Credible Disinflation Policy in a Dynamic Setting

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June 27, 1997

Abstract

This paper builds upon the analysis of Orphanides and Wilcox (1996) to evaluate optimal anti-inflation policy under a broader set of circumstances than considered in their work. We consider a monetary authority with two instruments–the funds rate and the discount rate–with the distinction that only movements of the latter are 'credible' alterations of the Fed's policy stance, reflecting reputational effects. The public forms expectations of inflation given realized inflation and the expected progress toward lower inflation, as evidenced by credible policy moves. Optimal policy is formulated in a stochastic, dynamic setting of the Tinbergen-Theil framework. The presence of a "cost-of-change" penalty on the sequence of discount rate adjustments generates expected trajectories for targets and policy instruments which differ considerably from those lacking such a penalty.

1. Introduction

Over the past decade, a considerable literature has been developing on central banks' appropriate stance toward inflation. As inflation in many industrialized countries has been brought under control, and in many cases reduced to low single-digit levels, attention has been focused on the policy actions that might be used to preserve that degree of control and effectively cope with any reflationary pressures. The roles of inflationary expectations and institutional arrangements which perpetuate a given rate of inflation, low or high, have been highlighted in this literature. Models of central bank behavior have been constructed with the consensus that while central banks can affect (or even peg) short-term nominal interest rates, they have a much looser grip on long-term nominal rates. Likewise, while monetary policy may have a sizable effect on real interest rates in the short run, it may only be able to influence real rates in the long run through the establishment of a credible policy which complements the real economy's productivity growth and fiscal stance.

While the central bank's definition of its targets, intermediate targets, and instruments has attracted attention from many researchers, much of this literature has either attempted to construct a tractable analytical model of central bank behavior or has been occupied with the empirical estimation of reaction functions, or revealed preferences of the central bank. The simplicity required of a tractable model has often led to the development of one-period, single-target, single-instrument dynamic models, with little relation to the empirical realities of "long and variable lags" in the effects of monetary policy on the real economy. Likewise, empirical reaction functions can reveal some information about central bankers' preferences, but do not arise from a well-specified optimization problem.

In this paper, we take a first step toward bridging this gulf by constructing a simple macro model incorporating two central bank policy instruments in an explicitly dynamic setting, and employing that model as a set of nonlinear constraints on a stochastic optimal policy problem. This "Tinbergen-Theil" approach, recently recommended by Blinder (1997) as a guide for monetary policymakers, requires an explicit identification of the targets, policy instruments, and of the dynamic relationships linking them. Using numerical solution techniques, we are able to examine the interactions of multiple targets: not only the macroeconomic objectives, but also constraints on the policy instruments which reflect some of the stylized facts of central bank behavior. Although this work is preliminary, it reflects our goal of using computational economics to analyze policymaking frameworks too complex for analytical interpretation. With this approach, we can consider the differential effects of various policies upon the credibility of monetary policy and the revision of economic agents' expectations.

The plan of the paper is as follows. Section 2 reviews a selection of relevant literature, while the following section presents the econometric model and illustrates its properties. In Section 4, the optimal policy experiments are presented, and implications drawn for the interactions between the set of targets and monetary policy responses. Section 5 concludes the paper, and provides a sketch of further research.

2. Review of the Literature

The recent literature on the conduct of monetary policy is vast, and we can only acknowledge the major influences on our work in this paper. The view of monetary policy as a process with identified goals and inherent constraints, rather than a sequence of discretionary actions, underlies the papers in several recent conference volumes. The Boston Fed's "Goals, Guidelines and Constraints Facing Monetary Policymakers" (1994) and the Kansas City Fed's symposium on "Achieving Price Stability" (1996) contain papers highlighting the importance of the design of monetary policy and its interaction with the real economy. John Taylor's article in the 1994 Boston volume specifically addresses the interactions between the "implicit" real interest rate specified by monetary policymakers and the economy's equilibrium real interest rate. William Poole's paper in the same volume is concerned with the choice facing monetary policymakers in using interest rates or monetary aggregates as their instrument. He finds reason for concern for the de facto abandonment of the aggregates as the instrument of policy, arguing that heavy reliance on the funds rate has rendered the financial markets hypersensitive to minor shifts (real or perceived) in the Fed's policy stance. Poole relates that "...the overwhelming majority of large changes in bond yields arise in response to actions by the monetary authorities and to releases of routine economic data." (1994, p.106) This generates, in his view, a situation where "...the Fed cannot use the behavior of interest rates to provide useful information on how it should adjust the federal funds rate. The bond market today tells the Fed what the market thinks the Fed is going to do." (1994, p.108) Thus, monetary policy actions must be taken with a clear understanding of how market participants' expectations will respond to those signals: a point also highlighted by former Fed Vice-Chair Alan Blinder, who claims

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that "Maintaining a long time horizon is perhaps the principal raison d'etre for central bank independence. Yet a central banker who takes his cues from the markets is likely to acquire the markets' short time horizon." (1997, p.15)

In the Kansas City Fed's 1996 symposium on "Achieving Price Stability," Mervyn King considers how central banks should achieve price stability, drawing on his experience at the Bank of England. He considers the distinction between a central bank's ex ante inflation target and its discretionary response to shocks, and suggests that "...in general, it is not optimal to move immediately to a regime of price stability unless that regime can be made fully credible by institutional or other changes." (1996, pp. 57-58) His rationale for that conclusion is based on the hypothesis that there are costs of disinflation, increasing more than proportionally in the rate of disinflation, related to private agents' ability to determine whether a regime change has actually taken place. (This argument is very closely related to that put forth by Kozicki and Tinsley (1996a,1996b) in their moving-endpoint models). King argues that "...expectations are likely to be influenced by the commitment to price stability among the public at large" (1996, p. 58) and suggests that central bank behavior can only influence that commitment. Private agents must learn about the new economic environment, and the central bank must learn about agents' revised behavior. In King's view, "pure rational expectations models are not a good basis on which to base policy because they ignore the process of learning." (1996, p.79) From his viewpoint, a successful model of monetary policy must take the learning process into account.

Finally, we must acknowledge the sizable contributions of two recent papers to our work. A paper by Orphanides and Wilcox (1996) introduces the "opportunistic approach" to disinflation: the concept that the Fed may actively combat inflation only when it threatens to increase, and otherwise should wait for external circumstances (e.g. recessions) that will bring the inflation rate

down. This approach leads to a switching strategy, in which the Fed acts to stabilize output when inflation is low, but moves to fight inflation when it is high. The definitions of "low" and "high" are state-dependent in their model, so that there is no fixed rule defining the policy response to current conditions. Orphanides and Wilcox provide an appealing argument for this mixed strategy: that is, for the central bank's concern for both output and employment. They suggest that the policymakers incur a "first-order loss from output deviations even when output is close to potential, and yet only a second-order loss from inflation deviations when inflation is close to its target." (1996, p.22) They provide what is characterized as a highly speculative rationale for this ordering, in that "The deleterious effects of inflation are mainly allocative in nature..." while "...employment is an all-or-nothing proposition...", providing "the basis for treating deviations of output from potential as imposing first-order costs on the policymaker." (1996, p.23) They extend their model to consider more realistic aspects of aggregate demand and supply, but their model is cast in a single-period framework. We take a number of elements of their model as a starting point in developing the model presented in the next section.

A second important paper is Roszbach's (1997) work, in which he estimates central bank reaction functions in the presence of adjustment costs. This approach is designed to quantify the rationale for central banks' apparent reluctance to utilize frequent policy changes (in his case, in the Swedish short-term funds rate target) as an outcome of a fixed cost-of-change. Since it is virtually costless for a central bank to intervene in financial markets, the cost referred to here is expectational: central bank policy that gives the appearance of being muddled or directionless will undermine central bank credibility, as market participants do not receive clear signals of the policy stance. The outcome of his analysis is a limited dependent variable model, where the observed changes

are expressed as threshold crossings, effected only when the policy instrument is far from its optimal value. Roszbach achieves considerable success in predicting the probability of the target rate being raised, lowered, or kept constant. These empirical findings reinforce the importance of policies' impact on expectations formation, and the likely behavior of a central bank with concerns for its reputation in the financial markets. Although our model does not generate the discrete responses predicted by Roszbach's findings, it permits us to evaluate the effects of a more complex "cost-of-change" specification on the vigor of optimal policy actions.

3. A Model of Monetary Policy

This section presents the framework we have developed to evaluate the dynamic interactions between anti-inflation monetary policy, inflation expectations, and two monetary policy instruments. The model necessarily abstracts from many elements of a complete and interdependent treatment of expectations formation and policy design, but has been constructed to focus on key elements of this process in the context of historical evidence. The model consists of three estimated behavioral equations and a number of identities linking two monetary instruments to the targets of policy. Unlike Orphanides and Wilcox'original work (1996), which is purely analytical, and their followup paper (Orphanides et al., 1996), which performs stochastic simulations in an historical setting, we utilize the model in a closed-loop optimal policy exercise, in which explicit penalties are applied to deviations from target values, and optimal feedback rules derived for the policy instruments.

The policy environment modelled here is one in which Federal Reserve actions can control short-term nominal interest rates, but cannot control real rates. As one of the major innovations of this model, we consider two instruments of

monetary policy, the Federal funds rate and the discount rate, both of which are assumed to have effects on the financial sector, but differ in their capability to affect inflation expectations.

We make use of well-known stylized facts about the discount rate: that it has been altered infrequently, in small increments, and almost never subjected to a reversal (e.g. Baum and Karasulu, 1998). These empirical regularities, coupled with the description of the discount rate by researchers inside and outside the Fed as an instrument with sizable "announcement effects," lead us to a description of policy in which a discount rate change is viewed as a credible signal of the stance of policy, precisely because such a signal is infrequently and cautiously emitted. The usual recursion of expectations applies: market participants are aware that the central bank wishes to preserve the credibility of its stance in the market (as Roszbach describes). Since the central bank will rationally exhibit restraint in its actions, signalling a policy shift only when it is justified by macroeconomic or financial conditions, market participants will rationally expect that an observed signal is meaningful–all the more so if such signals are few and far between.

The funds rate, on the other hand, is viewed as an instrument which has direct effects on the financial sector, but is far less credible as a signal of policy stance. Realistically, changes in the funds rate may reflect either the Fed's doing or their acquiescence to market forces–or even an allowed response to a threat to financial system stability. In the stylized environment of our model, we consider that changes in the funds rate are much less effective in influencing inflationary expectations than are discount rate changes, as market participants can readily observe such changes, but are faced with a signal extraction problem in deciding whether a rate change reflects a permanent policy shift or a transitory response to market conditions.

We begin development of the formal model with a stylized Phillips curve, or

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inverted aggregate supply curve:

$$\pi = f\left[y, \pi^e\right] + \epsilon \tag{3.1}$$

where π is inflation, π^e is expected inflation, and y is the percentage GDP gap (the deviation of output from potential output, as a difference of logarithms). The aggregate supply relation is inverted to generate the level of inflation consistent with the GDP gap and expected inflation. Inflation is modeled as a persistent phenomenon, implicitly reflecting contract terms and menu costs of price adjustment (along the lines of Fuhrer and Moore, 1995). In estimation of this equation, we allow for persistence with a partial adjustment mechanism, and do not constrain the effect of expected inflation to be unitary.

The second element of the model is the specification of expected inflation, which we express as:

$$\pi^e = f\left[\pi, (d - r^*)\right] + \eta \tag{3.2}$$

Expected inflation is taken to depend on actual inflation and the difference between the Fed's discount rate, d, and the economy's long-run equilibrium real rate, r^*. This proxies in a crude sense for the differing effects of the discount rate and the Fed funds rate on inflationary expectations. In this simple form of the model, the discount rate is a credible signal, and the Fed's anti-inflationary efforts will manifest themselves by moving the discount rate closer to the economy's long-run equilibrium real rate as inflation is eradicated. As long as the difference is positive, inflationary expectations are stimulated. In an elaboration of this framework, we might allow both the discount and the Fed funds rates to affect inflationary expectations, but with differential effects. In estimation of this equation, we express expected inflation as a convex combination of its lagged value and the remaining terms.

The third element of the model is an aggregate demand relation, specifying the GDP gap:

$$y = f\left[r - r^*\right] + \nu \tag{3.3}$$

Aggregate demand, as proxied by the GDP gap, responds negatively to movements in the real rate of interest r relative to its equilibrium long-run level. In estimation of this equation, we allow for persistence in the adjustment of aggregate demand to changes in the real rate and shocks to demand.

To close the model, the current real rate r is determined by the Fisher equation:

$$r = F - \pi^e \tag{3.4}$$

where F is the Federal funds rate.

3.1. Empirical Estimates

We now present the estimated relationships of our model, which were fit to monthly data over the 1975-1995 period. The GDP gap (defined as $Y = 100\log\left(\frac{GDP}{GDP^p}\right)$, where the GDP^P series was derived from Congressional Budget Office estimates) was converted to monthly form via RATS procedure *distribute* using model *rwar1*.[1] All other series in the model were available at a monthly frequency in the DRI Basic Economics database. The long-run equilibrium real rate was proxied by its average value (2.178%) over the twenty-year sample period, calculated from the Fisher equation, where inflationary expectations were

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[1]The *distribute* procedure converts data from a lower to a higher frequency. Model *rwar1* uses an ARIMA(1,1,0) model to represent the underlying timeseries, and generates the higher-frequency values subject to the constraint that their sum (or, in the case of flow variables, average) equals that of the original series (Estima, 1992).

generated by projecting inflation on twelve lags of inflation and the policy instruments.

The inflation equation (3.1) was implemented as a first-order autoregression, augmented with lagged expected inflation and the current GDP gap in both level and absolute value. In fitting this equation, there was clear evidence of asymmetry in the relation between the gap and the inflation rate, depending on the sign of the gap. The specification presented here allows for that asymmetry by incorporating the absolute value of the gap:

$$\pi_t = \alpha y_t + \beta \pi_{t-1}^e + \delta \pi_{t-1} + \gamma |y_t| + \epsilon_t \tag{3.5}$$

The estimates of this equation, generated by ordinary least squares with Hansen-White standard errors, are presented in Table 1.

TABLE 1. INFLATION EQUATION, 1975:01-1995:12.

Parameter	Est. Coeff.	Std. Error	t-statistic
α	0.39252	0.20141	1.95
β	0.51155	0.08015	6.38
δ	0.41533	0.08148	5.10
γ	0.44340	0.23631	1.88
$\overline{R}^2$=0.591	SER=2.37		

The expected inflation equation (3.2) was expressed as a convex combination of last period's expected inflation–allowing for persistence in expectations–and a second term, incorporating lagged inflation and the discount rate–equilibrium real rate differential:

$$\pi_t^e = \lambda \pi_t + (1 - \lambda) \left[\rho_1 \pi_{t-1}^e + \rho_2 (d_t - r_t^*) \right] + \eta_t \tag{3.6}$$

Given the nonlinear parameter constraint, this equation was estimated with nonlinear least squares with Hansen-White standard errors. The estimates are presented in Table 2.

TABLE 2. EXPECTED INFLATION EQUATION, 1975:01-1995:12.

Parameter	Est. Coeff.	Std. Error	t-statistic
λ	0.62556	0.05266	11.88
ρ_1	0.71385	0.06815	10.47
ρ_2	0.27832	0.09180	3.03
$\overline{R}^2$=0.857	SER=1.15		

The aggregate demand equation (3.3) was estimated as a fourth-order autoregression, augmented with the differential between the current real rate and the long-run equilibrium real rate:

$$y = \upsilon_1 y_{t-1} + \upsilon_2 y_{t-2} + \upsilon_3 y_{t-3} + \upsilon_4 y_{t-4} + \theta\left(r_t - r_t^*\right) + \nu_t \qquad (3.7)$$

The estimates of this equation, generated by ordinary least squares with Hansen-White standard errors, are presented in Table 3. The maximum modulus of the autoregressive part of the equation is 0.988, rendering the equation dynamically stable.

TABLE 3. GDP GAP EQUATION, 1975:01-1995:12

Parameter	Est. Coeff.	Std. Error	t-statistic
υ_1	3.90128	0.02516	155.05
υ_2	-5.75578	0.07458	-77.17
υ_3	3.80615	0.07418	51.31
υ_4	0.95184	0.02475	-38.45
θ	-0.00795	0.00292	-2.72
$\overline{R}^2$=1.000	SER=0.001		

To evaluate the performance of the entire model, an ex ante dynamic simulation was performed for 1996:1-1999:12. Monetary policy instruments were held at their 1995:12 values over the 48 month horizon. The resulting trajectories of the model's variables were sensible, with a modest loss of output (no more than 0.5% of GDP at the end of the horizon) and a modest reduction in realized

and expected inflation. In summary, the model would appear to be reasonably well behaved, and capable of being used in an out-of-sample period to reflect the interactions of the real and financial sectors in the context of anti-inflation policy.

4. Optimal Monetary Policy and Inflation Expectations

The framework in which we pose an optimal policy problem is that developed by Chow (1975, 1981) as an elaboration of the stochastic and dynamic linear-quadratic-Gaussian (LQG) optimal control framework. In a standard LQG exercise, the expectation of a multiperiod quadratic loss function is minimized, subject to the constraints posed by a linear econometric model, with stochastic elements arising from Gaussian errors:

$$W = \sum_{t=1}^{T} (y_t - a_t)' K_t (y_t - a_t) \tag{4.1}$$

subject to the constraints of the model:

$$y_t = A_t y_{t-1} + C x_t + b_t + \epsilon_t \tag{4.2}$$

where y_t is a state vector containing both endogenous and policy variables, x_t is a vector of policy variables, and ϵ is a vector of Gaussian errors. Higher-order lags of both endogenous and policy variables are included as additional elements of the state vector via the introduction of appropriate identities, so that a system of arbitrary order may be reduced to first order. The target values in the a_t vector may be specified for both endogenous and policy variables in Chow's framework, obviating the need for a second quadratic form in the policy instruments.

The solution is achieved by solution of the matrix Riccati equation, applying Bellman's principle of dynamic programming to generate optimal feedback

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rules for each period in the horizon. We might consider this framework unduly restrictive in terms of both the loss function and the econometric model. On the one hand we might often want to penalize deviations from targets in a non-quadratic (e.g. asymmetric) manner; and on the other hand we might have a model which is essentially nonlinear, expressed as:

$$y_t = \Phi\left(y_t, y_{t-1}, x_t, z_t\right) + \epsilon_t \tag{4.3}$$

Extensions to Chow's algorithm permit both of these generalizations by generating linearizations of a nonlinear model around each point on the target trajectory. A nonlinear model may contain complicated (and even nondifferentiable, or noncontinuous) functions of the underlying variables, which may then be targeted, allowing for nonquadratic functions of the variables of interest in the loss function. With these generalizations of the LQG framework, we may consider a quite realistic setting for the interactions of policy instruments and goals.

The key elements of such an optimal policy problem are the relative weights applied to the components of the state vector. The state vector for this model consists of current through third-order lags on the Y variables (which, with identities and definitions, are 11 in number) and current through second-order lags on the two X variables. In a stochastic optimal policy exercise, the existence of more than two targeted elements of the state vector ensures that not all targets will be hit even in an expected sense. The expected multiperiod loss may be considered similar to a measure of mean square error (MSE), including both a "bias" term (indicating the magnitude in which the targets were not hit) and a "variance" term derived from the estimated variance-covariance matrix of the equations' error processes. Although in this framework the parameters are taken as given at their point estimates (i.e. there is no multiplicative uncertainty) the

presence of additive uncertainty will generate expected loss even when "bias" is zero.

The loss function applied in this problem is an extension of that used by Orphanides and Wilcox (1996, p.7) in their model. They include three terms in their single-period loss function, $\ell_a = (\pi - \tilde{\pi})^2 + \gamma y^2 + \psi |y|$: the squared deviation of realized inflation from an inflation target, the squared GDP gap, and the absolute value of the GDP gap. The presence of the intermediate target for inflation, which they treat as merely a constant fraction of last period's inflation, and the weighting of both square and absolute value of the GDP gap give rise to their "opportunistic approach" to anti-inflation policy. They also demonstrate that the inflation term in their loss function is mathematically equivalent to targeting both the level of and changes in the rate of inflation: in terms of the control literature, applying both proportional and derivative control.

In our policy problem, we are facing a multiperiod horizon, and have two instruments to work with. Our loss function takes additional factors into account, reflecting the more complex setting in which these instruments interact. The primary innovation in our multiple-instrument setting is the specification of loss associated with discount rate changes. As discussed above, we assume that changes in the discount rate are viewed as credible signals of policy stance by the public. The maintenance of this credibility relies upon the Fed's willingness to alter the instrument infrequently and to avoid reversals, or "whipsaw" actions. Although we cannot directly model the degree of credibility attached to a signal in the expectations formation process, we can take the maintenance of credibility into account in the loss function. The Fed, aware of the value of a credible signalling mechanism, should be unwilling to reduce this value through haphazard manipulations of the discount rate. Thus, we construct a "cost-of-change"

variable (C) which takes into account both recent changes in the discount rate as well as the consistency of those changes, penalizing reversals more heavily than changes which merely reflect a trend, such as successive increases (decreases). The C function is defined as:[2]

$$cc_t = \left|\log\left(\frac{d_t}{d_{t-1}}\right)\right| + 0.5\left|\log\left(\frac{d_t}{d_{t-2}}\right)\right| + 0.333\left|\log\left(\frac{d_t}{d_{t-3}}\right)\right|$$

$$rc1_t = \begin{cases} \left|\log\left(\frac{d_t}{d_{t-1}}\right) - \log\left(\frac{d_{t-1}}{d_{t-2}}\right)\right|, & (d_t - d_{t-1})(d_{t-1} - d_{t-2}) < 0 \\ 0, & (d_t - d_{t-1})(d_{t-1} - d_{t-2}) \geq 0 \end{cases}$$

$$rc2_t = \begin{cases} 0.5\left|\log\left(\frac{d_t}{d_{t-2}}\right) - \log\left(\frac{d_{t-2}}{d_{t-3}}\right)\right|, & (d_t - d_{t-2})(d_{t-2} - d_{t-3}) < 0 \\ 0, & (d_t - d_{t-2})(d_{t-2} - d_{t-3}) \geq 0 \end{cases}$$

$$C_t = cc_t + rc1_t + rc2_t \tag{4.4}$$

This functional form might be considered an unnecessarily complicated alternative to a very simple penalty function, such as Roszbach's fixed cost of change (1997, p.7) which merely penalizes any Δd_t (Δi in his framework) equally in the central bank's optimization problem.[3] As Roszbach points out, Rudebusch (1995) analyses Fed funds rate target changes with a hazard model, and finds duration dependence, or an autocorrelated structure of target rate changes. A fixed cost applied to *any* single rate change cannot take into account the dynamics of the rate setting process, nor the duration of a particular setting; nor can it vary the penalty based on the magnitude of the change–even though the nature of observed changes suggests that both dynamics and magnitude are taken into account in observed policy actions.

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[2]The function is not defined for nonpositive d. In the optimal policy experiments, a test for this condition is applied, and C_t is set to a large positive value if a nonpositive d is encountered.

[3]Roszbach's penalty function allows for asymmetric penalties depending on the sign of the change in the funds rate, but does not take the magnitude of change into account.

The cost of change function (4.4) allows for both the dynamics and the magnitude of rate changes, taking four periods' values of d_t into account, with recent changes being weighted more heavily in the cc_t component. To abstract from the level of rates, the cost components are expressed in terms of a percentage change in the rate. An additional empirical regularity is captured by the rc terms. When discount rate changes have been relatively frequent, they have been of near-identical magnitude and similar duration. That is, the *pattern* of change is quite predictable, with $\dot{d}$ nearly constant over intervals of, say, one month. This predictable, stately reduction in the rate, although taking the form of a sequence of changes, could have been viewed as a very credible, consistent policy. A reversal in this sequence would presumably have diminished the credibility of the Fed's stance. Thus the rc terms in (4.4) are designed to penalize reversals, and to place a heavier penalty on market shifts in the rate of change of d_t. The C_t function takes the sequence of d_t values into account, and proxies the threat to the central bank's credibility posed by a given sequence of policy actions. Although this specification is ad hoc, it is meant to capture elements of the dynamic policy process that cannot be approached with single-period models and/or fixed cost-of-change penalties.

The value of the cost-of-change variable C_t is then targeted as an element of the loss function. To gain an understanding for the workings of the cost-of-change function, we plot its components (change cost and reversal cost, $rc1+rc2$) and their sum for the estimation period, 1975:1-1995:12, in Figure 1, and the total cost versus the discount rate in Figure 2. We may note that the Fed's actions, involving quite infrequent and consistent changes in the discount rate, correspond to a quite low cost throughout much of the historical period.

Two other elements derived from the policy instruments are targeted: the spread between the funds rate and the discount rate, and the spread between

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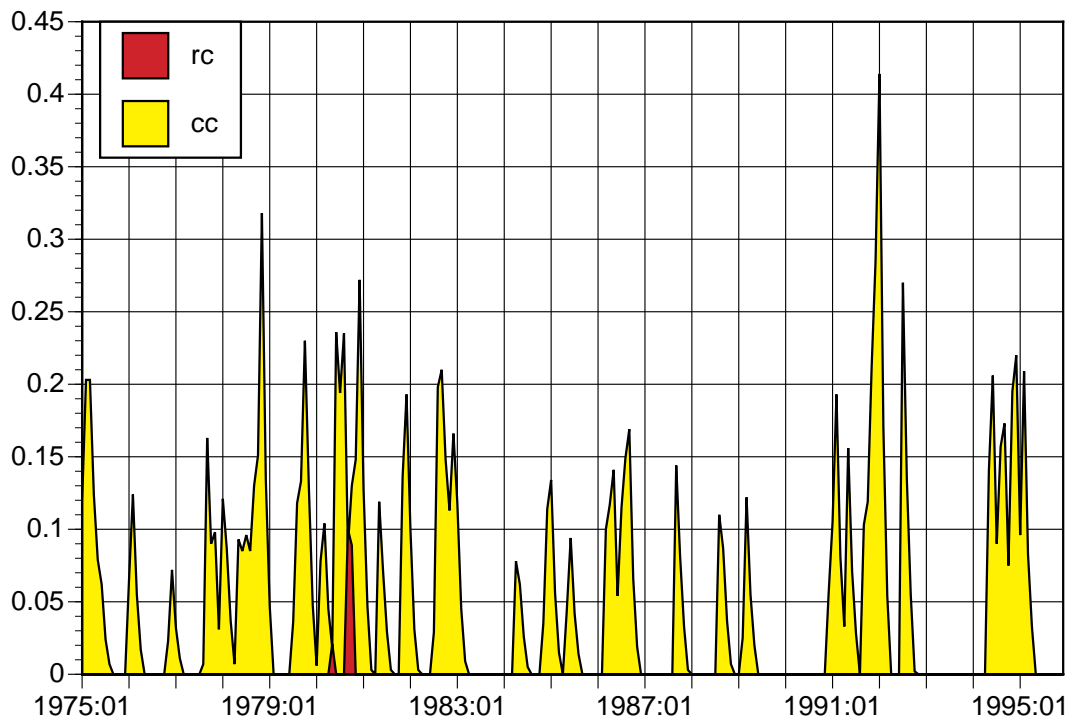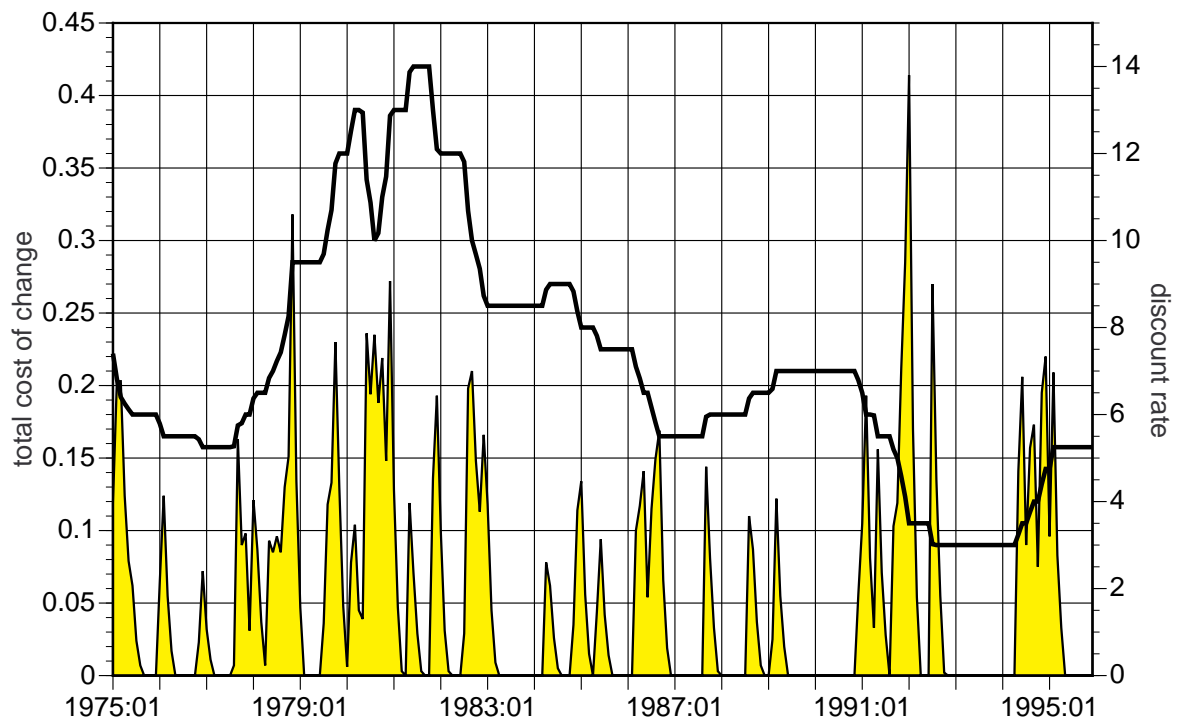Figure 1. Cost of Change Function for the Discount Rate, 1975-1995

Figure 2. Cost of Change Function and the Discount Rate, 1975-1995

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the current real rate and its long-run target. The rationale for the former relates to the mechanism by which the discount rate will provide incentives for member bank borrowing. The funds rate-discount rate spread is generally positive, reflecting that discount window borrowing is a privilege, and that "excessive" use of the discount windows will invite scrutiny. Therefore, banks with the need for reserves will turn to the Fed funds and repurchase agreement (RP) markets, paying a premium to conduct transactions free of this scrutiny. To model this empirical regularity, we target the funds rate-discount rate spread at 30 basis points, a value consistent with recent historical evidence. The latter spread– that between the current real rate and its long-run value–is targeted merely as a direct effect to speed convergence of the real rate to its long-run value.

The weights placed on various elements of the state vector y_t are the diagonal elements of the K_t matrix in (4.1).[4] It should be noted that, first, only relative weights matter, and second, that the magnitude of the variables affects the appropriate magnitude of the weights. In the policy experiment, the heaviest weight of 1.0 is placed on the deviation of current inflation from its target value, reflecting the Fed's primary concern with the reduction of inflation. Following Orphanides and Wilcox (1996), the intermediate target for inflation is taken to be 0.5 times last period's inflation, so that the long-run target for inflation is zero. Lower weights of 0.25 are applied to both the gap and the absolute value of the gap, with values chosen to generate some tension between anti-inflation and macroeconomic objectives. The cost-of-change function is also targeted with a weight of 1.0 to enhance the constrast between solutions in which it plays a role and those in which it is ignored. The two spreads mentioned above are each targeted at 0.10.

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[4]Although the framework allows for both a time-varying K matrix and off-diagonal elements in the K matrix, neither option is used here.

The outcome of the policy experiment is a set of optimal feedback rules which express the appropriate settings for the two instruments as a linearized function of the prior period's state vector:

$$\widehat{x}_t = G_t y_{t-1} + g_t \tag{4.5}$$

In this closed-loop optimal policy setting, the optimal policy is not expressed in terms of values for the instruments, but rather rules by which the instruments would be determined, contingent on economic conditions. The linearized certainty-equivalent trajectory for each of the instruments may be derived by ignoring the stochastic elements of the problem and applying the feedback rules to each period's values for the state vector. These certainty-equivalent trajectories may then be examined to judge the qualitative nature of the optimal policy solution.

To analyze the importance of the constraint we have placed on the discount rate via the cost-of-change function, we conducted another policy experiment in which zero weight was placed on the cost-of-change variable C_t. We may then compare the means, variances, and trajectories of each endogenous and policy variable, with and without this constraint on the discount rate, to determine its effect. We must note that the cost-of-change is not the only term in the loss function influencing use of the discount rate, as the (much lower) weight on the Fed funds–discount rate spread also influences the latter rate's use.

4.1. Results of the Optimal Policy Experiments

The optimal policy experiments, conducted over the 48 months 1996:1-1999:12, differed only in the specification of the cost-of-change variable's penalty. We first consider the experiment in which there was no penalty attached to movements of the discount rate. The descriptive statistics for the model's key variables are

given in the first two columns of Table 4. Certainty-equivalent trajectories of the gap, the current real rate, the inflation rate, the expected inflation rate, the two policy instruments and their spread are presented in Figures 3 through 9.

TABLE 4. DESCRIPTIVE STATISTICS FROM

OPTIMAL POLICY EXPERIMENTS

	No Penalty		Penalty		Test $\mu_{NP} = \mu_P$
	Mean	Std. Dev.	Mean	Std. Dev.	p-value
y_t	0.184	0.128	0.147	0.118	0.00
r_t	2.499	0.144	2.554	0.273	0.15
π_t	0.560	0.623	0.685	0.767	0.00
π_t^e	0.247	0.453	0.446	0.640	0.00
$(F_t - d_t)$	1.043	0.913	0.903	0.703	0.00
F_t	2.747	0.397	3.000	0.763	0.00
d_t	1.704	0.611	2.097	0.399	0.00
C_t			0.159	0.176	

The largest penalty in the loss function is placed on deviations of inflation, π_t, from its target value. The mean inflation rate is fractionally (but significantly) lower in the no-cost-of-change (NCC) experiment; Figure 6 illustrates that the adjustment to lower inflation is more gradual in the NCC case, converging on a similar rate of inflation after about 24 months. The second macroeconomic objective, the GDP gap y_t, is slightly higher in the NCC experiment (see Figure 3). The gap responds more quickly with the cost-of-change (CC) penalty in place. In both experiments, it declines from its initial condition of 0.315 (i.e. actual output at approximately 100.315% of potential in 1995:12), but the expected trajectories do not converge until 36 months have passed. The insignificant difference in the real interest rate (r_t) trajectories' means does not reflect the considerable perturbation that r experiences at the outset of the CC experiment's horizon; however, after 18 months, the trajectories (as shown in Figure 4) are

virtually identical. This perturbation reflects the optimal trajectory of the Fed funds rate (F_t), illustrated in Figure 8, which starts off 160 basis points higher in the presence of the cost-of-change. Without the cost-of-change, the optimal policy objectives are pursued by a rapid decline in the funds rate from its initial value of 5.6%. The funds rate is lower on average by about 25 basis points in the NCC scenario, and considerably less volatile. The discount rate, as shown in Figure 9, shows markedly different behavior in the two experiments. Without the constraint on its alteration, it is unrealistically reduced from its initial condition of 5.25% to 0.54%, and steadily increased over most of the horizon. In the presence of the cost-of-change penalty, the optimal discount rate trajectory is much less smooth, and follows a path which converges on its NCC alternative only after 30 months. The mean discount rate under the CC scenario is considerably higher, and its variance only about two-thirds of the NCC alternative. This latter statistic might be difficult to accept from observation of the graph, but it accurately reflects the rapid decline in the first 12 months followed by relatively little motion after the 18 month point.

What have we learned from examination of these experimental results? First of all, we may note that the two experiments reflect differing emphases on the inflation and output goals in a rather interesting manner. When the cost-of-change is introduced, the gap is more rapidly driven toward the potential output goal, at the cost of a slower convergence to the zero inflation target. As in the Orphanides/Wilcox "opportunistic approach," some current progress toward lower inflation is sacrificed in order to achieve more rapid convergence to potential output.[5] Second, we must consider that in the presence of the cost-of-change

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[5]This comparison would be more comprehensible from a textbook context if a negative GDP gap was being eradicated–that is, if the economy was operating below potential. The historical data we have used to initialize the experiments show the U.S. economy operating slightly beyond potential at the outset of the experiments–i.e. $y_t > 0$. However, the same tradeoff between progress toward zero inflation and potential output applies for a positive GDP gap.

function (4.4), the optimal trajectories for both policy instruments are quite different than in the absence of that element of loss, even though the constraint directly applies to the discount rate alone. Third, although the expected trajectory of the discount rate (and of its first difference) reflects the cost of change, it does not generate the discrete behavior that one observes in the historical record. Movements of the discount rate are still encountered in each period of the experiment; many of them are very small, but the presence of the cost-of-change function does not generate the threshold behavior discussed by Roszbach (1997) in a similar context.

The latter point deserves further attention. One might think that truly discrete behavior of the optimal discount rate–with changes only being made when the pressure for change surpasses some disequilibrium level–could be generated readily by a much simpler structure than (4.4), such as Roszbach's fixed cost applied to a change of any magnitude. However, in the context of our model, that structure is unworkable, as is the increase of the penalty weight on our C_t variable (which is capable of generating a similar effect). Unworkable, in the sense that with a non-trivial fixed cost of change, or a high weight on C_t, the dynamic optimization problem cannot be solved due to nonconvergence. Convergence in Chow's nonlinear optimization context must be achieved on two levels: first, the nonlinear model must be solved for a trial trajectory of the control variables for each point on the trajectory by Gauss-Seidel methods; and second, the optimal trajectories must converge. That is, in each major iteration of the entire control problem, a $T-$sequence of expected trajectories of the state vector is generated. To test convergence, an appropriate norm is applied to the difference between the current iterations' sequence and the previous iteration's sequence, and that normed value compared to a convergence criterion. Unless and until the successive iterations' expected trajectories have stabilized, convergence has not been

achieved. A non-trivial fixed cost-of-change value or a sizable weight on C_t has the effect of discouraging both types of convergence in this framework.

The model's inability to generate stable optimal trajectories with a cost-of-change sufficient to generate discrete behavior is an outcome of its simplicity. We have modeled the impact of the policy variables on the key target–inflation–asymmetrically, with the discount rate having a direct impact on inflationary expectations, and the Fed funds rate having no direct influence on that variable. This implies that, in a dynamic setting, there is an inexorable conflict between holding the discount rate fixed and making any progress toward the macroeconomic goals, which generates a dynamic instability in the model as augmented with the optimal policy functions (4.5). The obvious (but nontrivial) solution is to allow the Fed funds rate to transmit a signal, albeit with a low signal-to-noise ratio, enriching agents' expectation formation mechanism to embody a signal extraction problem.

Although many additional insights might be gleaned from analysis of the optimal policy experiments and variations on those experiments, it should be evident that many elements of the interactions between fiscal discipline, expectations of policy actions, and the policies chosen by the Fed are captured in this dynamic setting, in a manner that cannot be meaningfully captured in a single-period model. Although the model presented here is simplistic and stylized, it demonstrates some of the key dynamic elements of the policy process.

5. Conclusions

In this paper, we have stressed the importance of credibility of monetary policy through the introduction of a "credible signal" in the form of the discount rate. The historical manipulation of this policy instrument is consistent, we believe, with the underlying concept embedded in our cost-of-change function: that

market participants value a credible signal, and the Fed acts so as to maintain that credibility in manipulating an instrument with a powerful "announcement effect." Although the model constructed here is incapable of generating discrete behavior of the policy instrument, we believe that the dynamic setting is more likely to provide a useful framework in which to construct a more elaborate model which will be more faithful to the stylized facts of the monetary policy process.

In further development of this framework, we believe that the Orphanides and Wilcox "opportunistic" approach, which underlies much of the model presented in this paper, and our own prior work on modelling discount policy may be fruitfully combined with Roszbach's emphasis on threshold behavior of monetary policy tools. The concept of threshold behavior, in which action is only taken when a threshold is reached, is attractive in the context of a policy instrument whose use incurs a sizable cost. It may be feasible to combine our ad hoc specification of a cost-of-change function in this paper with an econometric approach to threshold modelling in order to generate a more realistic representation of monetary policymakers' actions.

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Figure 3. GDP Gap, with and without cost of change
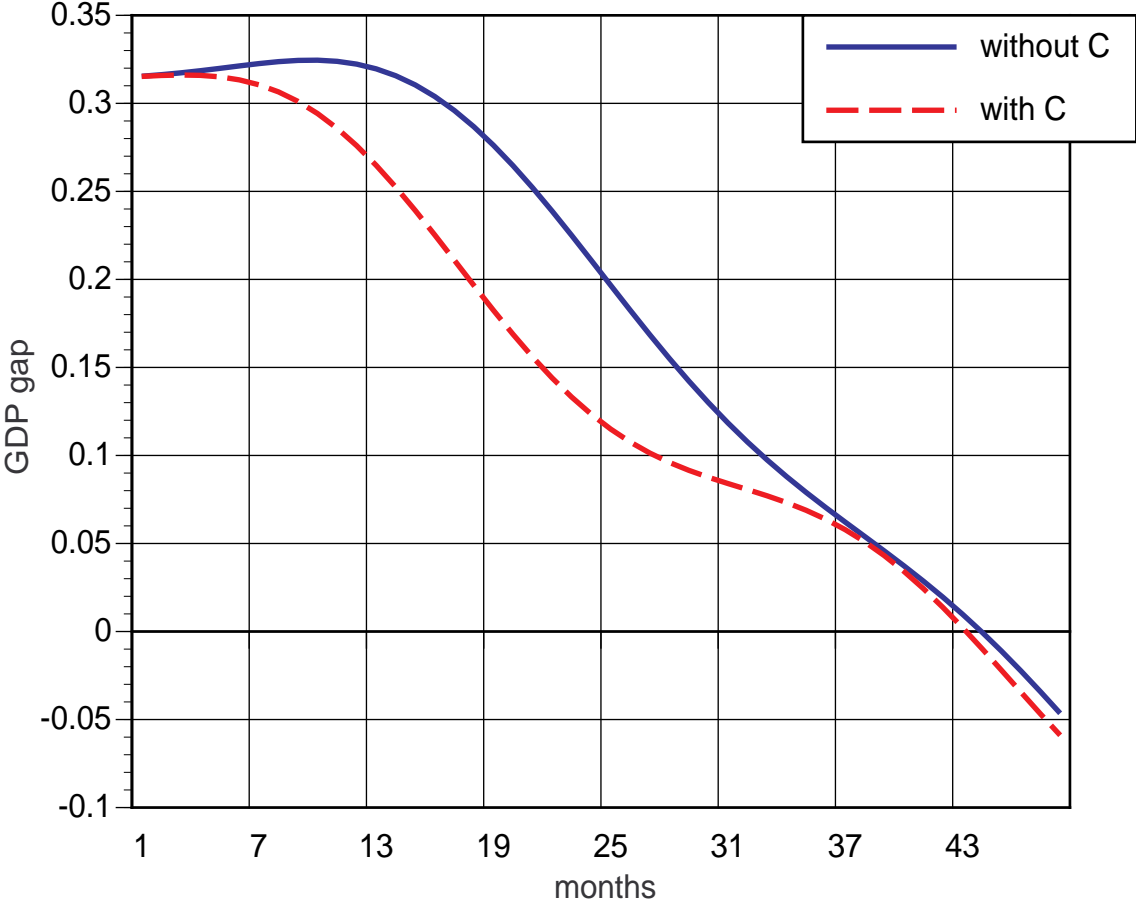
Figure 4. Current real interest rate, with and without cost of change
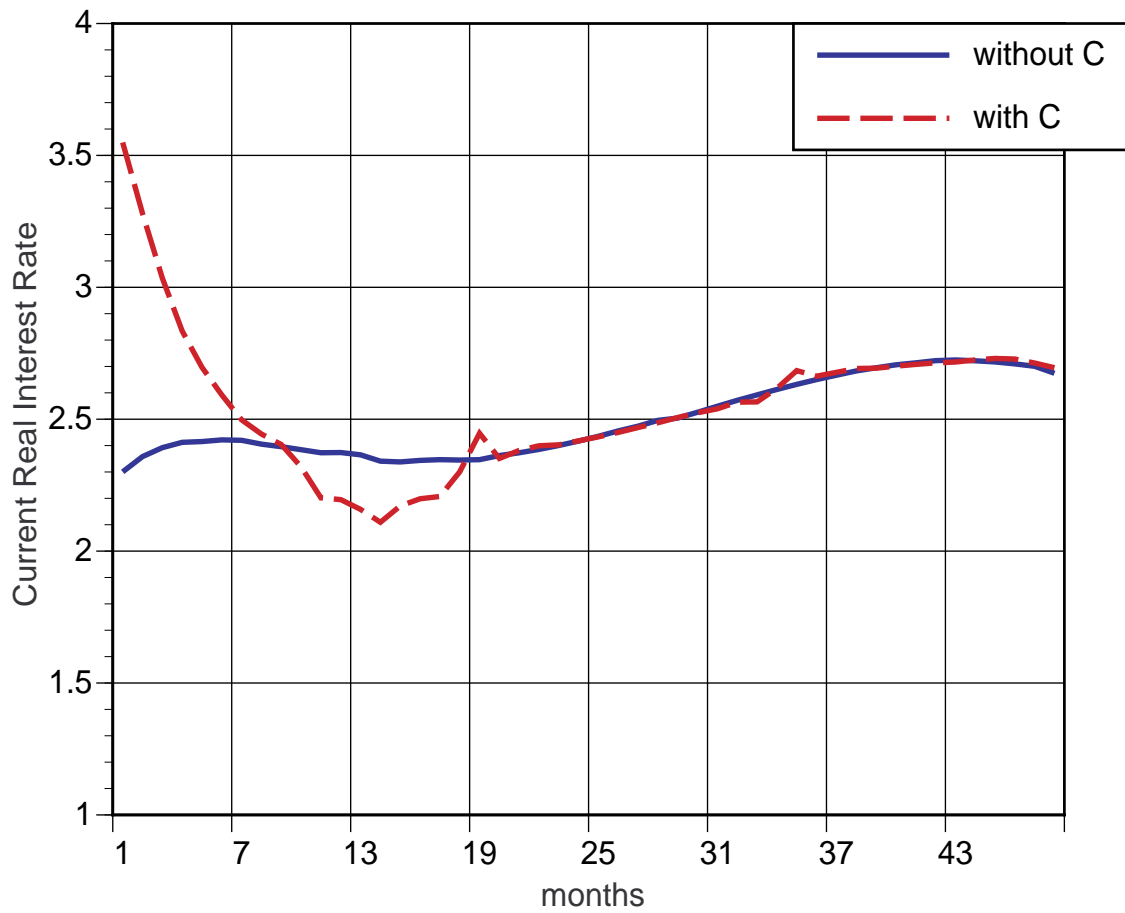
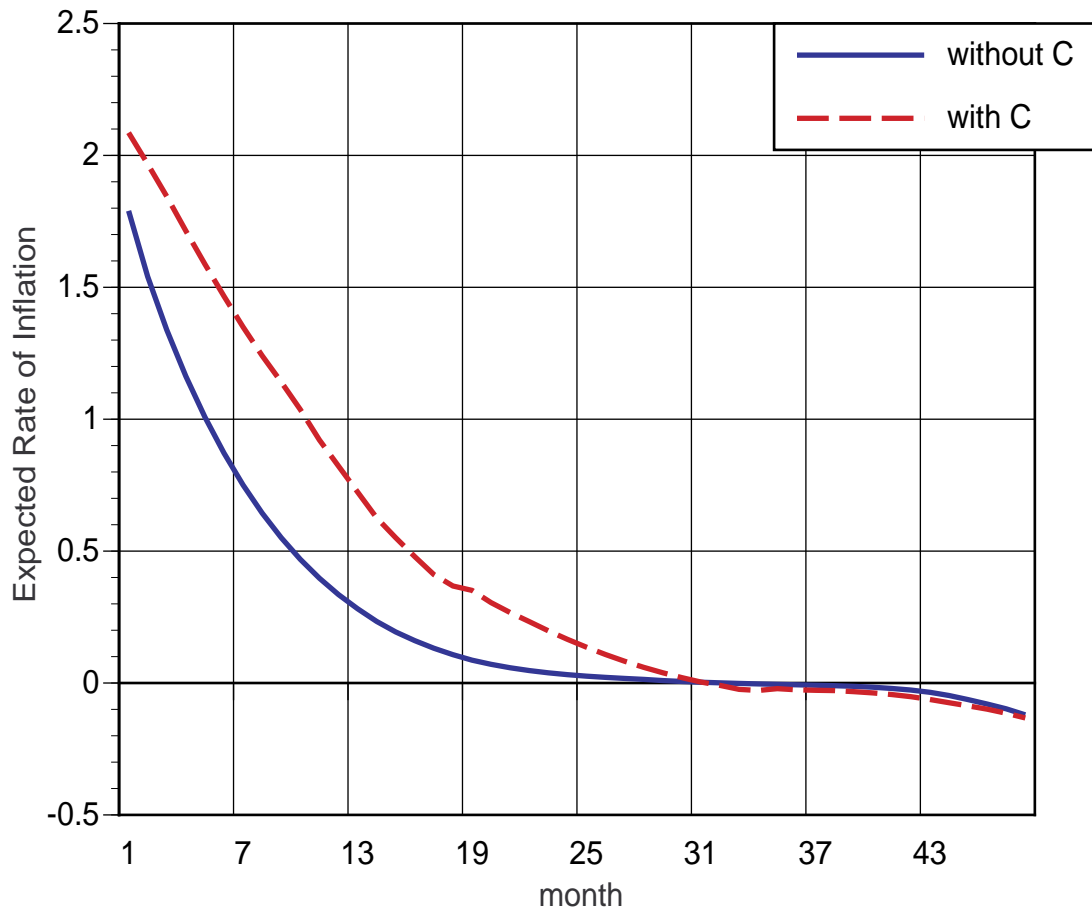Figure 5. Expected rate of inflation, with and without cost of change

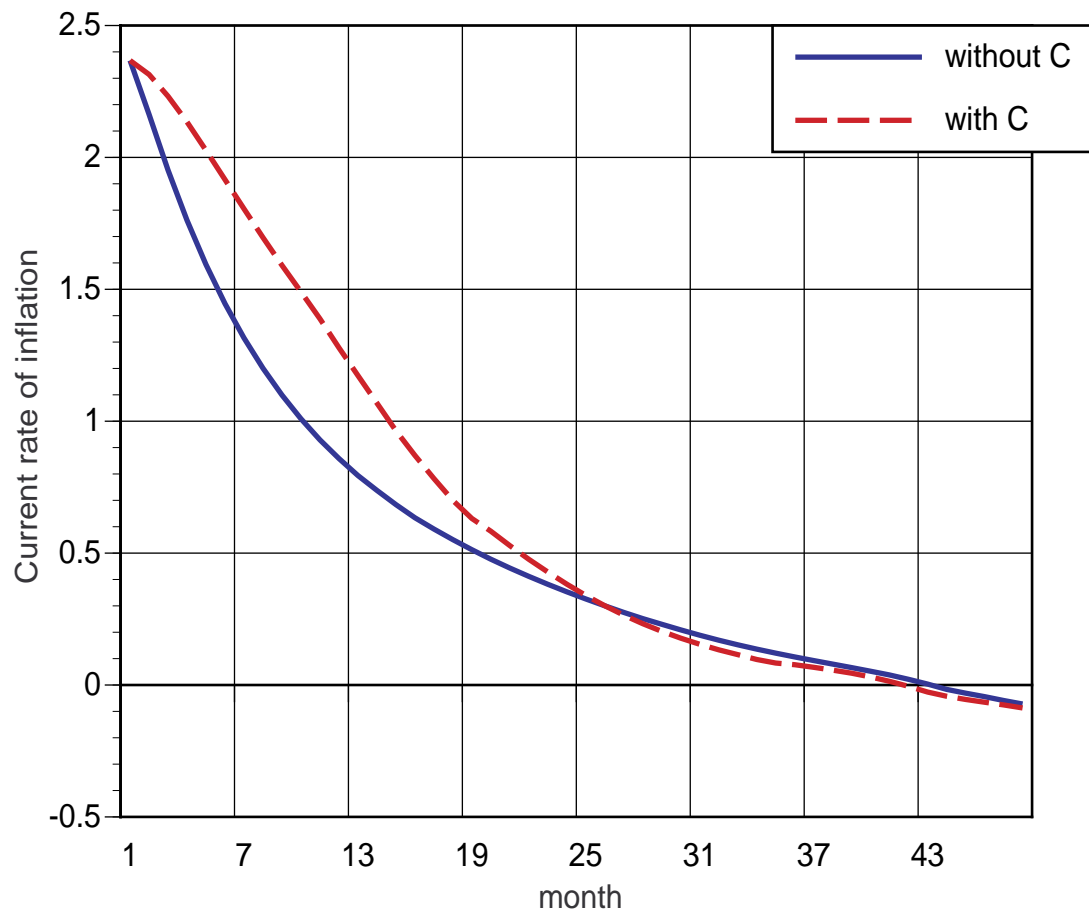Figure 6. Current rate of inflation, with and without cost of change

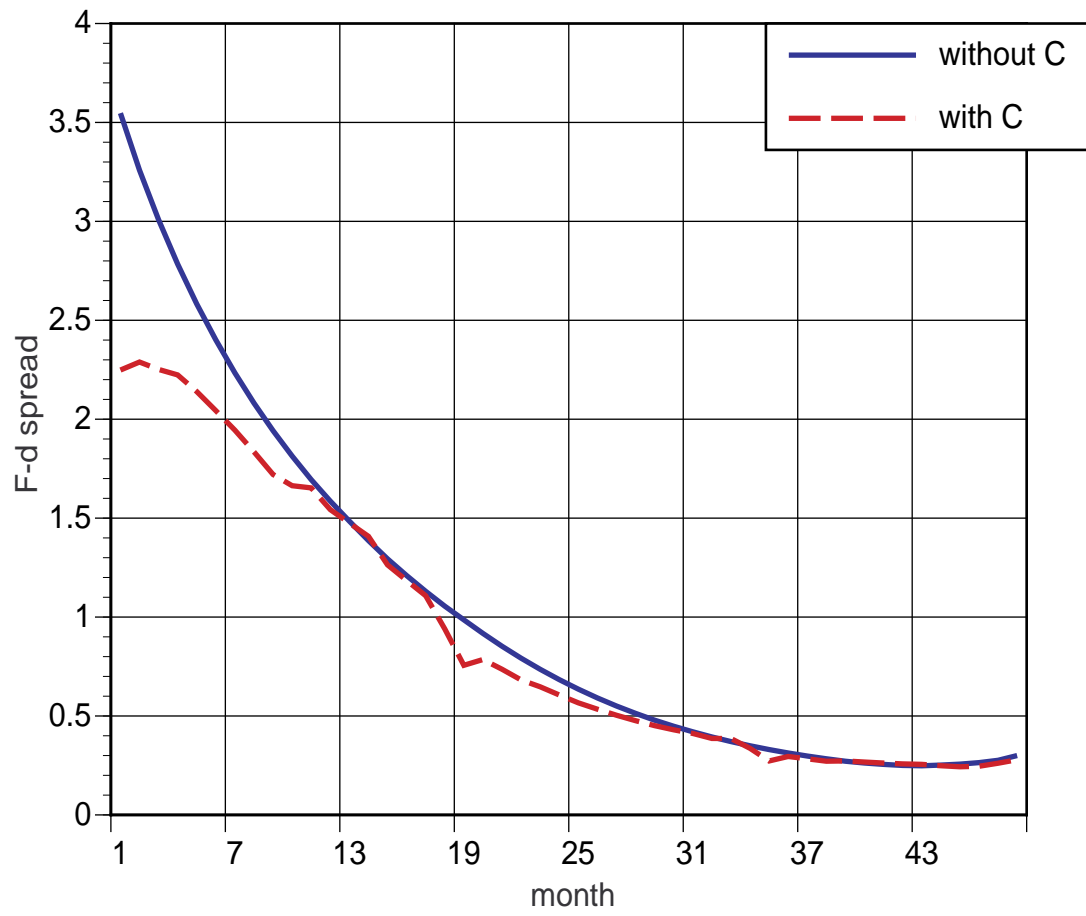Figure 7. (F-d) spread, with and without cost of change

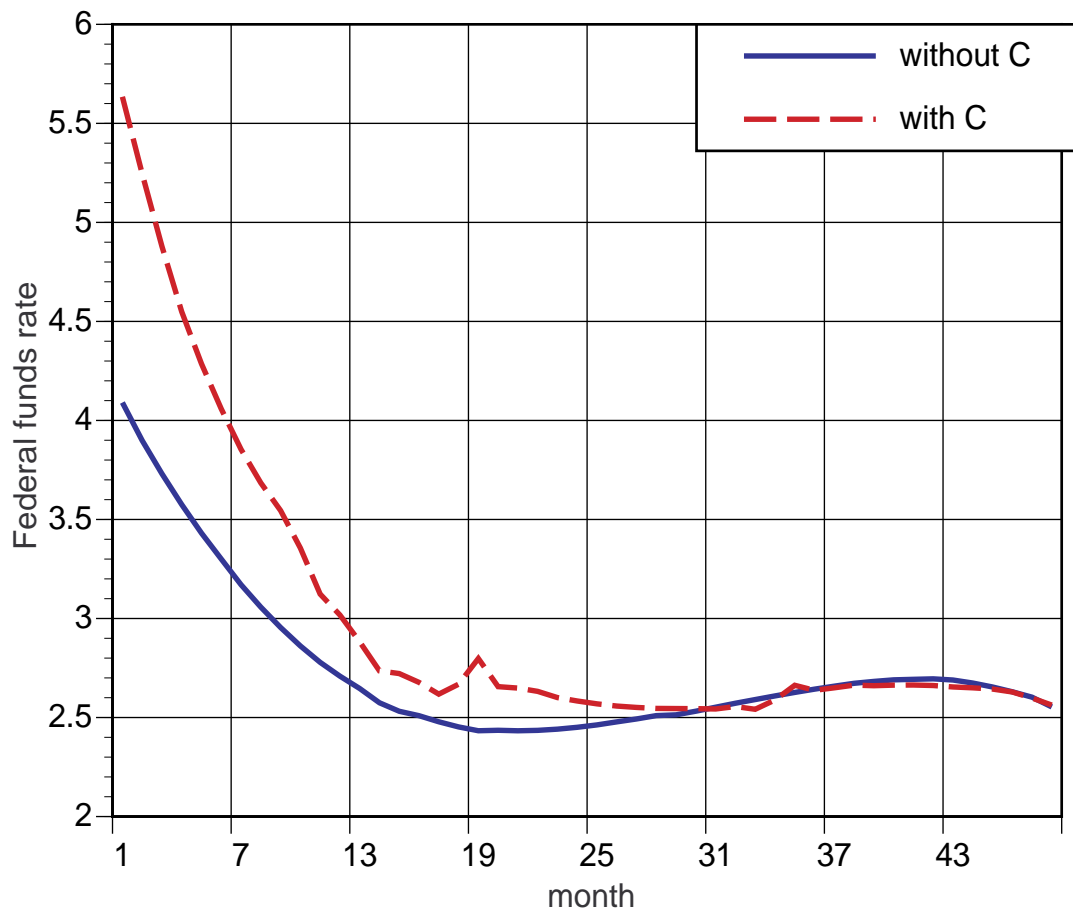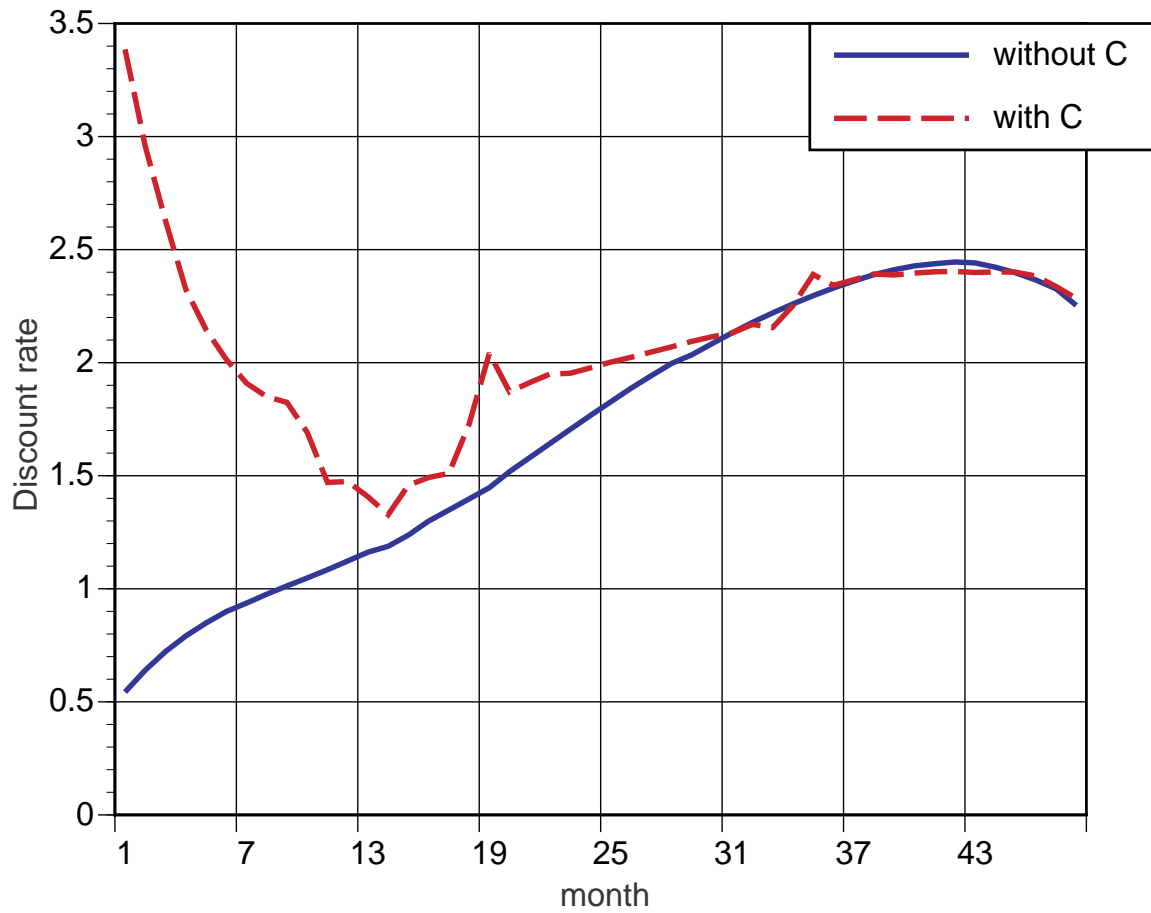Figure 8. Federal funds rate, with and without cost of change

Figure 9. Discount rate, with and without cost of change

Figure 10. Cost of change of discount rate