(j) \geq 0 \text{ for all } j \in \mathcal{L} & \\ & \alpha(i)y(s(i), G) + \beta(i) - c(s(i)) - \tfrac{1}{2} r\sigma^2\alpha(i)^2 \geq 0 \text{ for all } i \in \mathcal{I} - \mathcal{L} & (IR) \end{aligned}$$

Since $w(j)$ and $\beta(i)$ do not affect the incentive compatibility constraints, the individual rationality constraints must be binding at the optimum. Otherwise, the HQ's expected total profit can be increased by reducing $w(j)$ or $\beta(i)$. Substitute constraints (IR) into the objective function. Then

$$\max_{\{\alpha(i)\}_i,p} \int_{\mathcal{L}} y(T-g(j),G)dj + \int_{\mathcal{I}-\mathcal{L}} [y(s(i),G) - c(s(i)) - \frac{1}{2} r\sigma^2 \alpha(i)^2] di \quad (OP-HQ')$$

$$s.t. \quad (OP-s), (OP-G) \quad (IC)$$

3.2. Uniformity of high-powered and low-powered contracts

We make additional assumptions about the production and cost functions.

Assumption 3 $\lim_{s\to 0} y_s \equiv \frac{\partial y(s,G)}{\partial s} = \infty$, *and* $\lim_{G\to 0} y_G \equiv \frac{\partial y(s,G)}{\partial G} = \infty$.

This assumption is made to avoid possible complications of corner solutions.

Assumption 4 $\frac{c'(s)}{y_s(s,G)}$ *is convex in* s.

The assumption holds if the cost function is sufficiently convex. In the case that the production function is Cobb-Douglas and the cost function is $c = (s+g)^\lambda$, it is satisfied if λ is sufficiently large. $\frac{c'(s)}{y_s(s,G)}$ is the ratio of the marginal cost, to the marginal benefit, of the sales effort. The assumption implies that the solution to $(OP-s)$, $s(i)$, is concave in $\alpha(i)$.

Assumption 5 *The marginal product of goodwill effort,* $y_G \equiv \frac{\partial y(s,G)}{\partial G}$, *is increasing and concave in* s.

This assumption says that effort s and G are complementary and the return to s in enhancing the marginal product of G diminishes.

To characterize the optimal contracts, we first have:

Proposition 1 *The HQ should (1) request the same level of goodwill effort from all managers of L units, and (2) offer the same high-powered contract to all managers of H units.*

All proofs not provided in the main text are contained in the appendix.

The intuition for the first result is quite straightforward. While the HQ may request different levels of goodwill effort from different L units, the managers' total efforts remain the same, i.e., $s(j) + g(j) = T$ for $j \in \mathcal{L}$. Suppose that the HQ requests two levels of goodwill effort, g_1 and g_2, respectively from, p_1 and p_2 measure of the managers of L units, where $p = p_1 + p_2$ and $G = p_1 g_1 + p_2 g_2$. Then, by the concavity of $y(T - g, G)$ with respect to $T - g$, the HQ could obtain more profit by requesting same level of goodwill effort, $\bar{g}$, from all managers of L units, where $\bar{g} = \frac{p_1}{p_1+p_2} g_1 + \frac{p_2}{p_1+p_2} g_2$ and $(p_1 + p_2)\bar{g} = G$.

The second result of Proposition 1 is more involved. This is because, in contrast to the case of L units, both the sales and total efforts by the managers of H units are directly affected by the high-powered incentive contracts. So is the HQ's choice of goodwill level. Instead of offering a single high-powered contract, the HQ could offer two contracts, one less high-powered and the other more high-powered, under which one group of managers would decrease their sales effort and the other increase their sales effort. When the cost function is not convex enough, the decrease in sales effort is not too much while the increase in the sales effort is quite a lot, and the HQ could benefit by offering different high-powered contracts. However, when the cost function is sufficiently convex, specifically, when Assumption 4 is satisfied, offering the same high-powered contract increases the average sales effort of managers of H units. When s and G are complementary as Assumption 5 says, this also increases the HQ's incentive to choose high level of goodwill by increasing the marginal benefit of goodwill effort.[14]

Empirical studies by Lafontaine and Shaw (1996) show that, while franchise contracts vary from one franchisor to another, they are extremely uniform across franchisees, and rather stable over time, within any particular franchise company. McAfee and Schwartz (1994) offer a market-based explanation for this phenomena. Specifically, in their model, after signing a contract with one franchisee, a franchisor is tempted to offer another

[14]Technically, for choice of high-powered incentive contracts, the HQ's optimization problem is constrained, and in general not concave. However, with Assumption 4, we can show that the HQ's optimization problem for any given G is concave. Assumption 5 further ensures that the HQ's choice of G is also well-behaved. Taken together, we have the second result of Proposition 1.

franchisee a contract with a lower royalty fee to undercut the first franchisee and therefore obtain a higher lump-sum fee. Knowing the franchisor has such opportunistic behavior, the first franchisee is reluctant to accept the contract and the franchisor is thus better off by commiting to a uniform contract for all franchisees. Note that in our set-up, each manager is a local monopoly and McAfee and Schwartz's argument is no longer applicable. Our model thus offers an alternative and purely technological explanation for the uniformity of franchise contracts.

3.3. Equilibrium outcomes given the uniform contracts

Proposition 1 greatly simplifies the contract design problem (as described in Section 3.1). Let (w) be the low-powered contract offered by the HQ to all managers of L units, and g is the goodwill effort. Let (α, β) be the high-powered contract offered by the HQ to all managers of H units, and s is the corresponding sales effort.

At $t = 1$, given p, (w) and (α, β), H managers choose s and the HQ picks g. Since $\lim_{s \to 0} y_s = \infty$, y is concave and c is convex, for $\alpha > 0$, H manager's optimal choice of s is characterized by

$$\alpha y_s(s, G) - c'(s) = 0. \qquad (FOC - s).$$

Define the response function of s to G, $s = s(G; \alpha, p)$, to be the solution to $(FOC - s)$. Since s and G are complementary in the production function, i.e., $y_{sG} > 0$, a higher G makes the sales effort of the H manager more productive and therefore increases his choice of s. A larger α gives the H manager a higher share of revenue and thus increases his choice of s. That is,

Lemma 1 *The response function s increases in G and α, and is independent of p.*

The HQ's objective function at $t = 1$ becomes

$$\Pi = p[y(T - g, G) - w] + (1 - p)[(1 - \alpha)y(s, G) - \beta], \quad \text{where} \quad G = pg.$$

Since y is concave, it can be easily shown that Π is concave in g if $p \neq 0$ and Π is independent of g if $p = 0$. Furthermore, as $\lim_{s \to 0} y_s = \infty$ and $\lim_{G \to 0} y_G = \infty$, for $p > 0$, the HQ's optimal choice of g is an interior solution and is characterized by

$$py_G(T-\frac{G}{p},G)-y_s(T-\frac{G}{p},G)+(1-p)(1-\alpha)y_G(s,G)=0, \qquad (FOC-G)$$

where the left hand side of the equation is equal to $\frac{\partial \Pi}{\partial G}$. Define the response function of G to s, $G=G(s;\alpha,p)$, to be the solution to $(FOC-G)$. Again, the complementarity between s and G implies that G increases with s. As α increases, the HQ gets a smaller share of revenue and thus chooses a lower G. The effect of p on G depends on how p affects the marginal profit of G. Therefore, we have,

Lemma 2 *The response function G increases in s and decreases in α. It increases with p if and only if*

$$\frac{\partial^2 \Pi}{\partial p \partial G}=y_G^L+gy_{sG}^L-\frac{g}{p}y_{ss}^L-(1-\alpha)y_G^H>0.$$

Thus, at $t=1$, given p, (w) and (α,β), equilibrium (s,G) are determined simultaneously by $(FOC-s)$ and $(FOC-G)$; they represent the point of intersection of the two response functions, s and G, defined above. It is shown in the Appendix that there exists a unique equilibrium (s,G) which are functions of (α,p).

By the concavity of y, it is a standard exercise to show that the equilibrium is stable, and that s and G are differentiable functions of (α,p) when $\alpha>0$ and $p>0$.

Since the response function s is independent of p, how equilibrium s and G change with p depends on how the response function G depends on p. As Figure 1 illustrates, equilibrium s and G both increase (decrease, respectively) with p if the response curve G moves rightward (leftward, respectively).

⋆ ⋆ ⋆ Figure 1 ⋆ ⋆ ⋆

By Lemmas 1 and 2, the response function s increases with α but the response function G decreases with α. However, the effect of α on equilibrium s and G are ambiguous. In summary, we have:

Lemma 3 *For $\alpha > 0$ and $p > 0$: (1) Equilibrium s and G are differentiable functions of (α, p). (2) Equilibrium s and G both increase (or decrease) with p if $\frac{\partial^2 \Pi}{\partial p \partial G} > 0$ (or $\frac{\partial^2 \Pi}{\partial p \partial G} < 0$). (3) The effects of α on equilibrium s and G are usually ambiguous.*

As we attempt to investigate the optimality of contract mix (i.e., $\alpha^* > 0$ and $p^* \in (0, 1)$), it is important to analyze the boundary case in which $\alpha = 0$ or $p = 0$. The first step of the analysis is to properly define the case of $\alpha = 0$ and that of $p = 0$. Specifically, we define the case of $\alpha = 0$ and $p < 1$ to be the limiting case of $\alpha > 0$ and $p < 1$. It should be stressed that, even when α approaches zero, L units ($\alpha = 0$) are distinct from H units ($\alpha = 0+$); the managers of H units choose $s = T$ and $g = 0$, while the managers of L units follow the instruction of the HQ on the choice of (s, g) from set $\mathcal{O} = \{(s, g) : s + g = T\}$. Therefore, the case of $\alpha = 0$ and $p < 1$ is different from the case of $p = 1$, in which all units are L units and all managers follow the instruction of the HQ. For the boundary case of $p = 0$, we have $G = 0$. However, we define the value of g at $p = 0$ to be the limit of the optimal g as $p \to 0$. With these definitions, we can show:

Lemma 4 *(1) s and G are continuous at $\alpha = 0$. (2) When $p = 0$, $s(\alpha, 0)$ is uniquely determined by $(FOC - s)$ and is continuous in α. (3) Note that Lemma 4 does not imply that $s(\alpha, p)$ is continuous at $p = 0$; $\lim_{p \to 0} s(\alpha, p)$ may be different from $s(\alpha, 0)$. This is because at $p = 0$, $G = 0$ and $\lim_{G \to 0} y_G = \infty$.*

To conclude, note that, at $t = 0$, the HQ's objective function becomes

$$\Pi(\alpha, p) = py(T - \frac{G}{p}, G) + (1 - p)[y(s, G) - c(s) - \frac{1}{2} r \sigma^2 \alpha^2], \tag{1}$$

where s and g are the equilibrium defined by $(FOC - s)$ and $(FOC - G)$, and functions of (α, p). By Lemmas 3 and 4, $\Pi(\alpha, p)$ is continuous when $p > 0$ and differentiable when $\alpha > 0$ and $p > 0$. The HQ chooses α and p to maximize its objective function, which we turn to in the next two subsections.

3.4. Characterization of the high-powered contract

When the HQ chooses α, it considers several factors. First, as α increases, H managers are subject to more risk and therefore need to be compensated more. Second, the value of α affects the HQ's incentives to provide goodwill effort and H manager's incentives to expend sales effort. Differentiate Π with respect to α and simplify the derivative with $(FOC - s)$ and $(FOC - G)$. We have, for $\alpha > 0$,

$$\frac{d\Pi}{d\alpha} = -(1-p)r\sigma^2\alpha + \alpha(1-p)y_G^H\frac{dG}{d\alpha} + (1-p)(1-\alpha)y_s^H\frac{ds}{d\alpha}, \tag{2}$$

where a function with a superscript H means that it is evaluated at (s, G). On the right hand side of (2), the first term captures the risk factor and is negative, and the next two terms describe the effects of α on Π through G and s, respectively.

By Lemma 3, the signs of $\frac{dG}{d\alpha}$ and $\frac{ds}{d\alpha}$ are usually ambiguous, and so is the sign of $\frac{d\Pi}{d\alpha}$. However, when $\alpha \to 1$, G becomes independent of s as shwon in $(FOC - G)$; we then have $\frac{dG}{d\alpha} < 0$ as is illustrated by Figure 2. Therefore, as $\alpha \to 1$, the first two terms on the RHS of (2) are negative and the third term becomes negligible, even though the sign of $\frac{ds}{d\alpha}$ remains ambiguous; or $\frac{d\Pi}{d\alpha} < 0$ as $\alpha \to 1$, which implies that the optimal α is less than 1.

When $\alpha \to 0$, the first two terms on the RHS of (2) become negligible, and the sign of $\frac{ds}{d\alpha}$ is determined by that of the third term or $\frac{ds}{d\alpha}$. From $(FOC - s)$, we know that $s = T$ at $\alpha = 0$. Furthermore, as α increases from zero, $\alpha y_s(s, G)$ of $(FOC - s)$ becomes positive, which implies that $s > T$ at $\alpha = 0+$. Therefore, we have $\frac{ds}{d\alpha} > 0$ as $\alpha \to 0$. Consequently, $\frac{d\Pi}{d\alpha} > 0$ as $\alpha \to 0$, which implies that the optimal α is greater than 0.

Proposition 2 *At the optimum,* $\alpha \in (0, 1)$.

3.5. Characterization of contract mix

By equation (1), the HQ's total profit is $\Pi = p\pi^L + (1-p)\pi^H$, where $\pi^L \equiv y(T - \frac{G}{p}, G)$ is the HQ's expected profit from a L unit, and $\pi^H \equiv y(s, G) - c(s) - \frac{1}{2}r\sigma^2\alpha^2$ is the HQ's expected profit from a H unit. If π^L and π^H were independent of p, then either a high-powered contract or a low-powered contract would be offered by the HQ to all its units (namely, either $p = 0$ or $p = 1$). Contract mix (namely, $p \in (0, 1)$) would not be optimal unless with probability zero that $\pi^L = \pi^H$.

In this model, however, goodwill is a public good; in addition, only managers with a low-powered contract (or L units) can provide goodwill effort (recall that $G = pg$ where p is the measure of L units and g is goodwill effort by a L manager). This seems to justify the existence of L units as they have positive externality on other units of the company. On the other hand, L managers, because of the low-powered contract, expend a lower level of total effort than H managers ($s(j) + g(j) = T$ for $j \in \mathcal{L}$, whereas $s(i) \geq T$ for $i \in \mathcal{I} - \mathcal{L}$), which makes the L units less profitable. Intuition therefore suggests that the HQ offers a high-powered contract to some units and a low-powered contract to the others.

To investigate the conditions under which the HQ's optimal choice of p is within 0 and 1, we first consider the first-order derivative of Π with respect to p for the case of $p > 0$. Simplifying by $(FOC - s)$ and $(FOC - G)$, we have,

$$\frac{d\Pi}{dp} = (\pi^L - \pi^H) + (1-p)\alpha y_G^H \frac{\partial G}{\partial p} + (1-p)(1-\alpha) y_s^H \frac{\partial s}{\partial p} + g y_s^L, \tag{3}$$

where a function with a superscript H (L, respectively) means that it is evaluated at (s, G) ($(T - \frac{G}{p}, G)$, respectively). The four terms in (3) represent different effects of increasing p.

When there is one more L unit, the HQ gains π^L but loses π^H. The first term on the RHS of (3) captures this direct effect. Proposition 3 says that this direct effect is negative.

Proposition 3 *For any given p, $\pi^L < \pi^H$ at the corresponding HQ's optimal choice of α, denoted by $\alpha^*(p)$. In particular, $\pi^L < \pi^H$ holds in equilibrium.*

Intuitively, there are two reasons for the result of Proposition 3. One is that, to induce goodwill effort, the HQ is constrained to offering the low-powered incentive contract to L managers. As a result, L managers expend a lower level of total effort than H managers. The other reason is that H managers free ride on L managers for goodwill and therefore they are able to put more effort on sales. Both reasons stem from the public good nature of goodwill effort.

L units showing lower profits on the book than H units does not mean that there should be none or very few of L units. L units are important because they are the only providers of goodwill effort, which not only increases profits of all units, L or H, directly, but also affects the productivity of sales effort in H units. The second and the third terms in (3) capture, respectively, the effect of p on Π through total stock of goodwill and that through the level of sales effort of H managers. The signs of the second and the third terms are yet to be determined. They, respectively, are the same as those of $\frac{\partial G}{\partial p}$ and $\frac{\partial s}{\partial p}$, which we discussed in Lemma 3 for the case of $p > 0$.

Having more L units to provide goodwill also mitigates inefficient substitutions between sales and goodwill efforts in L units. When $p \neq 1$, the burden of providing a given level of goodwill, G, is unevenly borne by different units of the company; only L units provide goodwill. Because the sales effort has decreasing returns to scale, the cost of L units devoting less effort to sales exceeds the benefit of H units being able to exert more effort in sales (I don't understand this statement). With more L units, the cost of such inefficient substitutions between tasks is reduced. This substitution effect is captured by the fourth term in (3) and is always positive.

Thus far, we have analyzed $\frac{d\Pi}{dp}$ for the case of $p > 0$. To establish conditions for the optimality of contract mix (namely, $p^* \in (0,1)$), it is essential to address the boundary case of Π at $p = 0$.

Note that the optimality of $p^* > 0$ is straightforward, if $y(s, 0) = 0$ for all s. Specifically, at $p = 0$, $G = 0$ and $\Pi = 0$. At $p > 0$, $G > 0$ and $\Pi > 0$. Therefore, the optimal p must be positive. Intuitively, if the revenue vanishes in the absence of goodwill, then the HQ must have the L units to provide the essential input.

The following analysis, however, is focused on the scenario that $y(s, 0) > 0$. It is then not so apparent that the optimal p has to be positive. In that case, the sufficient conditions for $p^* > 0$ are (1) the limiting value of $\Pi(\alpha', p)$ as $(\alpha', p) \to (\alpha, 0)$ is no lower than that of $\Pi(\alpha, 0)$, and (2) $\frac{d\Pi}{dp} > 0$ as $p \to 0$.

Lemma 5 shows that the low bound of $\lim_{(\alpha',p)\to(\alpha,0)} \Pi(\alpha', p)$ is greater than $\Pi(\alpha, 0)$, thus ensuring that part (1) of the sufficient conditions is met.

Lemma 5 $\liminf_{(\alpha',p)\to(\alpha,0)} \Pi(\alpha', p) \geq \Pi(\alpha, 0)$. *Furthermore, if* $y(s, 0) = 0$ *for all* s*, then* $\lim_{(\alpha',p)\to(\alpha,0)} \Pi(\alpha', p) = \Pi(\alpha, 0)$.

To address part (2) of the sufficient conditions (or the asymptotic properties of $\frac{d\Pi}{dp}$ as $p \to 0$), we further assume that

Assumption 6 *If* $y(s, 0) > 0$ *for* $s > 0$*, then* $y_{sss} \geq 0$ *and* $\lim_{(s,G)\to(0,0)} y_s(s, G) = \infty$.

Assumptions 2, 3, 5, and 6 (all about the revenue function) are satisfied by the functions of $y(s, G) = c_0 s^a G^b + c_1 s^\delta + c_2 G^\eta$, with c_0, c_1, and c_2 nonnegative, which include Cobb-Douglas functions as special cases. For these revenue functions, Assumption 4 is also satisfied for sufficiently large λ if the cost function is $c(s + g) = (s + g)^\lambda$.

Note that, in terms of the importance of G to the revenue function, Assumption 6 imposes much less restrictions than $y(s, 0) = 0$ does. The two conditions in Assumption 6 are concerned with the effects of s on y. $\lim_{(s,G)\to(0,0)} y_s(s, G) = \infty$ means that, even with very small G, the marginal revenue of sales effort is still very large when the level of s is low. In this case, it is very costly to ask a few L units to concentrate almost all their effort on goodwill. Therefore, the substitution effect of having $p > 0$, $gy_s(T - g, G)$, is sufficiently large to dominate the negative direct effect, $\pi^L - \pi^H$. $y_{sss} \geq 0$ says that y_s is weakly convex. It enables us analyze the properties of $\frac{\partial^2\Pi}{\partial p\partial G}$ as $p \to 0$. We can prove that $\frac{\partial^2\Pi}{\partial p\partial G} \to \infty$ as $p \to 0$, which implies by Lemma 3 $\frac{\partial s}{\partial p} > 0$ and $\frac{\partial G}{\partial p} > 0$. This in turn implies that the second and the third terms in $\frac{d\Pi}{dp}$ are positive as $p \to 0$. All taken together, we have $\frac{d\Pi}{dp} \to \infty$ as $p \to 0$.

To summarize, we have:

Proposition 4 *It is optimal for the company to have some L units.*

Having spelled out the conditions for $p^* > 0$, we turn to the question of when it is optimal for the HQ to have some H units, namely, $p^* < 1$. With L units providing the goodwill, the HQ can offer managers of the remaining units a high-powered contract ($\alpha > 0$) and thereby elicit high sales effort from them. Whether or not high-powered contracts should be offered to some units depends on the magnitudes of their costs and benefits.

One cost of high-powered contracts is that they subject the managers to risks, resulting in inefficient risk sharing between the risk neutral HQ and the risk averse managers. Therefore, it is more likely for the HQ to have some H units when the managers are less risk averse (with smaller r) or when there is less uncertainty (smaller σ).

The main benefit of high-powered contracts is that they elicit high sales effort from H managers. When the marginal costs of sales effort are lower, it is easier to induce it and thus the benefit of high-powered contracts is higher. Therefore, it becomes more likely for the HQ to have some H units.

To formally establish these two predictions, note that at $p = 1$, $(FOC - p)$ becomes

$$\frac{d\Pi}{dp} = (\pi^L - \pi^H) + gy_s^L,$$

of which the first term is negative and the second term is positive. $(FOC - G)$ becomes

$$y_G(T - G, G) - y_s(T - G, G) = 0,$$

which implies that G does not depend on α. Therefore, α is chosen to

$$\begin{array}{rcll} \pi^H & = & \max & y(s, G) - Kc(s) - \frac{1}{2}r\sigma^2\alpha^2 \\ & & s.t. & \alpha y_s(s, G) - Kc'(s) = 0 \qquad (IC) \end{array}$$

where K is a cost parameter. Standard exercise shows that

Lemma 6 *When $p = 1$, the optimal $s > T$. $\frac{d\pi^H}{dr} = -\frac{1}{2}\sigma^2\alpha^2 < 0$, $\frac{d\pi^H}{d\sigma} = -r\sigma\alpha^2 < 0$ and $\frac{d\pi^H}{dK} = -c(s) - \lambda c'(s) < 0$, where λ is the Lagrange multiplier of the constraint and is positive.*

Since at $p = 1$, $\pi^L = y(T - G, G)$ and $gy_s^L = Gy_s(T - G, G)$ do not depend on σ or K, it follows from (FOC-p) and Lemma 6 that,

Proposition 5 *It is more likely for the HQ to have some H units when the managers are less risk averse (r is smaller), or when the uncertainty about its revenue (measured by σ) is smaller, or when marginal cost of effort (measured by K) is smaller.*

Proposition 5 gives one set of conditions for the HQ to have H units, namely, H units are attractive enough. A complementary set of conditions is that L units do not perform very well. With low-powered incentive contracts, the level of total effort is T. If T is very small, the profits from L units are very low. Therefore, it is necessary for the HQ to offer some high-powered contracts so that more effort can be induced. To confirm this intuition, we have:

Lemma 7 *When $p = 1$, if $y(s, 0) > 0$ for $s > 0$ and $\lim_{(s,G)\to(0,0)} G y_G(s, G) = 0$, then $\frac{d\pi}{dp} |_{p=1} < 0$ for sufficiently small T.*

It follows that,

Proposition 6 *If $y(s, 0) > 0$ for $s > 0$ and $\lim_{(s,G)\to(0,0)} G y_G(s, G) = 0$, then the HQ should have some H units when T is small enough.*

4. Multiple Ownership Arrangements

The above analysis has addressed the first distinguishing feature of franchise, namely, both high-powered and low-powered incentive contracts are used. What remains to be explained is why managers with the high-powered incentive contract own some or all of their units' physical assets whereas those with the low-powered incentive contract do not. In this section, we first discuss a stylized fact of franchise, namely, incompleteness of ex ante incentive contracts and the consequent contract renegotiation. We then show that those different-powered contracts can only be made renegotiation-proof by the corresponding ownership arrangements.

4.1 Contractual incompleteness and residual rights of control

A basic assumption in Sections 2 and 3 is that contracts are complete. The HQ offers the low-powered contract to some managers who provide the goodwill effort, and the high-powered contract to others who expend all their effort on sales. Once these contracts are written, the HQ is no longer needed for carrying out the business.[15] In this complete-contracting framework, it does not matter whether the units' physical assets are owned by the managers or the HQ.[16]

In reality, the retail markets for the HQ's brand name products and services are significantly uncertain. The HQ needs to develop and maintain its brand name in response to market changes. For example, when new scientific studies reveal that some of the food ingredients are not healthy enough, the HQ (of a fast-food business) may choose to replace those ingredients. To write complete (ex ante) contracts with the managers, it requires the HQ to foresee all possible future contingencies and devise corresponding strategies for the managers, which is very costly if not impossible. As a result, "many of the standards with which a franchisee must comply will not even be articulated until well after the contract has been signed," and "the key characteristic of the franchise contract is its incompleteness (Hadfield (1990))."

Rather than writing complete (ex ante) contracts, the HQ retains the residual rights of control to make business decisions as ex ante unforeseen contingencies arise (Grossman and Hart (1986), Hart and Moore (1990), Hart (1995)). In particular, the HQ may propose some minimum quality standard (MS) ex post for the managers.[17] An examination of the McDonald's franchise contract reveals that the HQ has substantial residual control rights, and that its franchises are required for "strict adherence to licensor's *standards and policies as they exist now and as they may be from time to time modified* (italics added)." A recent example is the outbreak of mad cow disease and subsequent McDonald's decision of not using UK beef.

While the residual rights of control greatly facilitates the HQ to develop and maintain its brand name in response to market changes, it also gives the HQ an opportunity to hold

[15] Note that contracts could be complete though the goodwill and sales efforts are not verifiable. See Hart (1995).

[16] Throughout the paper, we assume that the HQ owns the goodwill stock. This is in fact an optimal arrangement. We will further discuss this after Proposition 9.

[17] The MS is another production input besides the goodwill and sales efforts, and is indispensable for maintaining the brand name. Although MS is not contractible ex ante, it can be enforced if it is agreed upon by both the HQ and the manager ex post.

up the managers ex post. A HQ may "be enforcing a minor or curable contract violation not to promote the quality of its franchisees but to achieve some other, opportunistic goal at the franchisee's expense, " or abuse its power "in order to transfer the franchises to more profitable franchisees or to convert the outlets to company ownership (Hadfield (1990))".

It should be stressed that the objective of this paper is not to probe why franchise contracts are often incomplete. There is an extensive body of legal studies on this issue (see Hadfield (1990) and references therein). Among the reasons suggested are the importance of brand name products and services, the uncertainty of the retail markets, and the need for quick responses to possible market changes. See also Anderlini and Felli (1994), Hart (1995), Maskin and Tirole (1996), and Segal (1995) for the theoretical foundations of the incomplete-contracts approach. What this paper attempts to show is that, given the contractual incompleteness in franchise and the consequent contract renegotiation, the different-powered ex ante contracts can only be made renegotiation-proof by the multiple ownership arrangements of the units' physical assets.

4.2 Contract renegotiation

In the presence of contractual incompleteness and the HQ's residual rights of control, contract dispute between the HQ and the managers is inevitable. The managers could complain about the unreasonable MS, while the HQ insists on the importance of the MS. To make things even worse, contractual incompleteness makes it difficult to verify which party is at fault, and impossible to implement contingency-based penalty for breach of contract. This implies that ex ante contracts could be renegotiated ex post.[18]

Accordingly, the model of Section 2 is modified as follows. At $t = 0$, the incentive contracts (both high-powered and low-powered) are written and the contract mix ratio (p) is chosen. At $t = 1$, tasks g and s are chosen by the managers in the H units, and by the HQ in the L units. What is different from the time line in Section 2 is that, at $t = 2$,

[18] In general, the HQ could propose some minimum quality standard which is very costly to the manager, or brings extra benefit to the HQ, or both. Renegotiation of ex ante contracts under such circumstances is much more complicated.

uncertainty is resolved and the contract renegotiation could be initiated.[19]

We adopt MacLeod and Malcomson's (1993) framework to analyze the renegotiation game. Specifically, the renegotiation lasts for a period of unit length, which is divided into N stages. Each stage has length 1/N. In stage n, (n = 1, 2, ..., N), at $n.0$, nature chooses either the HQ with probability π or the manager with probability $1-\pi$ to make an offer. At $n.1$, the chosen party offers a revenue-sharing contract $RS(n)$.

The renegotiation game is continued at $n.2$ when the other party responds by taking the outside option (henceforth "O"), or accepting the offer ("A"), or rejecting the offer ("R"). If "O" is chosen, the game ends. If "R" (or "A") is chosen, the two parties decide whether to trade under $RS(n-1)$ (or $RS(n)$) at $n.3$ and $n.4$ sequentially, and the game continues with $RS(n-1)$ (or $RS(n)$) as the standing contract for the next stage of renegotiation. See Figure 3 of MacLeod and Malcomson (1993) for details.

In MacLeod and Malcomson (1993), both no-trade and outside option could be the triggers for contract renegotiation. In our model, however, trade under the mutually beneficial revenue-sharing contract (such as $RS(n)$ and $RS(n-1)$) is preferred by both parties to no-trade; and the only possible trigger for contract renegotiation is outside option. Let u denote the HQ's stage payoff under the original revenue-sharing contract, and v denote that of the manager. Let u_0 be the HQ's stage payoff from the outside option, and v_0 be that of the manager. We obtain the following equilibrium of the renegotiation game.

Proposition 7 (MacLeod and Malcomson (1993)) *Suppose $R \equiv u+v > u_0+v_0$. The payoffs to the HQ and the manager in any markov perfect equilibrium of the renegotiation game are given by U and V, where r is the interest rate,*

$$rV = R - rU, \tag{4}$$

$$rU = \begin{cases} R - v_0, & \textit{when } v_0 > v, \\ u_0, & \textit{when } u_0 > u, \\ u, & \textit{otherwise.} \end{cases} \tag{5}$$

[19] As long as the incentive contract is not renegotiated after the uncertainty is resolved, the argument for linear contracts still applies (Holmstrom and Milgrom (1987)).

Proposition 7 shows that, whenever the manager (or the HQ) has higher payoff from the outside option than from the original revenue-sharing contract, renegotiation occurs and the manager (or the HQ) obtains his outside option payoff under the new revenue-sharing contract.

4.3 How to make the ex ante contracts renegotiation-proof?

As shown in Section 4.2, in the presence of contractual incompleteness, the incentive contracts could be renegotiated thereby distorting the players' ex ante incentive. In this subsection, we first assume away any *contractual remedies* for the event of outside option, and show that appropriate ownership arrangements of the unit's physical asset can make the incentive contracts renegotiation-proof. We then conclude our analysis by showing that, even optimally chosen, the contractual remedies cannot mimic what the ownership arrangements do. Taken together, we can explain not only multiple ownership arrangements, but also their correspondence with the different-powered contracts.

Assume for the time being that there are no contractual remedies for the outside option, and the players' payoffs under the outside option are determined solely by the ownership arrangements of the units' physical assets. Note that, in contrast to Grossman and Hart (1986), the managers' efforts are not human capital in our model. Once the goodwill and sales efforts are made by the manager of a unit, the former is attached to the HQ's brand name while the latter is embedded in the unit's physical asset. Suppose that the HQ owns the unit's physical asset. In this case, the HQ can deny the manager access to the company goodwill and the effort-embedded physical asset. Thus the manager does not have any outside outside. In contrast, the HQ has the outside option of capturing all the revenue, namely, $y(s, G)$. Suppose instead that the manager owns the unit's physical asset. In this case, the manager can deny the HQ access to the effort-embedded physical asset. The HQ's outside option is $y(0, G)$. Implicitly assumed is that it is very costly for the HQ to hire another manager who would then expend the sales effort.[20] The manager,

[20] This is because the project (or business opportunity) lasts for a limited period, or with the track record of taking the outside option the HQ has difficulty in convincing a new manager.

on the other hand, cannot get access to the company goodwill; and his outside option is to provide generic goods and services, namely, $y(s, 0)$.[21]

Having examined the relation between the outside option and the ownership arrangements of the unit's physical asset, we turn to the question of whether ownership arrangements can make the optimal incentive contracts renegotiation-proof. Recall from Section 3 that, in the complete-contracting framework, the HQ offers the high-powered contract (α^*, β^*) to some managers and the low-powered contract (w^*) to the others. The sales effort by the H manager (s^*), the goodwill effort by the L manager (g^*) and the company goodwill stock (G^*) are jointly determined by (FOC-s), (FOC-G) and (3) (equation (3) is not FOC anymore!).

Consider first the low-powered contract. The HQ's payoff under the contract is $y(T - g^*, G^*)$, while the manager's is 0. Suppose that the manager owns the unit's physical asset. Since $y(T - g^*, 0) > 0$,[22] the manager gets higher payoff from the outside option than from the original contract, and the low-powered contract would be renegotiated (Proposition 7). Suppose instead that the HQ owns the unit's asset. Then neither the HQ nor the manager gets higher payoff from the outside option than from the original contract, thereby ensuring the low-powered contract renegotiation-proof. In summary, we have:

Proposition 8 *The low-powered incentive contract is renegotiation-proof if and only if the HQ owns the unit's physical asset.*

Consider next the high-powered contract. The HQ's payoff under the contract is $(1 - \alpha^*)y(s^*, G^*)$, while the manager's is $\alpha^* y(s^*, G^*)$. Suppose that the HQ owns the unit's asset. Since $\alpha^* \in (0, 1)$ (Proposition 2), the HQ gets higher payoff from the outside option than from the original contract (namely, $y(s^*, G^*) > (1 - \alpha^*)y(s^*, G^*)$), and the high-powered contract would be renegotiated (Proposition 7). Suppose instead that the

[21] To bring about the main idea, we focus on the expected terms only. It is shown in the Appendix that our main results still hold when the uncertainty terms are taken into consideration.

[22] The argument leading to (FOC-G) in Section 3 shows that $g^* < T$ or $T - g^* > 0$.

manager owns the unit's asset. Then the high-powered contract is renegotiation-proof, if

$$\alpha^* y(s^*, G^*) > y(s^*, 0),$$

$$(1-\alpha^*) y(s^*, G^*) > y(0, G^*).$$

The two conditions ensure that neither the HQ nor the manager gets higher payoff from the outside option than from the original contract.

For simplicity of analysis, we consider the following class of the revenue functions:

$$y(s, G) = z(s, G) + k_1 \mu(s) + k_2 \nu(G),$$

where $z(0, G) = 0$, $z(s, 0) = 0$, $\mu(0) = 0$, $\nu(0) = 0$, $\frac{\partial^2 z}{\partial s \partial G} > 0$, z, μ and ν are increasing and concave. The above two conditions thus become:

$$\alpha^* y(s^*, G^*) > y(s^*, 0) = k_1 \mu(s^*),$$

$$(1-\alpha^*) y(s^*, G^*) > y(0, G^*) = k_2 \nu(G^*).$$

And we have:

Proposition 9 *For k_1 and k_2 sufficiently small, the high-powered incentive contract is renegotiation-proof if and only if the manager owns the unit's physical asset.*

In $y(s, G)$, $z(s, G)$ is the component that captures the complementarity between s and G. When k_1 and k_2 are small, $z(s, G)$ is the dominating component of $y(s, G)$, and s and G are strongly complementary. Therefore, Proposition 9 means that manager ownership of the unit's physical asset makes the high-powered contract renegotiation-proof when the sales and goodwill efforts are sufficiently complementary.

Propositions 8 and 9 are established under the assumption that the HQ owns the brand name in which goodwill effort is embedded. This is in fact an optimal arrangement. Following the logic of analysis leading to Proposition 8, we can show that managers with the low-powered contract should not own any asset, including the brand name. If managers with the high-powered contract have some claim over the ownership of the brand name, they will want to renegotiate the low-powered contract, because they are not given any of L units' revenue under the contract. Therefore, the HQ should be the sole owner of the brand name.

One may well ask the following question: can contractual remedies mimic the ownership arrangement to ensure the high-powered contract renegotiation-proof? Specifically, while the HQ owns both the unit's physical asset and company goodwill, it could write an ex ante contract which stipulates a payment from the HQ to the manager in the event of the outside option.[23] Note that, when the outside option is taken, the actual sales revenue is no longer verifiable. This is because the HQ would hire another manager on a fixed-wage contract to finalize the production, and it is impossible for the court to verify the actual revenue without the help from the HQ or the new manager. Thus, the HQ's payment to the manager can only be a fixed payment depending on the expected sales revenue.

The ideal contractual remedies should (1) ensure neither party initiates the outside option, and (2) provide each party the effort incentive. To meet the first criterion, the HQ's payment to the manager should be set to the latter's expected payoff from the ex ante contract. Otherwise, either the HQ or the manager would have higher payoff from the outside option than from the ex ante contract. However, if the manager has such an outside option, he would prefer to shirk on the job and initiate the contract renegotiation. As a result, the equilibrium effort and expected sales revenue are lower than those under the optimal ownership arrangement.

Underlying the above argument is an important feature of the contractual remedies, namely, there is no loss of surplus when the outside option is taken. In contrast, under the optimal ownership arrangement for the high-powered contract, there is loss of surplus upon the outside option (namely, $y(s^*, 0) + y(0, G^*) < y(s^*, G^*)$) which is sufficient to ensure the contract renegotiation-proof (Proposition 9).

Presumably, in the contractual remedies, the parties involved can give money away when the outside option is taken. Then the optimal contractual remedies would stipulate

[23] The low-powered contract is renegotiation-proof so long as there is *no* contractual remedies at all.

that, once the outside option is taken, the HQ pays the manager $y(s^*, 0)$ and makes a donation of $y(s^*, G^*) - y(s^*, 0) - y(0, G^*)$ to a third party. However, under some reasonable circumstances, the optimal contractual remedies is inferior to the optimal ownership arrangement. Specifically, suppose that, after the manager makes the sales and goodwill efforts, he may have to quit the business for some benign (family) reasons with certain probability. If the manager owns the unit's physical asset, he could sell his asset to a third party and get his expected payoff $\alpha^* y(s^*, G^*)$.[24] In order for the contractual remedies to reproduce the outcome under the optimal ownership arrangement and thus avoid the unnecessary loss to the third party, the contract should pay the manager $\alpha^* y(s^*, G^*)$ when she leaves for benign reasons, but pay the manager $y(s^*, 0)$ and the third party $y(s^*, G^*) - y(s^*, 0) - y(0, G^*)$ when the HQ and the manager can not agree on the minimum standard. However, this contract can not prevent renegotiation if the reason for the manager's leave can not be verified without the help of the HQ or the manager. The HQ can order the manager to accept $\alpha^* y(s^*, G^*)$, leave the company, and cite a benign reason for her leave. The manager will not disobey the order because otherwise she can only get $y(s^*, 0)$. The third party can not prevent it either because it can not be verified that the HQ and the manager have separated under unfriendly terms. Therefore, under such optimally chosen contractual remedies, there is still no loss of surplus when the outside option is taken, which makes the contractual remedies inferior to the optimal ownership arrangement as shown earlier.

In summary, given there is severe contractual incompleteness in franchise, the different-powered contracts can only be made renegotiation-proof by the corresponding ownership arrangements.

5. Conclusion

It is challenging to the recent developments in the theory of the firm that there is both

[24] See Kaufmann and Lafontaine (1994) for empirical evidence on the franchisees' sales of their assets. In general, the partner's right to sell his/her asset is well protected by law, though the sale often needs to be first offered to other partners (see for example Lynch (1989)).

contract mix and multiple ownership structure in franchise. With the observation that system-wide goodwill and unit-specific sales activity are crucial to a franchise company, we construct a multi-task model in which one task has the feature of public good and the other has that of private good. We show that, when the two tasks are complementary, the principal should offer a fixed-wage contract to some agents and a revenue-sharing contract to the remaining agents. In addition, by incorporating the stylized feature of contractual incompleteness in franchise and possible ex post renegotiation, we show that the different-powered ex ante contracts can only made renegotiation-proof by the corresponding ownership arrangements as in franchise.

This paper thus provides the first theory that explains both contract mix and multiple ownership structure in franchise. More importantly, it adopts and extends important features from both the multi-task theory of the firm and the incomplete-contract theory of the firm. On the one hand, by incorporating the task of public-good nature, it makes it possible for the multi-task model to explain the optimality of contract mix for *ex ante homogeneous* agents. On the other hand, it considers ex ante contracts in settings of contractual incompleteness and explores other roles of ownership structures. In particular, the optimality of contract mix in the presence of a multi-task framework implies *multiple* ownership arrangements of complementary assets.

References

[1] Anderlini, Luca. and Felli, L. "Incomplete Written Contracts: Undescribable States of Nature." *Quarterly Journal of Economics* 1994, 1085-1124.

[2] Brickley, James A. and Dark, F. H. "The Choice of Organizational Form: The Case of Franchising." *Journal of Financial Economics* 1987, 401-420.

[3] Gallini, Nancy T., and Lutz, Nancy A. "Dual Distribution and Royalty Fees in Franchising." *Journal of Law, Economics and Organization* 1992, 471-501.

[4] Grossman, Sanford J. and Hart, Oliver D. "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration." *Journal of Political Economy* August 1986, 691-719.

[5] Hadfield, Gillian K. "Problematic Relations: Franchising and Law of Incomplete Contracts." *Stanford Law Review* 1990, 927-992.

[6] Hart, O. *Firms, Contracts, and Financial Structure.* Oxford University Press, 1995.

[7] Hart, Oliver, and John Moore. "Property Rights and the Nature of the Firm," *Journal of Political Economy* 1990, 1119-1158.

[8] Hart, Oliver, Shleifer, Andrei, and Vishny, Robert W. "The Proper Scope of Government: Theory and an Application to Prisons." Mimeo, Harvard, 1996.

[9] Holmstrom, Bengt. "Moral Hazard in Teams." *Bell Journal of Economics* 1982, 324-40.

[10] Holmstrom, Bengt and Milgrom, Paul. "Aggregation and Linearity in the Provision of Intertemporal Incentives." *Econometrica* March 1987, 55(2), 303-28.

[11] ______. "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership and Job Design." *Journal of Law, Economics and Organization* 1991, 7, Special Issue, 24-52.

[12] ______. "The Firm as an Incentive System." *American Economic Review* September 1994, 84(4), 972-91.

[13] Kaufmann, Patrick J. and Lafontaine, F. "Cost of Control: The Source of Economic Rents for McDonald's Franchisees." *Journal of Law and Economics* 1994, 417-453.

[14] Kostecka, Andrew. *Franchising in the Economy, 1985-1987.* Washington, D.C.: U.S. Department of Commerce, 1987.

[15] Lafontaine, Francine. "Agency Theory and Franchising: Some Empirical Results." *Rand Journal of Economics* 1992, v23 n2, 263-283.

[16] ______. "Contractual Arrangements as Signaling Devices: Evidence from Franchising." *Journal of Law, Economics and Organization* 1993, 256-289.

[17] ______. and Shaw, Kathryn L. "The Dynamics of Franchise Contracting: Evidence from Panel Data," NBER Working Paper 5585, May 1996.

[18] Love, John F. *McDonald's: Behind the Arches.* Bantam Book: New York, 1986.

[19] Lynch, Robert. *The Practical Guide to Joint Ventures and Corporate Alliances: How to Form, How to Organize, How to Operate.* New York: Wiley, 1989.

[20] MacLeod, W. Bentley and Malcomsom, James M. "Investment, Holdup, and the Form of Market Contracts." *American Economic Review* September 1993, 811-837.

[21] Maskin, Eric and J. Tirole. "Unforeseen Contingencies, Property Rights, and Incomplete Contracts." mimeo, Harvard, 1996.

[22] McAfee, R.P. and M. Schwartz (1994): "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity." *American Economic Review* 210-230.

[23] Segal, Ilya. "Complexity and Renegotiation: A Foundation for Incomplete Contracts." Mimeo, Harvard, 1996.

[24] Shelton, John P. "Allocative Efficiency vs. 'X-Efficiency': Comment." *American Economic Review* 1967, 1252-58.

Appendix

Proof of Proposition 1: The HQ should (1) request the same level of goodwill effort from all managers of L units, and (2) offer the same high-powered contract to all managers of H units.

(1) Let $\{g(j)\}_{j\in\mathcal{L}}$ be a profile of goodwill effort levels and $g \equiv \int_{\mathcal{L}} g(j)dj/d(\mathcal{L})$, the average of $\{g(j)\}_{j\in\mathcal{L}}$. Since $y(s,G)$ is concave in s, $y(T-g,G)$ is concave is g. Then by Jensen's Inequality,

$$\int_{\mathcal{L}} y(T-g(j),G)dj \leq \int_{\mathcal{L}} y(T-g,G)dj.$$

Therefore, the solution to program $(OP-G)$ is the choose $g(j)$ to be a constant. That is, The HQ should request the same level of goodwill effort from all managers of L units.
(2) Let us consider program $(OP-HQ')$. Assumptions 2 and 3 say that $y(s,G)$ is concave in s and $\lim_{s\to 0} y_s(s,G)=\infty$. Therefore, incentive compatibility constraint $(OP-s)$ can be replaced by

$$\alpha(i)y_s(s(i),G) = c'(s(i)).$$

By part (1) of this proposition, incentive compatibility constraint $(OP-G)$ becomes

$$g = \arg\max_g py(T-\frac{G}{p},G) + \int_{\mathcal{I}-\mathcal{L}}[(1-\alpha(i))y(s(i),G)-\beta(i)]di,$$

where, $G=pg$. It is easy to show that, since $y(s,G)$ is concave in (s,G),

$$py(T-\frac{G}{p},G) + \int_{\mathcal{I}-\mathcal{L}}[(1-\alpha(i))y(s(i),G)-\beta(i)]di$$

is concave in G. Its derivative with respect to G is

$$py_G(T-\frac{G}{p},G) - y_s(T-\frac{G}{p},G) + \int_{\mathcal{I}-\mathcal{L}}(1-\alpha(i))y_G(s(i),G)di,$$

which decreases with G, by Assumption 3, goes to ∞ as $G\to 0$, and goes to $-\infty$ as $G\to pT$. Therefore, $(OP-G)$ can be replaced by

$$py_G(T-\frac{G}{p},G) - y_s(T-\frac{G}{p},G) + \int_{\mathcal{I}-\mathcal{L}}(1-\alpha(i))y_G(s(i),G)di = 0. \qquad (FOC-G)$$

The objective function of program $(OP-HQ')$ is now

$$py(T-g,G) + \int_{\mathcal{I}-\mathcal{L}}[y(s(i),G)-c(s(i))-\frac{1}{2}r\sigma^2\alpha(i)^2]di,$$

which is also concave in G and the derivative of which with respect to G is positive for G satisfying constraint $(FOC-G)$. Therefore, program $(OP-HQ')$ can be rewritten as

$$\begin{array}{lll} \max & py(T-g,G) + \int_{\mathcal{I}-\mathcal{L}}[y(s(i),G)-c(s(i))-\frac{1}{2}r\sigma^2\alpha(i)^2]di & (OP-HQ') \\ s.t. & py_G(T-\frac{G}{p},G) - y_s(T-\frac{G}{p},G) + \int_{\mathcal{I}-\mathcal{L}}(1-\alpha(i))y_G(s(i),G)di \geq 0 & (IC-G) \\ & \alpha(i)y_s(s(i),G) = c'(s(i)) & (IC-s) \end{array}$$

When we change the equality sign in $(FOC - G)$ to $\geq$ in $(IC - G)$, we expand the feasible region of program $(OP - HQ')$ to the left along the G-direction, as the left hand side of $(FOC - G)$ decreases with G. This does not change the optimum because the objective function of $(OP - HQ')$ decreases with G in the expanded feasible region. Let $\phi(s, G) \equiv c'(s)/y_s(s, G)$. Then $(IC - s)$ implies $\alpha(i) = \phi(s(i), G)$, which by Assumption 4 is convex in $s(i)$. Substitute $\alpha(i) = \phi(s(i), G)$ into the objective function and constraint $(IC - G)$. Then the integrand in the objective function,

$$y(s(i), G) - c(s(i)) - \frac{1}{2} r\sigma^2 \alpha(i)^2,$$

becomes a concave function of $s(i)$. The integrand in constraint $(IC - G)$,

$$(1 - \alpha(i)) y_G(s(i), G),$$

is also concave in $s(i)$ in the convex range $\{s(i) : c'(s(i))/y_s(s(i), G) \leq 1\}$, because $(1 - \alpha(i))$ is non-negative, concave and decreasing in $s(i)$, and $y_G(s(i), G)$ is, by Assumption 5, positive, concave and increasing in $s(i)$; the product of two non-negative concave functions is concave if one of them is increasing and the other decreasing. Given a profile of sales effort levels,

$\{s(i)\}_{i \in \mathcal{I} - \mathcal{L}}$, let $s \equiv \int_{\mathcal{I} - \mathcal{L}} s(i) di / d(\mathcal{I} - \mathcal{L})$, the average of $\{s(i)\}_{i \in \mathcal{I} - \mathcal{L}}$. Then, for any given G, Jensen's inequality implies

$$\int_{\mathcal{I} - \mathcal{L}} [y(s(i), G) - c(s(i)) - \frac{1}{2} r\sigma^2 \alpha(i)^2] di \leq \int_{\mathcal{I} - \mathcal{L}} y(s, G) - c(s) - \frac{1}{2} r\sigma^2 \alpha^2 di,$$

and

$$\begin{aligned} & p y_G(T - \tfrac{G}{p}, G) - y_s(T - \tfrac{G}{p}, G) + \textstyle\int_{\mathcal{I} - \mathcal{L}} (1 - \alpha(i)) y_G(s(i), G) di \\ \leq \; & p y_G(T - \tfrac{G}{p}, G) - y_s(T - \tfrac{G}{p}, G) + \textstyle\int_{\mathcal{I} - \mathcal{L}} (1 - \alpha) y_G(s, G) di, \end{aligned}$$

where $\alpha = \phi(s, G)$. Therefore, choosing the same α and s for all managers of H units is better than choosing different ones. That is, the HQ should offer the same high-powered contract to all managers of H units. ■

Proof of Lemma 4: (1) s and G are continuous at $\alpha = 0$. (2) When $p = 0$, $s(\alpha, 0)$ is uniquely determined by $(FOC - s)$ and is continuous in α.

(1) We first prove that $\lim_{(\alpha, p') \to (0, p)} s(\alpha, p') = s(0, p) = T$. By $(FOC - s)$,

$$\alpha y_s(s(\alpha, p'), G) = c'(s(\alpha, p')).$$

Therefore,

$$0 \leq c'(s(\alpha, p')) - c'(s(0, p)) = \alpha y_s(s(\alpha, p'), G) \leq \alpha y_s(T, T) \to 0$$

as $\alpha \to 0$. The last inequality holds because $s(\alpha, p') \geq T$, $G \leq T$, and $y_s(s, G)$ increases with G and decreases with s. Since $c'' > 0$ as $t > T$, $(c')^{-1}$ is continuous in $[0, \infty)$ with

$(c')^{-1}(0) = T$. Therefore, $c'(s(\alpha, p')) - c'(s(0, p)) \to 0$ implies $\lim_{(\alpha,p')\to(0,p)} s(\alpha, p') = s(0, p) = T$, i.e., s is continuous at $\alpha = 0$.

When $p > 0$,

$$\frac{\partial^2 \Pi}{\partial G^2} = p y^L_{GG} - 2 y^L_{sG} + \frac{1}{p} y^L_{ss} + (1-p)(1-\alpha) y^H_{GG} < 0$$

by $(FOC - G)$ and the concavity of y, where a function with a superscript H (L, resp.) means that it is evaluated at (s, G) ($(T - \frac{G}{p}, G)$, resp.). Therefore $(FOC - G)$ implies G is a differentiable function of (s, α, p) when $p > 0$, by Implicit Function Theorem.

When $p = 0$,

$$G(\alpha, p) - G(0, 0) = pg \to 0.$$

(2) Now we prove that $s(\alpha, 0)$ is continuous in α. $s(\alpha, 0)$ is defined by

$$\alpha y_s(s(\alpha, 0), G) = c(s(\alpha, 0)).$$

If $y_s(T, 0) = 0$, then $y_s(s, 0) = 0$ for all $s \geq T$ by the concavity of y, and therefore, $s(\alpha, 0) = T$ for all α.

If $y_s(T, 0) \neq 0$, then for $\alpha > 0$, $s(\alpha, 0) > T$ and thus $c''(s(\alpha, 0)) > 0$. Implicit Function Theorem then implies that $s(\alpha, 0)$ is differentiable with respect to α. The continuity of $s(\alpha, 0)$ at $\alpha = 0$ is implied by $\lim_{\alpha' \to 0} c'(s(\alpha', 0)) = \alpha' y_s(s(\alpha', 0)) = 0$. ∎

Proof of Proposition 2: At the optimum, $\alpha \in (0, 1)$.

We first prove the result for the very special case of $p = 1$. In this case, $(FOC - G)$ becomes

$$y_G(T - G, G) - y_s(T - G, G) = 0,$$

which implies that G does not depend on α. Therefore, α is chosen to

$$\begin{aligned} \pi^H &= \max_{\alpha,s} \quad y(s, G) - c(s) - \tfrac{1}{2} r\sigma^2\alpha^2 \\ & \qquad s.t. \quad \alpha y_s(s, G) - c'(s) = 0 \qquad (IC) \end{aligned}$$

The Lagrangian of the program is

$$L = y(s, G) - c(s) - \frac{1}{2} r\sigma^2\alpha^2 + \lambda[\alpha y_s(s, G) - c'(s)].$$

Differentiation yields

$$\begin{aligned} \tfrac{\partial L}{\partial \alpha} &= -r\sigma^2\alpha + \lambda y_s(s, G), \\ \tfrac{\partial L}{\partial s} &= (1-\alpha) y_s(s, G) + \lambda[\alpha y_{ss}(s, G) - c''(s)]. \end{aligned}$$

By the incentive compatibility constraint, $s \geq T > 0$. Therefore, $\frac{\partial L}{\partial s} = 0$. If $\alpha = 1$, then $\frac{\partial L}{\partial s} = 0$ implies $\lambda = 0$, which in turn implies $\frac{\partial L}{\partial \alpha} < 0$. This is a contradiction. If $\alpha = 0$, the incentive compatibility constraint implies that $s = T$. Therefore, $\frac{\partial L}{\partial s} = y_s(T, G) > 0$. This is again a contradiction. Therefore, $\alpha \in (0, 1)$.

Now, we consider the more general case of $p < 1$. At $\alpha = 1$, $(FOC - \alpha)$ becomes

$$\frac{d\Pi}{d\alpha} = -(1-p)r\sigma^2 + (1-p)y_G^H \frac{dG}{d\alpha}.$$

Apply the implicit function theorem to $(FOC - s)$ and $(FOC - G)$. We have, at $\alpha = 1$,

$$\frac{dG}{d\alpha} = \frac{1}{|J|}(1-p)y_G^H(y_{ss}^H - c''),$$

where, the Jacobian matrix

$$J = \begin{pmatrix} -c''(s) & 0 \\ 0 & py_{GG}^L - 2y_{sG}^L + \frac{1}{p}y_{ss}^L \end{pmatrix} + \begin{pmatrix} \alpha y_{ss}^H & \alpha y_{sG}^H \\ (1-p)(1-\alpha)y_{sG}^H & (1-p)(1-\alpha)y_{GG}^H \end{pmatrix}$$

is positive definite; it is the sum of a positive-definite matrix and a semipositive- definite matrix. Therefore, at $\alpha = 1$, $\frac{dG}{d\alpha} < 0$, which implies that $\frac{d\Pi}{d\alpha} < 0$. Thus the optimal α is not 1.

At $\alpha = 0$, we cannot use the above argument anymore because the Jacobian becomes singular. We need to utilize the interaction between α and p and therefore some results from the next subsection. Suppose $\alpha = 0$, then $s = T$ by $(FOC - s)$. Thus $\frac{\partial s}{\partial p} = 0$ and $(FOC - p)$ becomes

$$\begin{aligned} \tfrac{d\Pi}{dp} &= \pi^L - \pi^H + gy_s^L \\ &= y(T-g, G) - y(T, G) + gy_s(T-g, G) > 0, \end{aligned}$$

as $g(s, G)$ is concave in s. This implies that the optimal $p = 1$. The first part of the proof shows that in this case the optimal α is not 0, a contradiction to our assumption that $\alpha = 0$. Therefore, $\alpha \in (0, 1)$ for the case of $p < 1$ also. ■

Proof of Proposition 3: For any given p, $\pi^L < \pi^H$ at the corresponding HQ's optimal choice of α, denoted by $\alpha^*(p)$. In particular, $\pi^L < \pi^H$ holds in equilibrium.

Note from the discussion preceding Lemma 4 that, when $\alpha = 0$, an H manager chooses $s = T$, and $\pi^H(\alpha = 0) \equiv \Pi^H(\alpha \to 0) = y(T, G)$. Now, we prove the result for two separate cases:
Case 1: $p < 1$

By the definition of π^L,

$$\pi^L(\alpha = \alpha^*(p)) = y(T - g(\alpha^*), pg(\alpha^*)) \leq \max_g y(T - g, pg).$$

Since $y(s, G)$ increases with s and $p < 1$,

$$\max_g y(T - g, pg) < \max_g py(T - g, pg) + (1-p)y(T, pg) = \pi(\alpha = 0).$$

By the definition of $\alpha^*(p)$,

$$\pi(\alpha = 0) \leq \pi(\alpha = \alpha^*(p)) = p\pi^L(\alpha = \alpha^*(p)) + (1-p)\pi^H(\alpha = \alpha^*(p)).$$

Combining the above three inequalities, we have:

$$\pi^L(\alpha = \alpha^*(p)) < p\pi^L(\alpha = \alpha^*(p)) + (1-p)\pi^H(\alpha = \alpha^*(p)),$$

which implies that $\pi^L(\alpha = \alpha^*(p)) < \pi^H(\alpha = \alpha^*(p))$.
Case 2: $p = 1$

In this case, $(FOC - G)$ implies that G, and thus π^L, is independent of α. Therefore, the optimal α maximizes π^H. Then $\pi^H(\alpha = \alpha^*(p)) \geq \pi^H(\alpha = 0) = y(T, G) > y(T - G, G) = \pi^L$. ■

Proof of Lemma 5: $\liminf_{(\alpha',p)\to(\alpha,0)} \Pi(\alpha', p) \geq \Pi(\alpha, 0)$. *Furthermore, if* $y(s, 0) = 0$, *then* $\lim_{(\alpha',p)\to(\alpha,0)} \Pi(\alpha', p) = \Pi(\alpha, 0)$.

At $(\alpha, p = 0)$, $G = 0$ and $s(\alpha, p = 0)$ is determined by

$$\alpha y_s(s, 0) = c'(s).$$

At $(\alpha', p > 0)$, $G > 0$ by $(FOC - G)$ and $s(\alpha', p)$ is determined by

$$\alpha' y_s(s, G) = c'(s). \tag{A1}$$

Because $y_{sG} > 0$, it is easy to see that $s(\alpha', p) > s(\alpha', p = 0)$, for $p > 0$.

By the definition of Π,

$$\Pi(\alpha', p) = py(T - g, G) + (1-p)[y(s(\alpha', p), G) - c(s(\alpha', p)) - \frac{1}{2}r\sigma^2\alpha'^2], \tag{A2}$$

and

$$\Pi(\alpha, 0) = y(s(\alpha, 0), 0) - c(s(\alpha, 0)) - \frac{1}{2}r\sigma^2\alpha^2]. \tag{A3}$$

By (A2) and (A3) and rearrangement, we have,

$$\begin{aligned}\Pi(\alpha', p) - \Pi(\alpha, 0) &= p[y(T - g, G) - \Pi(\alpha, 0)] + \tfrac{1}{2}(1-p)r\sigma^2(\alpha^2 - \alpha'^2)\\ &+(1-p)[y(s(\alpha', 0), 0) - c(s(\alpha', 0)) - y(s(\alpha, 0), 0) + c(s(\alpha, 0))]\\ &+(1-p)[y(s(\alpha', p), G) - c(s(\alpha', p)) - y(s(\alpha', 0), 0) + c(s(\alpha', 0))].\end{aligned}$$

In the above equation, as $p \to 0$ and $\alpha' \to \alpha$, the first two terms go to 0. By Lemma 1(2), $s(\alpha, 0)$ is continuous in α and thus the third term goes to 0 as $\alpha' \to \alpha$. Therefore, the last term is crucial in determining the sign of $\Pi(\alpha', p) - \Pi(\alpha, 0)$. We want to show that the last term is non-negative.

$$\begin{aligned} & y(s(\alpha', p), G) - c(s(\alpha', p)) - y(s(\alpha', 0), 0) + c(s(\alpha', 0)) \\ = \; & y(0, G) + \textstyle\int_0^{s(\alpha',p)}[y_s(s, G) - c'(s)]ds - y(0, 0) - \int_0^{s(\alpha',0)}[y_s(s, 0) - c'(s)]ds \\ \geq \; & \textstyle\int_{s(\alpha',0)}^{s(\alpha',p)}[y_s(s, G) - c'(s)]ds & (A4) \\ \geq \; & \textstyle\int_{s(\alpha',0)}^{s(\alpha',p)}[y_s(s(\alpha', p), G) - c'(s(\alpha', p))]ds & (A5) \\ = \; & [s(\alpha', p) - s(\alpha', 0)](1 - \alpha')y_s(s(\alpha', p), G) \geq 0. & (A6) \end{aligned}$$

Inequality (A4) is because $y(0, G) \geq y(0, 0)$. Inequality (A5) holds because $y_s(s, G) - c'(s)$ decreases in s. Equation (A6) is by (A1). Therefore, $\liminf_{(\alpha', p) \to (\alpha, 0)} \Pi(\alpha', p) \geq \Pi(\alpha, 0)$.

When $y(s, 0) = 0$, by (A2),

$$
\begin{array}{ll}
& \limsup_{(\alpha', p) \to (\alpha, 0)} \Pi(p, \alpha') \\
\leq & \limsup_{(\alpha', p) \to (\alpha, 0)} py(T - g, G) + (1 - p)y(s(\alpha', p), G) - \frac{1}{2}(1 - p)r\sigma^2\alpha'^2 \\
\leq & -\frac{1}{2}r\sigma^2\alpha^2 \\
\leq & \Pi(0, \alpha).
\end{array}
$$

Combining this with the above result, we have $\lim_{(\alpha', p) \to (\alpha, 0)} \Pi(p, \alpha') = \Pi(0, \alpha)$. ■

Proof of Proposition 4: It is optimal for the company to have some L units.

If $y(s, 0) = 0$ for all s, we have argued in the text why the optimal p is positive. If $y(s, 0)$ is not always zero, then the concavity, the monotonicity, and the non- negativity of y implies that $y(s, 0) > 0$ for all $s > 0$.

We consider the limit of $\frac{d\Pi}{dp}$ as $p \to 0$. By $(FOC - p)$,

$$
\frac{d\Pi}{dp} = (\pi^L - \pi^H) + gy_s^L + (1 - p)\alpha y_G^H \frac{\partial G}{\partial p} + (1 - p)(1 - \alpha)y_s^H \frac{\partial s}{\partial p}.
$$

$(FOC - G)$ implies that,

$$
y_s(T - g, G) \geq (1 - p)(1 - \alpha)y_G(s, G).
$$

Since $s \geq T$ and $y_{sG} > 0$,

$$
(1 - p)(1 - \alpha)y_G(s, G) \geq (1 - p)(1 - \alpha)y_G(T, G),
$$

the right hand side of which $\to \infty$ because $G = pg \leq pT$, Assumption 3 says that $\lim_{G \to 0} y_G = \infty$, and Proposition 2 says that $\alpha < 1$. Therefore, $y_s(T - g, G) \to \infty$, which implies $g \to T$. Then, by Assumption 6, the substitution effect in $(FOC - p)$, $gy_s^L \to \infty$;

$$
\lim_{p \to 0} gy_s^L = T \lim_{p \to 0} y_s(T - g, G) = T \lim_{(s,G) \to (0,0)} y_s(s, G) = \infty.
$$

In $(FOC - p)$, $\pi^L - \pi^H$ is bounded. Then to determine the sign of $\frac{d\Pi}{dp}$ as $p \to 0$, it is sufficient to show that $\frac{\partial G}{\partial p} > 0$ and $\frac{\partial s}{\partial p} > 0$ as $p \to 0$. By Lemma 3, it suffices to show that $\frac{\partial^2 \Pi}{\partial p \partial G} > 0$. Substitute $(FOC - G)$ into $\frac{\partial^2 \Pi}{\partial p \partial G}$ and rearrange. Then

$$
(1 - p)\frac{\partial^2 \Pi}{\partial p \partial G} = y_G^L + (1 - p)gy_{sG}^L - \frac{(1 - p)}{p}gy_{ss}^L - y_s^L, \qquad (A7)
$$

in which only the last term is negative. By Assumption 6, y_s is weakly convex. Then

$$
y_s(T, G) - y_s(T - g, G) \geq gy_{ss}(T - g, G),
$$

in which $y_s(T-g,G) \to \infty$. Therefore,

$$-gy_{ss}^L = -gy_{ss}(T-g,G) \to \infty.$$

Rearranging ($A7$) yields

$$\begin{aligned} (1-p)\tfrac{\partial^2 \Pi}{\partial p \partial G} &= y_G^L + (1-p)gy_{sG}^L - \tfrac{(1-2p)}{p} gy_{ss}^L - gy_{ss}(T-g,G) - y_s^L \\ &\geq y_G^L + (1-p)gy_{sG}^L - \tfrac{(1-2p)}{p} gy_{ss}^L - y_s(T,G) \to \infty. \end{aligned}$$

In summary, we have shown that $\frac{d\Pi}{dp} \to \infty$ as $p \to 0$. Therefore, the optimal p is positive unless the value of Π at $p=0$ is higher than $\lim_{p\to 0} \Pi$, which Lemma 5 excludes. This completes the proof of the Proposition. ■

Proof of Lemma 6: When $p=1$, *the optimal* $s > T$. $\frac{d\pi^H}{dr} = -\frac{1}{2}\sigma^2\alpha^2 < 0$, $\frac{d\pi^H}{d\sigma} = -r\sigma\alpha^2 < 0$ *and* $\frac{d\pi^H}{dK} = -c(s) - \lambda c'(s) < 0$, *where* λ *is the Lagrange multiplier of the constraint and is positive.*

The Lagrangian of the program that chooses the optimal α and s is

$$L = y(s,G) - Kc(s) - \frac{1}{2} r\sigma^2\alpha^2 + \lambda[\alpha y_s(s,G) - Kc'(s)].$$

In the proof of Proposition 2, we showed that the optimal $\alpha \in (0,1)$ and $\frac{\partial L}{\partial \alpha} = 0$. Therefore, $\lambda > 0$ and the optimal $s > T$.

By the envelope theorem, we have, $\frac{d\pi^H}{dr} = -\frac{1}{2}\sigma^2\alpha^2 < 0$, $\frac{d\pi^H}{d\sigma} = -r\sigma\alpha^2 < 0$ and $\frac{d\pi^H}{dK} = -c(s) - \lambda c'(s) < 0$. ■

Proof of Lemma 7: When $p=1$, *if* $y(s,0) > 0$ *for* $s > 0$ *and* $\lim_{(s,G)\to(0,0)} Gy_G(s,G) = 0$, *then* $\frac{d\pi}{dp}\mid_{p=1} < 0$ *for sufficiently small* T.

As $T \to 0$, $\pi^L = y(T-g,T) \to y(0,0)$. When $p=1$, $gy_s^L = Gy_s(T-G,G)$. By (FOC-G), This is $Gy_G(T-G,G)$, which, by the assumption of the Lemma, approaches 0 as both G and T go to 0.

$$\begin{aligned} \pi^H &= \max_\alpha \quad y(s,G) - c(s-T) - \tfrac{1}{2} r\sigma^2\alpha^2 \\ & \quad\; s.t. \quad \alpha y_s(s,G) - c'(s-T) = 0. \end{aligned}$$

As $T \to 0$, π^H approaches,

$$\begin{aligned} \pi^H(T=0) &= \max_\alpha \quad y(s,0) - c(s) - \tfrac{1}{2} r\sigma^2\alpha^2 \\ & \quad\; s.t. \quad \alpha y_s(s,0) - c'(s) = 0, \end{aligned}$$

which is independent of T and greater than $y(0,0)$. Therefore,

$$\lim_{T\to 0} \frac{d\pi}{dp}\mid_{p=1} = y(0,0) - \pi^H(T=0) < 0,$$

that is, $\frac{d\pi}{dp}\mid_{p=1} < 0$ for sufficiently small T. ■

Proof of Proposition 9: For k_1 and k_2 sufficiently small, the high-powered incentive contract is renegotiation-proof if and and if the manager owns the unit's physical asset.

Consider the HQ's optimization problem

$$\begin{array}{lll} \max & py(T-\frac{G}{p},G)+(1-p)[y(s,G)-c(s)-\frac{1}{2}r\sigma^2\alpha^2] & (OP-HQ) \\ s.t. & \alpha y_s(s,G)-c'(s)=0 & (FOC-s) \\ & py_G(T-\frac{G}{p},G)-y_s(T-\frac{G}{p},G)+(1-p)(1-\alpha)y_G(s,G)=0 & (FOC-G) \end{array}$$

where $y(s,G)=z(s,G)+k_1\mu(s)+k_2\nu(G)$. Let the solution to program $(OP-HQ)$ be denoted with a superscript *. We want to show that, for sufficiently small k_1 and k_2,

$$\alpha^*y(s^*,G^*) > y(s^*,0) = k_1\mu(s^*). \tag{A8}$$

The proof for the second inequality above Proposition 8 is similar.

If the solution to $(OP-HQ)$, (α^*,p^*,s^*,G^*), is continuous in (k_1,k_2), then, as $(k_1,k_2)\to(0,0)$, $\alpha^*y(s^*,G^*)\to\alpha^0y(s^0,G^0)$, where (α^0,s^0,G^0) is the equilibrium at $(k_1,k_2)=(0,0)$. By Proposition 2, $\alpha^0y(s^0,G^0)>0$. As $(k_1,k_2)\to(0,0)$, $k_1\mu(s^*)\to 0$. Therefore, $(A8)$ holds for sufficiently small k_1 and k_2. Unfortunately, it is not easy to show the continuity of the equilibrium because program $(OP-HQ)$ is in general not concave.

To prove the inequality, we first perform an exercise similar to the proof of Proposition 1(2). Let $\phi(s,G)\equiv c'(s)/y_s(s,G)$. Then $(FOC-s)$ becomes $\alpha=\phi(s,G)$. By Assumption 4, ϕ is convex in s. Substitute $\alpha=\phi(s,G)$ into the objective function and constraint $(FOC-G)$ in $(OP-HQ)$. Then $(OP-HQ)$ becomes

$$\begin{array}{lll} \max & py(T-\frac{G}{p},G)+(1-p)[y(s,G)-c(s)-\frac{1}{2}r\sigma^2\phi(s,G)^2] & (OP-HQ) \\ s.t. & py_G(T-\frac{G}{p},G)-y_s(T-\frac{G}{p},G)+(1-p)(1-\phi(s,G))y_G(s,G)\geq 0 & (FOC-G) \end{array}$$

The reason why we can change the equality in $(FOC-G)$ to inequality is the same as that offered in the proof of Proposition 1(2). Now, given (p,G), $(OP-HQ)$ is a concave program that chooses the optimal s. The solution $s=s(p,G,k_1,k_2)$ is differentiable. Substitute the solution into the objective function. We have an unconstrained optimization problem[25]

$$\max_{p,G} f(p,G,k_1,k_2), \tag{A9}$$

where f is differentiable. Again, we don't know whether or not the solution to $(A9)$ is continuous in (k_1,k_2).

Define

$$\mathcal{S}\equiv\{(p,G,k_1,k_2):(p,G)=\arg\max_{p,G} f(p,G,k_1,k_2)\}.$$

We claim that $\mathcal{S}$ is a closed set. Suppose this is not true. Then there exists a sequence $(p_n,G_n,k_{1n},k_{2n})\in\mathcal{S}$ such that $\lim_{n\to\infty}(p_n,G_n,k_{1n},k_{2n})=(p_0,G_0,k_{10},k_{20})$ but (p_0,G_0,k_{10},k_{20}) is not in $\mathcal{S}$. There exists (p',G') such that

$$f(p_0,G_0,k_{10},k_{20}) < f(p',G',k_{10},k_{20}). \tag{A10}$$

[25]The constraint that $p\in[0,1]$ does not affect the argument and is thus omitted.

Let $\epsilon \in (0, \frac{1}{2}[f(p', G', k_{10}, k_{20}) - f(p_0, G_0, k_{10}, k_{20})])$. Since f is continuous, for sufficiently large n,

$$|f(p_n, G_n, k_{1n}, k_{2n}) - f(p_0, G_0, k_{10}, k_{20})| < \epsilon,$$

and

$$|f(p', G', k_{1n}, k_{2n}) - f(p', G', k_{10}, k_{20})| < \epsilon.$$

$(A10)$ then implies that

$$f(p_n, G_n, k_{1n}, k_{2n}) < f(p', G', k_{1n}, k_{2n}),$$

which contradicts with the fact that $(p_n, G_n, k_{1n}, k_{2n}) \in \mathcal{S}$. Therefore, $\mathcal{S}$ is a closed set.

Now, we want to show that

$$\liminf_{(k_1,k_2)\to(0,0)} \alpha^* y(s^*, G^*) > 0. \qquad (A11)$$

Suppose, on the contrary, $\liminf_{(k_1,k_2)\to(0,0)} \alpha^* y(s^*, G^*) = 0$. Then, there exists a sequence $(p_n, G_n, k_{1n}, k_{2n}) \in \mathcal{S}$ such that $\lim_{n\to\infty}(p_n, G_n, k_{1n}, k_{2n}) = (p_0, G_0, 0, 0)$ and $\alpha_0 y(s_0, G_0) = 0$, where $s_0 = s(p_0, G_0, 0, 0)$ and $\alpha_0 = \phi(s_0, G_0)$. Since $\mathcal{S}$ is a closed set, $(p_0, G_0, 0, 0) \in \mathcal{S}$ and thus $(\alpha_0, p_0, s_0, G_0)$ is an equilibrium for the case of $(k_1, k_2) = (0, 0)$. Proposition 2 implies that $\alpha_0 y(s_0, G_0) \neq 0$. This is a contradiction. Therefore, $(A11)$ holds. ■

References

[1] Anderlini, Luca. and Felli, L. "Incomplete Written Contracts: Undescribable States of Nature." *Quarterly Journal of Economics* 1994, 1085-1124.

[2] Brickley, James A. and Dark, F. H. "The Choice of Organizational Form: The Case of Franchising." *Journal of Financial Economics* 1987, 401-420.

[3] Gallini, Nancy T., and Lutz, Nancy A. "Dual Distribution and Royalty Fees in Franchising." *Journal of Law, Economics and Organization* 1992, 471-501.

[4] Grossman, Sanford J. and Hart, Oliver D. "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration." *Journal of Political Economy* August 1986, 691-719.

[5] Hadfield, Gillian K. "Problematic Relations: Franchising and Law of Incomplete Contracts." *Stanford Law Review* 1990, 927-992.

[6] Hart, O. *Firms, Contracts, and Financial Structure.* Oxford University Press, 1995.

[7] Hart, Oliver, and John Moore. "Property Rights and the Nature of the Firm," *Journal of Political Economy* 1990, 1119-1158.

[8] Hart, Oliver, Shleifer, Andrei, and Vishny, Robert W. "The Proper Scope of Government: Theory and an Application to Prisons." Mimeo, Harvard, 1996.

[9] Holmstrom, Bengt. "Moral Hazard in Teams." *Bell Journal of Economics* 1982, 324-40.

[10] Holmstrom, Bengt and Milgrom, Paul. "Aggregation and Linearity in the Provision of Intertemporal Incentives." *Econometrica* March 1987, 55(2), 303-28.

[11] ______. "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership and Job Design." *Journal of Law, Economics and Organization* 1991, 7, Special Issue, 24-52.

[12] ______. "The Firm as an Incentive System." *American Economic Review* September 1994, 84(4), 972-91.

[13] Kaufmann, Patrick J. and Lafontaine, F. "Cost of Control: The Source of Economic Rents for McDonald's Franchisees." *Journal of Law and Economics* 1994, 417-453.

[14] Kostecka, Andrew. *Franchising in the Economy, 1985-1987.* Washington, D.C.: U.S. Department of Commerce, 1987.

[15] Lafontaine, Francine. "Agency Theory and Franchising: Some Empirical Results." *Rand Journal of Economics* 1992, v23 n2, 263-283.

[16] ______. "Contractual Arrangements as Signaling Devices: Evidence from Franchising." *Journal of Law, Economics and Organization* 1993, 256-289.

[17] ______. and Shaw, Kathryn L. "The Dynamics of Franchise Contracting: Evidence from Panel Data," NBER Working Paper 5585, May 1996.

[18] Love, John F. *McDonald's: Behind the Arches.* Bantam Book: New York, 1986.

[19] Lynch, Robert. *The Practical Guide to Joint Ventures and Corporate Alliances: How to Form, How to Organize, How to Operate.* New York: Wiley, 1989.

[20] MacLeod, W. Bentley and Malcomsom, James M. "Investment, Holdup, and the Form of Market Contracts." *American Economic Review* September 1993, 811-837.

[21] Maskin, Eric and J. Tirole. "Unforeseen Contingencies, Property Rights, and Incomplete Contracts." mimeo, Harvard, 1996.

[22] McAfee, R.P. and M. Schwartz (1994): "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity." *American Economic Review* 210-230.

[23] Segal, Ilya. "Complexity and Renegotiation: A Foundation for Incomplete Contracts." Mimeo, Harvard, 1996.

[24] Shelton, John P. "Allocative Efficiency vs. 'X-Efficiency': Comment." *American Economic Review* 1967, 1252-58.

Appendix

Proof of Proposition 1: The HQ should (1) request the same level of goodwill effort from all managers of L units, and (2) offer the same high-powered contract to all managers of H units.

(1) Let $\{g(j)\}_{j\in\mathcal{L}}$ be a profile of goodwill effort levels and $g \equiv \int_{\mathcal{L}} g(j)dj/d(\mathcal{L})$, the average of $\{g(j)\}_{j\in\mathcal{L}}$. Since $y(s,G)$ is concave in s, $y(T-g,G)$ is concave is g. Then by Jensen's Inequality,

$$\int_{\mathcal{L}} y(T-g(j),G)dj \leq \int_{\mathcal{L}} y(T-g,G)dj.$$

Therefore, the solution to program $(OP-G)$ is the choose $g(j)$ to be a constant. That is, The HQ should request the same level of goodwill effort from all managers of L units.
(2) Let us consider program $(OP-HQ')$. Assumptions 2 and 3 say that $y(s,G)$ is concave in s and $\lim_{s\to 0} y_s(s,G)=\infty$. Therefore, incentive compatibility constraint $(OP-s)$ can be replaced by

$$\alpha(i)y_s(s(i),G)=c'(s(i)).$$

By part (1) of this proposition, incentive compatibility constraint $(OP-G)$ becomes

$$g=\arg\max_g py(T-\frac{G}{p},G)+\int_{\mathcal{I}-\mathcal{L}}[(1-\alpha(i))y(s(i),G)-\beta(i)]di,$$

where, $G=pg$. It is easy to show that, since $y(s,G)$ is concave in (s,G),

$$py(T-\frac{G}{p},G)+\int_{\mathcal{I}-\mathcal{L}}[(1-\alpha(i))y(s(i),G)-\beta(i)]di$$

is concave in G. Its derivative with respect to G is

$$py_G(T-\frac{G}{p},G)-y_s(T-\frac{G}{p},G)+\int_{\mathcal{I}-\mathcal{L}}(1-\alpha(i))y_G(s(i),G)di,$$

which decreases with G, by Assumption 3, goes to ∞ as $G\to 0$, and goes to $-\infty$ as $G\to pT$. Therefore, $(OP-G)$ can be replaced by

$$py_G(T-\frac{G}{p},G)-y_s(T-\frac{G}{p},G)+\int_{\mathcal{I}-\mathcal{L}}(1-\alpha(i))y_G(s(i),G)di=0. \qquad (FOC-G)$$

The objective function of program $(OP-HQ')$ is now

$$py(T-g,G)+\int_{\mathcal{I}-\mathcal{L}}[y(s(i),G)-c(s(i))-\frac{1}{2}r\sigma^2\alpha(i)^2]di,$$

which is also concave in G and the derivative of which with respect to G is positive for G satisfying constraint $(FOC-G)$. Therefore, program $(OP-HQ')$ can be rewritten as

$$\begin{array}{lll} \max & py(T-g,G)+\int_{\mathcal{I}-\mathcal{L}}[y(s(i),G)-c(s(i))-\frac{1}{2}r\sigma^2\alpha(i)^2]di & (OP-HQ') \\ s.t. & py_G(T-\frac{G}{p},G)-y_s(T-\frac{G}{p},G)+\int_{\mathcal{I}-\mathcal{L}}(1-\alpha(i))y_G(s(i),G)di\geq 0 & (IC-G) \\ & \alpha(i)y_s(s(i),G)=c'(s(i)) & (IC-s) \end{array}$$

When we change the equality sign in $(FOC-G)$ to $\geq$ in $(IC-G)$, we expand the feasible region of program $(OP-HQ')$ to the left along the G-direction, as the left hand side of $(FOC-G)$ decreases with G. This does not change the optimum because the objective function of $(OP-HQ')$ decreases with G in the expanded feasible region. Let $\phi(s,G) \equiv c'(s)/y_s(s,G)$. Then $(IC-s)$ implies $\alpha(i)=\phi(s(i),G)$, which by Assumption 4 is convex in $s(i)$. Substitute $\alpha(i)=\phi(s(i),G)$ into the objective function and constraint $(IC-G)$. Then the integrand in the objective function,

$$y(s(i),G)-c(s(i))-\frac{1}{2}r\sigma^2\alpha(i)^2,$$

becomes a concave function of $s(i)$. The integrand in constraint $(IC-G)$,

$$(1-\alpha(i))y_G(s(i),G),$$

is also concave in $s(i)$ in the convex range $\{s(i): c'(s(i))/y_s(s(i),G)\leq 1\}$, because $(1-\alpha(i))$ is non-negative, concave and decreasing in $s(i)$, and $y_G(s(i),G)$ is, by Assumption 5, positive, concave and increasing in $s(i)$; the product of two non-negative concave functions is concave if one of them is increasing and the other decreasing. Given a profile of sales effort levels,

$\{s(i)\}_{i\in\mathcal{I}-\mathcal{L}}$, let $s\equiv\int_{\mathcal{I}-\mathcal{L}} s(i)di/d(\mathcal{I}-\mathcal{L})$, the average of $\{s(i)\}_{i\in\mathcal{I}-\mathcal{L}}$. Then, for any given G, Jensen's inequality implies

$$\int_{\mathcal{I}-\mathcal{L}}[y(s(i),G)-c(s(i))-\frac{1}{2}r\sigma^2\alpha(i)^2]di \leq \int_{\mathcal{I}-\mathcal{L}} y(s,G)-c(s)-\frac{1}{2}r\sigma^2\alpha^2 di,$$

and

$$\begin{array}{rl} & py_G(T-\frac{G}{p},G)-y_s(T-\frac{G}{p},G)+\int_{\mathcal{I}-\mathcal{L}}(1-\alpha(i))y_G(s(i),G)di \\ \leq & py_G(T-\frac{G}{p},G)-y_s(T-\frac{G}{p},G)+\int_{\mathcal{I}-\mathcal{L}}(1-\alpha)y_G(s,G)di, \end{array}$$

where $\alpha=\phi(s,G)$. Therefore, choosing the same α and s for all managers of H units is better than choosing different ones. That is, the HQ should offer the same high-powered contract to all managers of H units. ■

Proof of Lemma 4: (1) s and G are continuous at $\alpha=0$. (2) When $p=0$, $s(\alpha,0)$ is uniquely determined by $(FOC-s)$ and is continuous in α.

(1) We first prove that $\lim_{(\alpha,p')\to(0,p)} s(\alpha,p')=s(0,p)=T$. By $(FOC-s)$,

$$\alpha y_s(s(\alpha,p'),G)=c'(s(\alpha,p')).$$

Therefore,

$$0\leq c'(s(\alpha,p'))-c'(s(0,p))=\alpha y_s(s(\alpha,p'),G)\leq \alpha y_s(T,T)\to 0$$

as $\alpha\to 0$. The last inequality holds because $s(\alpha,p')\geq T$, $G\leq T$, and $y_s(s,G)$ increases with G and decreases with s. Since $c''>0$ as $t>T$, $(c')^{-1}$ is continuous in $[0,\infty)$ with

$(c')^{-1}(0) = T$. Therefore, $c'(s(\alpha, p')) - c'(s(0,p)) \to 0$ implies $\lim_{(\alpha,p')\to(0,p)} s(\alpha, p') = s(0,p) = T$, i.e., s is continuous at $\alpha = 0$.

When $p > 0$,

$$\frac{\partial^2 \Pi}{\partial G^2} = p y^L_{GG} - 2 y^L_{sG} + \frac{1}{p} y^L_{ss} + (1-p)(1-\alpha) y^H_{GG} < 0$$

by $(FOC - G)$ and the concavity of y, where a function with a superscript H (L, resp.) means that it is evaluated at (s, G) ($(T - \frac{G}{p}, G)$, resp.). Therefore $(FOC - G)$ implies G is a differentiable function of (s, α, p) when $p > 0$, by Implicit Function Theorem.

When $p = 0$,

$$G(\alpha, p) - G(0, 0) = pg \to 0.$$

(2) Now we prove that $s(\alpha, 0)$ is continuous in α. $s(\alpha, 0)$ is defined by

$$\alpha y_s(s(\alpha, 0), G) = c(s(\alpha, 0)).$$

If $y_s(T, 0) = 0$, then $y_s(s, 0) = 0$ for all $s \geq T$ by the concavity of y, and therefore, $s(\alpha, 0) = T$ for all α.

If $y_s(T, 0) \neq 0$, then for $\alpha > 0$, $s(\alpha, 0) > T$ and thus $c''(s(\alpha, 0)) > 0$. Implicit Function Theorem then implies that $s(\alpha, 0)$ is differentiable with respect to α. The continuity of $s(\alpha, 0)$ at $\alpha = 0$ is implied by $\lim_{\alpha' \to 0} c'(s(\alpha', 0)) = \alpha' y_s(s(\alpha', 0)) = 0$. ■

Proof of Proposition 2: At the optimum, $\alpha \in (0, 1)$.

We first prove the result for the very special case of $p = 1$. In this case, $(FOC - G)$ becomes

$$y_G(T - G, G) - y_s(T - G, G) = 0,$$

which implies that G does not depend on α. Therefore, α is chosen to

$$\begin{aligned} \pi^H \quad = \quad & \max_{\alpha, s} \quad y(s, G) - c(s) - \tfrac{1}{2} r \sigma^2 \alpha^2 \\ & s.t. \quad \alpha y_s(s, G) - c'(s) = 0 \qquad (IC) \end{aligned}$$

The Lagrangian of the program is

$$L = y(s, G) - c(s) - \frac{1}{2} r \sigma^2 \alpha^2 + \lambda[\alpha y_s(s, G) - c'(s)].$$

Differentiation yields

$$\begin{aligned} \tfrac{\partial L}{\partial \alpha} &= -r\sigma^2 \alpha + \lambda y_s(s, G), \\ \tfrac{\partial L}{\partial s} &= (1 - \alpha) y_s(s, G) + \lambda[\alpha y_{ss}(s, G) - c''(s)]. \end{aligned}$$

By the incentive compatibility constraint, $s \geq T > 0$. Therefore, $\frac{\partial L}{\partial s} = 0$. If $\alpha = 1$, then $\frac{\partial L}{\partial s} = 0$ implies $\lambda = 0$, which in turn implies $\frac{\partial L}{\partial \alpha} < 0$. This is a contradiction. If $\alpha = 0$, the incentive compatibility constraint implies that $s = T$. Therefore, $\frac{\partial L}{\partial s} = y_s(T, G) > 0$. This is again a contradiction. Therefore, $\alpha \in (0, 1)$.

Now, we consider the more general case of $p < 1$. At $\alpha = 1$, $(FOC - \alpha)$ becomes

$$\frac{d\Pi}{d\alpha} = -(1-p)r\sigma^2 + (1-p)y_G^H \frac{dG}{d\alpha}.$$

Apply the implicit function theorem to $(FOC - s)$ and $(FOC - G)$. We have, at $\alpha = 1$,

$$\frac{dG}{d\alpha} = \frac{1}{|J|}(1-p)y_G^H(y_{ss}^H - c''),$$

where, the Jacobian matrix

$$J = \begin{pmatrix} -c''(s) & 0 \\ 0 & py_{GG}^L - 2y_{sG}^L + \frac{1}{p}y_{ss}^L \end{pmatrix} + \begin{pmatrix} \alpha y_{ss}^H & \alpha y_{sG}^H \\ (1-p)(1-\alpha)y_{sG}^H & (1-p)(1-\alpha)y_{GG}^H \end{pmatrix}$$

is positive definite; it is the sum of a positive-definite matrix and a semipositive- definite matrix. Therefore, at $\alpha = 1$, $\frac{dG}{d\alpha} < 0$, which implies that $\frac{d\Pi}{d\alpha} < 0$. Thus the optimal α is not 1.

At $\alpha = 0$, we cannot use the above argument anymore because the Jacobian becomes singular. We need to utilize the interaction between α and p and therefore some results from the next subsection. Suppose $\alpha = 0$, then $s = T$ by $(FOC - s)$. Thus $\frac{\partial s}{\partial p} = 0$ and $(FOC - p)$ becomes

$$\begin{aligned} \tfrac{d\Pi}{dp} &= \pi^L - \pi^H + gy_s^L \\ &= y(T-g,G) - y(T,G) + gy_s(T-g,G) > 0, \end{aligned}$$

as $g(s, G)$ is concave in s. This implies that the optimal $p = 1$. The first part of the proof shows that in this case the optimal α is not 0, a contradiction to our assumption that $\alpha = 0$. Therefore, $\alpha \in (0, 1)$ for the case of $p < 1$ also. ■

Proof of Proposition 3: For any given p, $\pi^L < \pi^H$ at the corresponding HQ's optimal choice of α, denoted by $\alpha^*(p)$. In particular, $\pi^L < \pi^H$ holds in equilibrium.

Note from the discussion preceding Lemma 4 that, when $\alpha = 0$, an H manager chooses $s = T$, and $\pi^H(\alpha = 0) \equiv \Pi^H(\alpha \to 0) = y(T, G)$. Now, we prove the result for two separate cases:
Case 1: $p < 1$

By the definition of π^L,

$$\pi^L(\alpha = \alpha^*(p)) = y(T - g(\alpha^*), pg(\alpha^*)) \le \max_g y(T - g, pg).$$

Since $y(s, G)$ increases with s and $p < 1$,

$$\max_g y(T-g,pg) < \max_g py(T-g,pg) + (1-p)y(T,pg) = \pi(\alpha = 0).$$

By the definition of $\alpha^*(p)$,

$$\pi(\alpha = 0) \le \pi(\alpha = \alpha^*(p)) = p\pi^L(\alpha = \alpha^*(p)) + (1-p)\pi^H(\alpha = \alpha^*(p)).$$

Combining the above three inequalities, we have:

$$\pi^L(\alpha = \alpha^*(p)) < p\pi^L(\alpha = \alpha^*(p)) + (1-p)\pi^H(\alpha = \alpha^*(p)),$$

which implies that $\pi^L(\alpha = \alpha^*(p)) < \pi^H(\alpha = \alpha^*(p))$.
Case 2: $p = 1$

In this case, $(FOC - G)$ implies that G, and thus π^L, is independent of α. Therefore, the optimal α maximizes π^H. Then $\pi^H(\alpha = \alpha^*(p)) \geq \pi^H(\alpha = 0) = y(T, G) > y(T - G, G) = \pi^L$. ■

Proof of Lemma 5: $\liminf_{(\alpha',p)\to(\alpha,0)} \Pi(\alpha', p) \geq \Pi(\alpha, 0)$. *Furthermore, if* $y(s, 0) = 0$, *then* $\lim_{(\alpha',p)\to(\alpha,0)} \Pi(\alpha', p) = \Pi(\alpha, 0)$.

At $(\alpha, p = 0)$, $G = 0$ and $s(\alpha, p = 0)$ is determined by

$$\alpha y_s(s, 0) = c'(s).$$

At $(\alpha', p > 0)$, $G > 0$ by $(FOC - G)$ and $s(\alpha', p)$ is determined by

$$\alpha' y_s(s, G) = c'(s). \tag{A1}$$

Because $y_{sG} > 0$, it is easy to see that $s(\alpha', p) > s(\alpha', p = 0)$, for $p > 0$.

By the definition of Π,

$$\Pi(\alpha', p) = py(T - g, G) + (1-p)[y(s(\alpha', p), G) - c(s(\alpha', p)) - \frac{1}{2} r\sigma^2\alpha'^2], \tag{A2}$$

and

$$\Pi(\alpha, 0) = y(s(\alpha, 0), 0) - c(s(\alpha, 0)) - \frac{1}{2} r\sigma^2\alpha^2]. \tag{A3}$$

By (A2) and (A3) and rearrangement, we have,

$$\begin{aligned}\Pi(\alpha', p) - \Pi(\alpha, 0) = {} & p[y(T - g, G) - \Pi(\alpha, 0)] + \tfrac{1}{2}(1-p)r\sigma^2(\alpha^2 - \alpha'^2) \\ & +(1-p)[y(s(\alpha', 0), 0) - c(s(\alpha', 0)) - y(s(\alpha, 0), 0) + c(s(\alpha, 0))] \\ & +(1-p)[y(s(\alpha', p), G) - c(s(\alpha', p)) - y(s(\alpha', 0), 0) + c(s(\alpha', 0))].\end{aligned}$$

In the above equation, as $p \to 0$ and $\alpha' \to \alpha$, the first two terms go to 0. By Lemma 1(2), $s(\alpha, 0)$ is continuous in α and thus the third term goes to 0 as $\alpha' \to \alpha$. Therefore, the last term is crucial in determining the sign of $\Pi(\alpha', p) - \Pi(\alpha, 0)$. We want to show that the last term is non-negative.

$$\begin{aligned} & y(s(\alpha', p), G) - c(s(\alpha', p)) - y(s(\alpha', 0), 0) + c(s(\alpha', 0)) \\ = {} & y(0, G) + \textstyle\int_0^{s(\alpha',p)}[y_s(s, G) - c'(s)]ds - y(0, 0) - \int_0^{s(\alpha',0)}[y_s(s, 0) - c'(s)]ds \\ \geq {} & \textstyle\int_{s(\alpha',0)}^{s(\alpha',p)}[y_s(s, G) - c'(s)]ds & (A4) \\ \geq {} & \textstyle\int_{s(\alpha',0)}^{s(\alpha',p)}[y_s(s(\alpha', p), G) - c'(s(\alpha', p))]ds & (A5) \\ = {} & [s(\alpha', p) - s(\alpha', 0)](1 - \alpha')y_s(s(\alpha', p), G) \geq 0. & (A6)\end{aligned}$$

Inequality (A4) is because $y(0, G) \geq y(0, 0)$. Inequality (A5) holds because $y_s(s, G) - c'(s)$ decreases in s. Equation (A6) is by (A1). Therefore, $\liminf_{(\alpha', p) \to (\alpha, 0)} \Pi(\alpha', p) \geq \Pi(\alpha, 0)$.

When $y(s, 0) = 0$, by (A2),

$$
\begin{array}{rl}
& \limsup_{(\alpha', p) \to (\alpha, 0)} \Pi(p, \alpha') \\
\leq & \limsup_{(\alpha', p) \to (\alpha, 0)} py(T - g, G) + (1 - p)y(s(\alpha', p), G) - \frac{1}{2}(1 - p)r\sigma^2\alpha'^2 \\
\leq & -\frac{1}{2}r\sigma^2\alpha^2 \\
\leq & \Pi(0, \alpha).
\end{array}
$$

Combining this with the above result, we have $\lim_{(\alpha', p) \to (\alpha, 0)} \Pi(p, \alpha') = \Pi(0, \alpha)$. ■

Proof of Proposition 4: It is optimal for the company to have some L units.

If $y(s, 0) = 0$ for all s, we have argued in the text why the optimal p is positive. If $y(s, 0)$ is not always zero, then the concavity, the monotonicity, and the non- negativity of y implies that $y(s, 0) > 0$ for all $s > 0$.

We consider the limit of $\frac{d\Pi}{dp}$ as $p \to 0$. By $(FOC - p)$,

$$
\frac{d\Pi}{dp} = (\pi^L - \pi^H) + gy_s^L + (1 - p)\alpha y_G^H \frac{\partial G}{\partial p} + (1 - p)(1 - \alpha)y_s^H \frac{\partial s}{\partial p}.
$$

$(FOC - G)$ implies that,

$$
y_s(T - g, G) \geq (1 - p)(1 - \alpha)y_G(s, G).
$$

Since $s \geq T$ and $y_{sG} > 0$,

$$
(1 - p)(1 - \alpha)y_G(s, G) \geq (1 - p)(1 - \alpha)y_G(T, G),
$$

the right hand side of which $\to \infty$ because $G = pg \leq pT$, Assumption 3 says that $\lim_{G \to 0} y_G = \infty$, and Proposition 2 says that $\alpha < 1$. Therefore, $y_s(T - g, G) \to \infty$, which implies $g \to T$. Then, by Assumption 6, the substitution effect in $(FOC - p)$, $gy_s^L \to \infty$;

$$
\lim_{p \to 0} gy_s^L = T \lim_{p \to 0} y_s(T - g, G) = T \lim_{(s, G) \to (0, 0)} y_s(s, G) = \infty.
$$

In $(FOC - p)$, $\pi^L - \pi^H$ is bounded. Then to determine the sign of $\frac{d\Pi}{dp}$ as $p \to 0$, it is sufficient to show that $\frac{\partial G}{\partial p} > 0$ and $\frac{\partial s}{\partial p} > 0$ as $p \to 0$. By Lemma 3, it suffices to show that $\frac{\partial^2 \Pi}{\partial p \partial G} > 0$. Substitute $(FOC - G)$ into $\frac{\partial^2 \Pi}{\partial p \partial G}$ and rearrange. Then

$$
(1 - p)\frac{\partial^2 \Pi}{\partial p \partial G} = y_G^L + (1 - p)gy_{sG}^L - \frac{(1 - p)}{p}gy_{ss}^L - y_s^L, \tag{A7}
$$

in which only the last term is negative. By Assumption 6, y_s is weakly convex. Then

$$
y_s(T, G) - y_s(T - g, G) \geq gy_{ss}(T - g, G),
$$

in which $y_s(T-g,G) \to \infty$. Therefore,

$$-gy^L_{ss} = -gy_{ss}(T-g,G) \to \infty.$$

Rearranging $(A7)$ yields

$$\begin{aligned}(1-p)\tfrac{\partial^2\Pi}{\partial p \partial G} &= y^L_G + (1-p)gy^L_{sG} - \tfrac{(1-2p)}{p}gy^L_{ss} - gy_{ss}(T-g,G) - y^L_s \\ &\geq y^L_G + (1-p)gy^L_{sG} - \tfrac{(1-2p)}{p}gy^L_{ss} - y_s(T,G) \to \infty.\end{aligned}$$

In summary, we have shown that $\frac{d\Pi}{dp} \to \infty$ as $p \to 0$. Therefore, the optimal p is positive unless the value of Π at $p=0$ is higher than $\lim_{p\to 0}\Pi$, which Lemma 5 excludes. This completes the proof of the Proposition. ■

Proof of Lemma 6: When $p=1$, *the optimal* $s > T$. $\frac{d\pi^H}{dr} = -\frac{1}{2}\sigma^2\alpha^2 < 0$, $\frac{d\pi^H}{d\sigma} = -r\sigma\alpha^2 < 0$ *and* $\frac{d\pi^H}{dK} = -c(s) - \lambda c'(s) < 0$, *where* λ *is the Lagrange multiplier of the constraint and is positive.*

The Lagrangian of the program that chooses the optimal α and s is

$$L = y(s,G) - Kc(s) - \frac{1}{2}r\sigma^2\alpha^2 + \lambda[\alpha y_s(s,G) - Kc'(s)].$$

In the proof of Proposition 2, we showed that the optimal $\alpha \in (0,1)$ and $\frac{\partial L}{\partial \alpha} = 0$. Therefore, $\lambda > 0$ and the optimal $s > T$.

By the envelope theorem, we have, $\frac{d\pi^H}{dr} = -\frac{1}{2}\sigma^2\alpha^2 < 0$, $\frac{d\pi^H}{d\sigma} = -r\sigma\alpha^2 < 0$ and $\frac{d\pi^H}{dK} = -c(s) - \lambda c'(s) < 0$. ■

Proof of Lemma 7: When $p=1$, *if* $y(s,0) > 0$ *for* $s > 0$ *and* $\lim_{(s,G)\to(0,0)} Gy_G(s,G) = 0$, *then* $\frac{d\pi}{dp}\mid_{p=1} < 0$ *for sufficiently small* T.

As $T \to 0$, $\pi^L = y(T-g,T) \to y(0,0)$. When $p=1$, $gy^L_s = Gy_s(T-G,G)$. By (FOC-G), This is $Gy_G(T-G,G)$, which, by the assumption of the Lemma, approaches 0 as both G and T go to 0.

$$\begin{aligned}\pi^H &= \max_\alpha \quad y(s,G) - c(s-T) - \tfrac{1}{2}r\sigma^2\alpha^2 \\ & \qquad s.t. \quad \alpha y_s(s,G) - c'(s-T) = 0.\end{aligned}$$

As $T \to 0$, π^H approaches,

$$\begin{aligned}\pi^H(T=0) &= \max_\alpha \quad y(s,0) - c(s) - \tfrac{1}{2}r\sigma^2\alpha^2 \\ & \qquad s.t. \quad \alpha y_s(s,0) - c'(s) = 0,\end{aligned}$$

which is independent of T and greater than $y(0,0)$. Therefore,

$$\lim_{T\to 0}\frac{d\pi}{dp}\mid_{p=1} = y(0,0) - \pi^H(T=0) < 0,$$

that is, $\frac{d\pi}{dp}\mid_{p=1} < 0$ for sufficiently small T. ■

Proof of Proposition 9: For k_1 and k_2 sufficiently small, the high-powered incentive contract is renegotiation-proof if and and if the manager owns the unit's physical asset.

Consider the HQ's optimization problem

$$\begin{array}{lll} \max & py(T-\frac{G}{p},G)+(1-p)[y(s,G)-c(s)-\frac{1}{2}r\sigma^2\alpha^2] & (OP-HQ) \\ s.t. & \alpha y_s(s,G)-c'(s)=0 & (FOC-s) \\ & py_G(T-\frac{G}{p},G)-y_s(T-\frac{G}{p},G)+(1-p)(1-\alpha)y_G(s,G)=0 & (FOC-G) \end{array}$$

where $y(s,G)=z(s,G)+k_1\mu(s)+k_2\nu(G)$. Let the solution to program $(OP-HQ)$ be denoted with a superscript *. We want to show that, for sufficiently small k_1 and k_2,

$$\alpha^*y(s^*,G^*)>y(s^*,0)=k_1\mu(s^*). \qquad (A8)$$

The proof for the second inequality above Proposition 8 is similar.

If the solution to $(OP-HQ)$, (α^*,p^*,s^*,G^*), is continuous in (k_1,k_2), then, as $(k_1,k_2)\to(0,0)$, $\alpha^*y(s^*,G^*)\to\alpha^0y(s^0,G^0)$, where (α^0,s^0,G^0) is the equilibrium at $(k_1,k_2)=(0,0)$. By Proposition 2, $\alpha^0y(s^0,G^0)>0$. As $(k_1,k_2)\to(0,0)$, $k_1\mu(s^*)\to 0$. Therefore, $(A8)$ holds for sufficiently small k_1 and k_2. Unfortunately, it is not easy to show the continuity of the equilibrium because program $(OP-HQ)$ is in general not concave.

To prove the inequality, we first perform an exercise similar to the proof of Proposition 1(2). Let $\phi(s,G)\equiv c'(s)/y_s(s,G)$. Then $(FOC-s)$ becomes $\alpha=\phi(s,G)$. By Assumption 4, ϕ is convex in s. Substitute $\alpha=\phi(s,G)$ into the objective function and constraint $(FOC-G)$ in $(OP-HQ)$. Then $(OP-HQ)$ becomes

$$\begin{array}{lll} \max & py(T-\frac{G}{p},G)+(1-p)[y(s,G)-c(s)-\frac{1}{2}r\sigma^2\phi(s,G)^2] & (OP-HQ) \\ s.t. & py_G(T-\frac{G}{p},G)-y_s(T-\frac{G}{p},G)+(1-p)(1-\phi(s,G))y_G(s,G)\geq 0 & (FOC-G) \end{array}$$

The reason why we can change the equality in $(FOC-G)$ to inequality is the same as that offered in the proof of Proposition 1(2). Now, given (p,G), $(OP-HQ)$ is a concave program that chooses the optimal s. The solution $s=s(p,G,k_1,k_2)$ is differentiable. Substitute the solution into the objective function. We have an unconstrained optimization problem[25]

$$\max_{p,G} f(p,G,k_1,k_2), \qquad (A9)$$

where f is differentiable. Again, we don't know whether or not the solution to $(A9)$ is continuous in (k_1,k_2).

Define

$$\mathcal{S}\equiv\{(p,G,k_1,k_2):(p,G)=\arg\max_{p,G} f(p,G,k_1,k_2)\}.$$

We claim that $\mathcal{S}$ is a closed set. Suppose this is not true. Then there exists a sequence $(p_n,G_n,k_{1n},k_{2n})\in\mathcal{S}$ such that $\lim_{n\to\infty}(p_n,G_n,k_{1n},k_{2n})=(p_0,G_0,k_{10},k_{20})$ but (p_0,G_0,k_{10},k_{20}) is not in $\mathcal{S}$. There exists (p',G') such that

$$f(p_0,G_0,k_{10},k_{20})<f(p',G',k_{10},k_{20}). \qquad (A10)$$

[25]The constraint that $p\in[0,1]$ does not affect the argument and is thus omitted.

Let $\epsilon \in (0, \frac{1}{2}[f(p', G', k_{10}, k_{20}) - f(p_0, G_0, k_{10}, k_{20})])$. Since f is continuous, for sufficiently large n,

$$|f(p_n, G_n, k_{1n}, k_{2n}) - f(p_0, G_0, k_{10}, k_{20})| < \epsilon,$$

and

$$|f(p', G', k_{1n}, k_{2n}) - f(p', G', k_{10}, k_{20})| < \epsilon.$$

$(A10)$ then implies that

$$f(p_n, G_n, k_{1n}, k_{2n}) < f(p', G', k_{1n}, k_{2n}),$$

which contradicts with the fact that $(p_n, G_n, k_{1n}, k_{2n}) \in \mathcal{S}$. Therefore, $\mathcal{S}$ is a closed set.

Now, we want to show that

$$\liminf_{(k_1,k_2)\to(0,0)} \alpha^* y(s^*, G^*) > 0. \tag{A11}$$

Suppose, on the contrary, $\liminf_{(k_1,k_2)\to(0,0)} \alpha^* y(s^*, G^*) = 0$. Then, there exists a sequence $(p_n, G_n, k_{1n}, k_{2n}) \in \mathcal{S}$ such that $\lim_{n\to\infty}(p_n, G_n, k_{1n}, k_{2n}) = (p_0, G_0, 0, 0)$ and $\alpha_0 y(s_0, G_0) = 0$, where $s_0 = s(p_0, G_0, 0, 0)$ and $\alpha_0 = \phi(s_0, G_0)$. Since $\mathcal{S}$ is a closed set, $(p_0, G_0, 0, 0) \in \mathcal{S}$ and thus $(\alpha_0, p_0, s_0, G_0)$ is an equilibrium for the case of $(k_1, k_2) = (0, 0)$. Proposition 2 implies that $\alpha_0 y(s_0, G_0) \neq 0$. This is a contradiction. Therefore, $(A11)$ holds. ■