

TAKEOVER DEFENSES AND DILUTION:

A WELFARE ANALYSIS

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## **Takeover Defenses and Dilution: A Welfare Analysis**

### **Abstract**

Existing theory suggests that, in an unregulated market for corporate control, the level of takeovers is suboptimal because shareholders do not receive the full benefit from them. However, existing theory neglects that the threat of takeover may divert managerial effort from productive to defensive activities. This paper shows that, when this is considered, takeovers may in fact be excessive.

*Key Words:* Merger and Acquisition; Cost of takeover; Asset dilution

*JEL Classification:* G34

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## TAKEOVER DEFENSES AND DILUTION: A WELFARE ANALYSIS\*

### 1. Introduction

It is commonly argued that takeovers increase firm value by replacing a poor management team with a strong one, and that takeover *threats* enhance firm value by encouraging incumbent managers to be more productive. Grossman and Hart (1980a; G&H hereafter) formalize these intuitions. In their model, shareholders decide on the amount of surplus (the “level of dilution”) that goes to the successful raider. The higher this amount, the greater the incidence of successful raids and the greater the productive effort the manager expends. In choosing the level of dilution, shareholders weigh the greater incidence of successful raids, and the increased managerial effort that would derive from this, against the increased surplus that would go to the raiders. They view this surplus as a cost, but from the point of view of society it is simply a transfer from shareholders to raiders. According to this line of reasoning, the level of dilution chosen by shareholders and the resulting number of takeovers are both lower than optimal.

The above argument overlooks a potentially very important consideration, however. Faced with an increased threat of takeover, a manager may respond by strengthening takeover defenses, in addition to or perhaps instead of increasing productive effort. The next section reviews the extensive empirical evidence documenting not only that managerial defensive effort is quantitatively important but also that it can significantly reduce firm value.

Does the G&H conclusion — that the level of takeovers generated by the market for corporate control is too low — still hold when defensive effort is taken into account? That is the central question to be addressed in this paper. We show that the G&H result does *not* carry through given this further consideration. Thus, the unregulated level of takeovers may be excessive. Excessive takeover activity occurs when a marginal increase in dilution increases the probability of a takeover sufficiently to benefit

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shareholders, but by stimulating defensive effort increases the costs of takeovers — which are ignored by the shareholder — enough to reduce the resulting social surplus.

Our model generates several important policy insights. Most obviously, it adds weight to arguments for increased regulation of takeover activity, or at least weakens arguments for decreased regulation or for laissez-faire policy. But our model also points to the importance of considering the effects of regulation on defensive restructuring, which previous literature has tended to ignore. Should regulation encourage or discourage managerial defenses against takeover, or should the appropriate policy depend on the nature of the defensive activity? Recently, for example, a U.S. Appeals Court ruled that expenses incurred in defending against a takeover should be tax deductible in the same way as are ordinary business expenses. Was this decision a wise one?

One of the principal contributions of G&H was to identify a fundamental problem in share tendering during the takeover process, and to indicate how it can be overcome by dilution. Suppose that: (i) shareholders are atomistic; (ii) there is no uncertainty about firm value and the cost of takeover; (iii) a takeover costs the raider  $c$  per share *prior* to takeover; and (iv) non-tendering shareholders receive their share of the post-takeover value of the firm,  $v$ , which exceeds the exogenous pre-takeover value of the firm,  $q$ . If the raider offers  $p < v$  for shares, the rational atomistic shareholder will refuse to tender her shares, reasoning that her action has no effect on the outcome and that if the raid is successful she does better when she has not tendered her shares. To offer  $p > v - c$  for shares is unprofitable for the raider. Thus, no shares will be tendered owing to a free-rider problem. With dilution, however, the raider is permitted to expropriate some of the firm's assets upon takeover, thereby reducing the post-raid share value to  $v - \Phi$ , where  $\Phi$  is the level of dilution. Then if  $p > v - \Phi$  and  $p > q$ , the shareholder will tender her shares, and if  $p < v - c$ , the takeover is profitable for the raider. Now suppose that  $v$  and/or  $q$  are random variables, whose values are realized after  $\Phi$  is decided upon. Then an increase in  $\Phi$  increases the probability of takeover.

Dilution is key to the takeover mechanism considered in this paper. Other mechanisms to circumvent the free-rider problem also exist. Bradley, Desai, and Kim (1988), for instance, model an alternative mechanism to overcome the free-rider problem — front-end-loaded tender offers — whereby the raider offers a higher price to those shareholders who tender their shares early. Most corporate

charters have fair-price amendments, however, which preclude such price discrimination.<sup>1</sup> Shleifer and Vishny (1989) solve the free-rider problem by allowing the raider to acquire a substantial proportion of shares before formally announcing her takeover intention. But disclosure laws under the Williams Act require the raider to reveal her takeover intention.<sup>2</sup> In contrast, dilution is legal, voluntary, straightforward, and widespread. Thus, we consider our focus on dilution to highlight the welfare tradeoffs from takeovers to be well justified.

The remainder of the paper is organized as follows. Section 2 documents empirically the importance of dilution and managerial defense in the market for corporate takeovers. Section 3 describes the model, which incorporates managerial resistance to takeovers. Section 4 analyzes the simplest case, where both the firm's potential (post-takeover) value and the cost of takeover are known with certainty. Section 5 extends the analysis to a more realistic scenario, where both the cost of takeovers and the potential value of the firm are uncertain.<sup>3</sup> Concluding remarks, including discussion of policy insights and directions for future research, compose section 6.

## **2. Dilution and Defensive Restructuring**

### **2.1. Dilutions and the Corporate Charter**

Dilution is the extent to which a raider can exclude minority (non-tendering) shareholders from acquiring post-raid gains. A two-tier front-end loaded tender offer is an effective mechanism for dilution. A raider structures a bid promising to buy a certain percentage of the firm at a first-tier price and after acquiring majority control offers a lower price for the remaining shares. Such a mechanism dilutes the property rights of shareholders who do not tender shares immediately.<sup>4</sup> Alternatively the raider may be able to effect dilution (for minority shareholders) by transferring assets from the acquired firm to another

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<sup>1</sup> Over 85% of Fortune 500 companies have such provisions in their corporate charters.

<sup>2</sup> Firms can however acquire up to 5% of a firm's shares before 13-D filing, i.e., before publicly declaring the purpose of acquiring stocks.

<sup>3</sup> The intermediate cases, where only the cost of takeover is uncertain and where only the potential value of the firm is uncertain, are available in Chakraborty and Arnott (1994).

<sup>4</sup> Mesa Petroleum's two-tier bid for General American Oil Company is a good example of how two-tier bids help small companies finance acquisitions of large companies (Lipton et. al (1989)).

firm owned by the raider at a steep discount –below market value.<sup>5</sup> Other diluting practices may also involve supplying over-priced inputs to the target or buying under-priced products from the target.

Corporate charters do not contain a dollar figure for permissible dilution. The maximum dilution level is instead determined by the company’s disclosure and appraisal requirement policy (after a takeover).<sup>6</sup> Changing these policies usually requires the board’s consent. A majority of Fortune 500 companies have amended their charter provisions to include “fair price amendments.” These ensure that all shares be bought at the same price, reducing a raider’s ability to discriminate between tendering and non-tendering shareholders. Similarly, a firm might curb potential dilution by incorporating minority shareholders’ rights to dissent and appraisal in the corporate charter.<sup>7</sup> Thus corporate charters and security laws can effectively set limits on dilution. What should these limits be?<sup>8</sup>

Dilution is not necessary for takeovers. Scharfstein (1988) shows how a value-maximizing raid without dilution can take place when raiders are better informed than shareholders. Similarly, Bagnolli and Lipman (1989) demonstrate how free-riding incentives can be overcome when there are pivotal shareholders. But shareholders cannot depend on the presence of an informed raider or a pivotal shareholder to make value-maximizing raids occur. In contrast, dilution clauses can be written into corporate charters (or other takeover-related laws), ensuring that the presence and magnitude of dilution are public information. The voluntary adoption and transparency of such clauses ensure that they will hold under *all* circumstances, making them a reliable and effective instrument to encourage raids.

Finally, it has been argued that dilution clauses are largely redundant since, whatever the level of dilution, executive compensation can be structured so that managers operate in the shareholders’ interest. The proliferation of stock options in managerial compensation would appear to support this argument. However, empirical evidence on the sensitivity of executive compensation to firm performance is

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<sup>5</sup> A good example of such an asset transfer can be found in Texas Air’s acquisition of Eastern. Eastern’s computer reservation system which was valued (by independent analysts) at \$300-\$400 million was transferred to Texas Air for about \$100 million (Swoboda (1989), Bradsher (1990)).

<sup>6</sup> The Williams Amendment to the Securities Exchange Act of 1934, by recognizing the possibility of “minority squeeze out,” provides a legal basis for restricting dilution.

<sup>7</sup> G&H (1980b) provides an extensive analysis of how provisions in the corporate charter affect monetary levels of dilution.

<sup>8</sup> In this paper dilution always refers to the maximum permissible exclusion of the minority shareholders’ property rights. Also, for tractability, we restrict our model to a relatively narrow definition of dilution — dilution-related clauses in the corporate charter. Dilution in its broader sense refers to *all* actions that dilute property rights of the target shareholders and is not limited to corporate charters.

tenuous. Jensen and Murphy (1990 a,b) investigated over 3000 CEOs from 1974 to 1988 and found that a \$1000 change in corporate value corresponds to a change in CEO compensation of only \$2.59 to \$3.25.<sup>9</sup> Yermack (1995) finds little evidence to suggest that stock options are designed to reduce expected agency costs.<sup>10</sup> Moreover, the recent emergence of executive equity swaps (Bolster et al. (1996)) and the use of zero-cost collars ((Bettis et al. (1999)) is bound to reduce the effectiveness of stock options.<sup>11</sup> Hence creating an environment (through specific corporate by-laws) that facilitates takeovers is an attractive and reliable way to align managers' incentives to shareholders' interests.

## 2.2. Defensive Restructuring under Threat of Raid

Given the importance of managerial defense to our model, we present empirical evidence on the magnitude of such defenses. Dann and DeAngelo (1988) document defensive adjustment in the asset and ownership structure of 36% of exchange-listed targets of hostile bids for the period 1980-1983. Their evidence points to extensive corporate restructuring aimed at creating barriers specific to the hostile bidder, and indicates that defensive restructuring results in a decline in shareholder wealth. Palepu and Wruck (1993) report similar findings for firms that have undertaken leveraged payouts in anticipation of a takeover.<sup>12</sup> Bagwell (1992) explains that the cost of a takeover can be substantially increased by share repurchasing strategies. Since shareholders not willing to sell back their shares are systematically those who place a higher value on them, repurchase agreements increase the cost of a takeover for a potential raider. Denis (1990) and Denis and Denis (1991) provide empirical evidence of the effectiveness of share-repurchasing strategies in thwarting raids. Defensive restructuring should become even more important after the recent Appeals Court ruling noted earlier allowing resources "directed toward

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<sup>9</sup> They also report that the pay-for-performance relationship weakened over the period of study and that CEO compensation was no more sensitive to firm performance than was compensation for hourly and salaried employees.

<sup>10</sup> Hall and Liebman (1998) document a significant rise in the sensitivity of pay to performance in the 1990's, when CEOs' holding of stock and stock options are considered as a part of the compensation package. However, they note that the current relationship between CEO pay and firm performance remains weak.

<sup>11</sup> An executive equity swap is a privately traded contract through which executives can reduce their exposure to equity holding while maintaining their voting rights: "A typical executive equity swap transaction specifies a certain number of shares under the contract with the executive receiving quarterly interest payment from an alternative investment of similar notional value, such as a floating-rate security. The executive, in turn, pays the derivative dealer any dividend paid by the firm on the shares he/she owns. At the end of the life of the swap, normally a few years later, the executive pays the dealer the accumulated gains on the stock and receives the compensation for the losses." (Bolster et al. (1996), p.101)

<sup>12</sup> Leveraged payout in this study refers to borrowing money to pay out large special dividends or to buy back 30% or more of the company stock. For a detailed analysis of such restructuring see Jensen (1986, 1993).

defeating a hostile takeover by exploring alternative capital transactions” to be deemed ordinary business expenditure even if they fail to deter a takeover.<sup>13</sup>

### 3. The Model

Ideally, one would like to model the dynamic process in which a firm continually faces the threat of takeover. To keep the analysis manageable, however, we attempt to capture the essential economic features of the problem with a stylized single-period model.

We view the firm as some machine that can be employed in a number of ways. The firm's incumbent manager decides on the machine's use. The more productive effort he puts into researching alternative uses,  $a \geq 0$ , the better his choice and consequently the greater the machine's output,  $q(a)$ ; thus  $q'(a) > 0$ . We term  $q(a)$  the *status quo value of the firm*. For whatever reason, perhaps experience, perhaps privileged information, perhaps patented production processes, there may be another agent, the raider, who can make more productive use of the machine. The machine's output under the raider is denoted by  $\tilde{V}$ , the *potential value of the firm* (a tilde denotes a random variable). To acquire the machine, the raider must incur an out-of-pocket takeover cost,  $\tilde{C}$ . For the takeover to be successful, the raider must have her tender price,  $T$ , accepted by the shareholders. When the takeover is successful, the incumbent manager is fired. The manager therefore works to reduce the probability of takeover. He can do this by increasing either productive effort or defensive effort,  $b \geq 0$ , both of which are noncontractible. An increase in defensive effort increases the expected cost of takeover. Shareholders, who are risk-neutral and have access to common knowledge, and who therefore behave identically, have a single instrument at their disposal, the *dilution factor*, or level of dilution,  $\Phi$ , which is a lump sum payable to the raider by the non-tendering shareholders in the event of a successful takeover.

The cost of takeover and the potential value of the firm are uncertain in the general case; all other variables and functions are deterministic. The sequence of moves by the agents and the timing of the resolution of uncertainty are as follows:

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<sup>13</sup> Wall Street Journal, July 10, 1997, page B10, “Appeals Court rules costs incurred in battling takeovers are deductible,” by Tom Herman.



1. The corporate charter, here characterized by  $\Phi$ , is drawn up by shareholders. At this time, only the distributions of  $C$  and  $V$  are known.
2. Taking  $\Phi$  as given and knowing only the distributions of  $C$  and  $V$ , the manager maximizes his expected utility by choosing  $a$  and  $b$ .
3. The potential value of the firm and the cost of takeover are realized and become common knowledge.
4. The raider chooses whether to make a tender offer. If she chooses not to make a tender offer, she incurs no takeover cost. In this situation, we say that the takeover is thwarted. If she does make an offer, it will be the one that is accepted and takeover occurs. The raider incurs the realized takeover cost and receives her realized payoff,  $V - T - C$ , where  $T$  is the tender price.
5. If the takeover does not occur, the manager receives a salary that increases with the firm's output —  $s(q)$  with  $1 > s'(q) > 0$ ,<sup>14</sup> while the representative shareholder obtains the residual output,  $\hat{q} \equiv q - s(q)$ . If the takeover occurs, the manager receives nothing and the shareholder receives  $T$ .

We now specify the decision problems of the various agents, working backwards. The first decision is that of the representative shareholder, to accept or reject a tender offer. If the takeover is unsuccessful, she receives  $\hat{q}$  whether or not she tenders her shares; if the takeover is successful, she receives  $T$  if she tenders her shares and  $V - \Phi$  otherwise. We assume there are no pivotal shareholders. Consequently, shareholders view the outcome of the tendering process as exogenous and will not offer their shares if the tender price is below the status quo value of the firm,  $\hat{q}$ . Then a shareholder will accept an offer with  $T \geq \max[V - \Phi, \hat{q}]$ .

The second decision problem is that of the raider, whether to make a tender offer and at what price. If she does make a tender offer, it will be at the lowest successful price; thus  $T = \max[V - \Phi, \hat{q}]$ .

Also the raider's payoff from a successful tender offer must be nonnegative:

$V - T - C = V - \max[V - \Phi, \hat{q}] - C = \min[\Phi, V - \hat{q}] - C \geq 0$ . Note that when a takeover occurs, all shareholders tender their shares — there are no minority shareholders. Thus  $\Phi$  is never actually paid to

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<sup>14</sup>Note that the salary function is exogenous. More complete models would take into account how perturbations to the salary function would alter the manager's choices of productive and defensive effort, or would make the salary function endogenous.

the raider. Rather, the threat of dilution induces all shareholders to tender their shares when a tender offer is made and when  $V - \Phi > \hat{q}$  allows the raider to acquire the firm at a reduced tender price.

The third decision problem is the manager's, how much productive and defensive effort to expend. We assume that the manager's utility function is  $U = (1 - P)s(q) - D(a, b)$ , where  $P$  is the probability of takeover and  $D(\cdot)$  is the disutility of effort, with  $D_a, D_b, D_{aa}, D_{bb}, D_{ab} > 0$ , and  $D(0, 0) = 0$ . From the previous paragraph, a successful takeover occurs if  $\min[\Phi, V - \hat{q}(a)] - C(b) \geq 0$ .<sup>15</sup> Thus,

$$P = \text{Prob}\left[\min[\Phi, \tilde{V} - \hat{q}(a)] - \tilde{C}(b) > 0\right] \equiv P(\Phi, a, b). \quad (1)$$

The manager's decision problem can be written as<sup>16</sup>

$$\max_{a, b} U(\Phi, a, b) = (1 - P(\Phi, a, b))s(q(a)) - D(a, b). \quad (2)$$

Denote the solution to the maximization problem,  $a = a(\Phi)$  and  $b = b(\Phi)$ , and define

$$\bar{P}(\Phi) \equiv P(\Phi, a(\Phi), b(\Phi)).$$

The fourth decision problem is the shareholders' choice of the dilution factor to incorporate into the corporate charter. If the takeover is successful, the shareholder receives  $T = \max[V - \Phi, \hat{q}(a(\Phi))]$ ; if it is not successful, she receives  $\hat{q}(a(\Phi))$ . Thus, the shareholder's problem is to

$$\max_{\Phi} S(\Phi) = \bar{P}(\Phi)\mathcal{E}^c\left[\max[\tilde{V} - \Phi, \hat{q}(a(\Phi))]\right] + (1 - \bar{P}(\Phi))\hat{q}(a(\Phi)), \quad (3)$$

<sup>15</sup> Our cost function captures the manager's ability to increase the cost of takeover by expending defensive effort to restructure the firm. Managers may also choose to increase the cost of takeover by implementing a variety of other policies. For example, a manager could choose to issue poison pill security or enact a variety of anti-takeover amendments to increase the cost of takeover. A generalized cost function such as  $KC(b)$ , with  $K$  representing a scaling factor that measures the effectiveness of these policies, can capture these considerations. A manager then chooses productive effort ( $a$ ), defensive effort ( $b$ ) and  $K$  to maximize utility. For the sake of simplicity, we do not explicitly model the choice of  $K$ . Our cost function highlights the impact of managerial defensive effort on the cost of takeover given a particular defensive technology. Our goal is to highlight the impact of defensive restructuring, which (unlike  $K$ ) is difficult to monitor or regulate.

<sup>16</sup>We assume here that managers do not require higher salaries with a higher level of dilution. In Appendix 1, we show that the qualitative results of our model remain unchanged when there is a managerial participation constraint.

One could also allow for the manager to be paid partially in stock. If he were paid a prespecified proportion of the stock, for example, his decision problem would become

$$\max_{a, b} U(a, b, \Phi) = (1 - P(\Phi, a, b))(s(q(a)) + \eta\hat{q}(a)) + P(\Phi, a, b)\eta\mathcal{E}^c\left[\max(\tilde{V} - \Phi, \hat{q}(a))\right] - D(a, b),$$

where  $\eta$  is the proportion of the stock he owns. Different managerial compensation schemes would result in different functions  $a(\Phi)$  and  $b(\Phi)$ , but, as long as the manager does not own so much stock that he exerts zero defensive effort, would not alter the qualitative characteristics of the problem.

where  $S(\Phi)$  is the return to shareholders as a function of  $\Phi$  and  $\mathcal{E}^c[\cdot]$  is the expectations operator conditional on takeover. The shareholder's choice of dilution factor involves a tradeoff between three effects. With managerial effort fixed, an increase in dilution increases the probability of raid, which by itself is to the shareholder's benefit. But higher dilution also gives the raider a larger portion of the surplus from a raid, which hurts the shareholder. Finally, an increase in dilution causes the manager to alter both productive and defensive effort. As we shall see, this effect can go either way.

Having characterized equilibrium, we shall investigate its (constrained) efficiency. In order to keep our analysis comparable to G&H, we assume that society's objective is to maximize the expected net value (expected value of the firm net of the expected takeover cost) of the firm, which equals the sum of expected returns to shareholders and the raider.<sup>17</sup> Then the socially optimal level of  $\Phi$  solves

$$\max_{\Phi} W(\Phi) = \bar{P}(\Phi)\mathcal{E}^c[\tilde{V} - \tilde{C}(b(\Phi))] + (1 - \bar{P}(\Phi))\hat{q}(a(\Phi)). \quad (4)$$

#### 4. No Uncertainty

As noted earlier, the choice of dilution factor is a complex incentive problem. A change in  $\Phi$  will generally affect the manager's choices of productive and defensive effort, the raider's choice of tender price, and the shareholder's tendering decision. In choosing the level of dilution, the shareholder maximizes share value subject to the various incentive constraints. We start by examining the manager's choice of productive and defensive effort, given a level of dilution, when there is no uncertainty concerning either takeover cost or the potential value of the firm.

##### 4.1 The Manager's Problem

We shall proceed by examining the manager's choice of efforts *conditional on thwarting takeover* and then *conditional on not thwarting takeover*. We shall then obtain his unconditional choice of efforts as a function of  $\Phi$ .

Conditional on thwarting the takeover bid, the manager's problem is, from (2),

$$\max_{a,b} U(a,b) = s(q(a)) - D(a,b)$$

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<sup>17</sup> In Appendix 1, we have modified this assumption by including the manager's loss of utility in the social welfare function. Doing so does not alter the qualitative results of our analysis.

$$s.t. \quad \min[\Phi, V - \hat{q}(a)] - C(b) \leq 0, \quad (5)$$

where  $C(b)$  is the cost of takeover as a function of  $b$ , with  $C' > 0$  and  $C(0) = 0$ . The constraint requires that  $a$  and  $b$  be such that takeover is thwarted. We provide a geometric analysis of the problem. To rule out economically uninteresting technical complications, we assume that the functions are such that  $\operatorname{argmax}_a [s(q(a)) - D(a, b)]$  is continuous, positive, and finite for all  $b$  in the relevant range. We also assume that the potential value of the firm exceeds the status quo value when there is no threat of takeover.

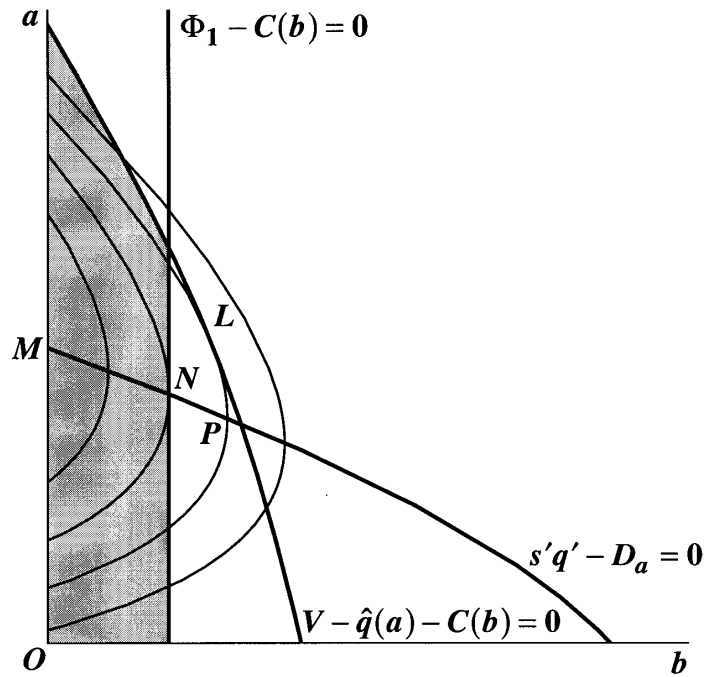


Figure 1: The Manager's Problem with No Uncertainty

Figure 1 displays the relevant loci in  $b$ - $a$  space. The unlabeled contours are managerial indifference curves. The point  $M$  is the equilibrium when there is no threat of takeover: defensive effort is zero and productive effort maximizes the manager's utility,  $s(q(a)) - D(a, 0)$ . The locus of points at which the level of productive effort optimizes unconstrained managerial utility, as a function of the level of defensive effort, is  $s'q' - D_a = 0$  (since  $D_{ab} > 0$ , this locus slopes downward). Above this locus,

from the manager's perspective both marginal productive and defensive effort are bad, so indifference curves are negatively sloped; below the locus, marginal productive effort is good while defensive effort is bad, making the indifference curves positively sloped. The no-takeover constraint in (5) corresponds to the requirement that  $(b, a)$  lie to the right of either  $\Phi - C(b) = 0$  or  $V - \hat{q}(a) - C(b) = 0$  or both. The locus  $\Phi - C(b) = 0$  is vertical, while the locus  $V - \hat{q}(a) - C(b) = 0$  slopes downwards. The feasible region for  $(b, a)$  given  $\Phi = \Phi_1$  is shown as the nonshaded area.

The manager can thwart a takeover in two qualitatively different ways. He can increase the cost of takeover so that it exceeds the dilution factor, such that the constraint  $\Phi \leq C(b)$  binds; we call this the *defensive strategy*. Or he can increase productive effort and the cost of takeover so that the status quo value of the firm exceeds its potential value net of takeover cost, such that  $V - \hat{q}(a) - C(b) \leq 0$  binds; we term this the *offensive strategy*. The manager prefers the defensive strategy for lower values of  $\Phi$  and the offensive strategy for higher values.

In Figure 1,  $\Phi$  is sufficiently low that the manager chooses to thwart the takeover defensively rather than offensively. With this strategy, the manager chooses N, the point of intersection of  $\Phi_1 - C(b) = 0$  and  $s'q' - D_a = 0$ , and a local increase in dilution decreases productive effort, increases defensive effort, and makes the manager worse off (i.e.  $\frac{da}{d\Phi} < 0$ ,  $\frac{db}{d\Phi} > 0$ , and  $\frac{dU}{d\Phi} < 0$ ).

There is a critical level of dilution,  $\hat{\Phi}$ , above which the offensive strategy dominates the defensive strategy. Under the offensive strategy, the manager chooses L, the point of maximum utility along  $V - \hat{q}(a) - C(b) = 0$ , and an increase in dilution does not affect the manager's level of productive effort, defensive effort, or level of utility (i.e. for  $\Phi > \hat{\Phi}$ ,  $\frac{da}{d\Phi} = 0$ ,  $\frac{db}{d\Phi} = 0$ , and  $\frac{dU}{d\Phi} = 0$ ).

At  $\hat{\Phi}$ , the manager is indifferent between the two strategies for thwarting takeover. Let P denote the point at which  $s'q' - D_a = 0$  intersects the indifference curve through L; then  $\hat{\Phi} = C(b_p)$ . Observe that as  $\Phi$  is increased, at  $\hat{\Phi}$  productive effort increases discontinuously and defensive effort decreases discontinuously. The intuition is that productive effort suddenly becomes more valuable since at the margin it is thwarting the takeover, while defensive effort suddenly becomes less valuable since it is now sharing with productive effort the task of thwarting the takeover rather than doing the job alone.

Suppose, instead, that the manager does not thwart the takeover. Since the raid is successful, the manager gets fired. We assume that he then receives a level of utility corresponding to the indifference curve through the origin  $U_0$ . Then he has no incentive to exert either productive or defensive effort. The way Figure 1 is drawn, whatever the level of  $\Phi$ , the manager chooses to thwart the takeover since the minimum utility from doing so,  $U_L$ , exceeds  $U_0$ . Suppose instead  $U_0 > U_L$ . Then for low levels of dilution (for  $\Phi < \check{\Phi}$ , where  $\check{\Phi}$  is the level of dilution for which  $\Phi - C(b)$  is tangent to  $U_0$ ), the manager would choose to thwart the takeover defensively, while for higher levels of dilution he would choose not to thwart the takeover.

We are now in a position to solve the shareholder's and the planner's choices of dilution. We refer to the two levels of dilution as the *privately* and *socially* optimal levels of dilution.

#### 4.2 Optimal Dilution from the Shareholder's and the Planner's Perspectives

*Proposition 1:* If  $U_L > U_0$  (so that the manager chooses to thwart the raid for all levels of dilution), the socially and privately optimal levels of dilution coincide.

*Proof:* Since  $U(a(\Phi), b(\Phi)) > U_0$  for all  $\Phi$ , raids will not succeed and the objective functions of the planner and the shareholder coincide. Hence both will choose that level of dilution which maximizes firm value under incumbent management. The optimal dilution in this case is either zero if  $a_M > a_L$  or greater than  $\hat{\Phi}$  if  $a_M < a_L$ . ■

Since the manager thwarts takeover whatever the level of dilution, the optimal level of dilution from both the shareholder's and society's perspective is that which maximizes the manager's productive effort and hence the status quo value of the firm. This will never entail a level of dilution such that the manager thwarts the raid defensively, since any such level of dilution would result in the value of the firm being lower than the no-dilution value ( $\hat{q}(a_M)$ ). If  $a_M < a_L$ , the optimal level of dilution is such that the manager thwarts the raid offensively,  $\Phi > \hat{\Phi}$ . In this case, shareholders incorporate dilution even though the raid does not succeed since the threat of raid forces the manager to work harder. If  $a_M > a_L$ , the optimal level of dilution is zero.

*Proposition 2:* If  $U_L < U_0$  (so that there are levels of dilution where the manager finds it in his interest to let the raid succeed), then the socially optimal level of dilution is greater than or equal to the privately optimal level.

*Proof:* The raid occurs with  $\Phi > \check{\Phi}$  and not otherwise. The optimal level of dilution contingent on a raid not occurring is zero. With this level of dilution, the return to both society and shareholder is  $\hat{q}(a_M)$ . When a raid occurs, the return to society exceeds  $\hat{q}(a_M)$  (since  $V > \hat{q}(a_M)$  has been assumed) and the return to the shareholder is  $V - \Phi$ . Thus the planner always wants the takeover to occur, which is achieved by setting  $\Phi > \check{\Phi}$ . Shareholders want the takeover to occur with the minimum level of dilution consistent with takeover,  $\check{\Phi}$ , if  $V - \check{\Phi} > \hat{q}(a_M)$ . In all other situations, shareholders want the takeover to be thwarted and no dilution. ■

The reason the socially and privately optimal levels of dilution may differ in this case is the managerial incentive constraint. The first-best scenario entails the raid occurring with no managerial resistance. To eliminate managerial resistance requires setting the level of dilution above  $\check{\Phi}$ . But with the level of dilution set infinitesimally above  $\check{\Phi}$ , shareholder utility is  $V - \check{\Phi}$ , which may be less than with no dilution,  $\hat{q}(a_M)$ . From the perspective of society, the dilution needed to eliminate managerial resistance is merely a transfer from the shareholder to the raider. But from the shareholder's perspective, the dilution is a cost.

In the certainty case, therefore, the socially optimal dilution factor is greater than or equal to the privately optimal dilution factor. This implies that if government intervention is merited, it should entail facilitating takeovers. We shall see in the next section, however, that this result does not extend to the situation where the cost of takeover is uncertain.

## 5. V and C Both Stochastic

We now look at the case where the potential value of the firm and the cost of the takeover are stochastic. To simplify somewhat, we assume that  $\tilde{V}$  and  $\tilde{C}$  are statistically independent and that their realizations are bounded below by zero and are unbounded above. Let  $f(V)$  be the p.d.f. of  $V$ , with  $F(V)$  the corresponding c.d.f.; and  $g(C, b)$  be the p.d.f. of  $C$  conditional on  $b$ , with  $G(\cdot)$  the

corresponding c.d.f. We assume that an increase in defensive effort increases the probability that takeover cost exceeds  $C$  for all  $C > 0$ , i.e.,  $G_b(C, b) < 0$  for all  $b$  and for all  $C > 0$ . We assume furthermore that  $D_a(0, b) = D_b(a, 0) = 0$ , implying that the manager always chooses to exert some productive and defensive effort, which allows us to ignore nuisance corner solutions.

The analysis of the certainty case was complicated by having to consider three régimes of managerial behavior, one for each of the qualitative outcomes: takeover thwarted defensively, takeover thwarted offensively, and takeover not thwarted. The analysis with stochasticity is in some ways simpler, since the uncertainty blends the three régimes and smoothes managerial behavior.

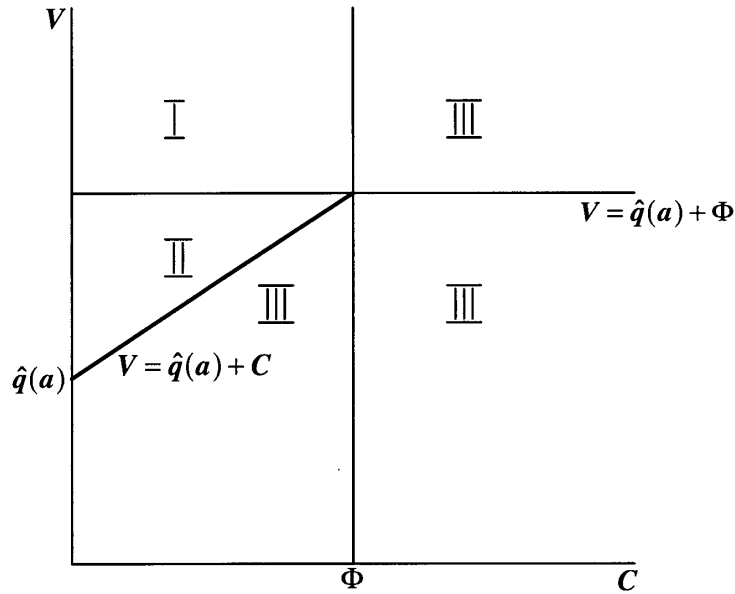


Figure 2: Both  $V$  and  $C$  Stochastic

Figure 2 displays the regions of  $V$  and  $C$  where the three outcomes occur. In region I,  $(V - \Phi > \hat{q}(a)) \cap (C < \Phi)$ . Since  $V - \Phi > \hat{q}(a)$ , if a takeover occurs the tender price is  $V - \Phi$  and the raider receives  $\Phi - C$ ; and since  $\Phi > C$ , takeover will occur. Thus, in region I takeover occurs with a tender price  $T = V - \Phi$ , the shareholder receives  $V - \Phi$ , the raider  $\Phi - C$ , and society  $V - C$ . In region II,  $(V - \Phi < \hat{q}(a)) \cap (C < V - \hat{q}(a))$ ; consequently, takeover occurs at a tender price of  $\hat{q}(a)$ , so that the



shareholder receives  $\hat{q}(a)$ , the raider  $V - \hat{q}(a) - C$ , and society  $V - C$ . In region III, no takeover occurs; consequently, the shareholder and society receive  $\hat{q}(a)$  and the raider nothing.

We shall proceed in the same way as in section 4, by first examining the manager's problem and then the optimal dilution factor from the perspective of the shareholder and of society.

### 5.1 The Manager's Problem

From (2), the manager's expected utility is

$$U = (1 - P(\Phi, a, b))s(q(a)) - D(a, b),$$

where  $1 - P$  is the probability that takeover does *not* occur. From above, the probability that takeover does not occur is the probability that  $V$  and  $C$  are in region III. Thus, from Figure 2

$$1 - P(\Phi, a, b) = \int_0^\Phi \int_0^{C+\hat{q}(a)} dFdG + (1 - G(\Phi, b)).$$

Combining the previous two equations gives the following expression for the manager's expected utility:

$$U = \left( \int_0^\Phi \int_0^{C+\hat{q}(a)} dFdG \right) s(q(a)) + (1 - G(\Phi, b))s(q(a)) - D(a, b), \quad (9)$$

which gives the probability of  $V$  and  $C$  being in region III (implying no takeover), times the manager's remuneration with no takeover, minus the disutility of effort. Equation (9) can be simplified somewhat to

$$U = \left\{ \int_0^\Phi F(C + \hat{q}(a))g(C, b)dC + (1 - G(\Phi, b)) \right\} s(q(a)) - D(a, b). \quad (10)$$

The corresponding first-order conditions with respect to  $a$  and  $b$  are<sup>18</sup>

$$a: \left[ \int_0^\Phi f(C + \hat{q}(a))\hat{q}'(a)dG \right] s(q(a)) + \{ \} s'(q(a))q'(a) - D_a(a, b) = 0 \quad (11)$$

$$b: \left[ \int_0^\Phi F(C + \hat{q}(a))g_b(C, b)dC - G_b(\Phi, b) \right] s(q(a)) - D_b(a, b) = 0. \quad (12)$$

Both equations have straightforward interpretations. In (11), the first term is the reduction in the probability of takeover with a unit increase in productive effort, times the salary; the second is the probability of no takeover times the gain in the salary; and the third is the marginal disutility of productive

<sup>18</sup> The second-order conditions do not hold in general. Thus, there may be multiple local optima. Furthermore, even in the region a local optimum, the signs of  $\frac{da}{d\Phi}$  and  $\frac{db}{d\Phi}$  are in general ambiguous. The sign of  $\frac{dU}{d\Phi}$  is however unambiguously negative.

effort. In (12), the first term is the reduction in the probability of takeover with a unit increase in defensive effort, times the salary; and the second is the marginal disutility of defensive effort. The solution to (11) and (12) yields  $a(\Phi)$  and  $b(\Phi)$ . We now examine the optimal dilution factor from the shareholder's and from society's perspective. The shareholder's perspective on dilution is examined first.

### 5.2 Optimal Dilution from the Shareholder's Perspective

From Figure 2 and the subsequent discussion, the shareholder's maximization problem is

$$\max_{\Phi} S(\Phi) = \int_0^{\Phi} \int_{\Phi+\hat{q}(a)}^{\infty} (V - \Phi) dFdG + \int_0^{\Phi} \int_0^{\Phi+\hat{q}(a)} \hat{q}(a) dFdG + \int_{\Phi}^{\infty} \int_0^{\infty} \hat{q}(a) dFdG$$

or

$$\max_{\Phi} S(\Phi) = \hat{q}(a) + G(\Phi, b) \int_{\Phi+\hat{q}(a)}^{\infty} (V - \Phi - \hat{q}(a)) dF, \quad (13)$$

where  $S(\Phi)$  is expected shareholder return, and  $a$  and  $b$  are functions of  $\Phi$  per (11) and (12).

When takeover does not occur or occurs with a tender price of  $\hat{q}(a)$  (regions II and III in Figure 2), the shareholder receives  $\hat{q}(a)$ . If, however,  $V - \Phi > \hat{q}(a)$ , the shareholder receives  $V - \Phi$ . Thus, we may view the shareholder as always receiving  $\hat{q}(a)$ , and as receiving a bonus of  $V - \Phi - \hat{q}(a)$  in region I.  $\Phi_s^*$  is defined to be the optimal dilution factor from the shareholder's perspective.

### 5.3 Optimal Dilution from the Planner's Perspective

From Fig. 2 and the subsequent discussion, the planner's problem is

$$\max_{\Phi} W(\Phi) = \int_0^{\Phi} \int_{C+\hat{q}(a)}^{\infty} (V - C) dFdG + \int_0^{\Phi} \int_0^{C+\hat{q}(a)} \hat{q}(a) dFdG + \int_{\Phi}^{\infty} \int_0^{\infty} \hat{q}(a) dFdG, \text{ or}$$

$$\max_{\Phi} W(\Phi) = \hat{q}(a) + \int_0^{\Phi} \int_{C+\hat{q}(a)}^{\infty} (V - C - \hat{q}(a)) dFdG, \quad (14)$$

where  $W(\Phi)$  is the expected social return.

When takeover occurs the social return is  $V - C > \hat{q}(a)$ , and when it does not the social return is  $\hat{q}(a)$ . Thus we may regard the social return as being  $\hat{q}(a)$  plus an extra amount  $V - C - \hat{q}(a) > 0$  which is received when takeover occurs.  $\Phi_w^*$  is defined to be the optimal dilution factor from the planner's perspective.

#### 5.4 Comparison of $\Phi_S^*$ and $\Phi_W^*$

*Proposition 3:* When defensive effort is fixed, the socially optimal level of dilution ( $\Phi_W^*$ ) exceeds the privately optimal level ( $\Phi_S^*$ ) unless  $\Phi_W^* = \Phi_S^* = 0$ .

*Proof:* Suppose not. Then either  $\Phi_W^* < \Phi_S^*$  or  $\Phi_W^* = \Phi_S^*$ . Consider each case in turn.

- $\Phi_W^* < \Phi_S^*$

With defensive effort fixed, the RHS of (14) may be written as  $\hat{W}(\Phi, a(\Phi))$ . Then

$$W(\Phi_W^*) - W(\Phi_S^*) = \left[ \hat{W}(\Phi_W^*, a(\Phi_W^*)) - \hat{W}(\Phi_S^*, a(\Phi_W^*)) \right] + \left[ \hat{W}(\Phi_S^*, a(\Phi_W^*)) - \hat{W}(\Phi_S^*, a(\Phi_S^*)) \right]. \quad (15)$$

Now since  $\frac{\partial \hat{W}}{\partial \Phi} > 0$  from (14),  $\left[ \hat{W}(\Phi_W^*, a(\Phi_W^*)) - \hat{W}(\Phi_S^*, a(\Phi_W^*)) \right] < 0$ . Also, by the definition of  $\Phi_W^*$ ,

$W(\Phi_W^*) \geq W(\Phi_S^*)$ . These two results, along with (15), imply that  $\left[ \hat{W}(\Phi_S^*, a(\Phi_W^*)) - \hat{W}(\Phi_S^*, a(\Phi_S^*)) \right] > 0$ .

Then since  $\frac{\partial \hat{W}}{\partial a} > 0$  from (14),  $a(\Phi_W^*) > a(\Phi_S^*)$ .

The raider's return is  $R(\Phi) = W(\Phi) - S(\Phi)$ . Then, from (13) and (14),

$$R(\Phi) = \int_0^\Phi [M(C + \hat{q}(a)) - M(\Phi + \hat{q}(a))] dG, \quad (16)$$

where  $M(x) \equiv \int_x^\infty (V - x) dF$ . Write the equation for  $R(\Phi)$  as  $R(\Phi) = \hat{R}(\Phi, a(\Phi))$ . Then

$$R(\Phi_W^*) - R(\Phi_S^*) = \left[ \hat{R}(\Phi_W^*, a(\Phi_W^*)) - \hat{R}(\Phi_S^*, a(\Phi_W^*)) \right] + \left[ \hat{R}(\Phi_S^*, a(\Phi_W^*)) - \hat{R}(\Phi_S^*, a(\Phi_S^*)) \right]. \quad (17)$$

Since  $\frac{\partial \hat{R}}{\partial \Phi} > 0$  from (16), the first term in square brackets on the RHS of (17) is negative. And since

$\frac{\partial \hat{R}}{\partial a} < 0$  from (16) and since  $a(\Phi_W^*) > a(\Phi_S^*)$ , the second term in square brackets on the RHS of (17) is

also negative. Thus,  $R(\Phi_W^*) - R(\Phi_S^*) < 0$ , which together with  $W(\Phi_W^*) \geq W(\Phi_S^*)$  implies that

$S(\Phi_W^*) - S(\Phi_S^*) > 0$ . But this is inconsistent with the definition of  $\Phi_S^*$ .

- $\Phi_W^* = \Phi_S^*$

There are two possibilities to consider. Either the optimal dilution factors are both positive and finite or both zero. Consider first the situation where both are positive. From the definitions of  $\Phi_W^*$  and

$\Phi_S^*$ , and since  $R(\Phi) = W(\Phi) - S(\Phi)$ ,  $\left. \frac{dR(\Phi)}{d\Phi} \right|_{\Phi_w^*} = 0$ . Now  $\frac{dR(\Phi)}{d\Phi} = \frac{\partial \hat{R}(\Phi, a(\Phi))}{\partial \Phi} + \frac{\partial \hat{R}(\Phi, a(\Phi))}{\partial a} \frac{da}{d\Phi}$ .

From the earlier part of the proof,  $\frac{\partial \hat{R}}{\partial \Phi} > 0$  and  $\frac{\partial \hat{R}}{\partial a} < 0$ . Thus,  $\left. \frac{da}{d\Phi} \right|_{\Phi_w^*} > 0$ . But then since  $\frac{\partial \hat{W}}{\partial \Phi} > 0$

and  $\frac{\partial \hat{W}}{\partial a} > 0$ ,  $\left. \frac{dW}{d\Phi} \right|_{\Phi_w^*} = \left[ \frac{\partial \hat{W}}{\partial \Phi} + \frac{\partial \hat{W}}{\partial a} \frac{da}{d\Phi} \right]_{\Phi_w^*} > 0$ , which contradicts the definition of  $\Phi_w^*$ .

Next consider the case where  $\Phi_w^* = \Phi_S^* = 0$ . If  $\left. \frac{da}{d\Phi} \right|_{\Phi=0}$  is sufficiently negative, it is possible

that  $\left. \frac{dW}{d\Phi} \right|_{\Phi=0} < 0$  and  $\left. \frac{dS}{d\Phi} \right|_{\Phi=0} < 0$  and also  $\Phi_w^* = \Phi_S^* = 0$ . ■

The proof for Proposition 3 is global. The following intuition for the result uses a local argument. At the socially optimal level of dilution, a local increase in dilution must cause a *decrease* in productive effort; otherwise, the local increase in dilution would increase social welfare (both directly and via the increase in productive effort) which is inconsistent with dilution being at the socially optimal level. Since the raider benefits directly from an increase in the dilution factor and also from a decrease in productive effort (since this decreases the expected tender price she must pay), at the socially optimal level of dilution the raider's return is increasing in the dilution factor. But this implies that at the socially optimal level of dilution, the shareholders' return is decreasing in the dilution factor.

We now investigate the relationship between the socially optimal and the privately optimal levels of dilution when defensive effort is admitted. We first show that the above argument does *not* generalize to the situation where defensive effort is variable. To simplify, we consider the opposite extreme where productive effort is fixed at  $\bar{a}$  and ignore the possibility that  $\Phi_w^* = 0$ . An increase in dilution has two

effects on social welfare. The direct effect  $\left( \frac{\partial \hat{W}}{\partial \Phi} \right)$  increases social welfare by increasing the probability

of a value-enhancing takeover. The indirect effect operates through defensive effort  $\left( \frac{\partial \hat{W}}{\partial b} \frac{db}{d\Phi} \right)$ . At the

socially optimal level of dilution,  $\Phi_w^*$ , the sum of the two effects is zero. Since an increase in defensive

effort lowers social welfare ( $\frac{\partial \hat{W}}{\partial b} < 0$  from (14)), in the neighborhood of  $\Phi_w^*$  a small increase in dilution must induce an increase in defensive effort. Thus, the socially optimal dilution factor weighs the direct benefit from increased dilution against the increase in expected takeover cost from the induced increase in defensive effort. Now consider the effect on the raider of an increase in dilution in the neighborhood of the socially optimal dilution factor. She benefits directly from the increase in dilution ( $\frac{\partial \hat{R}}{\partial \Phi} > 0$ ) but is adversely affected by the increase in defensive effort ( $\frac{\partial \hat{R}}{\partial b} < 0$ ).<sup>19</sup> Hence, at the socially optimal level of dilution, an increase in dilution appears to have an ambiguous effect on the raider's utility and hence on shareholder utility. This is indeed the case. We have constructed examples in which the socially optimal level of dilution exceeds that chosen by shareholders, and others in which the socially optimal level of dilution is less than that chosen by shareholders.<sup>20</sup> Thus we have

*Proposition 4:* With managerial defensive effort variable, the level of dilution chosen by shareholders may *exceed* the socially optimal level.

Under what circumstances will the level of dilution chosen by shareholders exceed the socially optimal level? An exhaustive answer to this question would be very difficult to provide, because both the planner's and the shareholders' optimal dilution factors are solutions to non-concave programming problems.<sup>21</sup> Appendix 2 identifies conditions under which, at a local, interior socially optimal level of dilution, shareholders would benefit for an incremental increase in dilution, for the situation where productive effort is fixed and the potential value of the firm is known. Here we present a more intuitive explanation — *of why shareholders may benefit from an incremental increase in dilution above a locally*

$$\begin{aligned} \frac{\partial \hat{R}}{\partial b} &= \int_0^\Phi (M(C + \hat{q}(\bar{a})) - M(\Phi + \hat{q}(\bar{a}))) g_b(C, b) dC \\ &= \left[ (M(C + \hat{q}(\bar{a})) - M(\Phi + \hat{q}(\bar{a}))) G_b(C, b) \right]_0^\Phi + \int_0^\Phi (1 - F(C + \hat{q}(\bar{a}))) G_b(C, b) dC < 0. \end{aligned}$$

<sup>19</sup> The examples are available in the working paper version of the paper at: <http://FMWWW.bc.edu/EC-V/Arnott.fac.html>.

<sup>21</sup> We have searched unsuccessfully for a class of *smooth* cumulative distribution functions,  $G(C, b)$ , for which shareholders choose more dilution than the social planner for a set of parameter values, with the opposite occurring for the complementary set. Thus, we cannot give a precise characterization of the circumstances under which shareholders choose excessive dilution, nor, for the same reason, can we say whether these circumstances are “likely” or “unlikely”.

*socially optimal level* — through a simple numerical example.

To simplify, we assume that both the dilution factor and takeover costs take on discrete values. Let  $i$  index the dilution factor and  $j$  takeover costs, and let  $p_{ij}$  denote the probability that the takeover cost is  $C_j$  when the dilution factor is  $\Phi_i$ , which incorporates defensive effort. Takeover occurs when and only when  $V - \bar{q} - \Phi > 0$  and  $\Phi > C$ , at a tender price  $V - \Phi$ . The general expressions for social welfare, shareholder welfare, and raider welfare are, respectively,

$$W_i = \sum_{j \in J_i} p_{ij} (V - \bar{q} - C_j) + \bar{q}. \quad (18a)$$

$$S_i = \sum_{j \in J_i} p_{ij} (V - \bar{q} - \Phi_i) + \bar{q} \quad (18b)$$

$$R_i = \sum_{j \in J_i} p_{ij} (\Phi_i - C_j) \quad (18c)$$

where  $J_i$  is the set of  $j$  for which  $\Phi_i > C_j$ .

Now turn to the example given in Table 1A and 1B below. Table 1A displays the upper left-hand corner of the matrix of  $\{p_{ij}\}$  since only this submatrix is used in the example. The columns indicate the values of  $\Phi$ , the rows the values of  $C$ , and the cells the corresponding probabilities. The superscript “+” on each value of  $C$  in Table 1A indicates that in constructing this example we consider values of  $C$  infinitesimally greater than the corresponding integer values. Thus, for example, when  $\Phi = 1$ , the manager’s choice of defensive effort (which is treated implicitly) results in takeover costs being  $0^+$  with probability .05,  $1^+$  with probability .05, etc. The form of the probability distribution for  $C > 4^+$  is not shown since it does not affect the example. The example is constructed so that increased dilution affects the probability distribution of takeover costs in particular ways. First, it is implicitly assumed that an increase in  $\Phi$  stimulates defensive effect, which, per our assumption that  $G_b(C, b) < 0$ , implies that the probability that takeover costs are below a particular level falls; second, when  $\Phi$  increases from 2 to 3, the corresponding increase in defensive effort eliminates the possibility of a takeover being “cheap” ( $C = 0^+$  or  $1^+$ ).

Table 1B gives the values of the expected social surplus from takeover ( $W - \bar{q}$ ), the expected shareholder surplus from takeover ( $S - \bar{q}$ ), and the expected raider's surplus ( $R$ ) for integer values of  $\Phi$  from 0 to 4, which are computed using (18a) - (18c) and  $V - \bar{q} = 12$ . A sample calculation may clarify the procedure. When  $\Phi=2$ , takeover occurs when  $C = 0^+$  or  $C = 1^+$ ;  $C = 0^+$  and  $C = 1^+$  each occur with probability .0333; when  $C = 0^+$  or  $C = 1^+$ , shareholders receive  $V-2$ , and otherwise takeover does not occur and they receive  $\bar{q}$ ; thus, the expected shareholder surplus from takeover is

$$S - \bar{q} = (.0666)(V - \bar{q} - 2) = (.0666)(10) = .666.$$

		<u>1A</u>					
		$\Phi$	0	1	2	3	4
$C$	$0^+$	.10	.05	.0333	0	0	
	$1^+$	.10	.05	.0333	0	0	
	$2^+$	.10	.05	.0333	.075	0	
	$3^+$	.10	.05	.0333	.025	.08	
	$4^+$	.10	.05	.0333	.025	.02	

		<u>1B</u>					
		$\Phi$	0	1	2	3	4
$W - \bar{q}$	0	.60	.766	.75	.72		
$S - \bar{q}$	0	.55	.666	.675	.64		
$R$	0	.05	.10	.075	.08		

Table 1: Numerical example illustrating that shareholders may (locally) prefer a higher dilution factor than the planner.

Note: Table 1A displays only the upper left-hand corner of the matrix of  $\{p_{ij}\}$  since only this part of the matrix is used to establish the result. Table 1B displays the values of  $W - \bar{q}$ ,  $S - \bar{q}$ , and  $R$  that can be derived from Table 1A's submatrix.

The first column in Table 1A indicates that when there is no dilution, the probability that takeover costs are  $0^+$  is .10, as is the probability that takeover costs are  $1^+$  is also .10 and so on. The probability of takeover is the probability that dilution exceeds takeover costs, which equals zero. The expected surpluses from takeover for society, shareholders, and the raider are of course all zero. Also, since there is no threat of takeover, the manager expends zero defensive effort. Now raise dilution to  $\Phi = 1$ . If the manager were to continue to expend zero effort, takeover would occur with probability .1. It now pays the manager to exert some effort to defend against takeover. The example assumes that his optimal choice of effort changes the probability distribution of takeover costs such that the probability that

takeover costs are  $0^+$  is .05, that they are  $1^+$  is .05, and so on. Takeover occurs only when the dilution factor exceeds takeover costs, *viz.* when  $C = 0^+$ , which occurs with a probability of .05. The expected social surplus from takeover is .60, the probability of takeover (.05) times the social surplus when takeover occurs ( $V - \bar{q} - 0^+ = 12$ ); this is divided between expected shareholder surplus ( $.05(V - \bar{q} - \Phi) = .55$ ) and expected raider surplus ( $.05(\Phi - 0^+) = .05$ ). When  $\Phi$  is raised to 2, the manager further increases his defensive effort. This can be seen by noting that, for the range of  $C$  shown, the probability increases that takeover costs are above a given level. Here, takeover occurs when  $C = 0^+$  or  $1^+$ .  $W - \bar{q}$ ,  $S - \bar{q}$ , and  $R$  are calculated by applying (18a) – (18c). Table 1B indicates that all the expected surpluses rise when  $\Phi$  is raised from 1 to 2.

The example's central point of interest occurs when  $\Phi$  is raised from 2 to 3. The manager introduces additional defenses that eliminate the possibility of a cheap takeover but do not prevent the probability of takeover from increasing. Because the possibility of cheap takeovers is eliminated, expected takeover costs increase by more than the expected gross surplus from takeover, so that social surplus falls. But the probability of takeover increases sufficiently that the expected gross surplus from takeover increases by more than expected dilution payments, so that shareholders are better off. Thus shareholders favor the increase from  $\Phi=2$  to  $\Phi=3$ , even though it is socially undesirable, since their decision calculus does not account for the substantial increase in expected takeover costs resulting from the elimination of the possibility of cheap takeovers.

## 6. Conclusion

The takeover boom of the mid-1990s raised once again the question of whether the market for corporate control generates too many takeovers or too few. Pointing to the large gains accruing to shareholders through the takeover process, some have argued that increased takeover activity should be encouraged. Others, viewing these gains as illusory, have argued for increased oversight and regulation of the takeover process. This paper does not attempt to come to grips with the full complexity of the issue. Rather, it focuses on a class of takeover costs that previous theoretical research has tended to overlook — corporate restructuring by managers to defend against takeover. Evidence strongly suggests



that such activity is pervasive, quantitatively important, and detrimental to shareholders' long-term interests.

Managerial resistance obviously increases the costs of raids. But it also alters the private and social cost-benefit calculus of takeover. If defensive managerial effort is fixed, the level of takeover activity is too low, since shareholders regard the financial incentives given raiders to stimulate takeover activity as a cost while society views them as a transfer. We showed that admitting defensive restructuring by managers upsets this result, implying that an unregulated market for corporate control may generate excessive takeovers. Empirically our model suggests that evidence from gains from takeovers may be overstated because they fail to take managerial defensive effort into account. Prior to a takeover, a firm may experience “a huge diversion of managerial effort into devising ways to reduce vulnerability that did not grow out of managerial inefficiency.” (p. 215, Herman and Lowenstein (1988)). Such behavior is frequently observed but very difficult to successfully prosecute given the courts' reluctance to meddle in a firm's day-to-day operations under the “business judgement rule”. A successful takeover may then appear profitable when in fact it simply undoes the inefficiencies caused by the defensive restructuring.

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## Appendix 1

There are two reasonable objections to the model. The first is that the planner should take the manager's expected utility into account. The second is that the manager should realize that a higher dilution factor reduces his expected utility, and since he presumably has other employment opportunities, he should require a higher level of expected remuneration if the dilution factor is increased.

To address the first objection, suppose that the economic environment is the same in all respects as that in sections 5, where both  $V$  and  $C$  are stochastic. The only modification is that social welfare as a function of  $\Phi$  changes from  $W(\Phi)$ , as given by (14), to

$\hat{W}(\Phi) = W(\Phi) + \hat{U}(\Phi)$  where  $\hat{U}(\Phi)$  is the manager's expected utility as a function of  $\Phi$ . The inclusion of the manager's utility function in the social welfare function does not alter the behavior of the economy. Since the manager's effort choices are unchanged,

$\hat{U}(\Phi) = U(\Phi, a(\Phi), b(\Phi))$  where  $a(\cdot)$ ,  $b(\cdot)$  and  $U(\cdot)$  are the solution to (2). Because the economic environment remains the same, so too does the optimal level of dilution from the shareholders' perspective. But since  $\hat{U}'(\Phi) < 0$  — the manager's expected utility decreases in the dilution factor — the inclusion of managerial utility in social welfare unambiguously reduces the optimal level of dilution from the planner's perspective. Thus, the inclusion of managerial utility in social welfare makes it "more likely" that the level of dilution chosen by shareholders is excessive.

To address the second objection, suppose that there is a managerial participation constraint that  $\hat{U}(\Phi) \geq \bar{Z}$  for all levels of  $\Phi$ . We assume that the manager's compensation comprises two components. The first is the performance-contingent component considered in

the model presented in the body of the paper; with the disutility of effort taken into account, this component provides expected utility as a function of  $\Phi$  of  $\hat{U}(\Phi)$ , as defined above. The second component is independent of performance and depends only on  $\Phi$ ,  $Y(\Phi)$ . Shareholders have no incentive to provide more generous remuneration than is needed to satisfy the managerial participation constraint since doing so has no incentive effects. Hence

$$Y(\Phi) = \bar{Z} - \hat{U}(\Phi), \quad (\text{A1.1})$$

with  $Y'(\Phi) > 0$  since  $\hat{U}'(\Phi) < 0$ . Since the second component of remuneration is independent of performance, it does not affect the manager's choice of productive and defensive effort as a function of  $\Phi$ . Shareholder utility is therefore

$$\hat{S}(\Phi) = S(\Phi) - Y(\Phi), \quad (\text{A1.2})$$

where  $S(\Phi)$  is given by (13). Since  $Y'(\Phi) > 0$ , the optimal dilution factor from the shareholder's perspective is lowered by the inclusion of  $Y(\Phi)$ . Now consider social welfare.

With managerial utility *excluded* from social welfare,

$$\begin{aligned} \hat{W}(\Phi) &= \hat{S}(\Phi) + R(\Phi) \\ &= W(\Phi) - Y(\Phi) \\ &= W(\Phi) + \hat{U}(\Phi) - \bar{Z} \quad (\text{using (A1.1)}) \\ &= \hat{W}(\Phi) - \bar{Z}. \end{aligned} \quad (\text{A1.3})$$

where  $R(\Phi)$  is given by (16). Thus, inclusion of  $Y(\Phi)$  in managerial remuneration lowers the optimal dilution factor from the planner's perspective by the same amount as did the inclusion of managerial utility in the social welfare function without  $Y(\Phi)$ . Since

$$\hat{W}(\Phi) - W(\Phi) = \hat{S}(\Phi) - S(\Phi) = -Y(\Phi), \quad (\text{A1.4})$$

the inclusion of  $Y(\Phi)$  in managerial remuneration may lower the optimal dilution factor from the planner's perspective either more or less than that from the shareholders' perspective, and

may cause the planner's optimal dilution factor to be higher than the shareholders' where with the exclusion of  $Y(\Phi)$  it was lower and *vice versa*. With managerial utility *included* in social welfare:

$$\begin{aligned}\hat{\hat{W}}(\Phi) &= \hat{S}(\Phi) + R(\Phi) + \bar{Z} \\ &= S(\Phi) - Y(\Phi) + R(\Phi) + \hat{U}(\Phi) + Y(\Phi) \\ &= \hat{W}(\Phi).\end{aligned}\tag{A1.5}$$

Thus, with the inclusion of managerial utility in the social welfare function, the inclusion of  $Y(\Phi)$  in managerial compensation does not alter the optimal level of dilution from the planner's perspective because the planner then regards the payment of  $Y(\Phi)$  simply as a welfare-neutral transfer from shareholders to the manager.

	Y( $\Phi$ ) excluded	Y( $\Phi$ ) included
U not in social welfare	I	III
U in social welfare	II	IV

Pulling the results together, there are four cases to consider, which are numbered according to the Table. Note that case I was treated in the body of the paper. Letting roman numeral superscripts index the case, we have

$${}^I\Phi_w^* > {}^{II}\Phi_w^* = {}^{III}\Phi_w^* = {}^{IV}\Phi_w^*\tag{A1.6a}$$

and

$${}^I\Phi_s^* = {}^{II}\Phi_s^* > {}^{III}\Phi_s^* = {}^{IV}\Phi_s^*.\tag{A1.6b}$$

In the body of the paper, we argued that  ${}^I\Phi_s^*$  can exceed  ${}^I\Phi_w^*$  and in discussing case III that  ${}^{III}\Phi_s^*$  case exceed  ${}^{III}\Phi_w^*$ . It follows from the above inequalities that  ${}^{II}\Phi_s^*$  can exceed  ${}^{II}\Phi_w^*$ , and

that  ${}^{IV}\Phi_S^*$  can exceed  ${}^{IV}\Phi_W^*$ . Hence, the principal result of our paper, that in the presence of defensive effort shareholders may choose excessive dilution, is valid as well for the extensions considered in this appendix.



## Appendix 2

$$\frac{dW}{d\Phi} = 0 \text{ and } \frac{dS}{d\Phi} > 0 \text{ are not inconsistent}$$

To simplify the analysis, we deal with the situation in which the potential value of the firm is known with certainty, and in which productive effort is fixed at  $\bar{a}$ . Furthermore, defining  $\bar{q} \equiv \hat{q}(\bar{a})$ , it is assumed that there are potential gains from takeover:  $V > \bar{q}$ .

Under these assumptions, (14) reduces to

$$W(\Phi) = \int_0^\Phi (V - C - \bar{q})g(C, b(\Phi))dC + \bar{q}; \quad (\text{A2.1})$$

in words, social welfare equals  $\bar{q}$  plus the net (of takeover costs) social surplus from takeover.

At a local interior optimum of social welfare

$$\begin{aligned} \frac{dW}{d\Phi} &= (V - \Phi - \bar{q})g(\Phi, b(\Phi)) + \int_0^\Phi (V - C - \bar{q})g_b(C, b(\Phi))b'(\Phi)dC \\ &= (V - \Phi - \bar{q})g(\Phi, b(\Phi)) + \left[ (V - \Phi - \bar{q})G_b(\Phi, b(\Phi)) + \int_0^\Phi G_b(C, b(\Phi))dC \right] b'(\Phi) = 0. \end{aligned} \quad (\text{integration by parts}) \quad (\text{A2.2})$$

Since  $V - \Phi > \bar{q}$  the second term must be negative. The term in square brackets is negative, which implies that  $b'(\Phi) > 0$ . This result is intuitive. Raising  $\Phi$  holding defensive effort fixed, is unambiguously beneficial. Thus, at a local optimum in  $\Phi$ , defensive effort must be increasing in  $\Phi$ . Now rearrange (A2.2):

$$(V - \Phi - \bar{q})(g(\Phi, b(\Phi)) + G_b(\Phi, b(\Phi))b'(\Phi)) + \left( \int_0^\Phi G_b(C, b(\Phi))dC \right) b'(\Phi) = 0. \quad (\text{A2.2}')$$

That the second term is negative implies that the first is positive. The first term is  $V - \Phi - \bar{q}$  times the total derivative of the probability of takeover ( $G(\Phi, b(\Phi))$ ) with respect to  $\Phi$ . Thus, at a local

social optimum in  $\Phi$ , defensive effort must be increasing in  $\Phi$  but not so sharply as to cause the probability of takeover to be decreasing in  $\Phi$ .

Since  $W = S + R$ , explaining why  $\frac{dS}{d\Phi} > 0$  is possible at a local interior social optimum of the dilution factor is equivalent to explaining why  $\frac{dR}{d\Phi} < 0$  is possible. The latter is somewhat easier. An increase in  $\Phi$  has three effects on the raider: the gross return per raid — the level of dilution — is larger; the probability of takeover is higher; and due to the increase in defensive effort, expected costs of a takeover increase.  $\frac{dR}{d\Phi} < 0$  requires that the last effect dominate the former two, so that expected takeover costs increase by more than expected dilution,  $\Phi G$ .

Formally:

$$\begin{aligned} R(\Phi) &= \int_0^\Phi (\Phi - C)g(C, b(\Phi))dC \\ &= \Phi G(\Phi, b(\Phi)) - \int_0^\Phi Cg(C, b(\Phi))dC; \end{aligned} \tag{A2.3}$$

the first term on the RHS is expected dilution, the second term expected takeover costs.

Integrating the second term by parts yields

$$R(\Phi) = \int_0^\Phi G(C, b(\Phi))dC, \tag{A2.3'}$$

so that

$$\frac{dR(\Phi)}{d\Phi} = G(\Phi, b(\Phi)) + \int_0^\Phi G_b(C, b(\Phi))b'(\Phi)dC. \tag{A2.4}$$

Thus, the condition that expected takeover costs increase by more than expected dilution reduces

to the condition that  $-\int_0^\Phi G_b b' > G$ .

Define  $\int G = \mathcal{G}$ . Then combining (A2.2) and (A2.4) gives a necessary and sufficient

condition for  $\frac{dS}{d\Phi} > 0$  when  $\frac{dW}{d\Phi} = 0$ :

$$\left\{ \frac{-(V - \Phi - \bar{q})g}{(V - \Phi - \bar{q})G_b + \hat{G}_b} \right\}_{\Phi^*} = b'(\Phi^*) > \left\{ \frac{-G}{\hat{G}_b} \right\}_{\Phi^*}, \quad (\text{A2.5})$$

where  $\Phi^*$  denotes an interior local optimum of dilution.<sup>1</sup>

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<sup>1</sup> Please note that while we have successfully constructed an example where our required inequality is satisfied, the example does not address the issue of whether such condition is “likely” or “unlikely” to be satisfied.

### Appendix 3

#### Example with $\Phi_s^* > \Phi_w^*$

(Not for Publication)

- $G(C,b)$ ,  $g(C,b)$

$$G(C,b) = \begin{cases} 0 & \text{for } C < 1 - e^{-\alpha b} \\ \frac{1}{2} & \text{for } C \in [1 - e^{-\alpha b}, 1) \\ \frac{1}{2} + \frac{1}{2b}(C-1)^2 & \text{for } C \in [1, 1\frac{1}{2}) \\ \frac{1}{2} + \frac{1}{8b} & \text{for } C \in [1\frac{1}{2}, 3] \\ 1 & \text{for } C > 3 \end{cases}$$

$$g(C,b) = \begin{cases} 0 & \text{for } C < 1 - e^{-\alpha b} \\ \text{mass} & \text{at } C = 1 - e^{-\alpha b} \\ 0 & \text{for } C \in (1 - e^{-\alpha b}, 1) \\ \frac{1}{b}(C-1) & \text{for } C \in [1, 1\frac{1}{2}) \\ 0 & \text{for } C \in [1\frac{1}{2}, 3] \\ \text{mass} & \text{at } C = 3^+ \end{cases}$$

- $D(b) = db$
- $V - \bar{q} \equiv v = 3$ ,  $\bar{q} > 0$ , other parameters to be determined.

#### Manager's Choice of Defensive Effort

$\Phi < 1$ : The manager may choose to expend sufficient defensive effort to thwart the takeover of the mass at  $C = 1 - e^{-\alpha b}$ . If he does, he will expend just enough effort to deter takeover of the mass. If this results in his achieving a higher level of utility than by doing nothing, he will expend that level of effort; otherwise, he will expend zero effort. The minimum level of effort which deters takeover of the mass solves

$$e^{-\alpha b} = 1 - \Phi \Rightarrow b = -\frac{1}{\alpha} \ln(1 - \Phi).$$

Thus, if the manager chooses to deter takeover of the mass,

$$\begin{aligned} U &= s(\bar{q}) - db \\ &= s(\bar{q}) + \frac{d}{\alpha} \ln(1 - \Phi). \end{aligned}$$

If instead he chooses not to deter takeover,  $b=0$  and  $U=0$ . He will therefore choose

to deter takeover if  $\Phi < 1 - e^{-\bar{s}\alpha/d}$  ( $\bar{s} \equiv s(\bar{q})$ ) and not to otherwise. Thus,

$$b(\Phi) = \begin{cases} -\frac{1}{\alpha} \ln(1 - \Phi) & \text{if } \Phi < 1 - e^{-\bar{s}\alpha/d} \\ 0 & \text{if } \Phi \geq 1 - e^{-\bar{s}\alpha/d}. \end{cases} \quad (\text{A3.1})$$

$$\Phi \in [1, 1\frac{1}{2}]: \max_b (1 - G(\Phi, b))s(\bar{q}) - db$$

$$\Rightarrow \max_b \frac{1}{2} \left( 1 - \frac{1}{b} (\Phi - 1)^2 \right) s(\bar{q}) - db$$

$$b: \frac{1}{2b^2} (\Phi - 1)^2 s(\bar{q}) - d = 0$$

(and second-order conditions hold). Thus

$$b = (\Phi - 1) \sqrt{\frac{\bar{s}}{2d}}.$$

Define  $w \equiv \sqrt{\frac{\bar{s}}{2d}}$ . Then

$$b = w(\Phi - 1). \quad (\text{A3.2})$$

Note that in this region,  $b$  is increasing in  $\Phi$  because  $g(\cdot)$  and hence the marginal

benefit from defensive effort is increasing in  $\Phi$ .

$$\Phi \in [1\frac{1}{2}, 3]: \max_b (1 - G(\Phi, b))s(\bar{q}) - db$$

$$\Rightarrow \max_b \left( \frac{1}{2} - \frac{1}{8b} \right) \bar{s}(q) - db$$

$$b: \frac{1}{8b^2} \bar{s}(q) - d = 0$$

(and second-order conditions hold). Thus,

$$b = \sqrt{\frac{\bar{s}}{8d}} = \frac{w}{2}. \quad (\text{A3.3})$$

$\Phi > 3$ :  $T = \bar{q}$ . Shareholders tender their shares. The solution for  $b$  is given by (A3.3).

#### Shareholders' Choice of $\Phi$

$\Phi < 1$ : If  $\Phi < 1 - e^{-\bar{s}\alpha/d}$ , the manager will deter takeover in the mass at  $C = 1 - e^{-\alpha b}$ . No takeovers will occur.

If  $\Phi > 1 - e^{-\bar{s}\alpha/d}$ ,  $b=0$  and all firms in the mass will be taken over but no others.

It is in the shareholders' interest in this region to set  $\Phi$  as low as possible consistent with takeover of firms in the mass.

$$\begin{aligned} \max_{\Phi} S &= G(\Phi, b(\Phi))(V - \Phi - \bar{q}) + \bar{q} \\ &= \frac{1}{2}(V - \Phi - \bar{q}) + \bar{q} \quad \text{s.t.} \quad \Phi > 1 - e^{-\bar{s}\alpha/d} \\ \Rightarrow \Phi &= 1 - e^{-\bar{s}\alpha/d} \quad \text{and} \\ S &= \frac{1}{2}\left(V - 1 + e^{-\bar{s}\alpha/d}\bar{q}\right) + \bar{q}. \end{aligned} \quad (\text{A3.4})$$

$\Phi \in [1, 1\frac{1}{2}]$ :

$$\begin{aligned} \max_{\Phi} S &= G(\Phi, b(\Phi))(V - \Phi - \bar{q}) + \bar{q} \\ &= \left(\frac{1}{2} + \frac{1}{2b}(\Phi - 1)^2\right)(V - \Phi - \bar{q}) + \bar{q} \\ &= \frac{1}{2w}(w + (\Phi - 1))(V - \Phi - \bar{q}) + \bar{q} \quad (\text{using (A3.2)}) \end{aligned}$$

$$\Rightarrow \Phi = \begin{cases} 1.0 & \text{if } \frac{1+v-w}{2} \leq 1.0 \\ \frac{1+v-w}{2} & \text{if } \frac{1+v-w}{2} \in (1.0, 1.5] \\ 1.5 & \text{if } \frac{1+v-w}{2} > 1.5 \end{cases} \quad (\text{A3.5a})$$

and

$$S = \begin{cases} \frac{1}{2}(v-1) + \bar{q} & \text{if } \frac{1+v-w}{2} \leq 1.0 \\ \frac{1}{8w}(v+w-1)^2 + \bar{q} & \text{if } \frac{1+v-w}{2} \in (1.0, 1.5] \\ \frac{1}{2w}(w+.5)(v-1.5) + \bar{q} & \text{if } \frac{1+v-w}{2} \geq 1.5. \end{cases} \quad (\text{A3.5b})$$

$\Phi \in [1\frac{1}{2}, 3]$ :

$$\max_{\Phi} S = \left(\frac{1}{2} + \frac{1}{8b}\right)(V - \Phi - \bar{q}) + \bar{q}$$

Choose lowest  $\Phi$ ,  $\Phi=1.5$ .

$\Phi > 3$ :  $S = \bar{q}$

#### Planner's Choice of $\Phi$

$\Phi < 1$ : The planner would like as many takeovers as possible with minimal takeover costs.

Within this region of  $\Phi$ , both these objectives are met with  $\Phi \geq 1 - e^{-\bar{s}\alpha/d}$ , in which case  $b=0$  and

$$\begin{aligned} W &= \max_{\Phi} \int_0^{\Phi} (V - C - \bar{q})g(C, b(\Phi))dC + \bar{q} \\ &= \frac{v}{2} + \bar{q}. \end{aligned} \quad (\text{A3.6})$$

$\Phi \in [1, 1\frac{1}{2}]$ :

$$W = \max_{\Phi} \int_0^{\Phi} (V - C - \bar{q})g(C, b(\Phi))dC + \bar{q}$$

$$\begin{aligned}
&= \max_{\Phi} \frac{1}{2} \left( v-1 + e^{-\alpha w(\Phi-1)} \right) + \int_1^{\Phi} (v-C) \left( \frac{1}{b}(C-1) \right) dC + \bar{q} \\
&= \max_{\Phi} \frac{1}{2} \left( v-1 + e^{-\alpha w(\Phi-1)} \right) + \int_0^{\Phi-1} \frac{(v-C'-1)}{w(\Phi-1)} C' dC' + \bar{q} \quad (C' = C-1) \\
&= \max_{\Phi} \frac{1}{2} \left( v-1 + e^{-\alpha w(\Phi-1)} \right) - \frac{(\Phi-1)^2}{3w} + \frac{(v-1)(\Phi-1)}{2w} + \bar{q} \quad (A3.7)
\end{aligned}$$

$$\Phi: \frac{-\alpha w}{2} e^{-\alpha w(\Phi-1)} - \frac{2}{3w} (\Phi-1) + \frac{v-1}{2w} = 0 \quad (A3.8)$$

$\Phi \in [1\frac{1}{2}, 3]$ :

$$\begin{aligned}
W &= \max_{\Phi} \int_0^{\Phi} (V-C-\bar{q}) g(C, b(\Phi)) dC + \bar{q} \\
&= \max_{\Phi} \frac{1}{2} \left( v-1 + e^{-\frac{\alpha w}{2}} \right) + \int_1^{1\frac{1}{2}} (v-C) \frac{2}{w} (C-1) dC + \bar{q} \\
&= \max_{\Phi} \frac{1}{2} \left( v-1 + e^{-\frac{\alpha w}{2}} \right) - \frac{1}{12w} + \frac{v-1}{4w} + \bar{q} \quad (A3.9)
\end{aligned}$$

Since any  $\Phi$  in the region leads to the same number of takeovers and generates the same defensive effort, the planner is indifferent between  $\Phi$  in this region.

$\Phi > 3$ : (A3.9) applies. The planner is indifferent between  $\Phi$  for  $\Phi \geq 1.5$ .

#### Comparison of Shareholders' and Planner's Choice of $\Phi$

We shall not attempt an exhaustive examination. Rather, we shall determine a set of parameter values for which  $\Phi_s^* \in [1, 1\frac{1}{2})$ , and  $\Phi_w^* < 1$ .

- The best shareholders can do outside  $\Phi \in [1, 1.5)$  is  $S = \frac{1}{2} \left( v-1 + e^{-\frac{\bar{s}\alpha}{d}} \right) + \bar{q}$ .

First, we choose  $w=1.2$  (implying  $\frac{\bar{s}}{d} = 2.88$ ) and  $\alpha = 3$ . Then from (A3.5a) the  $\Phi \in [1, 1.5)$

which maximizes shareholder utility is  $\Phi=1.4$ . From (A3.5b),  $\Phi_s^* = 1.4$  iff



$$\frac{1}{8w}(v+w-1)^2 > \frac{1}{2}\left(v-1+e^{-\bar{s}\alpha/d}\right)$$

$$1.06^* > \frac{1}{2}\left(2+e^{-\bar{s}\alpha/d}\right)$$

$$.13^* > e^{-\bar{s}\alpha/d} = .000177.$$

• Next we solve for the level of dilution which maximizes social welfare. The maximum social welfare for  $\Phi \notin [1, 1.5]$  is  $W = \frac{v}{2} + \bar{q} = 2.5$ , which applies for any  $\Phi$  in the interval

$\left(1 - e^{-\bar{s}\alpha/d}, 1\right) = (.99982, 1)$ . In the interval  $\Phi \in (1, 1.5)$ ,  $W(\Phi)$  is convex. (From (A3.8))

$W'' = (\Phi) = \frac{(\alpha W)^2}{2} e^{-\alpha W(\Phi-1)} - \frac{2}{3W} > 0$  for  $\Phi < 1.6824$ ). Thus, the maximum (or supremum)

in this interval occurs at one or the other of the endpoints, and arithmetic gives that it occurs at

$\Phi = 1^+$  where  $W = \frac{v}{2} + \bar{q}$ . Thus, social welfare is maximized for any  $\Phi \in (.99982, 1]$ .

## Appendix 4

### Numerical Example

We consider a situation with  $V$  non-stochastic,  $C$  stochastic, and productive effort completely inelastic at  $\bar{a}$  (with  $\bar{q} \equiv q(\bar{a})$ ). Since  $T = \max[V - \Phi, \bar{q}]$ , from (13) and (14):

$$S(\Phi) = \begin{cases} (V - \bar{q} - \Phi)G(\Phi, b(\Phi)) + \bar{q} & \text{for } \Phi \leq V - \bar{q} \\ \bar{q} & \text{for } \Phi > V - \bar{q} \end{cases} \quad (\text{A4.1})$$

$$R(\Phi) = \begin{cases} \int_0^\Phi (\Phi - C)g(C, b(\Phi))dC & \text{for } \Phi \leq V - \bar{q} \\ \int_0^{V-\bar{q}} (V - \bar{q} - C)g(C, b(\Phi))dC & \text{for } \Phi > V - \bar{q} \end{cases} \quad (\text{A4.2})$$

$$= \begin{cases} \int_0^\Phi G(C, b(\Phi))dC & \text{for } \Phi \leq V - \bar{q} \\ \int_0^{V-\bar{q}} G(C, b(\Phi))dC & \text{for } \Phi > V - \bar{q}. \end{cases} \quad (\text{A4.2}')$$

And recall that  $W(\Phi) = S(\Phi) + R(\Phi)$ .

INSERT FIGURE A4.1 HERE

Figure A4.1: An Example

The example, shown in Figure A4.1, is constructed so that the maxima of  $S$  and  $W$  occur when  $V - \Phi > \bar{q}$  and hence where the tender offer is  $V - \Phi$ . Shareholder surplus is maximized at  $\Phi_S^* = 2\pi + .21$ , while social surplus is maximized at  $\Phi_W^* = 2\pi$ . Since the tender price is higher with  $\Phi_W^*$  than with  $\Phi_S^*$ , for shareholder surplus to be higher at  $\Phi_S^*$  than at  $\Phi_W^*$  the probability of takeover must be higher at  $\Phi_S^*$  than at  $\Phi_W^*$ . Social surplus equals shareholder surplus plus raider surplus. Since social surplus is higher at  $\Phi_W^*$  than at  $\Phi_S^*$  while shareholder surplus is higher at  $\Phi_S^*$ , raider surplus must be higher at  $\Phi_W^*$  than at  $\Phi_S^*$ . Now, raider surplus equals the probability of takeover times the expected surplus conditional on takeover. Since the probability of takeover is higher at  $\Phi_S^*$  than at  $\Phi_W^*$ , the raider's expected surplus conditional on takeover must be higher at  $\Phi_W^*$  than at  $\Phi_S^*$ . This scenario requires that defensive effort be higher at  $\Phi_S^*$  than at  $\Phi_W^*$  (but not so much higher that the probability of takeover is higher at  $\Phi_S^*$  than at  $\Phi_W^*$ ) and also that this higher defensive effort significantly increase expected takeover costs.

Put more succinctly: The optimal dilution factor from the shareholder's perspective trades off the tender price against the probability of takeover. Thus, the shareholder ignores the effect of increasing the dilution factor on expected takeover costs. The planner, however, takes this into

account. If, therefore, an increase in the dilution factor causes an increase in defensive effort, and if this increased defensive effort has a "small" effect on the probability of takeover but a "large" effect on expected takeover costs, the planner's optimal dilution factor may be smaller than the shareholder's.

The example is constructed to illustrate this scenario. We assume the following functional form for  $g(C,b)$ :

for  $\frac{k}{b} < \frac{1}{2}$ :

$$g(C,b) = \begin{cases} \frac{1}{4\pi} - b^\alpha \sin C & \text{for } C \in [0, 2\pi] \\ \frac{k}{b} & \text{for } C \in (2\pi, 2\pi + 1] \\ 0 & \text{for } C \in (2\pi + 1, V - \bar{q}] \\ \text{mass} & C = (V - \bar{q})^+ \end{cases} \quad (\text{A4.3a})$$

and for  $\frac{k}{b} > \frac{1}{2}$ :

$$g(C,b) = \begin{cases} \frac{1}{4\pi} - b^\alpha \sin C & \text{for } C \in [0, 2\pi] \\ \frac{k}{b} & \text{for } C \in \left(2\pi, 2\pi + \frac{b}{2k}\right] \\ 0 & \text{for } C > 2\pi + \frac{b}{2k}. \end{cases} \quad (\text{A4.3a}')$$

The corresponding functional forms for  $G(C,b)$  are:

for  $\frac{k}{b} < \frac{1}{2}$ :

$$G(C,b) = \begin{cases} \frac{C}{4\pi} - b^\alpha (1 - \cos C) & \text{for } C \in [0, 2\pi] \\ \frac{1}{2} + \frac{(C - 2\pi)k}{b} & \text{for } C \in (2\pi, 2\pi + 1] \\ \frac{1}{2} + \frac{k}{b} & \text{for } C > (2\pi + 1, V - \bar{q}] \\ 1 & \text{for } C > (V - \bar{q})^+ \end{cases} \quad (\text{A4.3b})$$

and for  $\frac{k}{b} > \frac{1}{2}$ :

$$G(C, b) = \begin{cases} \frac{C}{4\pi} - b^\alpha(1 - \cos C) & \text{for } C \in [0, 2\pi] \\ \frac{1}{2} + \frac{(C - 2\pi)k}{b} & \text{for } C \in \left(2\pi, 2\pi + \frac{b}{2k}\right] \\ 1 & \text{for } C > 2\pi + \frac{b}{2k}. \end{cases} \quad (\text{A4.3b}')$$

Let us refer to  $C \in [0, \pi]$  as low takeover costs,  $C \in (\pi, 2\pi]$  as medium takeover costs,  $C \in (2\pi, 2\pi + 1]$  as high takeover costs, and  $C = (V - \bar{q})^+$  as prohibitive takeover costs. The form of  $g(\cdot)$  is chosen so that an increase in defensive effort: i) transfers probability mass from low to medium takeover costs; ii) does not affect the sum of the probabilities that takeover costs are low and medium; and (for  $\frac{k}{b} < \frac{1}{2}$ ) iii) transfers probability mass from high to prohibitive takeover costs.

We now turn to the manager's choice of effort. We assume the cost-of-effort function to be

$$D(a, b) = nb. \quad (\text{A4.4})$$

Then the manager's maximization problem is

$$\max_b \left(1 - G(\min(V - \bar{q}, \Phi), b)\right) \bar{s} - nb. \quad (\text{A4.5})$$

There are three potential difficulties. First, the  $b$  which solves this problem could result in  $g(C, b)$  being negative for some  $C$ . Second, which of (A4.3b) or (A4.3b') applies is endogenous. And third, we wish (A4.5) to be well-behaved. These difficulties are dealt with by imposing the following parameter restrictions:

$$\left(\frac{2\alpha\bar{s}}{n}\right)^{\frac{\alpha}{1-\alpha}} \leq \frac{1}{4\pi} \quad \left(\frac{\bar{s}k}{n}\right)^{\frac{\alpha}{2}} \leq \frac{1}{4\pi} \quad \frac{nk}{\bar{s}} < \frac{1}{4} \quad \alpha \in (0, 1). \quad (\text{A4.6})$$

With these restrictions:

$$G(\min(V - \bar{q}, \Phi), b(\Phi)) = \begin{cases} \frac{\Phi}{4\pi} - \left( \frac{\bar{s}\alpha(1 - \cos\Phi)}{n} \right)^{\frac{\alpha}{1-\alpha}} (1 - \cos\Phi) & \text{for } \Phi \in [0, 2\pi] \\ \frac{1}{2} + \left[ \frac{nk}{\bar{s}} (\Phi - 2\pi) \right]^{\frac{1}{2}} & \text{for } \Phi \in (2\pi, 2\pi + 1] \\ \frac{1}{2} + \left( \frac{nk}{\bar{s}} \right)^{\frac{1}{2}} & \text{for } \Phi > 2\pi + 1 \end{cases} \quad (\text{A4.7})$$

$$b(\Phi) = \begin{cases} \left( \frac{\bar{s}\alpha(1 - \cos\Phi)}{n} \right)^{\frac{1}{1-\alpha}} & \text{for } \Phi \in [0, 2\pi] \\ \left( \frac{\bar{s}k}{n} \right)^{\frac{1}{2}} (\Phi - 2\pi)^{\frac{1}{2}} & \text{for } \Phi \in (2\pi, 2\pi + 1] \\ \left( \frac{\bar{s}k}{n} \right)^{\frac{1}{2}} & \text{for } \Phi > 2\pi + 1 \end{cases} \quad (\text{A4.8})$$

For  $\Phi \in [0, 2\pi]$  and  $C \leq \Phi$ :

$$G(C, b(\Phi)) = \frac{C}{4\pi} - \left( \frac{\bar{s}\alpha(1 - \cos\Phi)}{n} \right)^{\frac{\alpha}{1-\alpha}} (1 - \cos C) \quad (\text{A4.9a})$$

For  $\Phi \in [2\pi, 2\pi + 1]$  and  $C \leq \Phi$ :

$$G(C, b(\Phi)) = \begin{cases} \frac{C}{4\pi} - \left[ \left( \frac{\bar{s}k}{n} \right)^{\frac{1}{2}} (\Phi - 2\pi)^{\frac{1}{2}} \right]^{\alpha} (1 - \cos C) & \text{for } C \in [0, 2\pi] \\ \frac{1}{2} + \frac{(C - 2\pi)}{(\Phi - 2\pi)^{\frac{1}{2}}} \left( \frac{nk}{\bar{s}} \right)^{\frac{1}{2}} & \text{for } C \in (2\pi, \Phi] \end{cases} \quad (\text{A4.9b})$$

And for  $\Phi > 2\pi + 1$  and  $C \leq \min(V - \bar{q}, \Phi)$ :

$$G(C, b(\Phi)) = \begin{cases} \frac{C}{4\pi} - \left( \frac{\bar{s}k}{n} \right)^{\frac{\alpha}{2}} (1 - \cos C) & \text{for } C \in [0, 2\pi] \\ \frac{1}{2} + (C - 2\pi) \left( \frac{nk}{\bar{s}} \right)^{\frac{1}{2}} & \text{for } C \in (2\pi, 2\pi + 1] \\ \frac{1}{2} + \left( \frac{nk}{\bar{s}} \right)^{\frac{1}{2}} & \text{for } C \in (2\pi + 1, \min(V - q, \Phi)] \end{cases} \quad (\text{A4.9c})$$

Between  $\Phi = 2\pi$  and  $\Phi = 2\pi + 1$ , raising the dilution factor increases the probability of takeover (see (A4.8)) and stimulates defensive effort (see (A4.7)). The example is constructed so that, over this range of dilution, the increase in the probability of takeover is small relative to the increase in

