Accounting for Trends in the Almost Ideal Demand System

Serena Ng*
Department of Economics, Boston College
and
C.R.D.E., University of Montreal

June 1997

Abstract

Flexible functional forms of indirect utility and expenditure functions are frequently used in approximating the behavior of utility maximizing consumers to arrive at demand systems that can be easily estimated. A common finding in time series estimations of the Almost Ideal Demand System is strong persistence in the estimated residuals. This paper suggests two explanations for this result. First, the functions used to approximate total expenditure does not allow for the possibility of economic growth. Hence when the data on expenditure have trends, the inadequacy of the approximation results in residuals that are serially correlated. Second, when the economy grows and/or prices trend at different rates, Stone's price index provides a poor approximation to the theoretically appropriate price variable. The consequence is also reflected in the error term. Simulations are used to illustrate these arguments and cointegration is proposed as a guide to model specification.

Keywords: Demand Systems, Trends, Non-stationarity, Stone's price index.
JEL Classification: C3, D1.

*Financial support from SSHRC (Canada) and FCAR (Quebec) are gratefully acknowledged.
Correspondence: Department of Economics, Boston College, Chestnut Hill MA 02167. Email: serena.ng@bc.edu

Accounting for Trends in the Almost Ideal Demand System

June 1997

Abstract

Flexible functional forms of indirect utility and expenditure functions are frequently used in approximating the behavior of utility maximizing consumers to arrive at demand systems that can be easily estimated. A common finding in time series estimations of the Almost Ideal Demand System is strong persistence in the estimated residuals. This paper suggests two explanations for this result. First, the functions used to approximate total expenditure does not allow for the possibility of economic growth. Hence when the data on expenditure have trends, the inadequacy of the approximation results in residuals that are serially correlated. Second, when the economy grows and/or prices trend at different rates, Stone's price index provides a poor approximation to the theoretically appropriate price variable. The consequence is also reflected in the error term. Simulations are used to illustrate these arguments and cointegration is proposed as a guide to model specification.

Keywords: Demand Systems, Trends, Non-stationarity, Stone's price index.
JEL Classification: C3, D1.

1 Introduction

One of the basic principles of economics is that agents make their decisions on a rational basis. According to the theory of consumer choice, the demand of a utility maximizing consumer should satisfy i) the adding up of budget shares, ii) the negativity of compensated own price elasticities, iii) the symmetry of the Slutsky matrix, and iv) the homogeneity of degree zero with respect to prices and income. These hypotheses can be conveniently tested within the framework of demand systems.

Empirical demand systems began with the pioneer work of Stone (1953), and more flexible systems have since been developed. A commonly used model is the *Almost Ideal Demand System* of Deaton and Muellbauer (1980) [hereafter AIDS]. The AIDS has been applied to cross-section and time series data in a vast number of studies. However, from the original work of Deaton and Muellbauer (1980) to the recent work by Attfield (1991) and Ng (1995), the residuals of the AIDS are often found to be highly serially correlated and sometimes non-stationary. This problem was thought to be a consequence of mis-specified dynamics, and the AIDS has on occasions been estimated in first-differenced form.

This article suggests that the original AIDS is consistent with an economic environment in which there is no economic growth. Loosely speaking, the problem is that the functions used to approximate total expenditure depend on prices only and hence do not adequately capture the direct movements in expenditure due to economic growth. As well, Stone's price index is found to provide an inadequate approximation for the theoretical price index. In practice, the resolution to these problems amounts to finding proxies for trend growth such that the demand system is cointegrated in the sense of Engle and Granger (1987).

2 The Almost Ideal Demand System

The AIDS begins with the assumption the that household (the consumer unit) preferences belong to the PIGLOG class[1] and that preferences satisfy intertemporal separability so that once the consumption-saving decision is made, the remaining issue for the household is to allocate its total spending among n goods given prices $p_1...p_n$. Total expenditure for household h, is by definition, $X^h = \sum_{i=1}^{n} p_i q_i^h$, and the "needs corrected" expenditure for this household is X^h/K^h, where K^h is an index capturing the size, age, and other characteristics of the household. Now for a household whose utility level is u^h, we can approximate total expenditure $E^h(p, u^h)$ by flexible functions $a(p)$

[1]PIGLOG stands for price independent generalized logarithmic.

and $b(p)$. Suppressing time subscripts, we have:

$$logE^h(p,u^h) = (1-u^h)log[a(p)] + u^h log[b(p)], \tag{1}$$

where $p = (p_1, \ldots, p_n)$. As discussed in Deaton and Muellbauer (1980), $a(p)$ can be thought of as "poverty" or subsistence expenditure, while $b(p)$ is "affluence" or bliss expenditure. The demand functions $\partial logE^h(p,u^h)/\partial logp_i = q_i^h$ are first order local approximations to any set of demand functions derived from utility maximizing behavior. The budget share of good i is $w_i^h = p_i q_i^h / X^h$ $= \partial logE^h/\partial logp_i$. Deaton and Muellbauer (1980) assumed:

$$\begin{aligned} log[a(p)] &= a_0 + \sum_k \alpha_k logp_k + \frac{1}{2}\sum_k\sum_j \gamma^*_{kj} logp_k logp_j, \\ log[b(p)] &= log[a(p)] + \beta_0 \prod_k p_k^{\beta_k}. \end{aligned} \tag{2}$$

A little algebra yields

$$w_i^h = \alpha_i + \sum_j \gamma_{ij} logp_j + \beta_i log[(X^h/K^h)/P], \tag{3}$$

where $\gamma_{ij} = (\gamma^*_{ij} + \gamma^*_{ji})/2$, and $logP = \alpha_0 + \sum_k \alpha_k logp_k + \frac{1}{2}\sum_j\sum_k \gamma_{kj} logp_k logp_j$. The PIGLOG assumption then allows for exact aggregation over households so that the share equations for the market have the same form as those for a household. Let $\bar{X}$ be average expenditure and define K from $log(\bar{X}/K) = \sum_h X^h log(X^h/K^h)/\sum X^h$. Then

$$\bar{w}_i = \alpha_i + \sum_j \gamma_{ij} logp_j + \beta_i log[\bar{X}/(KP)], \tag{4}$$

where $\bar{w}_i$ is the average budget share for good i. Adding up then implies that $\sum_{i=1}^n \alpha_i = 1$, $\sum_{i=1}^n \beta_i = 0$ and $\sum_{i=1}^n \gamma_{ij} = 0$; homogeneity requires that $\sum_{j=1}^n \gamma_{ij} = 0$; Slustky symmetry holds when $\gamma_{ij} = \gamma_{ji}$, for all i, j, and negativity can be checked from the eigenvalues of the Slutsky matrix. These are propositions that the AIDS are set up to test. In practice, the share equation that is being estimated is

$$\bar{w}_i = \alpha_i + \sum_j^{n-1} \gamma_{ij} log[p_j/p_n] + (\sum_j^n \gamma_{ij}) logp_n + \beta_i log[\bar{X}/(KP^*)]. \tag{5}$$

where $logP^* = \sum_k w_k logp_k$ is Stone's index used to approximate $logP$.

3 Economic Growth

To see that there is an internal inconsistency in the AIDS when there is economic growth, we shall first suppose that prices are constant and there is no population growth, but that the real income of the economy grows at a constant rate g. We assume that economic growth has no distributive effects and hence the real income of all households grow at the same rate. We also assume that households have identical characteristics and hence $K^h = 1 \, \forall h$. Thus, the budget shares of the representative household coincide with those of the average household and the subscript h is dropped from all variables.

Since $X = E(p, u)$ for a utility maximizing household, (1) becomes

$$logX = (1 - u)log[a(p)] + ulog[b(p)]. \tag{6}$$

Since X is the level of income to be spent as determined by two stage budgeting and prices are constant by assumption, X grows only if real income grows. This can be thought of as the increase in real wage induced by technical progress. The time derivative of the left hand side is $\partial logX/\partial t = g$. However the time derivative of the right hand side is zero on the assumption that prices are constant. The two sides of the equation have the same time derivative only in a no growth economy with $g = 0$.

Another way to look at the problem is to note that the indirect utility function is of the form

$$v = F\left(\left[\frac{X}{a(p)}\right]^{1/b(p)}\right), \tag{7}$$

where $F(\cdot)$ is a monotone function. As specified, $a(p)$ and $b(p)$ are both constant since prices are constant by hypothesis. If the numerator grows but the denominator is constant, v will grow, implying that utility will increase as the economy grows irrespective of the household's consumption choice. Relaxing the assumption of constant prices to allow for stationary price movements or allowing X to be a random walk with drift will not change the thrust of the argument that in the presence of real economic growth, (6) is not a balanced equation in the sense that the left hand side and the right hand side do not share the same time derivative.

It is customary to perform steady state analysis of growing economies by adjusting the discount rate for growth and standardizing the choice variables by a trending variable. For example, when the period utility is $C^{1-\sigma}/(1-\sigma)$ and the discount factor is β, consumption is usually normalized by an index of technical progress and utility is discounted by $\beta(1+g)^{(1-\sigma)}$ so that the discounted present value of utility is bounded. See, for example, King, Plosser and Rebelo (1988). The problem discussed above requires a growth adjustment of the same nature. Two possibilities come to mind. First, we can "growth adjust" total expenditure following the same reasoning that we "needs adjust" the expenditure of a household via K. Denote this growth deflator by A. This leads to "effective household expenditure" $X/(AK)$. Alternatively, households can be thought of as adjusting their poverty and affluence expenditures for growth. We can thus replace $a(p)$ by a growth adjusted $\widetilde{a}(p)$ so that by construction, the subsistence and affluent level of expenditure are scaled by a growth factor. Both adjustments have the implicit effect of not letting households be fooled into feeling content just because the economy becomes richer.

To see that these two solutions yield identical share equations up to a scale factor, first note that if we growth adjust total expenditure by A, the left hand side of (1) becomes $E(p,u)/A$. It is then straightforward to show that (4) is replaced by

$$\begin{aligned} w_i &= \alpha_i + \sum_j \gamma_{ij} log p_j + \beta_i log[X/(AP)] \\ &= \alpha_i - \beta_i log A + \sum_j \gamma_{ij} log p_j + \beta_i log(X/P). \end{aligned} \tag{8}$$

Now suppose we let $\widetilde{a}(p) = log\widetilde{A} + a(p)$, where $log\widetilde{A}$ adjusts the subsistence and the affluent expenditures for growth. In other words, the AIDS approximation is given by $log\widetilde{X} = u\widetilde{a}(p) + (1-u)\widetilde{b}(p)$. We will then end up with an equation like (8) but with $log\widetilde{A}$ instead of $logA$ in the share equation. Obviously, if X and A follow the same growth path (and hence the first method is valid), $\widetilde{A}$ must also follow the same growth path if X and $\widetilde{X}$ were to have the same growth path. It follows that $log(\widetilde{A}) = \delta + log(A)$ for some $\delta \in R^1$. Hence the two adjustments lead to share equations that differ only by an unidentified constant. Note also that $log(A)$ is common to all the share equations. Since $\sum_i \beta_i = 0$ is required to satisfy the adding-up constraint, no other restrictions need be imposed on the demand system with the introduction of growth.

It can be seen from (8) that when $(X/P) \propto A$ with a constant factor of proportionality, the budget shares will be constant in the absence of price movements. This can be thought of as the stationary state of the economy. Out of the stationary state, the constant proportionality between (X/P) and A breaks down. When $(X/P) > A$, the share of luxury goods with $\beta_i > 0$ will rise while those of necessary goods with $\beta_i < 0$ will fall. Necessity and luxury are appropriately defined relative to the state of the growing economy. One question that arises in analyses of demand systems is that if the share of necessary (luxury) goods in households' budget falls (rises) as the economy becomes richer, nothing prevents the shares to be bounded between zero and one. Normalizing $a(p)$ and $b(p)$ by a growth factor have the effect of keeping the budget shares bounded within the unit simplex.

While we have used the AIDS to highlight the problem, the issue is inherent in demand systems derived from flexible functional forms of the expenditure function. The AIDS is a member of PIGLOG demand system, of which the translog system is also a member. PIGLOG demand systems are generated by indirect utility functions of the form $V(p,u) = G(p)[logE - log[g(p)]]$

with the property that $G(p) = G(\lambda p)$ and $g(p) = g(\lambda p)$.[2] The corresponding share equations are of the form $w_i = p_i g_i/g - p_i G_i[logE - log[g]]/G$, where g_i and G_i are the partial derivatives of g and G with respect to good i. Because total expenditure is unadjusted for growth, the fundamental imbalance remains in PIGLOG based demand systems.

The above analysis is based on partial equilibrium arguments. In a general equilibrium context, one would expect prices to rise in response to excess demand. This would only make total expenditure grow faster and inadequacy of the approximation for X provided by AIDS will remain. Nevertheless, the properties of the AIDS when prices have trends are interesting in their own right. This is now the subject of analysis.

4 Non-Stationary Prices

To analyze the implications of non-stationary prices, we now take the other extreme and assume that the real income does not grow but prices do. Now the approximated expenditure is $(1 - u)log[a(p)] + ulog[b(p)]$. The required time derivatives for a given u are:

$$\begin{aligned}
\partial log[a(p)]/\partial t &= M_1 + M_2 \\
M_1 &\equiv \sum_i \alpha_k \partial log[p_k]/\partial t, \\
M_2 &\equiv \sum_{k,j,k\neq j} \gamma_{kj}(log[p_k]\partial log[p_j]/\partial t + log[p_j]\partial log[p_k]/\partial t) + \sum_i \gamma_{ii} log[p_i]\partial log[p_i]/\partial t, \\
\partial log[b(p)]/\partial t &= M_3 \equiv \partial log[a(p)]/\partial t + (\sum_k \beta_k \partial log[p_k]/\partial t)\prod_k p_k^{\beta_k},
\end{aligned}$$

using the definition $\gamma_{ij} = 0.5(\gamma^*_{ij} + \gamma^*_{ji})$.

4.1 Case 1: All Prices Grow at the Same Rate

If the prices all grow at the same constant rate, say μ, $\partial logX/\partial t = \mu$ since $\sum_i w_i = 1$. The expenditure approximated by the AIDS will grow at the same rate if i) the adding up constraint holds (since it implies $M_1 = \mu$ and the second term of M_3 is zero) and ii) adding up, homogeneity, and symmetry holds (since this ensures that $M_2 = 0$). This result can also be seen from the time derivative of the right hand side of (4). The variable $log[P]$ grows at rate $\mu \sum_k \alpha_k$, which by adding-up, resolves to μ. It follows that X/P has no trend. By homogeneity, $\sum_j \gamma_{ij} = 0$. Hence, the shares will be non-trending in spite of prices growing. This result is to be expected since $a(p)$ and $b(p)$ are linear homogeneous of degree one and zero in p respectively, and hence when the conditions of adding-up and homogeneity are satisfied, households' budget shares are invariant to absolute price

[2] The AIDS takes $log[g(p)] = log(P)$ defined earlier, and with $G(p) = \prod p_k^{\beta_k}$. See Pollak and Wales (1992)

movements that have no relative price implications. If, however, homogeneity fails, some shares will drift up while others will drift down at rates proportional to μ. In spite of this, the share equations implied by the AIDS are data consistent in the sense that the left and the right hand side of (4) grow at the same rate. This is to be contrasted with the case of real economic growth of the previous section where the right but not the left hand side of the equation is trending, a consequence of omitting growth considerations in the approximating function.

4.2 Case 2: Relative Price Changes

Consider the time profile of the shares when prices grow at (possibly non-common) rates μ_i. This implies trends in relative prices, a phenomenon documented in Ng (1995) and Lewbel (1996a). Even if we impose symmetry, homogeneity and adding up, there is no reason to expect $\sum_i w_i \mu_i$ (the growth rate of actual expenditure) to equal $(1-u)[M_1 + M_2] + uM_3$ (the growth rate of the expenditure approximated by the AIDS). Thus, as seen from the share equation (4), the effects of prices growing at different rates on the share of good i are two fold: the direct effect of prices, as given by $\sum_k \alpha_k \mu_k$, and an indirect effect via $\partial log[X/P]/\partial t$. Note that the shares should exhibit time trends because of substitution by households across goods to take advantage of changing relative prices. This is consistent with optimizing behavior.

5 Econometric Implications and Simulations

Previous estimations of the AIDS have often found the residuals of the share equations to be highly serially correlated. Lewbel (1996b) presents a generalized composite commodity theorem to allow for aggregation without separability, and shows that the error term of the share equations should be a function of the within-group relative prices when demands are not separable. He finds that there is still substantial serial correlation in the residuals even when the within-group relative price movements are taken into account. In fact, his results indicate the residuals are sufficiently persistent to suggest the presence of a unit root. We offer two explanations for the strong persistence in the residuals. First, as suggested earlier, (8) is the appropriate model consistent with economic behavior when there is economic growth. It follows that regression models which omit the trends are misspecified. More precisely, the regression error will contain the omitted trend, A. To the extent that A is persistent, the residuals will be serially correlated. Furthermore, A is not orthogonal to the regressor because A and X share a common trend. The estimates of the demand system will be biased and inconsistent.

Second, Stone's price index, $log(P^*)$, can be a poor approximation for $log(P)$ when prices are non-stationary. It can be shown that the difference between the two is $0.5 \sum_j \sum_k \gamma_{kj} log p_k log p_j +$

$(\sum_k \beta_k log p_k) log(X/P)$.[3] This difference will have trends when real income grows and/or prices do not grow at a common rate. Since the error term of the share equations contains the approximation error $(logP^* - logP)$, it will inherit the trends and residual autocorrelation follows. It should be emphasized that the difference between P and P^* is a function of X/P also. Hence, growth in X due to real income will interact with growth in prices to determine the adequacy of approximation by Stone's price index.

To illustrate the first problem, we conduct a small simulation experiment. In theory, the AIDS should provide a good local approximation to any set of demand functions derived from utility maximizing behavior. Thus, we can generate data from demand functions for which the true expenditure elasticities can be easily calculated and then assess the ability of the AIDS in estimating these elasticities. This is an interesting exercise in its own right as little is known about the adequacy of using flexible functional forms in approximating economic behavior in practice.

For the purpose of the simulations, we continue to assume that all households are identical and hence the average budget shares and average household expenditure coincide with those of the individuals. Preferences are assumed to be generated by the indirect addilog utility [see Varian (1978)], with quantity demand given by

$$q_i(p_1, p_2, X) = \frac{a_i b_i X^{b_i} p^{-b_i - 1}}{\sum_{j=1}^{k} a_j b_j X^{b_j - 1} p_j^{-b_j}}. \tag{9}$$

The error term of the (correctly specified) share equations is therefore induced by approximating demand behavior derived from an indirect addilog utility by the AIDS.

In the simulation experiment we assume that there are two goods. Thus, there are three random variables whose stochastic properties need to be specified. Data for the two prices are generated as follows: $p_1 \sim N(1, .01^2)$ and $p_2 \sim N(1, .02^2)$. For total expenditure, we assume that $\bar{X}_t = d_t + s_t$, where the deterministic component is $d_t = d_0 + d_1 t$ and the stochastic component is $s_t = \rho s_{t-1} + v_t$, $v_t \sim iid(0, .05^2)$. The parameters are set for $d_0 = 5$, $a_1 = 0.4$ $a_2 = 0.6$, $b_1 = 0.3$ and $b_2 = 0.7$. Given (9), the expenditure elasticity of demand for good i evaluated at the mean can be calculated and is denoted $\epsilon_{i,X}$. We then estimate the share equations using (5) and compute the expenditure elasticity of demand (denoted $\hat{\epsilon}_{iX}$) implied by the AIDS model. Without loss of generality we only report results for good 1. We set the sample size to 200. The results for 1000 simulations are as follows:

[3] This assumes that the AIDS approximates households' behavior exactly An additional term $\sum_k u_k log[p_k]$ will appear if w_i given by (4) differs from the true share by u_i. See Buse (1995).

Table 1: Simulation Results: Prices Stationary

d_1	ρ	$D.W.$	ϵ_{1X}	$\hat{\epsilon}_{1X}$	σ_X	w_1	$\sigma_{\bar{p}}$	σ_{p^*}
0.0000	0.0000	2.0009	1.1010	1.2325	0.0499	0.7475	0.0090	0.0090
0.0000	0.3000	1.9604	1.1010	1.2323	0.0567	0.7476	0.0090	0.0090
0.0000	0.6000	1.7370	1.1009	1.2316	0.0812	0.7476	0.0090	0.0090
0.0000	0.9000	1.1890	1.1007	1.2306	0.1759	0.7482	0.0090	0.0090
0.0000	1.0000	1.4355	1.0957	1.2205	0.2685	0.7607	0.0090	0.0090
0.0100	0.0000	0.7245	1.0955	1.2205	0.5807	0.7608	0.0090	0.0090
0.0100	0.3000	0.7254	1.0955	1.2204	0.5779	0.7609	0.0090	0.0090
0.0100	0.6000	0.7448	1.0955	1.2203	0.5716	0.7609	0.0090	0.0090
0.0100	0.9000	0.8599	1.0953	1.2195	0.5313	0.7614	0.0090	0.0090
0.0100	1.0000	0.8587	1.0911	1.2104	0.6214	0.7718	0.0089	0.0089

Notes: $D.W.$ is the Durbin Watson statistic; ϵ_{1X} and $\hat{\epsilon}_{1X}$ are the actual and estimated expenditure elasticity of demand for good 1; σ_X is the standard deviation of total expenditure, X, w_1 is the mean budget share of good 1; $\sigma_{\bar{p}}$ is the standard deviation of the consumption deflator, defined as $X/(q_1+q_2)$; and σ_{p^*} is the standard deviation of Stone's price index, P^*.

When $d_1 = 0$, X is covariance stationary when $\rho < 1$ and has a stochastic trend when $\rho = 1$. The standard deviation of X reflects the effects of the trends on the variations in the series. When $d_1 = 0.01$, X is stationary around a deterministic time trend when $\rho < 1$ and becomes a unit root process with a drift of 0.01 when $\rho = 1$. As we can see from the results, the expenditure elasticities of demand for good 1 are estimated quite precisely; the actual expenditure elasticities are between 1.09 and 1.10 and are estimated to be between 1.21 and 1.23. However, the Durbin Watson statistic drifts away from 2 as the deterministic and stochastic trends in X dominate. It is not uncommon to find estimations of demand systems using time series data to report Durbin Watson statistics below one. Our analysis suggests the omission of growth in the approximating functions as a possible cause. The second to last column is the standard deviation of the consumption deflator, defined as $X/(q_1+q_2)$. The properties of this price index is extremely close to those of P^* (i.e. Stone's price index). Both series vary around one percent of the mean as expected since prices are assumed *i.i.d.*.

We now repeat the simulation exercise above, assuming the same stochastic process for X, but replacing the data generating process of prices by two independent random walks. Specifically, $\Delta p_{1t} \sim N(.01, .002^2)$, $\Delta p_{2t} \sim N(.005, .005^2)$, with initial conditions $p_{11} = p_{21} = 1$. Since p_1 drifts at a faster rate than p_2, there are trends in relative prices by construction.

Table 2: Simulation Results: Prices Non-Stationary with Non-Common Trends

d_1	ρ	$D.W.$	ϵ_{1X}	$\hat{\epsilon}_{1X}$	σ_X	w_1	$\sigma_{\bar{p}}$	σ_{p^*}
0.0000	0.0000	0.1221	1.1309	1.2676	0.0499	0.6774	0.4402	0.1447
0.0000	0.3000	0.1432	1.1309	1.2478	0.0567	0.6775	0.4397	0.1445
0.0000	0.6000	0.1630	1.1308	1.2340	0.0812	0.6776	0.4409	0.1448
0.0000	0.9000	0.1541	1.1306	1.2324	0.1759	0.6781	0.4397	0.1445
0.0000	1.0000	0.1100	1.1246	1.2877	0.2685	0.6928	0.4454	0.1459
0.0100	0.0000	0.8352	1.1245	1.1957	0.5807	0.6934	0.4496	0.1470
0.0100	0.3000	0.6338	1.1244	1.1994	0.5779	0.6934	0.4506	0.1472
0.0100	0.6000	0.3751	1.1244	1.2127	0.5716	0.6934	0.4494	0.1470
0.0100	0.9000	0.2033	1.1243	1.2273	0.5313	0.6938	0.4492	0.1469
0.0100	1.0000	0.1102	1.1193	1.2567	0.6214	0.7062	0.4537	0.1480

In these simulations, the mean for p_1 and p_2 are calibrated to be about the same as in Table 1 by reducing the size of the innovations to the random walk component. Note that now the variations in the consumption deflator is much higher than that for P^*. This is a consequence of the fact that Stone's index takes geometric averaging of the prices, and when prices are non-stationary, the resulting series grows slower than a price index based on arithmetic averaging (such as the consumption deflator). Note that the Durbin Watson statistic deteriorates relative to the results for stationary price in Table 1. Even when total expenditure is stationary, significant residual autocorrelation is found in the data. In many of these cases a formal test of cointegration cannot reject the null hypothesis of no cointegration. The degree of serial correlation evidently depends on the joint time series properties of X/P as explained earlier.

In this second set of simulations, prices and nominal expenditure do not evolve together, so that real expenditure is either rising or falling. We now consider the case when X, p_1 and p_2 all drift at the rate 0.01. In particular, $\Delta p_1 \sim N(.01, .002^2)$, $p_2 = p_1 + N(0, .001^2)$, so that p_1 and p_2 share the same deterministic and stochastic trend.

Table 3: Simulation Results: Prices Non-Stationary with Common Trends

d_1	ρ	$D.W.$	ϵ_{1X}	$\hat{\epsilon}_{1X}$	σ_X	w_1	$\sigma_{\bar{p}}$	σ_{p^*}
0.0100	0.0000	1.0714	1.1172	1.2093	0.5807	0.7101	0.5789	0.1742
0.0100	0.3000	0.7673	1.1172	1.2127	0.5779	0.7101	0.5790	0.1742
0.0100	0.6000	0.4263	1.1171	1.2212	0.5716	0.7102	0.5789	0.1742
0.0100	0.9000	0.1980	1.1169	1.2306	0.5313	0.7107	0.5790	0.1742
0.0100	1.0000	0.0644	1.1122	1.2456	0.6214	0.7225	0.5785	0.1741

By construction, prices and expenditure share the same deterministic trend but evolve around different stochastic trends. Residual autocorrelation continues to result from the use of Stone's price index. As seen by comparing the last two columns, Stone's price index remains significantly less variable than the consumption deflator. Although Deaton and Muellbauer suggest Stone's price index is adequate if prices are collinear, this appears not to be the case. As explained earlier, $log(P^*) - log(P)$ is also a function of X/P. Hence, if X and P are not collinear, use of Stone's price index will induce a trend.

6 Cointegration as a Guide for Specifying the AIDS

Including a trend in the share equations or first differencing the data have been used in the literature to account for the presence of serial correlation. The validity of these procedures will depend on the properties of the data generating process. For example, if the data are trend stationary, first differencing is not appropriate because the data will be over-differenced. A more structural solution to the problem is to model the growth deflator, A, directly. Although this variable is unobserved, the choice of A can be guided by the condition that the residuals from estimating the share equations should have neither deterministic nor stochastic trends. It should be emphasized that it is not sufficient for the estimated residuals to have no unit roots (stochastic trends). Deterministic trends must also be absent, for otherwise an inconsistency will remain between the growth in $log(X)$ and the approximating expenditure function. More precisely, the demand system should satisfy deterministic cointegration as defined in Campbell and Perron (1991).

Using post-war data for the U.S., Ng (1995) found that merely including polynomial time trends will not render the demand system cointegrated. This suggests that there is a stochastic trend in A. Proxies for A worth considering include the permanent component of an aggregate variable like disposable income. This can be constructed by applying the Hodrick-Prescott filter or the Beveridge-Nelson decomposition to disposable income, subject to the condition that A will indeed render the estimated residuals of the demand system stationary. Naturally, if X/P has segmented trends, A must also embody the breaks in the trend function.

We have abstracted from population growth in the analysis. This is because in practice, X is usually taken as per-capita expenditure, and hence expenditure is adjusted for by population growth. Indeed, the need for a growth deflator such as A is precisely to arrive at a concept of per-capita expenditure in *effective* terms. Other aspects of demographic changes might nevertheless have implications for the treatment of the A. For example, if we relax the assumption to allow K^h and A^h to vary across households, A will be defined from $log[\bar{X}/A] = \sum_h X_h log[X_h/(A^h K^h)]/\sum X_h$. Thus, A now embodies cross-section variations in household characteristics as well as a pure growth component. If the growth component dominates the cross-section variations (in terms of the degree of integration), then the cointegration criterion should remain useful in identifying growth component of the Almost Ideal Demand System. If economic and demographic changes both induce trends, we need to find trending demographic variables (such as the working age population and female participation rate) which, along with the growth trend, yield a cointegrated system. In such cases, at least two trending components will be necessary to render the demand system cointegrated.

7 Conclusion

This article argues that the original AIDS is ill-suited for analyzing non-stationary data. In particular, the approximating functions are not specified to allow for growth in real income. The consequence is an omitted trend component in empirical demand systems. This model misspecification is consistent with the frequent finding of residual autocorrelation when the demand system is estimated in level form. The approximation provided by Stone's price index is also shown to be inadequate when prices grow at different rates. The consequence is also reflected in the error term which is shown to be serially correlated. Simulations are used to illustrate these arguments. The thrust of this analysis is that time series estimations of demand systems must deal with non-stationarity in the data. We suggest using cointegration between budget shares, prices, and expenditure as a guide to empirical modelling of demand systems when the data are non-stationary.

References

Attfield, C. (1991), Estimation and Testing when Explanatory Variables are Endogenous, *Journal of Econometrics* **48**, 395–408.

Buse, A. (1995), Testing Homogeneity in the Linearized Almost Ideal Demand System, University of Alberta Discussion Paper 95-7.

Campbell, J. Y. and Perron, P. (1991), Pitfalls and Opportunities: What Macroeconomists Should Know About Unit Roots, *NBER Macroeconomic Annual*, Vol. 6, M.I.T. Press, pp. 141–201.

Deaton, A. S. and Muellbauer, J. (1980), An Almost Ideal Demand System, *American Economic Review* **70**, 312–326.

Engle, R. F. and Granger, C. (1987), Cointegration and Error-Correction: Representation, Estimation, and Testing, *Econometrica* **55**, 251–276.

King, R. G., Plosser, C. and Rebelo, S. (1988), Production, Growth and Business - I. The Neoclassical Model, *Journal of Monetary Economics* **21**, 195–232.

Lewbel, A. (1996a), Aggregation Without Separability: A Generalized Composite Commodity Theorem, *American Economic Review* **86**, 524–543.

Lewbel, A. (1996b), Large Demand Systems and Nonstationary Prices: Income and Price Elasticities for Many US Goods, mimeo Brandeis University.

Ng, S. (1995), Testing for Homogeneity in Demand Systems when the Regressors are Non-Stationary, *Journal of Applied Econometrics* **10**, 147–163.

Pollak, R. and Wales, T. (1992), *Deamdn System Specification and Estimation*, Oxford University Press.

Stone, J. (1953), *The Measurement of Consumers' Expenditure Behaviour in the United Kingdom: 1920-38*, Vol. 1, Cambridge University Press, Cambridge.

Varian, H. (1978), *Microeconomic Analysis*, W. W. Norton.