Concentration of Competing Retail Stores*

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Abstract

Geographical concentration of stores that sell similar commodities is pervasive. To analyze this phenomenon, this paper provides a simple two dimensional spatial competition model with consumer taste uncertainty. Given taste uncertainty, concentration of stores attracts more consumers since more variety means that a consumer has a higher chance of finding her favorite commodity (a market size effect). On the other hand, concentration of stores leads to fiercer price competition (a price cutting effect). The trade-off between these two effects is the focus of this paper. We provide a few sufficient conditions for the nonemptiness of equilibrium store location choices in pure strategies. We illustrate, by an example, that the market size effect is much stronger for small scale concentrations, but as the number of stores at the same location becomes larger, the price cutting effect eventually dominates. We also discuss consumers' incentives to visit a concentration of stores instead of using mail orders.

Keywords: consumer search, market size effect, price cutting effect, taste uncertainty.
JEL classification number: D4, L1, R1, R3.

1 Introduction

Concentration of car dealers is commonly observed in American suburbs. Similarly, one finds several fashionable apparel stores in a single shopping mall. In both cases, competitors' commodities are substitutes for each other, and a consumer typically buys only one unit. Thus, by concentrating at one location, competitive forces would drive down the prices of commodities. The questions we ask in this paper are: Why do stores concentrate at the same location? Why don't they keep some distance from others and monopolize the customers nearby?

The main idea of this paper can be roughly described in the following example: Consider a consumer who gets up on Sunday morning wondering if she should to get a new fancy car to replace her old Honda. She has some vague idea about those fancy cars, but she does not know how much she likes each of them (relative to their high prices) before she actually visits the dealers and tries them. Suppose that she expects that if she visits any one car dealer (BMW, Mercedes, Volvo, and so on), then the probability that she likes the cars sold by the dealer well enough to buy is $\frac{1}{4}$ (25%), and these probabilities are independently distributed. Then, if she visits a shopping center with BMW only, the probability of getting a buyable car is $\frac{1}{4}$ (25%), which is a little bit costly for wasting her precious Sunday. On the other hand, if a shopping center has Mercedes and Volvo together, then the probability of finding a buyable car is $\frac{7}{16}$ (43.7%), since the probability of *not* finding a buyable car at each dealer is $\frac{3}{4}$ (75%) and if she visits two dealers then the probability that she cannot find a buyable car at either dealer decreases to $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$. Given the increased chance of finding a car she likes, she may visit the two car dealer shopping center even though the location is a bit far away. If there are five car dealers together at a shopping center, the probability of finding a buyable car increases to $1 - \left(\frac{3}{4}\right)^5$ (76.3%), so that it is very likely that she will not waste her Sunday by visiting the shopping center. In such a case, she may not mind going to the shopping center although it may be far away from her house. Thus, concentration of car dealers can increase the size of the pie (*the market size effect due to taste uncertainty*) although close proximity may imply that they then compete with each other more vigorously (*the price cutting effect*). Therefore, if the former effect exceeds the latter effect, then car dealers can actually make higher profits under concentration than by staying alone to extract monopoly rents from the nearby customers.

We formalize this idea in a spatial oligopoly model with price competition in order to describe the trade-off between the market size effect and the price cutting effect. In order to determine the number of consumers who visit a given shopping center (the market size), it is necessary to determine the geographical area from which residents visit this shopping center (the market area). To pin down the market size via the market area, we need to introduce an explicit *spatial* structure into our model. The key assumption we use in this paper is that consumers do not know their exact tastes over commodities (consumer taste uncertainty). The structure of the model is as follows: Consumers are distributed over the plane and each consumer can buy at most one unit of a commodity at a shopping center by paying the commuting costs in addition to the price of the commodity. There are a finite

number of stores that decide their locations from the set of potential shopping centers (stage I). Consumers can observe the locations of stores, yet they know neither their willingnesses-to-pay for commodities nor the prices before they actually visit the stores.[1] Thus, when a consumer decides which shopping center to visit, she calculates the expected utility of searching commodities at each shopping center by taking commuting costs into account. For simplicity, we assume that each consumer chooses to visit at most one shopping center (stage II). Once she arrives at a shopping center, the commuting costs are sunk, and at no cost she can try every commodity sold at the same shopping center. Thus, she chooses to buy a commodity which gives her the highest (positive) surplus (her realized willingness-to-pay minus the price of a commodity) among the commodities sold at the shopping center. If no commodity gives her a positive surplus, she does not buy any commodity. Taking consumers' commodity choice behavior into account, stores compete with prices (stage III). If a consumer's willingness-to-pay distributions over different commodities are not perfectly correlated (statistical independence is assumed in this paper), the concentration of stores at a shopping center increases the expected utility from visiting there. This implies that consumers living far away may visit the shopping center, and its market area expands. However, since each consumer can choose the commodity which gives her the highest surplus among commodities available at the shopping center, stores may be forced to compete for customers by cutting prices. Thus, our model captures the trade-off between the two effects by featuring both an explicit geographical structure of the economy and price competition among stores.

Looking more closely at the mechanics of concentration of stores, we find that there are two distinct but interconnected incentives for stores to concentrate. First, as we noted in the example, there is *the market size effect due to taste uncertainty*: Concentration of stores increases the probability of a consumer finding a buyable commodity at the shopping center. Thus, a consumer's expected utility from shopping there increases, resulting in a larger market size at that shopping center. The second effect also operates through the increase in a consumer's expected utility: Concentration of stores sends to consumers a signal of lower prices at the shopping center. This increases a consumer's expected utility of choosing the shopping center, and the market size expands. This may be called *the market size effect due to the lower price expectation.* Thus, the consumer taste uncertainty and the imperfect information regarding prices give stores incentives to concentrate.[2]

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[1] The market structure is similar to Perloff and Salop (1985), Wolinsky (1986), and Fischer and Harrington (1996). Anderson and Renault (1997) synthesize the literature of product diversity and consumer search nicely.

[2] There is an additional incentive for stores to concentrate that is not through the expansion of market size: Suppose that there are two stores each of which has a mutually exclusive customer group. If each of them sells its commodity to its own customer group, then many consumers cannot find a buyable commodity, since each consumer has an access to only one type of commodity. However, if these two stores pool their customers, then the consumers' probability of finding a buyable commodity increases as long as consumers' willingnesses-to-pay are not perfectly correlated between two commodities. This implies that these two stores' per store sales and profits will be raised by pooling their consumers, if their prices are kept constant. This effect may be called *the consumer pooling effect.* I thank Parikshit Ghosh for helpful conversations on

In this paper, we first establish the existence of the third stage equilibrium and an inverse relationship between the number of stores at a shopping center and the equilibrium prices (the price cutting effect: Proposition 1). Then, we proceed to show that the radius of the market of a shopping center increases with the number of stores (the market size effect: Proposition 2). These two propositions show that our model captures the trade-off between these two effects. Moreover, we can show that the market size effect can be decomposed into the one due to taste uncertainty and the one due to the lower price expectation (Proposition 2). However, to establish the existence of subgame perfect equilibrium is more tricky. The main difficulty comes from store's location choice problem (stage I). By the very market size effect, if a shopping center has other stores then it is not profitable to open a store near the shopping center: All potential customers will visit the shopping center, and the new store cannot make any profit (the "urban shadow"). On the other hand, a store can make a positive profit, if it is opened right at the shopping center, or if it is opened far from any shopping centers. Thus, each store's profit function is not quasi-concave with respect to its location, and we cannot apply the standard fixed point argument to stores' location choice problem. We provide three existence theorems for a subgame perfect equilibrium in pure strategies, although the conditions are somewhat strong (Propositions 3, 4, and 5).

Then, we illustrate the relationship among the number of stores at a shopping center, equilibrium prices, market sizes, consumer's probability of finding a buyable commodity, and each store's profit by two numerical examples with the following simple structure: (i) consumers are uniformly distributed over the plane, and (ii) consumers' willingnesses-to-pay are uniformly distributed. The first example assumes that potential shopping centers are far from each other so that their markets would not be overlapped with each other. The most striking observation is that a marginal increase in the number of stores dramatically expands market size and each store's profit when there are a small number of stores at a shopping center (Table 1). Thus, the market size effect dominates the price cutting effect, and there is a strong incentive for stores to concentrate at the same shopping center. In this example, we can also fully characterize the set of subgame perfect equilibria. The result suggests that (a) there will be multiple (quasi-) homogenous shopping centers, and (b) there could be multiple (Pareto-ranked) subgame perfect equilibria due to the coordination problem (Proposition 6). The second example assumes that there are only two potential shopping centers, but their markets can be overlapped with each other. We observe that a symmetric equilibrium (the same number of stores at each shopping center) and/or clustering equilibria (all stores at the same shopping center) exist depending on the number of stores and the distance between two shopping centers. If the number of stores is relatively small and two shopping centers are very close to each other, then a symmetric equilibrium may vanish. On the other hand, if the number of stores is relatively large and if two shopping centers are not too close to each other, then clustering equilibria may vanish. When two types of equilibria coexist, a symmetric equilibrium tends to attain a higher profit for each store.

At the end of the paper, we extend the model and discuss how the presence of mail order companies may enhance the concentration of stores. At a shopping center, consumers can

this effect.

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try commodities before the purchase. We modify our model in order to allow the possibility of mail order shopping with return policies. This modified model suggests that consumer taste uncertainty can be the main reason why consumers actually visit stores instead of using mail order shopping. In a different extension, we discuss that owners of shopping malls can increase their rent revenues by restricting the number of stores that belong to the same category.

The rest of the paper is organized as follows: Section 2 provides a brief summary of the literature. Section 3 presents the formal model. Section 4 analyzes each subgame, and provides three existence theorems for a pure strategy subgame perfect Nash equilibrium. Section 5 provides simple examples which illustrate the relationship among the number of stores at a shopping center, equilibrium prices, market sizes, and each store's profits. Section 6 is devoted for the discussions on related issues: We note how our model can be extended to discuss a few related issues including the possibility of mail order shopping and shopping malls. Appendix collects the proofs of propositions and lemmas.

2   Summary of the Literature

Since past literature related to this paper is large, we concentrate on several most related papers.[3] These include Stahl (1982), Wolinsky (1983), Dudey (1990), and Fischer and Harrington (1996).[4] Stahl (1982) and Wolinsky (1983) assume that each type of consumers have different tastes over commodities, but they do not know which store sells their most preferred commodity. Consumers pick a shopping center to visit only by observing the number of stores at each shopping center. Both papers analyze clustering equilibria. However, these models cannot analyze the profit reducing force due to increased price competition since they either assume that there is no price (Stahl, 1982) or that each store charges the same price (Wolinsky, 1983). Thus, these models contain the market size effect due to taste uncertainty (and the consumer pooling effect), but they do not have any price related effects: neither the price cutting effect nor the market size effect due to the lower price expectation (see also Economides and Siow, 1988, for a related mechanism). Dudey (1990) considers a (homogeneous commodity) Cournot oligopoly model with finite numbers of consumers and stores. Each consumer has the same demand curve so that two consumers at the same shopping center means the demand curve is doubled in its scale. Consumers are uninformed about prices, but they choose the shopping center by inferring which shopping center has the lowest prices (the market size effect due to the lower price expectation). Thus, if a store chooses to locate alone, then the store loses all the customers since transportation costs are zero by

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[3]See Fujita and Thisse (1996) for a nice survey of the literature. There are interesting models that explain concentration of retail stores using quite different mechanisms. Rob (1993) uses capacity constraint and demand uncertainty to explain concentration of restaurants. Caplin and Leahy (1998) stress the importance of information externality in explaining a rapid (re)vitalization of a specific part of a city.

[4]In all of those papers including the current paper, the consumer's search behavior plays an essential role. For search theory, see Stigler (1961), Kohn and Shavell (1974), Stuart (1979), and Wolinsky (1986). The original idea without search behavior can be found in Eaton and Lipsey (1979).

assumption.[5] As a result, all stores concentrate at one location (for the case of the standard linear demand function). Note that Dudey's model has both the price cutting effect and the market size effect (although it is limited to the one due to the lower price expectation). The model closest to ours can be found in Fischer and Harrington (1996), although their main interest is different from the other papers including ours: They are interested in interindustry variation in the concentration of stores. They assume that there are two abstract locations: a "cluster" and a "periphery." Consumers can visit one of the two locations or both. If a consumer visits the cluster, then she can get information on all stores located there at a fixed cost. If she visit the periphery, she can search stores there at the same marginal cost per store. Using numerical examples, Fischer and Harrington (1996) illustrate that greater store concentration is associated with industry characterized by greater product variety in equilibrium. It turns out that we can relate our model to theirs by introducing outside opportunities for consumers (mail order shopping) into our model. We further discuss their theoretical contribution (nonemptiness of equilibrium) in Subsection 6.1.

Schulz and Stahl (1996) and Gehrig (1998) are also related to our model. Both papers utilize the taste uncertainty assumption to generate concentration of stores. Schulz and Stahl (1996) analyze how many stores enter the market in a model with one shopping center in which the market price increases with the number of stores, but the market size shrinks because of the price increase (see also Rosenthal, 1980). Gehrig (1998) analyzes competition between two shopping centers by using a spatial model. He specifically shows the existence of an equilibrium with two symmetric clusters for certain parameter values. In contrast to others, both papers assume that consumers know prices of commodities before searching. Thus, each stores chooses its price knowing that her price decision affects consumers' search decisions (and the market size).

The common feature that these papers and ours share is our assumption regarding information available to consumers: all of the above papers assume that consumers have imperfect information regarding the types (and the prices) of commodities sold by stores before they arrive the stores, which is the very source of the concentration of stores in those models. Since Hotelling (1929), there exists a huge literature on spatial and price competition with perfect information. However, in those models, there may not be an equilibrium in pure strategies, or even if it exists, there is no clustering equilibrium in most cases (see d'Aspremont et. al., 1979, and Bester et. al., 1996, among others). Stores tend to choose different locations.[6] A notable exception is de Palma et al. (1985): By employing a discrete choice model (see McFadden, 1981, and Anderson, de Palma, and Thisse, 1992), they introduce heterogeneous-taste consumers into the Hotelling model. De Palma et. al. show that there is a clustering equilibrium at the center even under perfect information, if consumers' tastes are sufficiently dispersed.[7] Bester (1989) analyzes a spatial model in which a consumer and a store play a

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[5]In his original paper (Dudey, 1989), the model contained transportation costs, but the results are essentially the same as no transportation cost case (Dudey, 1990, p 1095).

[6]See d'Aspremont, et. al. (1979), Economides (1989) and Kats (1995) for the existence of such equilibria. Anderson, de Palma, and Thisse (1992) has a complete literature survey.

[7]Note that mechanism of generating concentration of stores in the de Palma et al. model is very different

perfect information (noncooperative) bargaining game over the transaction price by taking the consumer's option to search into account. In his model, no pair of stores choose the same location since they make zero profits otherwise.

The literature on monopolistic competition is also motivationally related. Employing monopolistically competitive markets, Dixit and Stiglitz (1977) construct a model in which the number of commodities is endogenously determined and the more commodities are available, the higher utility levels consumers can enjoy. Using this mechanism, Krugman (1991) and others explain geographical concentration of economic activities (see Matsuyama, 1995, for a survey of the literature).

3   The Model

Let $N = \{1, 2, ..., n\}$ be the set of stores. There is a continuum of consumers, who are distributed over the (two-dimensional) plane $Z = \Re^2$ according to a density function $g : Z \to \Re_+$.[8] Each store can produce one type of indivisible good at the same marginal cost. We normalize the marginal cost to zero without loss of generality. Different stores sell different types of goods. Each consumer buys at most one unit of at most one indivisible commodity. Each consumer has identical ex ante preferences over the types of indivisible goods, but ex post willingnesses-to-pay can be different. Her willingness-to-pay for the indivisible commodity sold by each store $i \in N$ (type i commodity) is simply assumed to be a random variable $\mathbf{v}_i$ with probability density $f(v)$ on the interval $[0, b]$ where $0 < b < \infty$, and $\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n$ are identically and independently distributed (i.i.d.).[9] A vector $(v_1, v_2, ..., v_n)$ describes that the consumer's ex post willingness-to-pay for each commodity.[10] We assume that each consumer chooses at most one location to shop for simplicity. Although this assumption is not the most realistic one, it simplifies our analysis dramatically. A consumer needs to pay the commuting cost, which is linear (with coefficient t) in the Euclidean distance from her location to the chosen shopping location.[11] Although a consumer does not have information on commodities (her willingnesses-to-pay and the prices) ex ante, once she arrives

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from the one described by the market size effect due to taste uncertainty. In de Palma et al. (1985), even though consumers' tastes are differentiated, each consumer knows each store's commodity type and its price before her search. As a result, there is no benefit for stores to be close to other stores. The reason that a concentration of store at the center of the landscape occurs is that the central location is so attractive for stores that they want to stay there as long as competition does not bring down prices too much. Price competition force is weakened by sufficiently dispersed consumers' tastes.

[8]We use a two-dimensional spatial model in order to avoid discontinuities of profit functions (see also footnote 14).

[9]We assume that the law of large numbers applies to the case of a continuum of i.i.d. random variables (see Judd, 1985).

[10]The preference structure here is similar to the one in Fischer and Harrington (1996), although we do not assume that f is uniform. Kranton and Minehart (1998) also employ a similar setup in their auction/network paper. Bester (1992) analyzes the Bertrand equilibrium in a duopoly model with heterogenous consumers dispersed over characteristics. De Palma et. al. (1985) employ a discrete choice model and obtain an explicit solution.

[11]Results in Section 4 are not affected by assuming an increasing and convex transportation cost function.

at a shopping center she obtains this information on all the commodities sold at the shopping center without any additional search cost. Thus, if a consumer at location $a \in Z$ goes to location $d \in Z$ and buys a type i commodity with price p_i, then her payoff is $v_i - p_i - t\,\|d - a\|$.

Each store can choose its location from a *finite* set of locations (shopping centers) D embedded in the same location space for consumers ($D \subset Z$).[12] This assumption can be justified if there are zoning policies by the local authorities. Once stores' location decisions are made, their locations are described as a partition over shopping centers: $(N_d)_{d \in D}$, where N_d is the set of stores at shopping center d. Consumers can observe stores' locations $(N_d)_{d \in D}$. The game goes as follows:[13]

Stage I (store's location choice decision): Each store $i \in N$ chooses its location from the set D simultaneously.

Stage II (Consumers' shopping location decision): Knowing $(N_d)_{d \in D}$, each consumer chooses a shopping center from $D^* = D \cup \{\emptyset\}$ (she shops at a shopping center in D, or decides not to shop (choose $\emptyset$)). In making their decision, she infers the prices of commodities at $d \in D$ and calculates the expected utility from shopping at $d \in D$. Denoting the expected price of the type i commodity by p_i^e, the expected payoff of a consumer at $a \in Z$ who shops at d with $N_d \neq \emptyset$ can be written as:

$$EU_a(d; (N_{d'})_{d' \in D}) = E\left(\max_{i \in N_d} (\mathbf{v}_i - p_i^e)\right) - t\,\|d - a\|.$$

If she shops at d with $N_d = \emptyset$, then her expected payoff is $EU_a(d; (N_{d'})_{d' \in D}) = -t\,\|d - a\|$, since she finds no store to shop there. Of course, nobody chooses this option since it is dominated by the option of not visiting any shopping center ($\emptyset \in D^*$). A consumer chooses the location $d^*(a; (N_{d'})_{d' \in D}) \in D^*$ which attains the highest expected payoff:

(i) $d^*(a; (N_{d'})_{d' \in D}) = \arg\max_{d \in D} EU_a(d; (N_{d'})_{d' \in D})$,[14]

if $\arg\max_{d \in D} EU_a(d; (N_{d'})_{d' \in D}) \geq 0$, and

(ii) $d^*(a; (N_{d'})_{d' \in D}) = \emptyset$,

if $\arg\max_{d \in D} EU_a(d; (N_{d'})_{d' \in D}) < 0$ (she does not visit any shopping center).

We assume that each consumer perfectly foresees the prices when she makes the decision: $p_i^e = p_i$ (subgame perfection).

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[12]We assume finiteness of D for simplicity. As we discussed in Section 1, each store's profit function is not quasi-concave in its location even if D is convex.

[13]The order of moves by players would affect the equilibrium allocation (see for example, Anderson, de Palma, and Thisse, 1992, Dudey, 1993, and Ma and Burgess, 1993). We use our setup since it seems reasonable for the markets we are interested in and is tractable.

[14]We do not need to specify the tie breaking rule. It is not important since the measure of consumers who are indifferent between two options is zero under two dimensional spatial structure. (If the spatial structure is one dimensional then the d'Aspremont et al. problem occurs if the transportation cost function is linear. See d'Aspremont et al., 1979.)

Stage III (Price competition among stores at the same location): Given the population of consumers who shop at $d \in D$, denoted $G(d; (N_{d'})_{d' \in D}) \equiv \int_{\{a' \in Z: \, d^*(a'; (N_{d'})_{d' \in D}) = d\}} g(a) da$, stores in N_d decide their prices of commodities $(p_i)_{i \in N_d}$, and each consumer decides whether to buy a commodity as well as the type of commodity to buy after the realization of her willingness-to-pay $(v_i)_{i \in N_d}$. She buys the type i commodity only if $i \in \arg\max_{i \in N_d} (v_i - p_i)$ and $v_i - p_i \geq 0$. Note that the commuting costs are *sunk* when consumers make their shopping decisions.

Our equilibrium notion utilized in this paper is subgame perfect Nash equilibrium (SPNE). In the next section, we analyze equilibria of our game.

4 Equilibria of the Game

We proceed by backward induction.

4.1 Stage III

Consider a location d in D. Since consumers have already decided which locations to visit, the measure of consumers who visit d (the size of market at d) is already determined, and the size of the market at d is $G(d; (N_{d'})_{d' \in D})$. Since firms are symmetric, we focus our attention on symmetric Nash equilibria in the price competition (Bertrand) game. To guarantee the existence and the uniqueness of Nash equilibria having desirable property, we impose the following two conditions on the probability density:

Assumption 1. *The probability density f is log concave.*

This condition is used to guarantee the existence of an equilibrium by utilizing the Prékopa Theorem (see Bagnoli and Bergstrom, 1989, Caplin and Nalebuff, 1991, Dierker, 1991, and Anderson, de Palma, and Thisse, 1992). It is satisfied by a variety of commonly used probabilistic distributions.

Assumption 2. *The probability density f is continuously differentiable, and satisfies $f(v) > 0$ and $-2 \leq \frac{v f'(v)}{f(v)} \leq 1$ for every $v \in [0, b]$.*

This condition is more restrictive but is the simplest sufficient condition that guarantees the uniqueness of symmetric equilibrium (the lower bound) and an inverse relationship between the number of stores and the equilibrium prices (the upper bound). These require that $f(v)$ does not increase or decrease too fast. Of course, the uniform distribution satisfies Assumptions 1 and 2. Assumptions 1 and 2 will be maintained throughout the paper. The main result of this subsection is the following:

Let $k > 0$ be the number of stores at location d. There is a unique symmetric price equilibrium at which all stores charge p_k^*. Moreover, p_k^* is decreasing in k ($p_k^* > p_{k+1}^*$ for $k > 0$).

For the existence of a symmetric price equilibrium in stage III, we need only that v_i are identically but interdependently distributed according to a symmetric log concave density (see Theorem A.1 in the appendix). Propositions 3, 4, and 5 (below) can be proved under the same assumption as well. However, in order to show the uniqueness of symmetric price equilibrium and the inverse relationship between the number of stores and the equilibrium prices, we need independence of $\mathbf{v}_i$s and Assumption 2.

This proposition guarantees uniqueness of the symmetric price equilibrium in the third stage game, and shows the monotonicity of p_k^* in k (the price cutting effect). The reason why price competition does not lead stores to a zero price equilibrium can be seen in a two firm case. See Figure 1. Even if store j is charging a very low price, store i still can make profits by charging a higher price due to consumers' taste uncertainty. Store i can get customers in the shaded area of Figure 1 ($v_i - p_i \geq v_j - p_j$ and $v_i \geq p_i$). This is the reason that zero price equilibrium does not occur.[15] In the following, we select this unique symmetric price equilibrium as the outcome of each stage III subgame.[16]

4.2 Stage II

In this subsection, we analyze consumers' shopping location decision (stage II) which determines the market size of each location $G(d; (N_{d'})_{d' \in D})$ for any $d \in D$. First, we find which consumer can get a *positive* expected payoff from visiting location d, and later we consider consumers' choice between two locations $d, d' \in D$. The expected utility of a consumer who commutes to d where there are k stores is:

$$EU_a(d; (N_{d'})_{d' \in D}) = k \int_{p_k^*}^{b} (v - p_k^*) f(v) F(v)^{k-1} dv - t \| d - a \|.$$

Let $\mu(k, p_k^*) \equiv k \int_{p_k^*}^{b} (v - p_k^*) f(v) F(v)^{k-1} dv$ be the expected utility from shopping at a shopping center with k stores (no transportation costs). Let $r_k^* \equiv \frac{\mu(k, p_k^*)}{t}$. It is apparent that $EU_a(d; (N_{d'})_{d' \in D}) = \mu(k, p_k^*) - t \| d - a \| \geq 0$ holds if and only if $\| d - a \| \leq r_k^*$. Thus, r_k^* denotes the radius of area within which consumers can get *positive* expected payoffs by shopping at d when there are k stores at d (the potential market size). The significance of r_k^* is that consumers in the circle with its center at d and with radius r_k^* would potentially visit the shopping center d unless there is another shopping center d' which gives higher payoffs.

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[15]See de Palma et. al. (1985) and Bester (1992) as well for the relationship between dispersed tastes and positive markups.

[16]The proposition does not say that there is no asymmetric price equilibrium. If $k = 2$ and f is uniform, then we can easily show that the game is a supermodular game (see Milgrom and Roberts, 1990, Vives, 1990, and Milgrom and Shannon, 1994), and the symmetric price equilibrium is the unique equilibrium. However, in general, we cannot make such a statement. For example, with a general density function f, we can easily construct a game which violates supermodularity even if $k = 2$.

Thus, if all possible locations for stores are far apart from each other, the population in the circle determines the size of the market. We are interested in how r_k^* changes as k increases. To see this, we need to see how $\mu(k, p_k^*)$ changes as k increases. Actually, we can conveniently decompose the total change of $\mu(k, p_k^*)$ into two effects: the market size effects due to taste uncertainty and due to the lower price expectation:

$$\mu(k+1, p_{k+1}^*) - \mu(k, p_k^*) = [\mu(k+1, p_k^*) - \mu(k, p_k^*)] + [\mu(k+1, p_{k+1}^*) - \mu(k+1, p_k^*)].$$

The contents of the first bracket in the above equation denote **a market size effect due to taste uncertainty**, which is purely based on the consumers' benefits from having more variety of commodities. This effect is positive since increasing the number of options for a fixed price raises the expected utility. This effect is also described by Stahl (1982) and Wolinsky (1983). On the other hand, the contents of the second bracket is **a market size effect due to the lower price expectation**, which is described in Dudey (1990). This effect is also positive, since we know $p_k^* > p_{k+1}^*$ from Proposition 1. Consumers are attracted by the lower equilibrium prices at more concentrated shopping centers. Thus, the (total) market size effect (the LHS) contains these two effects discussed separately in the previous literature. Obviously, the (total) market size effect is positive as well, and we have the following proposition.[17]

The radius of potential market r_k^* is increasing in the number of stores k ($r_k^* < r_{k+1}^*$ for $k > 0$).

Potential customers of shopping center d are in the area of a circle centered at d with radius $r_{\#N_d}^*$. For each $d \in D$, we can draw circles. If $a \in Z$ does not belong to any circle, consumers at a do not visit any shopping center. If a belongs to only one circle, consumers there definitely visit that shopping center. If a belongs to multiple circles, then among the circles to which a belongs consumers visit the shopping center d that maximizes

$$EU_a(d; (N_{d'})_{d' \in D}) = \mu(\#N_d, p_{\#N_d}^*) - t \|d - a\| = t \left(r_{\#N_d}^* - \|d - a\| \right).$$

Thus, we can describe her choice by

$$d^*(a; (N_{d'})_{d' \in D}) \in \arg\max_{d \in D^*} t \left(r_{\#N_d}^* - \|d - a\| \right),$$

if $r_{\#N_{d^*}}^* \geq \|d^*(a; (N_{d'})_{d' \in D}) - a\|$.

4.3 Stage I

We denote the market area covered by stores at location $d \in D$ by $A(d; (N_{d'})_{d' \in D}) = \{a \in Z : d = d^*(a; (N_{d'})_{d' \in D})\}$. We also let R_k^* be the profit of a store when it shares the same location with other $k - 1$ stores (at the location there are k stores) and the market size is one. There

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[17]In the case of Schulz and Stahl (1996), the market size effect due to the lower price expectation is negative. For large ks this effect dominates the market size effect due to taste uncertainty, and the total market size effect becomes negative.

certainly exists a mixed strategy equilibrium in this game (in the first stage mixed strategies are used). However, in general, it is difficult to show the nonemptiness of equilibrium in pure strategies. Nonetheless, we can show the nonemptiness of equilibrium in pure strategies in the following three special cases: a non-overlapped market case (Proposition 3), a two shopping center case (Proposition 4), and the case where potential shopping centers are in a small area (Proposition 5). Note that we do not make any assumption on distribution of consumers $g(a)$.

First, we state Proposition 3. To guarantee non-overlapped market structure, we define the minimum distance between any pair of shopping centers: $\bar{r}_n \equiv \max_{k \in \{1,...,n\}} \left(r_k^* + r_{n-k}^* \right)$ (see Figure 3).

Suppose that D is a finite set which satisfies $\|d - d'\| \geq \bar{r}_n$ for any $d, d' \in D$ ($d \neq d'$). Then, there is an SPNE in pure strategies.

Although $\bar{r}_n$ is not a primitive, it can be uniquely calculated by utilizing unique p_k^* (and unique r_k^*). We can replace $\bar{r}_n$ by $\frac{2b}{t}$ (a condition on primitives). It is because the expected payoff is bounded above by b, and the radius of a shopping center is bounded above by $\frac{b}{t}$.

The proof is based on the potential function approach developed by Rosenthal (1973).[18] Unfortunately, this method applies only for the case of non-overlapped market (no interaction between firms at d and d' ($d \neq d'$)). The following proposition gives an alternative assumption which gives us an existence theorem:

If $\#D = 2$ (i.e., $D = \{d, d'\}$), then there is an SPNE in pure strategies.

The proof of this proposition is based on d'Aspremont et. al. (1983). The following proposition says that there is a clustering equilibrium if the area of D is small enough (see Figure 4).

Suppose that there is $d \in D$ such that $\|d - d'\| \leq r_{n-1}^* - r_1^*$ for any $d' \in D$. Then, there is an SPNE with a cluster of stores at d ($N_d = N$).

Note that Proposition 5 applies even if the area is really congested. Although the cluster of stores forces stores to cut down their prices and earn small profits, they cannot move to a nearby shopping center. No consumer would visit a deviated store, since the cluster of stores gives consumers higher expected utilities despite location differences. Of course, there could be more equilibria without clustering. However, this proposition says that if the populated area is small enough then there will be a clustering equilibrium. This proposition also says that if there are multiple potential shopping centers that satisfy the stated condition, then there are at least as many clustering equilibria as the number of those potential shopping centers. Since we assume a general population density over the plane, one of the shopping centers could be better than others in the sense that the shopping center can cover a more populated area. This is simply a coordination problem.[19] Now consider the situation that

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[18]See also Monderer and Shapley (1996) and Konishi, Le Breton, and Weber (1998). For an application in the field of industrial organization, see Slade (1994). Note that this proof could be extended to more general cases such as a multiple branch case by applying Rosenthal's (1973) technique.

[19]There is a huge literature on coordination problems. See Farrell and Saloner (1985), Katz and Shapiro (1985) and Arthur (1989) for network externalities, and Cooper and John (1988) for macroeconomic application.

there is another location d'' with small population size which is far away from d. Then, depending on the level of congestion at d, a store may want to move to d'' instead of staying at d with other stores.[20] Thus, we may say that there is a tendency for two close shopping centers to merge with each other, but an isolated shopping center can stay in business.

Note that the r_k^*'s and $\bar{r}_n$ are decreasing in the transportation parameter t. Thus, if t goes down, the condition in Proposition 3 becomes less likely to be satisfied, and the condition in Proposition 5 becomes more likely. This observation suggests us that as transportation costs decrease, concentration of stores may become more likely.

5 Examples: Double Uniform Distribution Assumption

In this section, we provide explicit calculations for a class of examples by assuming that $g : Z \to \Re_+$ is uniform distribution with $g(a) = 1$ for any $a \in Z$. The probability density function f is specified as uniform distribution over the interval $[0, 1]$ ($b = 1$). Transportation cost parameter t is normalized to one (t has only proportional scale effects on r_k^* and Π_k^*). We consider two special cases in which the SPNE is guaranteed to be nonempty: The first one is the case with non-overlapped markets (Proposition 3), and the other is the case with only two potential shopping centers (Proposition 4).

5.1 Non-Overlapped Markets

In this subsection, we assume that markets of different shopping centers $d, d' \in D$ do not overlap. This condition is satisfied, for instance, if $\|d - d'\| \geq 2$ for any $d, d' \in D$. Given the uniform distribution f, we can solve for p_k^* numerically (Lemma 1 below), which in turn determines r_k^* (Proposition 2). Equilibrium prices in the third stage are calculated as follows:

Under the hypotheses of this section, the equilibrium price p_k^*, the market size r_k^*, and each store's profit $\Pi_k^* = \left(\int_{a' \in \{a \in A : \|d - a\| \leq r_k^*\}} 1 da' \right) \times R_k^*$ for each $k = 1, 2, ...$ satisfy: (i) p_k^* is a solution of $-(p_k^*)^k - kp_k^* + 1 = 0$, (ii) $r_k^* = \frac{k}{(k+1)} (1 - p_k^* - (p_k^*)^2)$, and (iii) $\Pi_k^* = \pi(r_k^*)^2 (p_k^*)^2$. Moreover, the probability of no purchase (a consumer does *not* find a buyable commodity) is $prob(\text{No}) = (p_k^*)^k$.

The following table describes the equilibrium prices, the radius of the market, the equilibrium profits, and the probability of not finding a buyable commodity for each k:

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[20]This tendancy is observed in our numerical examples in Subsection 5.2.

k	p_k^*	r_k^*	Π_k^* ($\times 100$)	$prob$(No)
1	.5	.125	1.2272	50%
2	.41421	.27615	4.1104	17.2%
3	.32219	.4305	6.0439	3.34%
4	.24904	.55115	5.9187	0.385%
5	.19994	.6334	5.0385	0.032%
6	.16666	.69048	4.1602	-
7	.14286	.73214	3.4368	-
8	.125	.76389	2.8644	-
9	.11111	.78889	2.4137	-
10	.1	.80909	2.0566	-
13	.076923	.85165	1.3483	-
14	.071429	.8619	1.1907	-

Table 1

This table shows the trade-off between the price cutting effect and market size effect, and describes how the concentration of stores changes the stores' profits. We also attach a consumer's probability of not finding a buyable commodity ("-" in Table 1 denotes negligibly small numbers). Uniformity of g is needed to calculate Π_k^*'s. As we know from Propositions 1 and 2, price and market size move in different directions, but the market size expands sharply when k is small. As a result, the equilibrium profit goes up very quickly for small k's and attains maximum at $k = 3$. After that the profit starts to decline slowly. Till $k = 13$, the equilibrium profit is still more than in the monopoly case. This shows that the concentration effect is strong since stores make location decision noncooperatively. The probability that a consumer cannot find a buyable commodity goes down very quickly with k. This suggests that the market size effect is quite significant relative to the price cutting effect for small number of ks.

In the following, we characterize the equilibrium store distribution structure in our special case in order to see the implication for concentration of stores. Since markets are not overlapped and consumers are uniformly distributed, each store's profit is solely determined by how many competitors are at the same shopping center. Thus, without loss of generality, *a store distribution structure* can be essentially described by a list of integers $\{n_1, n_2, ..., n_\ell\}$, where (i) $n_j > 0$, (ii) $\sum_{j=1}^{\ell} n_j = n$, and (iii) ℓ is a positive integer. Actually, it is easy to characterize every Nash equilibrium in this particular case.

Under the hypotheses of this section, a store distribution structure $\{n_1, n_2, ..., n_\ell\}$ is an equilibrium store distribution structure if and only if (i) $\max\{n, 3\} \leq n_j \leq 13$ for any $j = 1, ..., \ell$, and (ii) $|n_j - n_k| \leq 1$ for any $j, k = 1, 2, ..., \ell$.

The first condition says that every shopping center has the profit maximizing number of stores ($k = 3$) or more if n is not less than three. The second condition asserts that equilibrium number of stores can differ at most by one among nonempty shopping centers

(so at most two sizes can be observed). Thus, in this particular case, we can claim that shopping centers will be more or less homogeneous and the number of stores at each shopping center would be no less than three (if $n \geq 3$). For example, if $n = 13$, there could be a few equilibria. One is a grand coalition $\{13\}$, others are $\{6,7\}$, $\{4,4,5\}$ and $\{3,3,3,4\}$. Obviously, the average profit is highest under $\{3,3,3,4\}$ and lowest under $\{13\}$. These equilibria are therefore Pareto ranked from the stores' perspectives. On the other hand, if $n = 14$, the grand coalition vanishes, and there are only two equilibria $\{7,7\}$, $\{4,5,5\}$ and $\{3,3,4,4\}$. The reason why $\{14\}$ is not an equilibrium is that every store has an incentive to move out the grand coalition. Then, can $\{1, 13\}$ be an equilibrium? The answer is no. Since $k = 2$ is much more profitable than $k = 13$, a store will move from the bigger shopping center to the smaller.

5.2   Two Potential Shopping Centers

In this subsection, we assume that there are only two potential shopping centers d and d' but there could be market overlap between them. Let δ be the distance between d and d'. We first set $n = 10$, and find equilibria for various δs. If $\delta \leq \bar{r}_{10} = 2r_5^* = 1.2668$, then markets may overlap with each other. Thus, we find equilibria in the cases of $\delta = 0.2$, 0.4, ..., 1.2. We find only two types of equilibria: a symmetric equilibrium (each shopping center has the same number of stores) or a clustering equilibrium (one of the shopping centers has all the stores). There is no asymmetric non-clustering equilibrium in this example. The numbers in the column describe each store's equilibrium profit times 100.

δ	symmetric eq	clustering eq
0.2	None	2.0566
0.4	3.5149	2.0566
0.6	3.9795	2.0566
0.8	4.4007	2.0566
1.0	4.7556	2.0566
1.2	5.0021	2.0566
1.2668∼	5.0385	2.0566

Table 2-1$(n = 10)$

The bottom row corresponds to the non-overlapped market case. In Table 2-1, we can make a few observations: First, each store's profit is higher in a symmetric equilibrium (if it exists). It is not surprising from the profit levels of the non-overlapped market case. Second, there is always a clustering equilibrium. It is because the number of stores is small. From Table 1, we know that even if there is no overlap, a ten store cluster gives each store a higher profit than standing alone (for any $n \leq 13$, $\Pi_n^* > \Pi_1^*$ holds). Thus, there is no incentive for stores to deviate from the cluster unilaterally (with overlap, a deviator may get even less than Π_1^*). Third, there is no symmetric equilibrium for $\delta = 0.2$. This is because the number of stores is pretty small. In a symmetric allocation, there are five stores each. If a store moves from one

shopping center to the other, then the allocation becomes six stores and four stores. Since the two shopping centers are very close to each other, the six store shopping center can steal a lot of customers from the four store shopping center (see Figure 5). This is the reason why there is no symmetric equilibrium in this case. This observation says that if the two shopping centers are very close and the number of stores is small, then incentive for stores to concentrate is stronger due to competition between shopping centers.

In the next table, we assume that $n = 20$. We focus on the case where $\delta \leq \bar{r}_{20} = 2r_{10}^* = 1.61818$. Again, we find only symmetric and clustering equilibria.

δ	symmetric eq	clustering eq
0.2	1.1897	0.6395
0.4	1.3486	0.6395
0.6	1.5024	0.6395
0.8	1.6481	None
1.0	1.7824	None
1.2	1.9008	None
1.4	1.9967	None
1.6	2.0551	None
1.61818~	2.0566	None

Table 2-2($n = 20$)

In this case, the number of stores is relatively large in the sense that a clustering allocation is less attractive for each store than standing alone, and there is no clustering equilibrium in the non-overlapped market case. There are two main differences from Table 2-1. First, there is a symmetric equilibrium in the case of $\delta = 0.2$. It is because a concentration of 11 stores is not much better than a concentration of 9 stores when competing for customers. Thus, two ten store shopping centers can coexist even when two shopping centers are very close to each other. Second, there is no clustering equilibrium for δ more than 0.8. It is because the clustering allocation is less attractive than standing alone. So, if the two shopping centers are far enough from each other, then a clustering allocation cannot be supported as an equilibrium.

We list two other cases for interested readers. First, there can be an asymmetric equilibrium. When $n = 14$ and $\delta = 0.2$, there are three types of equilibria: a symmetric equilibrium $\{7, 7\}$ (profit: 2.0162), clustering equilibria $\{14, 0\}$ (profit: 1.1907), and asymmetric equilibria $\{5, 9\}$ (profit: 2.3186 for 5 stores, 2.0265 for 9 stores). Interestingly, an asymmetric equilibrium Pareto-dominates a symmetric equilibrium and a clustering equilibrium in this particular case. Second, a clustering equilibrium may survive even without the sufficient condition in Proposition 5. By the argument in the case of $n = 10$, we know that a clustering equilibrium always exists when $n \leq 13$. However, if $n > 13$ and if the sufficient condition is violated, we do not know if a clustering equilibrium exists. The following example says that it may. When $n = 14$, the sufficient condition is $\delta \leq r_{13}^* - r_1^* = 0.7266$. We find that even if $\delta = 0.8$, there is a clustering equilibrium in this case. It is because a deviating store

from the cluster may not be able to attract many customers due to competition between two shopping centers (see Figure 6). [21]

6 Discussion

6.1 Outside Opportunities

In this subsection, we introduce outside opportunities such as mail order companies into the model. The presence of outside options raises the consumer's reservation utility, and hence affects the concentration of stores. We model a mail order market by adopting Wolinsky's (1986) framework. [22] Thus, a consumer can search commodities sequentially within the mail order market by purchasing and returning them. There is an additional transaction cost $c > 0$ for each purchasing and returning such as a waiting cost and/or a mailing cost. Our interest is how the presence of mail order shopping affects the degree of concentration of stores. The structure of mail order market turns out to be the same as the one of Fischer and Harrington's (1996) periphery market. Thus, we adopt their assumptions in this subsection. There is only one shopping center (a cluster) in the economy ($D = \{d\}$), and each consumer has mail order purchase as an outside option (a periphery). Her willingness-to-pay is an i.i.d. random variable drawn from a uniform distribution on $[0, 1]$ for any commodity sold at any store (irrespective of a mail order company or a store at the shopping center), and she cannot realize her willingness-to-pay for a commodity before visiting center or before the purchase if it is a mail order commodity. In the second stage of the game, each consumer chooses one of the following options: (i) choose a shopping center to visit first (and then move to mail order shopping if the realizations of willingnesses-to-pay are not good), (ii) choose mail order shopping (and stay in it: do a sequential search within the mail order market), and (iii) choose not to shop at all. If she purchase a commodity from a mail order company then she needs to pay the transaction cost c in addition to the equilibrium price in the mail order market $\bar{p}(c)$. If she returns a commodity, then $\bar{p}(c)$ is refunded and she only needs to pay c at this search. Kohn and Shavell (1974) and Wolinsky (1986) show that the optimal strategy in this mail order market is a sequential search with a fixed stopping rule. Let $R(c)$ be the critical value for $c > 0$: i.e., if the realized willingness-to-pay of the first commodity is more than $R(c)$, she stops searching and keeps it; otherwise she returns the commodity and continues to search with the same critical value $R(c)$. We have the following

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[21]This obsevation may not be robust for a larger number of stores case, since the cluster would have a weaker competitive power against a deviating store. Actually, if $n = 20$, the sufficient condition in Proposition 5 is $\delta \leq r_{19}^* - r_1^* = 0.772368$. However, we found that for $\delta = 0.772369$, there is already no clustering equilibrium. It is probably because a store's deviation would pay even if the resulting market area for the store after the deviation is very small (the profit level being at the cluster is too low).

[22]In this subsection, following Fischer and Harrington (1996), we assume that a consumer can try mail order commodities infinite times sequentially. Wolinsky (1986) actually analyzes the case where the number of stores is finite (Anderson and Renault, 1998, as well). The result in this subsection would not be affected much by changing this simplifying assumption. If the number of stores is finite, then $\tilde{u}(c)$ would be pushed down more for each c.

results in the mail order market.

(Wolinsky, 1986) For each commodity, the critical value is $R(c) = 1 - \sqrt{2c}$, the equilibrium price $\bar{p}$ satisfies $\bar{p}(c) = c + \sqrt{2c}$, and the expected utility of trying mail order shopping given c is $\tilde{u}(c) = 1 - c - 2\sqrt{2c}$.

Thus, each consumer is guaranteed to get an expected utility $\tilde{u}(c)$ by using mail order companies (the reservation utility level). This changes our analysis slightly: Let $D^{**} \equiv \{d\} \cup \{\emptyset\} \cup \{M\}$, where M denotes the mail order option. Note that $N_d = N$ since $D = \{d\}$. Thus, $d^*(a; (N_{d'})_{d' \in D})$ and $\arg\max_{d \in D} EU_a(d; (N_{d'})_{d' \in D})$ can be written as $d^*(a)$ and $EU_a(d)$, respectively. We need to modify the definition of $d^*(a) \in D^{**}$ in the following way (originally defined in the description of the game (in Stage II)):

(i') $d^*(a) = \{d\}$,

if $EU_a(d) \geq \max\{\tilde{u}(c), 0\}$ (if she visits the shopping center first),

(ii') $d^*(a) = \{M\}$,

if $\tilde{u}(c) \geq 0$ and $EU_a(d) < \tilde{u}(c)$ (she uses mail order shopping), and

(iii') $d^*(a) = \emptyset$,

if $\tilde{u}(c) < 0$ and $EU_a(d) < 0$ (she does not shop at all).

Note that even if a consumer chooses to shop at the shopping center, it *does not* mean that she does not use mail order shopping. If she does not find a commodity that gives her more than $\tilde{u}(c)$ she simply does not buy any commodity at d and tries mail order shopping. Stores at the shopping center take this possibility into account, and compete for customers. Thus, Lemma 1 needs to be modified as follows (the proof is similar to the one of Lemma 1, so omitted):

Suppose that $\tilde{u}(c) \geq 0$. Under the hypotheses of this subsection, p_k^*, r_k^*, and Π_k^* for each $k = 1, 2, \ldots$ satisfy: (i) p_k^* is a solution of $-(p_k^* + \tilde{u}(c))^k - k p_k^* + 1 = 0$, (ii) $r_k^* = \max\left\{0, \frac{1}{k+1}\left[k - (k+1)p_k^* - (p_k^*)^{k+1}\right] - \tilde{u}(c)\right\}$, and (iii) $\Pi_k^* = \pi(r_k^*)^2(p_k^*)^2$.

This modification changes the optimal number of stores at a location, namely increases the optimal number of stores. The profit maximizing number of stores given c, $\tilde{k}(c)$, is described in the following table:

c	$\tilde{u}(c)$	$\tilde{k}(c)$
0.0279	0.5	7
0.0393	0.4	6
0.0524	0.3	5
0.0671	0.2	4
0.0834	0.1	4
0.1010	0	3

Table 3

Notice that for a relatively small c, $\tilde{u}(c)$ is already very small. If c is more than 0.1010, then $\tilde{u}(c) < 0$ follows and no consumer use mail order companies (and so the previous result

holds, see Table 1). As c goes to zero (transaction costs of mail order become less and less costly), r_k^* and p_k^* decrease. As a result, stores have more incentive to concentrate at the same shopping center to provide greater variety of commodities to attract consumers who could have chosen mail orders. If $c \leq 0.0279$, shopping centers with less than three stores can attract no customers. Therefore, if $c = 0.0279$, at least three stores need to concentrate to compete with mail order companies, and the optimal number of stores is seven. Table 3 tells us the general tendency that the higher expected utility mail order companies offer, the more incentives for stores to agglomerate. The introduction of a mail order option makes possible the following: If a consumer is living at a remote location that is far from a small city with a limited variety of stores, then she is likely to use mail orders. On the other hand, if a consumer lives at another remote location that is far from a big city with a huge variety of stores, then she might rather visit the city to buy commodities.

Finally, we discuss Fischer and Harrington (1996). They analyze nonemptiness of *free entry equilibrium* in the model discussed in this subsection with an entry cost for each store. In a free entry equilibrium, no store at the shopping center wants to switch to a mail order company, and no mail order company wants to switch to a store at the shopping center. Moreover, every store in the market obtains nonnegative profit, and no other store wants to enter the market. Fischer and Harrington (1996) prove the nonemptiness of free entry equilibrium by using a model similar to d'Aspremont et al. (1983). It is possible to generalize their result to cover more general (non-uniform) willingness-to-pay distributions by utilizing our Propositions 1 and 2.

6.2   Sequential Search Over Shopping Centers

In this paper, we have assumed that consumers can search at most one shopping center. This is obviously an unsatisfactory assumption. There are papers that allow consumers' sequential search in various context (Diamond, 1971, and Anderson and Renault, 1998, among others). However, it turns out to be a difficult task to incorporate sequential search into a model with an explicit geographical structure when consumers are uninformed about prices and their willingnesses-to-pay. A consumer faces multiple options after she searches a shopping center.[23] Her decision is based not only on the vector of the realized willingnesses-to-pay in the past search, but also the location of her residence. Taking such a search behavior into account, stores need to decide their prices given other stores' prices. This is a complicated decision problem.

However, we can show that there is an incentive for stores to concentrate even in such a setting by an example. Consider a double uniform distribution example with only two stores and assume $D = Z$ (stores can locate anywhere: they can be as close to each other as they want, or they can choose the same location as before). Given the distance between two stores δ, we can calculate a symmetric equilibrium profit. If equilibrium profit is maximized

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[23]The options are as follows: (i) she buys one of the commodities sold at the shopping center, (ii) she goes home without buying any commodity, (iii) she continues to search elsewhere, and (iv) she goes back to one of (already visited) shopping centers in order to buy a commodity (and goes home).

at $\delta = 0$, then we can say that stores have an incentive to be together. If δ is more than $\frac{1}{4}$, markets are not overlapped and stores get the monopoly profits Π_1^* in Table 1 (recall $r_1^* = \frac{1}{8}$). If δ is less than $\frac{1}{4}$ yet more than $\frac{1}{8}$, then still no consumer searches sequentially even if she cannot find a buyable commodity at the store that she visited first. If δ is less than $\frac{1}{8}$, some consumers in the middle of two stores start to search both stores sequentially (potentially: if they could not find a buyable commodity at the first store). The region where such consumers reside expands gradually as δ becomes smaller. When δ reaches $\frac{1}{10}$, every consumer starts a sequential search potentially, and the market becomes more competitive. As δ falls even further, the number of consumers who quit searching after visiting the first store becomes less and less, and a consumer gets a higher expected utility from searching. This expands the market size. Moreover, the equilibrium price must go down due to fiercer competition, and the market size expands even more. It turns out that the market size effect is much bigger than the price cutting effect as before (Table 1), and the equilibrium profit is maximized at $\delta = 0$. The equilibrium price, market size, and profit at $\delta = 0$ are p_2^*, r_2^* and Π_2^* in Table 1, respectively. Therefore, store's concentration can be expected even if consumers can visit multiple shopping centers.

6.3 Shopping Malls As an Exclusion Device[24]

An example in the previous section ($n = 13$) says that there could be many equilibria in this game. And if $n = 5$, there is a unique equilibrium ($\{5\}$), but it is not an efficient allocation for stores. For example, the allocation $\{3, 2\}$ generates higher payoffs for all stores. However, this allocation is not an equilibrium since any store in the two store location wants to move to the other. The main problem here is free mobility of stores, which prevents stores from coordinating. Stahl (1982) and Dudey (1993) discuss the possibilities and impossibilities of coordination devices. In this subsection, we discuss through our model why shopping malls usually restrict the number of stores in the same category by using land-developers' entrepreneurship.[25]

Shopping malls are owned by entrepreneurs. Thus, it is for the owners' interest to maximize total rent revenue. Suppose that each store has reservation profit level Π (outside opportunity for stores: not specified here). Then, the owner of a shopping mall can extract $\Pi_k^* - \Pi$ from each store if she admits k stores in her mall. As a result, she can earn $k \times (\Pi_k^* - \Pi)$ as rent revenue. If $\Pi = 0$, then the maximum rent revenue will be attained by setting $k = 5$. However, if $\Pi = 2$, then $k = 4$ is optimal for her. Note that restriction on the number of stores effectively increase the profits of stores. This relaxes the coordination failure problem. Since the optimal number of stores for the stores at a shopping center are in general different from the one for the owner of the shopping center, it is not easy to see the welfare consequence. In any case, shopping malls' standard practice of restricting the number of the same type of stores can be justified in our framework.

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[24]I thank Michael Manove for suggesting me the argument in this subsection.

[25]See Henderson (1974) for the land-developers' entrepreneurship in the context of local public goods economy.

7   Appendix

7.1   Proof of Proposition 1

First, we prove the existence of a symmetric price equilibrium price p_k^* for each $k = 1, 2, ...$
We will show that no store has an incentive to deviate from p_k^* given that every other store
chooses p_k^*. Since we can prove the existence of equilibrium without assuming independence
of distribution, we define the joint probability density on the space $B^k = [0, b]^k$ ($k = 1, ..., n$).
Let $h^k : B^k \to \Re_+$ be probability density. We say $h^k(v^k)$ is *symmetric* iff $h^k(v^k) = h^k(\tilde{v}^k)$ for
any $v^k \in B^k$ and any permutation $\tilde{v}^k$ of v^k. The second condition represents the symmetry of
probability density. Let $R_i^k(p_i, p)$ be such that the profit that store i makes from market size 1
(Thus the profit of store i at shopping center d is $G(d; (N_{d'})_{d' \in D}) \times R_i^k(p_i, p)$) when other firms
at d is choosing p and store i chooses p_i, given that there are k stores at d. Given $p_i \in [0, b]$
and $p \in [0, b]$, the set of consumers who buy commodity i is described by $B_i^k(p_i, p) \equiv \{v^k \in
B^k : v_i \geq p_i$ and $v_i - p_i \geq v_j - p$ for any $j \neq i\}$. Thus, $R_i^k(p_i, p) = p_i \times \int_{B_i^k(p_i,p)} h^k(v^k)dv^k$.
We prove the existence of a symmetric equilibrium by utilizing Prékopa's theorem:

The Prékopa Theorem (Prékopa (1973)). *Let ψ be a probability density function on $\Re^m$
with convex support C. Take any measurable sets A_0 and A_1 in $\Re^m$ with $A_0 \cap C \neq \emptyset$ and
$A_1 \cap C \neq \emptyset$. For any $0 \leq \lambda \leq 1$, define $A_\lambda = (1 - \lambda)A_0 + \lambda A_1$, the Minkowski average of the
two sets.*[26] *If $\psi(\alpha)$ is log concave, then*

$$\log \int_{A_\lambda} \psi(\alpha)d\alpha \geq (1 - \lambda)\log \int_{A_0} \psi(\alpha)d\alpha + \lambda \log \int_{A_1} \psi(\alpha)d\alpha.$$

We prove the following theorem by utilizing the Prékopa theorem:

Theorem A.1. *There is a symmetric price equilibrium in the third stage if $h^k(v^k)$ is log
concave and has convex support on B^k and symmetric for any $k = 1, 2, ...$.*

It is easy to see that $B_i^k((1 - \lambda)p_i + \lambda p_i', p) \supseteq (1 - \lambda)B_i^k(p_i, p) + \lambda B_i^k(p_i', p)$ for any
$p, p_i, p_i' \in [0, b]$.[27] By using the Prékopa Theorem, we obtain:

$$\log \int_{B_i^k((1-\lambda)p_i+\lambda p_i',p)} h^k(v^k)dv \geq \log \int_{(1-\lambda)B_i^k(p_i,p)+\lambda B_i^k(p_i',p)} h^k(v^k)dv$$
$$\geq (1 - \lambda)\log \int_{B_i^k(p_i,p)} h^k(v^k)dv + \lambda \log \int_{B_i^k(p_i',p)} h^k(v^k)dv.$$

Thus, we conclude that $\int_{B_i^k(p_i,p)} f(v^k)dv^k$ is log concave in p_i. Since $\log R_i^k(p_i, p) = \log p_i +$
$\log \int_{B_i^k(p_i,p)} h^k(v^k)dv$, $R_i^k(p_i, p)$ is log concave in p_i as well. This implies that $R_i^k(p_i, p)$ is quasi-
concave in p_i. Since $h^k(v^k)$ is a density function, $R_i^k(p_i, p)$ is continuous. By the Maximum

---

[26]The Minkowski average A_λ is defined as all points of the form $x_\lambda = (1 - \lambda)x_0 + \lambda x_1$, with $x_0 \in A_0$,
$x_1 \in A_1$, and $0 \leq \lambda \leq 1$.

[27]The first inclusion could be strict if $p_i < p < p_i'$.

Theorem, the best response correspondence $\beta_i : [0, b] \rightarrow [0, b]$ is upper hemi-continuous. Quasi-concavity of R_i^k implies convex-valuedness of β_i. By the Kakutani Theorem, there is a fixed point $p_k^* \in [0, b]$ such that $p_k^* \in \beta_i(p_k^*)$.

This result is a special case of a result in Dierker (1991). We provided a simple proof of the result in this special case. Caplin and Nalebuff's (1991) technique is also very closely related. The easiest reference for this result is Theorem 6.3 (and the following discussion on page 168) in Anderson, de Palma, and Thisse (1992).

Now, we turn to the uniqueness and the inverse relationship between the number of stores and the equilibrium prices. From here on, we need statistical independence of $\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n$ and Assumption 2. We derive explicit formulas for $R_i^k(p_i, p)$. Since there is some difference between raising the price more than others and lowering the price less than others, we analyze these two cases separately. Let $R_i^{k+}(p_i, p)$ be store i's profit given that the market size is equal to 1 when $p_i \geq p$. This can be written as follows (see Figure 1):

$$R_i^{k+}(p_i, p) = p_i \int_{p_i}^{b} f(v_i) F(v_i - (p_i - p))^{k-1} dv_i,$$

where $F(u) = \int_0^u f(v_i) dv_i$. Note that $F(v_i - (p_i - p))$ denotes the probability that a consumer prefers type i commodity than type $j(\neq i)$ commodity given that her realization of $\mathbf{v}_i$ is v_i. Thus, $F(v_i - (p_i - p))^{k-1}$ denotes the probability that a consumer who prefers type i commodity to any other type at location d given that her realization of $\mathbf{v}_i$ is v_i. Next, we consider the other case. Let $R_i^{k-}(p_i, p)$ be store i's profit given that the market size is equal to 1 when other firms at d is choosing p and store i chooses $p_i \leq p$. This can be written as follows (see Figure 2):

$$R_i^{k-}(p_i, p) = p_i \int_{p_i}^{b-(p-p_i)} f(v_i) F(v_i - (p_i - p))^{k-1} dv_i + \int_{b-(p-p_i)}^{b} f(v_i) dv_i.$$

The last term shows that if $v_i \in [b - (p_i - p), b]$, then with probability one consumers buy type i goods $(f(v_i) \int_{b-(p-p_i)}^{b} 1^{k-1} dv_i)$. It is easy to see that $R_i^{k+}(p, p) = R_i^{k-}(p, p)$. Moreover, by letting $\varepsilon = p_i - p$ go to zero we have the following:

$$
\begin{aligned}
\varphi_k(p) &= \left. \frac{\partial R_i^{k+}(p_i, p)}{\partial p_i} \right|_{p_i = p} \\
&= \left. \frac{\partial R_i^{k-}(p_i, p)}{\partial p_i} \right|_{p_i = p} \\
&= \int_p^b f(v) F(v)^{k-1} dv - p f(p) F(p)^{k-1} - (k-1) p \int_p^b f(v)^2 F(v)^{k-2} dv.
\end{aligned}
$$

This is a continuous function in the interval $[0, b]$. Note that every symmetric price equilibrium must satisfy the first order condition $\varphi_k(p_k^*) = 0$. Thus, if $\varphi_k(p)$ is a monotonic function, then we can conclude that there is at most one p_k^* with $\varphi_k(p_k^*) = 0$, which is the

unique candidate for a symmetric equilibrium price. From Theorem A.1, there exists a symmetric equilibrium price, and p_k^* must be the unique symmetric equilibrium price. It is easy to see $\varphi_k(0) = \int_0^b f(v)F(v)^{k-1}dv > 0$ and $\varphi_k(b) = -bf(b) < 0$. Thus, we only need to show that φ_k is non-increasing. Differentiating φ_k we obtain:

$$
\begin{aligned}
\varphi_k'(p) &= -f(p)F(p)^{k-1} - f(p)F(p)^{k-1} - pf'(p)F(p)^{k-1} - (k-1)pf(p)^2F(p)^{k-2} \\
&\quad -(k-1)\int_p^b f(v)^2F(v)^{k-2}dv + (k-1)pf(p)^2F(p)^{k-2} \\
&= -2f(p)F(p)^{k-1} - pf'(p)F(p)^{k-1} - (k-1)\int_p^b f(v)^2F(v)^{k-2}dv \\
&= -f(p)F(p)^{k-1}\left(2 + \frac{pf'(p)}{f(p)}\right) - (k-1)\int_p^b f(v)^2F(v)^{k-2}dv.
\end{aligned}
$$

Thus, $\varphi_k'(p) < 0$ follows for any $p \in [0, b)$, since $-2 \leq \frac{pf'(p)}{f(p)}$ is guaranteed by Assumption 2 ($\varphi_k'(p) = 0$ would not happen because the last term of $\varphi_k'(p)$ is negative for any $p \in [0, b)$). Therefore, there is unique symmetric equilibrium price $p_k^* \in (0, 1)$ which satisfies $\varphi_k(p_k^*) = 0$.

Finally, we show $p_k^* > p_{k+1}^*$ for each $k = 1, 2, \dots$. By integrating the last term of $\varphi_k(p_k^*)$ by parts, we obtain:

$$
\begin{aligned}
\varphi_k(p_k^*) &= \int_{p_k^*}^b f(v)F(v)^{k-1}dv - p_k^* f(p_k^*)F(p_k^*)^{k-1} \\
&\quad -p_k^*\left\{\left[f(v)F(v)^{k-1}\right]_{p_k^*}^b - \int_{p_k^*}^b f'(v)F(v)^{k-1}dv\right\} \\
&= \int_{p_k^*}^b f(v)F(v)^{k-1}dv - p_k^* f(p_k^*)F(p_k^*)^{k-1} \\
&\quad -p_k^* f(b) + p_k^* f(p_k^*)F(p_k^*)^{k-1} + p_k^*\int_{p_k^*}^b f'(v)F(v)^{k-1}dv \\
&= \int_{p_k^*}^b f(v)F(v)^{k-1}dv + p_k^*\int_{p_k^*}^b f'(v)F(v)^{k-1}dv - p_k^* f(b) \\
&= \int_{p_k^*}^b f(v)F(v)^{k-1}\left(1 - \frac{p_k^* f'(v)}{f(v)}\right)dv - p_k^* f(b).
\end{aligned}
$$

If we can show $\varphi_{k+1}(p_k^*) < 0$, then we can show $p_{k+1}^* < p_k^*$ since $\varphi_{k+1}(p)$ is non-increasing in p. However, since $\varphi_k(p_k^*) = 0$, it suffices to show $\varphi_{k+1}(p_k^*) < \varphi_k(p_k^*)$. We demonstrate that in the following:

$$
\varphi_k(p_k^*) - \varphi_{k+1}(p_k^*) = \int_{p_k^*}^b f(v)F(v)^{k-1}(1 - F(v))\left(1 - \frac{p_k^* f'(v)}{f(v)}\right)dv.
$$

It is easy to see that $\varphi_k(p_k^*) - \varphi_{k+1}(p_k^*) \geq 0$ follows if $1 \geq \frac{p_k^* f'(v)}{f(v)}$ for any $v \geq p_k^*$. Assumption 2 supposes that $1 \geq \frac{v f'(v)}{f(v)}$ for any $v \in [0, b]$. Thus, for any $v > p_k^*$, $1 \geq \frac{v f'(v)}{f(v)} > \frac{p_k^* f'(v)}{f(v)}$ must follow if $f'(v) > 0$. If $f'(v) \leq 0$, then $1 - \frac{p_k^* f'(v)}{f(v)} > 0$ anyway. Therefore, we conclude $p_k^* > p_{k+1}^*$. This completes the proof of Proposition 1. ∎

7.2 Proof of Proposition 3

Since $\|d - d'\| \geq \bar{r}_n$ for any pair $d, d' \in D$ ($d \neq d'$), the assumption in the statement guarantees no overlap among market areas for any strategy configuration. This implies $A(d; (N_{d'})_{d' \in D}) = \{a \in A : \|d - a\| \leq r_{\#N_d}^*\}$. Given that it is easy to see that for each $d \in D$, for each k, we can determine the profits of each store at d : $\Pi_d(k) = R_k^* \int_{a \in A(d; (N_{d'})_{d' \in D})} g(a) da$. Let $\Psi((N_d)_{d \in D}) = \sum_{d \in D} \sum_{k=1}^{\#N_d} \Pi_d(k)$. We claim that the strategy configuration which maximizes Ψ is an SPNE: $(N_d^*)_{d \in D} \in \arg\max_{(N_d)_{d \in D}} \Psi((N_d)_{d \in D})$. Suppose not. Then, there are $d', d'' \in D$ with $d' \neq d''$ and $\Pi_{d'}(\#N_{d'}) < \Pi_{d''}(\#N_{d''} + 1)$. Now, consider a strategy configuration $(N_d')_{d \in D}$ which satisfies: (i) For any $d \neq d', d''$, $N_d' = N_d^*$, (ii) $N_{d'}' = N_{d'}^* \backslash \{i\}$, and (iii) $N_{d''}' = N_{d''}^* \cup \{i\}$ for some $i \in N_{d'}^*$. Then,

$$
\begin{aligned}
\Psi((N_d')_{d \in D}) &= \sum_{d \in D} \sum_{k=1}^{\#N_d'} \Pi_d(k) \\
&= \sum_{d \in D} \sum_{k=1}^{\#N_d^*} \Pi_d(k) - \Pi_{d'}(\#N_{d'}^*) + \Pi_{d''}(\#N_{d''}^* + 1) \\
&= \Psi((N_d^*)_{d \in D}) - \Pi_{d'}(\#N_{d'}^*) + \Pi_{d''}(\#N_{d''}^* + 1) \\
&> \Psi((N_d^*)_{d \in D}).
\end{aligned}
$$

This is a contradiction to the definition of $(N_d^*)_{d \in D}$. Therefore, $(N_d^*)_{d \in D}$ is an SPNE. ∎

7.3 Proof of Proposition 4

In this special case, the number of firms at d, say $k = \#N_d$, describe the pattern of strategy configurations, and payoffs are determined solely by k. The payoffs of stores at d and d' are described by

$$
\bar{\Pi}_d(k) = R_k^* F(d; (N_d, N_{d'})) = R_k^* \int_{a \in A(d; (N_d, N_{d'}))} g(a) da,
$$

$$
\bar{\Pi}_{d'}(k) = R_{n-k}^* F(d'; (N_d, N_{d'})) = R_{n-k}^* \int_{a \in A(d'; (N_d, N_{d'}))} g(a) da,
$$

where $A(d; (N_d, N_{d'})) = \{a' \in Z : \|d - a'\| \leq r_k^*, \ r_k^* - \|d - a'\| \geq r_{n-k}^* - \|d' - a'\|\}$, and $A(d'; (N_d, N_{d'})) = \{a' \in Z : \|d' - a'\| \leq r_{n-k}^*, \ r_{n-k}^* - \|d' - a'\| \geq r_k^* - \|d - a'\|\}$. Let us consider the case where $k = 0$ (all stores are at d', and no store is at d). If $\bar{\Pi}_{d'}(0) \geq \bar{\Pi}_d(1)$, then $k = 0$ is an equilibrium (of the first stage of the game). Thus, we assume $\bar{\Pi}_{d'}(0) < \bar{\Pi}_d(1)$.

Now, one store is moved to d ($k = 1$). If $\bar{\Pi}_{d'}(1) \geq \bar{\Pi}_d(2)$, then it is an equilibrium since a store at d does not want to move back to d' by the assumption. Thus, we assume $\bar{\Pi}_{d'}(1) < \bar{\Pi}_d(2)$. We move one additional store to d ($k = 2$). Again, if $\bar{\Pi}_{d'}(2) \geq \bar{\Pi}_d(3)$, then it is an equilibrium by the same reason, and we assume $\bar{\Pi}_{d'}(2) < \bar{\Pi}_d(3)$, and so on and so forth. If all of $k = 0, 1, 2$, ..., $n-1$ violate $\bar{\Pi}_{d'}(k) \geq \bar{\Pi}_d(k+1)$, then we conclude $\bar{\Pi}_{d'}(n-1) \leq \bar{\Pi}_d(n)$. This implies that if all stores choose d, then no store wants to deviate from the allocation. Hence, there is an SPNE in this game.■

7.4 Proof of Proposition 5

We only need to show that if all n stores are at d, no store wants to move to any $d' \neq d$ unilaterally. Suppose that store i moves to d' alone. Then, store i's potential market is a circle with its center at d' and radius r_1^*. However, the shopping center d's potential market area is a circle with its center at d and radius r_{n-1}^*, and the latter circle contains the former circle. This implies that store i cannot get any customer by this move, and it does not have an incentive to leave the cluster. Thus, there is an SPNE with a cluster at d.■

7.5 Proof of Lemma 1

We utilize the first order condition for profit maximization defined in the proof of Proposition 1. Given that $b = 1$ and $f(v) = 1$ for any $v \in [0, 1]$, we can write:

$$
\begin{aligned}
\varphi_k(p) &= \int_p^1 v^{k-1} dv - pp^{k-1} - (k-1)p \int_p^1 v^{k-2} dv \\
&= \left[\frac{v^k}{k} \right]_p^1 - p^k - p \left[v^{k-1} \right]_p^1 \\
&= \frac{1}{k} \left(-p^k - kp + 1 \right).
\end{aligned}
$$

Thus, p_k^* is implicitly defined by $\varphi_k(p_k^*) = 0$ for each k. We calculate $\mu(k, p_k^*)$ by utilizing this relationship.

$$
\begin{aligned}
\mu(k, p_k^*) &= k \int_{p_k^*}^1 v^{k-1}(v - p_k^*) dv \\
&= k \left[\frac{v^{k+1}}{k+1} - \frac{p_k^* v^k}{k} \right]_{p_k^*}^1 \\
&= k \left[\frac{1}{k+1} - \frac{p_k^*}{k} - \frac{(p_k^*)^{k+1}}{k+1} + \frac{(p_k^*)^{k+1}}{k} \right] \\
&= \frac{k}{k+1} \left[1 - p_k^* - (p_k^*)^2 \right].
\end{aligned}
$$

In the last transformation, used $\frac{1}{k}\left(-(p_k^*)^k - kp_k^* + 1\right) = 0$. Finally, we turn to the equilibrium profit Π_k^*. Note that $\Pi_k^* = \left(\int_{a' \in \{a \in A : \|d-a\| \le r_k^*\}} 1 da'\right) \times (R_k^*)$. It is easy to see the value in the parenthesis is $\pi(r_k^*)^2$. Thus, what is left is to calculate R_k^*. By using the expression in the proof of Proposition 1, we obtain:

$$R_k^* = R_i^{k+}(p_k^*, p_k^*) = p_k^* \int_{p_k^*}^1 (v)^{k-1} dv = p_k^* \left(\frac{1 - (p_k^*)^k}{k}\right) = (p_k^*)^2.$$

In the last transformation, we used $\frac{1}{k}\left(-(p_k^*)^k - kp_k^* + 1\right) = 0$. We completed the proof.■

7.6 Proof of Proposition 6

The proof is a variation of the one in Conley and Konishi (1998). It is easy to see that a store distribution structure which satisfies (i) and (ii) is an equilibrium store distribution structure. Thus, we concentrate on the other direction. First, note that the relevant range of k is $1 \le k \le 13$, since if $k \ge 14$ then a store tries to be independent since Z is assumed to be large enough. Second, notice that Π_k^* is single-peaked at $k = 3$ in the relevant range (actually, it is globally single-peaked). Third, Π_k^* increases in k very quickly till $k \le 3$ and goes down slowly. By using these observations, we will characterize the set of equilibria. Since it is trivial to see the statement is true for $n < 3$, we assume $n \ge 3$ in the following.

We claim that there are at most two sizes of shopping centers in an equilibrium. Suppose that there are three sizes $k < k' < k''$. If $k \ge 3$, then a store in a size k'' shopping center joins a size k shopping center. Thus, it cannot happen in the equilibrium. Thus, $k < 3$ holds. First, let us assume that $k = 2$. Then, $k'' \ge 4$ must hold. As a result, a store in a size k'' shopping center joins a size k shopping center. This implies $k = 1$. However, since a firm in a size k shopping center does not move to a size k' shopping center, we have $\Pi_1^* \ge \Pi_{k'+1}^* \ge \Pi_{k''}^*$. Since we know $\Pi_2^* > \Pi_1^*$, we conclude $\Pi_2^* > \Pi_{k''}^*$, and a store in a size k'' shopping center joins a size k shopping center. This is a contradiction. Hence, there are at most two sizes of shopping centers.

Now, let these two sizes be k, k' with $k < k'$. We only need to show (1) $k \ge 3$ and (2) $k' = k + 1$ to show (i) and (ii). For (1), let us assume $k < 3$. Since stores have no incentive to move, $\Pi_k^* \ge \Pi_{k'+1}^*$ and $\Pi_{k+1}^* \le \Pi_{k'}^*$ hold. However, since $k < 3$, we have $\Pi_k^* < \Pi_{k+1}^*$. Thus, $\Pi_{k'+1}^* \le \Pi_k^* < \Pi_{k+1}^* \le \Pi_{k'}^*$ must hold. However, since profits increase very quickly until 3 and decrease slowly after that (see Table 1), we cannot find any $k < 3$ and $k' \ge 3$ which satisfies these inequalities. Hence, (1) is proved. It is easy to see that (2) holds given (1): Suppose that $k < k + 1 < k'$. By (1), we know $\Pi_k^* > \Pi_{k+1}^* > \Pi_{k'}^*$. Thus, a store in a size k' shopping center moves to a size k shopping center.■

7.7 Proof of Lemma 2

For each commodity, the critical value is always $R(c)$, implicitly defined in the following equation (see Kohn and Shavell (1974) and Wolinsky (1986)):

$$\int_{R(c)}^{1} (v - R(c))\, dv = c.$$

By solving this equation, we obtain $R(c) = 1 - \sqrt{2c}$. Given this, Wolinsky (1986) finds that the equilibrium price $\bar{p}$ is solved as follows (his Proposition):

$$\bar{p}(c) = c + (1 - R(c)) = c + \sqrt{2c}.$$

The expected utility given c is:

$$
\begin{aligned}
\tilde{u}(c) &= -\bar{p}(c) + \left(\int_{R(c)}^{1} v\,dv - c \right) + R(c) \left(\int_{R(c)}^{1} v\,dv - c \right) + R(c)^2 \left(\int_{R(c)}^{1} v\,dv - c \right) + \dots \\
&= -\bar{p}(c) + \frac{1}{1 - R(c)} \left(\int_{R(c)}^{1} v\,dv - c \right) \\
&= -\bar{p}(c) + \frac{R(c) - R(c)^2}{1 - R(c)} \\
&= -\bar{p}(c) + R(c) \\
&= 1 - c - 2\sqrt{2c}.
\end{aligned}
$$

We have completed the proof.∎

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