

FORMATION OF HUB CITIES: TRANSPORTATION COST ADVANTAGE AND POPULATION AGGLOMERATION^{*†}

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Abstract

Many cities are located on rivers or coasts. This paper argues that such cities developed as transportation hubs or markets for interregional trade, since these locations provide better access (lower marginal transportation costs) to other regions. Local products are collected at such hubs, and interregional trade then takes place among these transportation hubs. As the volume of trade between hubs increases, more workers are needed in order to meet labor demand for shipping and handling commodities, resulting in population agglomeration at such hubs. This paper constructs a simple three location-identical consumer model, in which transportation hub and population agglomeration emerge endogenously. In contrast with much of the literature on city formation, we introduce no economies of scale into the model. Markets are assumed to be perfectly competitive and complete. Since prices are determined in equilibrium,

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transportation costs and routes are simultaneously determined in the system. Population agglomeration occurs solely because of location-specific production technologies (which generates gains from trade) and the differences in transportation technologies among locations (which determines the transportation routes). It is shown that a hub city emerges when transportation technologies are heterogeneous enough.

1 Introduction

Recently, the theory of endogenous city formation has attracted significant attention. Fujita and Ogawa (1982), Abdel-Rahman (1988), Krugman (1991a), Krugman (1991b), and Fujita, Mori and Krugman (1995) stress the importance of scale economies (and/or demand externalities) in explaining population agglomeration by using monopolistic competition models. More generally, Fujita and Thisse (1996) provide a survey of the whole literature on the agglomeration of economic activities generated by various kinds of positive externalities.¹ In these models, homogeneity of locations are usually assumed, and the locations of cities are determined by scale economies and transportation costs. Such models typically lead to multiple equilibria, and the locations of cities are often determined by historical accidents.

In real life, however, we often find that cities arise near rivers and coasts. It seems that many of these cities developed as transportation hubs or markets for interregional trade, since these locations provided better access to other regions. Thus, the geographical features of locations (differences in transportation costs relative to other regions) play an important role in determining the locations of cities (Atack and Passell (1994): Chapter 6). Cronon (1991) discusses how advantages in transportation costs to the East made Chicago a commercial and transshipment center in the 19th century Midwest region of the United States (Cronon (1991): Chapter 2).²

This paper tries to explain where and how a hub city emerges. In the mid-19th century, many cities in the Midwest developed essentially as ‘gateway cities’: that is, these cities imported manufacturing commodities from the East Coast, and exported the regional agricultural products to the East Coast such as corn, wheat, and lumber. Chicago serves as an excellent example (see Cronon (1991)). Chicago had cheap access to New York through waterways because of the Great Lakes and the Erie Canal.^{3 4} Thus, for farmers in

¹Jacobs (1969) and Glaeser et. al. (1992) among others assert that Marshallian externalities across industries (urbanization economies) are important. On the other hand, Henderson (1974, 1988) stresses the importance of Marshallian externalities within industries (localization economies).

²Of course, there are many other cities developed as transshipment centers. For example, many Mediterranean cities such as Venice, Barcelona, and Marseille prospered as interregional trade centers (between Middle East and Europe) based on their harbors in the Medieval times. Osaka in Japan is another good example.

³Atack and Passell (1994) report the costs of different transportation media in 1860 (Table 6.3). Cost per ton-mile for different media are as follows: road 15.00 or more, the Mississippi River 0.37, the Erie Canal 0.99, the Great Lakes 0.10, and the New York Central (railroad) 2.06.

⁴Before the Erie Canal (that connects Lake Erie and the Hudson River) was in operation (1825), the main

Illinois, it was beneficial to send their harvest to Chicago, since they could get more cash there.⁵ In the late 1840s, Chicago started to develop local railroad links and the Chicago-La Salle Canal (between the Illinois and Lake Michigan), and by the end of 1850s Chicago became the hub of railroad spokes in the Midwest. As a result, Chicago obtained great advantages in transportation technologies locally as well as interregionally. In the large area of the Midwest, it became much more attractive to send commodities via Chicago instead of other cities such as St. Louis. Huge amount of crops and lumber in the Midwest flowed into Chicago by local railroads, and were transshipped into sailboats bound for the East Coast (later, to the New York Central (railroad), especially in winters). As the volume of commodity transportation through Chicago increased, the numbers of workers and merchants increased as well, thus making Chicago an isolated giant in the Midwest.⁶

The mechanism generating population agglomeration at hub cities such as Chicago may be described in the following way: There are potentially two ways to send commodities from one location to another. One way is to send commodities directly, and another way is to send commodities indirectly through a third location. If the transportation costs are homogeneous, it is not beneficial to use an indirect route. On the other hand, if it is cheaper to send commodities via a third location instead of sending them directly to the destination, the volume of commodities transported through the third location increases. As a result, more workers are needed at that location in order to meet labor demand for transshipping and handling commodities, resulting in population agglomeration at such a location. We call this location a *hub city* in this paper.⁷

This paper intends to explain the emergence of a hub city by using a general equilibrium model with multiple locations in which transportation activities are explicitly modelled.

transportation route between the East Coast and the Midwest was through the ocean and the Mississippi (via New Orleans), since the ground transportation was prohibitively expensive due to poor road conditions (see Atack and Passell (1994)). The Erie Canal made the Midwest much closer to the East Coast than before.

⁵Cronon describes this as follows: “The farmers chose Chicago as their destination because they received more cash for their crops there, and because they could buy more and better supplies at lower prices. River towns in the interior — Peoria, Springfield, Vincennes, even St. Louis — did not have the cheap lake transportation to the east that gave Chicago its price advantage. ... Lester Harding reported in October 1847 that Chicago prices were ‘70^{cts} for spring and 80^{cts} to 85 for winter wheat,’ compared with fifty cents back home in Paw Paw, Illinois. ‘Farmers,’ he said ‘cannot grumble at these prices. ... There can be no better [market] any where in the Union.’ ” (Cronon (1991), page 60).

⁶In 1820, Chicago had only one quarter of the population of St. Louis (before the Erie Canal was open). However, in 1890, Chicago had over one million people, which was three times the population of St. Louis.

⁷The term ‘hub’ is used in various contexts in the literature. Hendricks, Piccione, and Tan (1995) and Starr and Stinchcombe (1992) characterize the profit maximizing transportation network choice in a monopolistic air carrier’s route choice problem. They show that a hub-spoke network is optimal under some conditions. Krugman (1991c) analyzes the relationship between concentration of manufacturing and transportation cost advantage in the context of international trade (no labor mobility). He finds that if a location has transportation cost advantage then the portion of workers who engage in manufacturing at the location increases. He calls this concentration of the manufacturing sector, the ‘hub’ effect. Mori and Nishikimi (1998) analyze the evolution of a transportation hub location by assuming scale economies in transportation. In all the above models, consumers are not allowed to move implicitly or explicitly.

We assume that transportation activities require labor input to capture the mechanism described above. It is *essential* to use a general equilibrium approach for the following reasons: To explain the emergence of a hub city, heterogeneous transportation costs are necessary, since they determine the transportation routes, and so the population distributions. However, transportation costs are not exogenous parameters. Since transportation activities require labor input, transportation costs are determined by transportation technologies as well as wage rates (input prices).⁸ But wage rates are in turn determined endogenously. Therefore, the transportation route, the locations of hubs, population distribution as well as the prices and transportation costs need to be determined simultaneously within a general equilibrium model.

This paper develops the simplest possible model that can generate a hub city. Transportation technologies are given as economic data, and we analyze which transportation route is selected as the cheapest route and where and how a hub city emerges in equilibrium. The model does not introduce any kind of increasing returns to scale technologies in production and transportation, and assumes perfectly competitive and complete markets to focus only on the role of heterogeneity of transportation technologies in population agglomeration.⁹ The structure of the model is as follows: There are three locations (called location 1, 2, and 3) and identical atomless consumers in the economy.¹⁰ Consumers provide labor and consume commodities, and they can choose their locations freely. To transport one unit of a commodity from one location to another, a constant amount of *labor* (at the originating location) is required as input. Because of this, a rise in the volume of commodity transportation through a location results in an increase in the population at that location. This assumption is crucial in generating a hub city. If we assume that transportation costs are paid by the transported commodity itself (the commodity melts away during the transportation: the so-called Samuelson ‘iceberg’ transportation costs), then even if the amount of commodities transported through a hub location becomes large, the population there does not increase since the demand for transportation workers at the hub location does not go up. To introduce gains from trade, we assume that different commodities are produced at different locations (location-specific production): that is, if a consumer decides to live at a location, she can only produce the commodity which is

⁸Formally, transportation activities can be regarded as production activities (see Schweizer, Varaiya, and Hartwick (1976)). In an economy with multiple locations, physically the same commodities at different locations must be regarded as different commodities (see Debreu (1954)). Sending wheat from Chicago to New York by using Chicago transportation workers can be interpreted as producing ‘wheat in New York’ from ‘wheat in Chicago’ and ‘labor in Chicago’ by using transportation technologies. Thus, transportation costs (the payments for workers) are endogenously determined by transportation technologies and the wage rates.

⁹Ideally speaking, transportation technologies should be determined endogenously within the model since some transportation systems are provided by the governments as public infrastructure (such as the Erie Canal and federally subsidized railroad links). In this paper, however, we assume that transportation technologies are exogenously determined to focus on the transportation route selection in equilibrium. Berliant and Konishi (1994) try to endogenize transportation system in an economy by using a collective choice mechanism (the ‘core’ allocation) given consumers’ mobility.

¹⁰Note that three is the minimum number which can generate a transportation hub.

specific to her location or can engage in the transportation of commodities departing from that location.¹¹ We assume that one unit of each commodity is produced by one unit of labor, and consumers value these three commodities symmetrically. These symmetry assumptions are made in order to focus only on the effect of heterogenous transportation technologies.¹²

We begin our analysis with the case where the transportation technologies between any pair of locations are the same. Of course, in this case, the population distribution at each location is completely symmetric, and commodities are traded directly between any pair of two locations (the “no hub” route: see Figure 1). We then reduce the labor input coefficients for transportation between locations 1 and 2 and between locations 1 and 3 at the same rate, while keeping the labor input coefficient for the transportation between locations 2 and 3 constant. Thus, we give location 1 an advantage in transportation technologies. If these reductions in the coefficients are small, then the no-hub route is still used. However, if the coefficients are reduced sufficiently, then the transportation route switches from the no-hub to the hub route (see Figure 2). That is, at some point, there is a switch in transportation route, resulting in a jump in the population at location 1. We call this effect the ‘hub effect’ in population agglomeration. We can show that the switch from the no-hub route to the hub route occurs once and only once in the process of improving the transportation technology at location 1 (Proposition 5). This is the main analytical result of the paper, and this property is useful for the computations of equilibria in numerical examples. We can also show that the population at the hub location is actually higher than that of other locations (a hub ‘city’) if the utility function belongs to the class of CES functions (Proposition 6). Changes in population distribution and the magnitude of hub effects are presented by numerical examples of CES utility functions with various elasticities of substitution. It is observed that (i) as the elasticity of substitution goes up the population at the hub location increases, and (ii) as handling cost for transshipment service increases, the population at the hub location increases.

Although the structure of the model is simple, we need to develop several technical innovations to prove our results. Since there are transportation costs in the economy, the prices of goods differ across locations even if these goods are physically identical. As a result, there are twelve goods (labor, commodities 1, 2, and 3, one for each location)

¹¹It seems important to discuss the relationship with Starrett’s (1978) ‘spatial impossibility theorem’. Starrett asserted that if there are (i) complete markets at every location, (ii) perfectly competitive markets, (iii) no moving cost, (iv) no externalities, and (v) locations are homogeneous, then there will not be agglomeration in economic activities. In our model, only condition (v) is violated due to location-specific production technologies. Externality models usually violate conditions (ii) and (iv). Previous competitive model such as Berliant and Wang (1993), Wang (1993), Ellickson and Zame (1991, 1994), and Berliant and Konishi (1994) violate at least conditions (i) and (v). Thus, this paper clarifies how far we can go by violating only condition (v). Fujita (1990) provides a nice survey on the literature on this theorem.

¹²If these symmetries are lost, it is obvious that efficiencies of production technologies and preferences over commodities affect population agglomeration: i.e., if commodity specific to a location is produced by an inefficient technology (the labor coefficient is large), and if consumers prefer the commodity to other commodities, then population agglomeration occurs at that location.

in the economy. Moreover, due to price differences, a consumer's behavior is dependent upon location although we assume identical consumers. Thus, in some senses, it is a heterogeneous agent model. Finally, we have highly nonlinear restrictions due to free mobility of consumers: at each location, consumers need to attain the same utility levels. We overcome these obstacles by utilizing some special features of the model.

Scale economies and demand externalities à la Krugman may be the main factors behind economic agglomeration especially in modern society. Due to technological progress, heterogeneity of transportation costs among locations also seem much less significant now. What we claim here is that heterogeneity of transportation costs *was* essential in determining the initial value of the dynamic system of population agglomeration. Thus, this paper complements the city formation literature with scale economies (see also a recent textbook by Bogart (1997)). Fujita and Mori (1996) analyze the 'lock-in' effects of 'port cities' by using a monopolistic competition model: i.e., port cities may remain to be big cities even after harbors became less important due to an improvement in the transportation system. The hub effect plants the 'seeds of cities', and the other factors (such as demand externalities) stressed by recent literature determine the (current) city structure in a country (or a region).¹³ Some cities take off and become metropolises (as Chicago), while others decline after they lose advantage in transportation technologies.¹⁴ For the possibility of combining the model in this paper with those models, see the concluding remarks.

Section 2 describes the model. Section 3 studies properties of equilibria. Section 4 provides numerical examples by using the family of CES utility functions. Section 5 concludes the paper. Section 6 collects all the proofs of the propositions stated in Section 3.

2 The Model

There are three locations in the economy $\{1, 2, 3\}$ and three commodities $\{1, 2, 3\}$. There is a unit mass of identical (atomless) consumers. Each consumer can move without cost, but she has to choose one location as her residence. Population distribution needs to satisfy the following equation:

$$N^1 + N^2 + N^3 = 1, \tag{1}$$

where N^j denotes population at location j . A consumer is endowed with one unit of labor irrelevant of her residence. However, if she chooses location $j \in \{1, 2, 3\}$, then she can work only at location j . Thus, formally, labor in different locations is labor of different types, and each consumer's endowment is dependent on her choice of location. In other words, if a consumer chooses location j , then her endowment is one unit of labor at location j . To describe location-specific production technology, we assume that commodity j can be

¹³Kim (1996) stresses that the establishment of wholesale markets played an important role in the development of big cities. Since the approach in this paper explicitly deals with commodity flows, his idea together with our model may be used to explain the development of cities.

¹⁴For example, Albany and Buffalo were very big cities when the Erie Canal was the cheaper mode of transportation. However, later as railroads and highways became dominant, these cities shrunk in size.

produced only at location j . Production at each location exhibits constant returns to scale: One unit of commodity j is produced by one unit of labor at location j . Commodities can be transported across locations by using labor. Transportation of one unit of (any) commodity from location j to location k requires t_{jk} units of labor at location j .¹⁵ For simplicity, we assume that $t_{jk} = t_{kj}$. Commodity transportation from location j to k via location ℓ requires labor input at location ℓ for transaction purposes, in addition to labor inputs at locations j and ℓ for transportation purposes.¹⁶ In such a case, for each unit of commodity, s_ℓ units of labor are also needed at location ℓ as transaction costs. Transaction costs could be thought of as handling costs at railroad stations, harbors, or marketplaces. Commodity and labor markets in the economy are complete. That is, at each location, there are markets for labor and all commodities. However, since there are transportation costs, the price of a commodity at location j differs from the price at location k . The commodity price vector at location j is denoted by $p^j = (p_1^j, p_2^j, p_3^j)$, where p_k^j represents the price of commodity k at location j . Throughout the paper, superscripts denote locations and subscripts denote commodities. Since one unit of commodity j is produced by one unit of labor at location j , the wage rate at location j , w^j , satisfies $w^j = p_1^j$. As a result, consumers choose their locations and consumption bundles by observing (p^1, p^2, p^3) .

A consumer's utility function is $U(x_1, x_2, x_3)$, where x_j denotes the amount of consumption of commodity j . A consumer's utility function is not affected by her location choice, we assume that her consumption takes place at her location only: i.e., even if she is a transportation worker and travels, she buys and consumes her consumption bundle only at her location. Since we want to treat every commodity symmetrically, we assume that $U(\cdot)$ is symmetric (i.e., for any $x_1, x_2, x_3 \in \mathbb{R}_+$, $U(x_1, x_2, x_3) = U(x_j, x_k, x_\ell)$ for any $j, k, \ell \in \{1, 2, 3\}$ with $j \neq k, \ell$ and $k \neq \ell$). To obtain clear-cut analytical results, we assume that $U(\cdot)$ is strictly quasi-concave, monotonic, linearly homogeneous, C^2 differentiable in a suitable sense, and satisfies (weak) boundary conditions.¹⁷ The boundary conditions together with monotonicity require that a consumer consumes a positive amount of every commodity to get positive utility, which guarantees gains from trade.

Our analysis below proceeds in the following way. For analytical simplicity, we will let locations 2 and 3 be symmetric ($t_{12} = t_{13}$), since our focus is primarily to show that a transportation hub emerges and population agglomeration occurs if transportation tech-

¹⁵This assumption is made for the sake of simplicity. We can assume that labor at both j and k is used in transporting commodities from j and k with any fixed coefficients.

¹⁶We introduce transshipping (handling) costs to explain mercantile cities.

¹⁷The formal definitions are as follows: *Strict quasi-concavity*: for any $t \in (0, 1)$, for any $x, x' \in \mathbb{R}_{++}^3$ such that $U(x) = U(x')$, $U(x) < U(tx + (1-t)x')$ holds. *Monotonicity*: for any $x, x' \in \mathbb{R}_+^3$ such that $x \ll x'$, $U(x) < U(x')$, where $x \ll x'$ means $x_i < x'_i$ for any $i = 1, 2, 3$. *Linear homogeneity*: for any $x \in \mathbb{R}_+^3$, for any $t > 0$, $U(tx) = tU(x)$. *Weak boundary conditions*: for any $x \neq 0$ with $x_i = 0$ for some $i = 1, 2, 3$, the set $\{x' \in \mathbb{R}^3 : U(x') \geq U(x)\}$ has a unique support at any boundary point. In our context, the differentiability of U means that the underlying preference by U is C^2 differentially strictly convex (see Mas-Colell, Whinston, and Green (1995): page 94). This assumption simply says that the utility function U is second-order differentiable and the Hessian matrix is negative definite at any $x \in \mathbb{R}_{++}^3$. Thus, actually, C^2 differentially strict convexity implies that U is strictly quasi-concave. The CES utility functions satisfy these restrictions except for the linear ($\sigma = 1$) and the Leontief ($\sigma = -\infty$) utility functions.

nologies are heterogeneous enough.¹⁸ We start with the completely symmetric case where $t_{12} = t_{13} = t_{23} = \bar{t}$, and reduce $t_{12} = t_{13}$ (denoted by t) simultaneously, yet t_{23} is kept constant at \bar{t} . Thus, locations 2 and 3 are always in a symmetric situation. Next, we investigate how improvement in the transportation technology at location 1 affects the equilibrium allocation (prices and population distribution). We show that at some point in the process of the transportation technology improvement, the population at location 1 jumps up due to the change in the transportation route (from the no-hub route to the hub route: a hub effect).

3 Properties of Equilibrium

In this section, we study the properties of equilibrium. First of all, the strict quasi-concavity of U implies that consumers' consumption vectors are the same if they live at the same location. This property is useful to define an equilibrium. Let $p^j = (p_1^j, p_2^j, p_3^j)$ and $x^j = (x_1^j, x_2^j, x_3^j)$ for any $j = 1, 2, 3$, and let $\theta^2, \theta^3 \in [0, 1]$, where θ^j ($j = 2, 3$) is the ratio of the amount of commodity j sent from j to k ($k \neq j$, and $k = 2, 3$) via location 1 to the total amount of commodity j sent from j to k .¹⁹ At each location j , a consumer maximizes her utility given the price vector at j :

$$x^j = \arg \max_{x \in \mathbb{R}_+^3} U(x) \quad \text{s.t.} \quad p^j \cdot x^j = \sum_{i=1}^3 p_i^j x_i^j = p_j^j \quad \text{for any } j = 1, 2, 3. \quad (2)$$

If a consumer chooses location j , then she earns $w^j = p_j^j$ and she faces the commodity price vector $p^j = (p_1^j, p_2^j, p_3^j)$. Since in the equilibrium nobody wants to move to another location, the utility levels at three locations are the same, and this is expressed by the following condition:

$$U(x^1) = U(x^2) = U(x^3). \quad (3)$$

The specification of our production and transportation technologies require price arbitrage conditions. Since commodity 1 is sent to locations 2 and 3 directly and commodities 2 and 3 are also sent to location 1 directly, we have the following conditions:

$$p_2^1 = (1+t)p_2^2, \quad p_3^1 = (1+t)p_3^3, \quad p_1^2 = p_1^3 = (1+t)p_1^1. \quad (4)$$

The first condition can be interpreted as follows: Recall that *the wage rate at location j is equal to the price of commodity j at location j* due to the specification of our production technology. To provide one unit of commodity 2 at location 1, we need p_2^2 for production

¹⁸The symmetry between locations 2 and 3 simplifies our analysis. If this assumption is dropped, the uniqueness result in Proposition 5 could be lost in general even if the utility function satisfies the nice properties listed above. As a result, we need to deal with multiple equilibrium problem, and we will not be able to provide clear-cut analytical results even though quantitatively similar results may be obtained.

¹⁹Note that an indirect route will not be used in transporting commodity 1 from location 1 to location 2 or 3. It is because $t_{12} = t_{13} = t$ holds: an indirect route is simply more costly.

(1 unit of labor at location 2) and $t \times p_2^2$ for transportation (t units of labor at location 2). In total, we need to pay $(1+t)p_2^2$ for commodity 2 at location 1. Other two conditions are interpreted similarly. The following conditions are for the rest of the cases:

$$p_\ell^k = \min \left\{ (1+\bar{t})p_\ell^\ell, (1+t)p_\ell^\ell + (t+s)p_1^1 \right\}, \quad (5)$$

where $\ell, k = 2, 3$ and $\ell \neq k$. These conditions simply say that the cheaper transportation route determines the price of commodity 2 (3) at location 3 (2). The first element in the brace of equation (5) represents the price of commodity ℓ at k when the no-hub route is used, while the second element represents the price of commodity ℓ at k when the hub route is used. For the latter case, the production cost is p_ℓ^ℓ , the transportation cost from location ℓ to 1 is $t \times p_\ell^\ell$, the handling cost at location 1 is $s \times p_1^1$, and the transportation cost from location 1 to k is $t \times p_1^1$. Summing them up, we obtain $(1+t)p_\ell^\ell + (t+s)p_1^1$. The following equations describe the restrictions on the choices of transportation routes due to the relative costs of two routes:

$$\theta^\ell = 0 \quad \text{if} \quad p_\ell^k = (1+\bar{t})p_\ell^\ell \leq (1+t)p_\ell^\ell + (t+s)p_1^1, \quad (6)$$

$$\theta^\ell = 1 \quad \text{if} \quad p_\ell^k = (1+t)p_\ell^\ell + (t+s)p_1^1 \leq (1+\bar{t})p_\ell^\ell, \quad (7)$$

$$\theta^\ell \in [0, 1] \quad \text{if} \quad p_\ell^k = (1+\bar{t})p_\ell^\ell = (1+t)p_\ell^\ell + (t+s)p_1^1, \quad (8)$$

where $\ell, k = 2, 3$ and $\ell \neq k$. These three conditions say that we have $\theta^j = 0$ if the no-hub route is strictly better than the hub route for commodity j transportation, and $\theta^j = 1$ if the hub route is strictly better than the no-hub route for commodity j transportation. We can have an interior $\theta^j \in (0, 1)$, only when both routes minimize transportation costs. Finally, we describe labor market clearing condition at each location. The condition at location 1 is:

$$N^1 x_1^1 + (1+t)N^2 x_1^2 + (1+t)N^3 x_1^3 + \theta^2(s+t)N^3 x_2^3 + \theta^3(s+t)N^2 x_3^2 = N^1. \quad (9)$$

The *RHS* of equation (9) represents the labor supply at location 1: there are N^1 consumers at location 1, and the labor supply at 1 is N^1 . The *LHS* of equation (9) represents the labor demand at location 1. There are two types of labor demand: labor demand for the production of commodity 1 and that for the transportation of commodities. To satisfy the former demand, we need $N^1 x_1^1 + N^2 x_1^2 + N^3 x_1^3$ units of labor. The latter demand again consists of two types: (i) the labor required to transport commodity 1 to locations 2 and 3, and (ii) the labor required to transport and handle commodities from locations 2 and 3 to locations 3 and 2, respectively. The second type of labor demand shows up only when location 1 becomes a transportation hub ($\theta^j > 0$). To transport one unit of commodity 1 from location 1 to j ($j = 2, 3$), t units of labor is needed. As a result, $t(N^2 x_1^2 + N^3 x_1^3)$ units of labor are needed for the transportation of commodity 1 (type (i)). Similarly, $(s+t)(\theta^3 N^2 x_3^2 + \theta^2 N^3 x_2^3)$ units of labor are needed for the handling and transportation of commodities 2 and 3 (type (ii)). By summing these terms up, we can get the *LHS* of equation (9). The following two equations (10) and (11) represent the labor market

clearing conditions at locations 2 and 3, respectively. These equations can be interpreted similarly:

$$(1+t)N^1x_2^1 + N^2x_2^2 + (1-\theta^2)(1+\bar{t})N^3x_2^3 + \theta^2(1+t)N^3x_2^3 = N^2, \quad (10)$$

$$(1+t)N^1x_3^1 + (1-\theta^3)(1+\bar{t})N^2x_3^2 + \theta^3(1+t)N^2x_3^2 + N^3x_3^3 = N^3. \quad (11)$$

An *equilibrium* is a list $(N^1, N^2, N^3; p^1, p^2, p^3; x^1, x^2, x^3; \theta^2, \theta^3)$, which satisfies conditions (2)-(11) together with condition (1): The following proposition gives us the foundations of our analysis.

Proposition 1 *There exists an equilibrium in the economy for any $t \in [0, \bar{t}]$, and every equilibrium is Pareto efficient.*²⁰

We say that locations 2 and 3 are *symmetric in consumption and price vectors* in an equilibrium, if and only if $p_2^1 = p_3^1, p_2^2 = p_3^2, p_1^2 = p_1^3, p_2^3 = p_3^3, x_2^2 = x_3^2, x_2^3 = x_3^3, x_2^1 = x_3^1$. We say that locations 2 and 3 are *symmetric* in an equilibrium, if and only if locations 2 and 3 are symmetric in consumption and price vectors, and moreover $N^2 = N^3$, and $\theta^2 = \theta^3$. The following result simplifies the analysis greatly:

Proposition 2 *Locations 2 and 3 are symmetric in consumption and price vectors in every equilibrium. Moreover, even if $\theta^2 \neq \theta^3$ and $N^2 \neq N^3$ in an equilibrium, there exists an equilibrium in which locations 2 and 3 are symmetric.*

This proposition asserts that in every equilibrium locations 2 and 3 are symmetric in consumption and price vectors under symmetric transportation costs ($t_{12} = t_{13}$), and if either the no-hub route or the hub route is exclusively used (not mixed), then the equilibrium is symmetric in locations 2 and 3. Moreover, even if there is an asymmetric mixed equilibrium, there always exists a symmetric equilibrium. This gives us a strong justification to study only symmetric equilibria. We can define a symmetric equilibrium (in locations 2 and 3) by using the symmetry conditions: A *symmetric equilibrium* (at locations 2 and 3) is a list $(N; p^1, p^2; x^1, x^2; \theta)$, which satisfies the following conditions:

$$x^j = \arg \max_{x \in \mathbb{R}_+^3} U(x) \quad \text{s.t.} \quad p^j \cdot x^j = p_j^j \quad \text{for any } j = 1, 2, \quad (12)$$

$$U(x^1) = U(x^2), \quad (13)$$

$$p_2^1 = (1+t)p, \quad p_1^2 = (1+t)\bar{p}, \quad (14)$$

$$p_3^2 = \min \{(1+\bar{t})p, (1+t)p + (t+s)\bar{p}\} \quad (15)$$

$$\theta = 0 \quad \text{if} \quad p_3^2 = (1+\bar{t})p \leq (1+t)p + (t+s)\bar{p}, \quad (16)$$

$$\theta = 1 \quad \text{if} \quad p_3^2 = (1+t)p + (t+s)\bar{p} \leq (1+\bar{t})p + (t+s)\bar{p}, \quad (17)$$

²⁰We can prove the equivalence between the core and the equilibrium, which is a much stronger result than the first welfare theorem. However, in this paper, we only need to utilize the first welfare theorem.

$$\theta \in [0, 1] \quad \text{if} \quad p_3^2 = (1 + \bar{t})p = (1 + t)p + (t + s)\bar{p}, \quad (18)$$

$$Nx_1^1 + (1 + t)(1 - N)x_1^2 + \theta(s + t)(1 - N)x_3^2 = N, \quad (19)$$

$$(1 + t)Nx_2^1 + \left(\frac{1 - N}{2}\right)x_2^2 + \{(1 - \theta)(1 + \bar{t}) + \theta(1 + t)\} \left(\frac{1 - N}{2}\right)x_3^2 = \left(\frac{1 - N}{2}\right), \quad (20)$$

where $N = N^1$, $p = p_2^2$ and $\bar{p} = p_1^1$. The next proposition states that there will be a continuum of equilibria with various population distributions, if the costs of the no hub and the hub routes are the same. More precisely, the proposition states that if there is a mixed equilibrium ($\theta \in (0, 1)$) for some $t = t_{12} = t_{13}$, then there are no-hub and hub equilibria, and mixed equilibria with any combination of the two transportation routes for the same price vectors and the same commodity consumption vectors. Note that the equilibrium consumption vectors differ from location to location. Thus, it is not obvious that the same consumption (and price) vectors still compose an equilibrium for a different transportation route, since a switch in transportation routes changes labor demand at each location, resulting in a different population distribution.

Proposition 3 *Suppose that in an equilibrium $(N; p^1, p^2; x^1, x^2; \theta)$, $p_3^2 = (1 + \bar{t})p = (1 + t)p + (t + s)\bar{p}$ is satisfied. Then, for any $\theta' \in [0, 1]$, $(N'; p^1, p^2; x^1, x^2; \theta')$ composes an equilibrium for some $N' \in (0, 1)$.²¹*

The following proposition asserts that the wage rate at location 1 is always lower than the wage rates at other locations (Note that \bar{p} and p are also the wage rates at location 1 and locations 2 and 3, respectively). Due to the advantage in transportation technologies, location 1 is more attractive for consumers. However, the boundary conditions require that each location has a positive population. Thus, the wage rate must be lower at location 1 than at other locations.²²

Proposition 4 *For any $t \in (0, \bar{t})$, $p > \bar{p}$ holds in the equilibrium.*

The following proposition is the main result of this paper, and gives us a full characterization of the equilibrium set of the economy. This result and the method of the proof are crucial to the numerical analysis in the next section. We say that equilibrium is *essentially unique* if and only if every equilibrium in the economy has the same price and consumption vectors (when \bar{p} is normalized).

²¹Actually there are equilibria in which locations 2 and 3 are not symmetric in population and θ^j . There exists a continuum of equilibria with any combination of θ^2 and θ^3 (with the same consumption and price vectors).

²²Note that we assume that there is no land in the model for simplicity, so consumers do not value land. This is the reason that the wage rates in cities are lower than the ones in rural area. This result is common to models with no land in utility (see Fujita and Mori (1996) and Fujita, Krugman, and Mori (1995)). In cities, rents are much higher than in rural area, and the residents in cities need higher wages to be compensated. If rent payments are subtracted, the city wage rates may be lower than the rural wage rates.

Proposition 5 *Suppose that $s \leq \bar{t}$ and $s \leq 1$ hold. Then, there exists $t^* \in (0, \bar{t})$ which satisfies the following:*

- (i) *for any $t \in (t^*, \bar{t}]$, the equilibrium is unique and uses the no-hub route exclusively,*
- (ii) *for $t = t^*$, there exists an equilibrium for any combination of the hub and the no-hub routes (for any $\theta \in [0, 1]$) and the equilibrium is essentially unique, and*
- (iii) *for any $t \in [0, t^*)$, the equilibrium is unique and uses the hub route exclusively.*

This proposition has some interesting and somewhat surprising implications. First, when we reduce t from \bar{t} to zero, the equilibrium transportation routes jump from the no-hub to the hub route at t^* *once for all*.²³ Second, unless $t = t^*$, the equilibrium is *unique* and either the no-hub route or the hub route is used exclusively. Third, at $t = t^*$ any kind of mix of the two transportation routes (including nonmixed routes) composes an equilibrium for the same consumption and price vectors. In such cases, population at location 1 can be anything between N_{nh} (the population at location 1 under the no-hub route) and N_h (the population at location 1 under the hub route). Since it is easy to see that N_{nh} is smaller than N_h (given the same consumption vectors), it follows that the population at location 1 jumps up at $t = t^*$. Thus, we can say that a switch from the no-hub to the hub route generally increases the population at location 1. This effect can be called a ‘hub effect’. Next proposition shows that if the utility function belongs to the family of CES functions, the hub location actually has more population than other locations (a hub ‘city’).

Proposition 6 *Suppose that the utility function belongs to the family of CES functions. Then, for any $t \in (0, t^*)$, $N > \frac{1}{3}$ in the equilibrium.*

Since the total population in the economy is one, N is more than $\frac{1}{3}$ means that location 1 has more population than other locations. Although it is in general unknown if N is more than $\frac{1}{3}$ even when the hub route is used, it seems that the same result applies for a much wider class of utility functions.²⁴ In the following section, we provide numerical examples for cases in which the utility function is a CES function.

4 Numerical Examples

Even though we imposed nice properties on the utility function, it is still difficult to obtain comparative static results analytically due to the complex structure of the model. Thus, in this section, we provide several numerical examples using the family of CES utility functions: i.e., $U(x_1, x_2, x_3) = (x_1^\sigma + x_2^\sigma + x_3^\sigma)^{\frac{1}{\sigma}}$, where $\sigma < 1$ ($\sigma = 1$ means the

²³Note that if $s = 0$, then $t^* > \frac{\bar{t}}{2}$ holds. It is because $(1 + \bar{t})p = (1 + t)p + (t + s)\bar{p}$ holds at $t = t^*$, and Proposition 4 asserts $\bar{p} < p$.

²⁴It seems that the shape (curvatures) of an indifference curve matters in general for this result. However, to get $N > \frac{1}{3}$ for $t \in (0, t^*)$, it is *sufficient* to have $x_1^1 \geq x_2^2$ in the equilibrium (see Lemma 6 in the appendix). Actually, a CES utility function guarantees $x_1^1 = x_2^2$.

utility function is linear and the (weak) boundary conditions are violated). We use the expenditure functions and the compensated demand functions to calculate the equilibrium. This approach is especially convenient since condition (13) requires that the utility level must be the same at all locations. The equilibrium systems are described by equations (23)-(25) for the no-hub case, and by equations (27)-(29) for the hub case. (See the appendix). From Propositions 1 (the first welfare theorem) and 5, the resulting utility levels in these two cases are equal at t^* , and for $t < t^*$ the hub route is used while for $t^* < t$ the no-hub route is used. Thus, by calculating the equilibria in the two economies with the routes fixed, we can determine equilibrium allocations in our original economy. First, we analyze how the equilibrium population at location 1 depends upon the value of t and the elasticity of substitutions σ by setting $s = 0$ to give the least chance for population agglomeration at the hub. We then provide an example with a positive s , and check how population at the hub is affected by fixing σ at zero (Cobb-Douglas utility function).

The exercises are described as follows: We set the parameter values at $s = 0$ and $\bar{t} = 0.5$, and we investigate how equilibrium changes by changing t from 0 to $0.5 (= \bar{t})$. We provide examples for $\sigma = -\infty$ (Leontief), $-10, -1, 0$ (Cobb-Douglas), and 0.5 (see Figures 3-7). In each figure, the circled, the crossed, and the solid curves represent the no-hub route, hub route, and the actual equilibria, respectively. Obviously, at $t = 0$ or $t = 0.5$, the population at location 1, N , is $\frac{1}{3}$.²⁵ In each case, $t^* = 0.26$ and N jumps up by about 0.04 (the hub effect). The population N at t^* when the hub route is used increases monotonically with the value of σ . However, the sign of population change when we reduce t is different when the no-hub route is used ($t \in (t^*, \bar{t})$). If $\sigma < -1$, then N goes down, and if $\sigma > -1$, N increases by a reduction of t . When $\sigma = -1$, the population N is constant at location 1. This is because of the following reasons. Since the transportation costs go down, the transportation sector does not demand as much labor at location 1 as before. This reduces the population at location 1. We may call this *a labor saving effect* due to the reduction in transportation cost. On the other hand, there is *a substitution effect* at work since commodity 1 becomes relatively cheap at all locations due to a reduction in t (see Proposition 4). The magnitude of this effect is related to the elasticity of substitution. Thus, N increases or decreases depending on the relative magnitudes of *the labor saving effect* and *the substitution effect*. When $\sigma < -1$, the labor saving effect dominates, resulting in a reduction in N . In particular, if $\sigma = -\infty$ (Leontief), the substitution effect vanishes, and only the labor saving effect is present. When $\sigma > -1$, the opposite is true, resulting in an increase in N . As a result, at $t^* = 0.26$, N is increasing in σ . When $\sigma = 0.5$, $N = 0.4$ at $t^* = 0.26$, and location 1 has 33% more population than other locations.

Finally, we briefly describe the effect of increasing the value of s by an example ($s = 0.3$: see Figures 8) in the Cobb-Douglas utility case ($\sigma = 0$). We observe that t^* goes down as s increases. Also, when the hub route is used, N at t^* increases as s increases. For $s = 0$ case, population N when the hub route is used is given by 0.382 at $t^* = 0.26$. For $s = 0.3$, $t^* = 0.12$ and $N = 0.41$ at this point. From these observations, we can see that if handling

²⁵At $t = 0$ the economy goes back to the classical one by using the hub route since s is set at 0, and $N = \frac{1}{3}$ holds. At $t = 0.5$ the three locations are completely symmetric, and $N = \frac{1}{3}$ holds.

cost goes up, it becomes necessary to have a strong transportation cost advantage for a location to become a hub city, but the population of a hub city would be larger for the high handling cost case. This sounds reasonable and intuitive.

5 Concluding Remarks

This paper presents a simple general equilibrium model that explains the emergence of a hub city solely in terms of differences in transportation technologies. The key point of departure with respect to the past literature on city formation is the specification of transportation costs. In the previous literature, it is usually assumed that the transportation cost function is of the ‘iceberg’ type: i.e., transportation costs are manifested in the extent to which the commodity “melts away” during transportation. In this paper, we assume that the transportation of commodities incurs labor input at transportation nodes (the departing locations). As a result, if large amount of commodities flow into a location that are transshipped to other locations, labor demand at that location goes up, resulting in population agglomeration. Clearly, if transportation costs are of the iceberg type, this mechanism does not work as is discussed in the introduction. Although our model looks simple, the characterization of equilibrium is still quite involved due to the fact that the model is a general equilibrium model with nonconvex consumption sets.²⁶

In our model, we assumed that there are no demand externalities and scale economies, in order to focus on the effects of differences in transportation costs on population agglomeration. However, it is possible that demand externalities and scale economies do indeed play important roles in causing economic agglomerations. Thus, it is natural to introduce demand externalities and/or scale economies into the model presented in this paper.²⁷ By introducing demand externalities à la Dixit and Stiglitz (1977) and Krugman (1991a), it is possible to explain a well-known mechanism of the development of cities, *import substitution* (see Mieszkowski (1979) and Mills and Hamilton (1994)). Import substitution means the following: As a region becomes more populated the imported manufacturing goods from other regions will be substituted gradually by the home production (of manufacturing goods) since the market becomes large enough to support home production.

²⁶Analytical results in the paper depend on some simplifying assumptions. We expect that we can obtain similar results in a more general model by using numerical simulations (it would be very difficult to obtain analytical results). With more than three locations in the economy, we expect that increased number of locations probably increases the amount of commodities flow into hub locations, resulting in more population agglomeration. It is also possible to allow imperfect specialization in production (in a Ricardian fashion), but analysis would be far more complicated as well.

²⁷Alternatively, we can also introduce Marshallian externalities within industries (localization economies: see Henderson (1988)) into our model. Henderson (1974) develops a general equilibrium model of system of cities in which each city specializes to the production of one commodity. City sizes are limited by land and commuting costs to the central district where production takes place in the fashion of monocentric city model by Mills (1967). Since Henderson’s model does not have transportation costs between cities and our model also has complete specialization (by location-specific technologies), it seems possible to combine these two models.

This phenomenon has been observed in many port cities in the Midwest in the last century.²⁸ Suppose that there are two types of commodities: one is an agricultural good traded in competitive markets, and the other is a group of manufacturing goods traded in monopolistically competitive markets à la Krugman (demand externalities).²⁹ Suppose that initially New York is the only place which produces manufacturing goods, and the Midwest locations are producing the agricultural product. Further suppose that a location has an advantage in transportation to New York. Then, by the hub city mechanism the population at that location increases (more transportation workers). The demand externalities of a larger market size attracts manufacturing firms, and the location becomes a large city.

6 Appendix

Here we collect all the proofs in Section 3.

6.1 Sketch of the Proof of Proposition 1

In this subsection, we sketch the proofs of nonemptiness and efficiency of equilibrium. A general equilibrium model with transportation costs and mobility of consumers is first analyzed by Schweizer, Varaiya, and Hartwick (1976), and we can easily interpret our transportation technologies as production technologies by generalizing their approach. The nonemptiness of equilibrium is shown by using their method in our simple setting with some detailed arguments.³⁰

To prove the efficiency of an equilibrium, we need to utilize ‘trading sets’ (McKenzie (1959)) instead of standard consumption sets. First, recall that the first welfare theorem is proved in the following way (see Mas-Colell, Whinston, and Green (1995)). Note all values are evaluated by the equilibrium prices:

1. Suppose, to the contrary, that an equilibrium allocation x is Pareto dominated by a feasible allocation y .
2. For each consumer, the value of her consumption plan under y is not less than her *income under x* (the value of her endowment plus her profits under x), and there is at least one consumer who cannot afford her consumption plan under y with her income under x .

²⁸Holmes (1996) provides a model which describes the process of migration of an industry by introducing variation in the quality of final commodities.

²⁹The Dixit-Stiglitz-Krugman model is not quite tractable without the iceberg type transportation technology. Ottaviano and Thisse (1998) propose an alternative demand externality model that can accommodate non-iceberg type transportation technologies.

³⁰More generally, we can utilize the methods in Berliant and Konishi (1994), Ellickson and Zame (1994), Guo (1996) and Konishi (1996). However, none of the theorems are directly applicable to the present model.

3. The value of the total consumption under y is strictly more than the total income under x (the value of the total endowment plus the total profits under x).
4. Since firms are maximizing profits under x , the value of the total consumption under y is strictly more than the sum of the value of the total endowment and the total profits under y given the equilibrium price.
5. However, it contradicts the feasibility of y . Hence, any equilibrium allocation is Pareto efficient.

It is easy to see that the proof relies on the property that the value of the total endowment is the same under x and y . However, in our model, it is no longer true if we use consumption sets. Consumers' location assignment is a part of the description of the feasible allocation. If a consumer moves from Chicago to New York, the value of her endowment (wage income) changes since in general the wage rate in Chicago is different from the wage rate in New York. Thus, the statement in step 2 is no longer true and the proof breaks down. McKenzie's trading set normalizes the consumer's endowment at the origin, so a consumer's endowment income is always zero, and the statement in step 2 recovers. Hence, the first welfare theorem is still valid in our economy. A core equivalence theorem can be proved by applying a standard proof such as in Berliant and Konishi (1994). ■

6.2 Proof of Proposition 2

The proof utilizes the first welfare theorem, which is stated in Proposition 1. We assume that there is an equilibrium which is not symmetric in consumption vectors, and show that there is a feasible allocation which Pareto dominates the equilibrium. Then, by the first welfare theorem, we get a contradiction. Let $\mathbf{a} \equiv (N^1, N^2, N^3; p^1, p^2, p^3; x^1, x^2, x^3; \theta^2, \theta^3)$ be an asymmetric equilibrium allocation. That is, one (or more) of the followings is violated: $x_2^1 = x_3^1$, $x_2^2 = x_3^2$, $x_1^2 = x_3^2$, or $x_2^3 = x_3^3$. We will construct a new allocation, which is feasible and Pareto-dominates \mathbf{a} . Let an allocation $\mathbf{a}' \equiv (N^{1'}, N^{2'}, N^{3'}; x^{1'}, x^{2'}, x^{3'}; \theta^{2'}, \theta^{3'})$ be such that $N^{1'} = N^1$, $N^{2'} = N^{3'} = \frac{N^2 + N^3}{2}$, $x_1^{1'} = x_1^1$, $x_2^{1'} = x_3^{1'} = \frac{x_2^1 + x_3^1}{2}$, $x_2^{2'} = x_3^{2'} = \frac{N^2 x_2^2 + N^3 x_3^2}{N^2 + N^3}$, $x_1^{2'} = x_1^{3'} = \frac{N^2 x_1^2 + N^3 x_1^3}{N^2 + N^3}$, $x_3^{2'} = x_2^{3'} = \frac{N^2 x_3^2 + N^3 x_2^3}{N^2 + N^3}$, and $\theta^{2'} = \theta^{3'} = \frac{\theta^2 N^2 x_3^2 + \theta^3 N^3 x_2^3}{N^2 x_3^2 + N^3 x_2^3}$. By strict convexity of preferences, it is obvious that allocation \mathbf{a}' Pareto-dominates allocation \mathbf{a} as long as the consumption vectors are asymmetric (note that $U(x^1) = U(x^2) = U(x^3)$ applies from the equilibrium conditions). Now, we will show that \mathbf{a}' is a feasible allocation. By the definitions of N^j s and x_1^j s, $N^{1'} x_1^{1'} = N^1 x_1^1$, $N^{2'} x_1^{2'} + N^{3'} x_1^{3'} = \frac{N^2 + N^3}{2} \left(\frac{N^2 x_1^2 + N^3 x_1^3}{N^2 + N^3} + \frac{N^2 x_1^2 + N^3 x_1^3}{N^2 + N^3} \right) = N^2 x_1^2 + N^3 x_1^3$, and $\theta^{2'} N^{2'} x_2^{2'} + \theta^{3'} N^{3'} x_2^{3'} = \frac{\theta^2 N^2 x_3^2 + \theta^3 N^3 x_2^3}{N^2 x_3^2 + N^3 x_2^3} \times \frac{N^2 + N^3}{2} \times \left(\frac{N^2 x_3^2 + N^3 x_2^3}{N^2 + N^3} + \frac{N^2 x_3^2 + N^3 x_2^3}{N^2 + N^3} \right) = \theta^2 N^2 x_3^2 + \theta^3 N^3 x_2^3$. Then, from equation (9) it is easy to see that the labor market at location 1 is cleared under allocation \mathbf{a}' : i.e.,

$$N^{1'} x_1^{1'} + (1+t) N^{2'} x_1^{2'} + (1+t) N^{3'} x_1^{3'} + \theta^{2'} (s+t) N^{3'} x_2^{3'} + \theta^{3'} (s+t) N^{2'} x_2^{2'} = N^{1'}.$$

Thus, it only remains to be shown that the labor market at location 2 clears under allocation \mathbf{a}' (if labor market at location 2 clears then the one at location 3 also clears automatically by the symmetry relation in \mathbf{a}'): i.e.,

$$(1+t)N^{1'}x_2^{1'} + N^{2'}x_2^{2'} + (1-\theta^{2'})(1+\bar{t})N^{3'}x_2^{3'} + \theta^{2'}(1+t)N^{3'}x_2^{3'} = N^{2'}.$$

It is easy to see $N^{1'}x_2^{1'} = N^1 \left(\frac{x_2^1 + x_3^1}{2} \right)$, $N^{2'}x_2^{2'} = \frac{N^2 + N^3}{2} \times \frac{N^2x_2^2 + N^3x_3^3}{N^2 + N^3} = \frac{N^2x_2^2 + N^3x_3^3}{2}$, and $N^{3'}x_2^{3'} = \frac{N^2 + N^3}{2} \times \frac{N^2x_3^2 + N^3x_2^3}{N^2 + N^3} = \frac{N^2x_3^2 + N^3x_2^3}{2}$. Thus, we need to show that the following equality holds:

$$(1+t)N^1 \left(\frac{x_2^1 + x_3^1}{2} \right) + \left(\frac{N^2x_2^2 + N^3x_3^3}{2} \right) + (1+\bar{t}) \left(\frac{N^2x_3^2 + N^3x_2^3}{2} \right) - (\bar{t}-t) \left(\frac{\theta^2N^2x_3^2 + \theta^3N^3x_2^3}{2} \right) = \frac{N^2 + N^3}{2}.$$

However, the *RHS* (*LHS*) of this equation is the average of the *RHS* (*LHS*) of equations (10) and (11). Thus, allocation \mathbf{a}' is feasible. This implies that unless consumption vectors are the same, the allocation \mathbf{a}' Pareto-dominates the equilibrium allocation \mathbf{a} , which contradicts the first welfare theorem. This proves that all of the followings are satisfied: $x_2^1 = x_3^1$, $x_2^2 = x_3^3$, $x_1^2 = x_1^3$, or $x_2^2 = x_2^3$. Since we have differentiability of U , the indifference curves are smooth. As a result, the supporting prices of consumption bundles are uniquely determined. This together with the symmetry of U imply that the followings are also satisfied: $p_2^1 = p_3^1$, $p_2^2 = p_3^3$, $p_1^2 = p_1^3$, $p_2^3 = p_2^2$. This proves the first part of the proposition.

Given the symmetry of the consumption and price vectors, let an equilibrium satisfy $N^2 \neq N^3$ and $\theta^2 \neq \theta^3$. Again, we can construct a symmetric allocation $(N^{1'}, N^{2'}, N^{3'}; x^{1'}, x^{2'}, x^{3'}; \theta^{2'}, \theta^{3'})$, which is an feasible allocation by the same argument as before. Since the consumption vectors are the same $(x^{1'}, x^{2'}, x^{3'}) = (x^1, x^2, x^3)$, the supporting price vectors (p^1, p^2, p^3) are the same. Thus, $(N^{1'}, N^{2'}, N^{3'}; p^1, p^2, p^3; x^{1'}, x^{2'}, x^{3'}; \theta^{2'}, \theta^{3'})$ also has to be an equilibrium. ■

6.3 Proof of Proposition 3

Totally differentiating equations (19) and (20) with respect to N and θ , we obtain,

$$\left(x_1^1 - (1+t)x_1^2 - \theta(s+t)x_3^2 - 1 \right) dN + (s+t)(1-N)x_3^2 d\theta = 0, \quad (21)$$

$$\left(2(1+t)x_2^1 - x_2^2 - (1+\bar{t})x_3^2 + \theta(\bar{t}-t)x_3^2 + 1 \right) dN - (\bar{t}-t)(1-N)x_3^2 d\theta = 0. \quad (22)$$

We will show that these two equations are linearly dependent given condition $(1+\bar{t})p = (1+t)p + (t+s)\bar{p}$. This implies that if we adjust N and θ to satisfy equation (21) then equation (22) is also satisfied.³¹ Thus, we can find a feasible allocation

³¹Given $x_i^j > 0$ for any $i, j = 1, 2, 3$, it is easy to see that any equilibrium population at location 1 is in the open interval $(0, 1)$, and we can differentiate the equations with respect to N and θ . This is guaranteed by the weak boundary conditions.

for any $\theta' \in [0, 1]$. Specifically, if $(N; p^1, p^2; x^1, x^2; \theta)$ is an equilibrium, we can find an equilibrium $(N(\theta'); p^1, p^2; x^1, x^2; \theta')$ for any $\theta' \in [0, 1]$, where $N(\theta') = N(\theta) + \int_{\theta}^{\theta'} \left(\frac{(s+t)(1-N(\tilde{\theta}))x_3^2}{-x_1^1 + (1+t)x_1^2 + \tilde{\theta}(s+t)x_3^2 + 1} \right) d\tilde{\theta}$, where $N(\theta) = N$. Now, we show the linear dependence of the two equations. Multiplying $\bar{p} = p_1^1$ on both sides of equation (21) produces,

$$\left(\bar{p}x_1^1 - (1+t)\bar{p}x_1^2 - \theta(s+t)\bar{p}x_3^2 - \bar{p} \right) dN + (s+t)(1-N)\bar{p}x_3^2 d\theta = 0.$$

Note that consumers at locations 1 and 2 have the following budget constraints, respectively: $\bar{p}x_1^1 + 2(1+t)px_2^1 = \bar{p}$, and $(1+t)\bar{p}x_1^2 + px_2^2 + (1+\bar{t})px_3^2 = p$. Together with condition $(1+\bar{t})p = (1+t)p + (t+s)\bar{p}$, we obtain,

$$\left(-2(1+t)px_2^1 + px_2^2 + (1+\bar{t})px_3^2 - p - \theta(\bar{t}-t)px_3^2 \right) dN + (\bar{t}-t)(1-N)px_3^2 d\theta = 0.$$

Dividing both sides by p , we obtain equation (22). ■

6.4 Proof of Proposition 4

At location 1, the wage rate is \bar{p} and the price vector that a consumer faces is $p^1 = (\bar{p}, (1+t)p, (1+t)p)$, while at location 2, the wage rate is p , and the price vector that a consumer faces is $p^2 = ((1+t)\bar{p}, p, (1+\bar{t})p)$ if the no-hub route is used, and $p^2 = ((1+t)\bar{p}, p, (1+t)p + (t+s)p)$ if the hub route is used. Let $\rho = \frac{p}{\bar{p}}$. Then, the normalized price vectors for consumers at locations 1 and 2 are $r^1 \equiv \frac{p^1}{\bar{p}} = (1, (1+t)\rho, (1+t)\rho)$, and $r^2 \equiv \frac{p^2}{p} = (\frac{1+t}{\rho}, 1, \frac{1+\bar{t}}{\rho})$ if the no-hub route is used, $r^2 \equiv \frac{p^2}{p} = (\frac{1+t}{\rho}, 1, 1+t+\frac{t+s}{\rho})$ if the hub route is used, respectively. Suppose that $\rho \leq 1$. Then, $(1+t)\rho \leq \frac{1+t}{\rho}$, $(1+t)\rho < \frac{1+\bar{t}}{\rho}$, and $(1+t)\rho < 1+t+\frac{t+s}{\rho}$ hold. Thus, $(1, (1+t)\rho, (1+t)\rho) < (1, \frac{1+t}{\rho}, \frac{1+\bar{t}}{\rho})$ and $(1, (1+t)\rho, (1+t)\rho) < (1, \frac{1+t}{\rho}, 1+t+\frac{t+s}{\rho})$ hold for any $t \in (0, \bar{t})$. Symmetry of the utility function implies that a consumer's utility level at location 1 is higher than the one at location 2. Thus, all consumers would move to location 1. However, this cannot happen by the boundary conditions. Hence, $\rho > 1$. ■

6.5 Proof of Proposition 5

The proof of Proposition 5 is involved and requires several steps. We utilize Propositions 1-4 to prove this proposition. We first consider two hypothetical economies in which the transportation routes are ex ante determined and consumers cannot choose other routes: (I) the economy uses only the no-hub route, and (II) the economy uses only the hub route. We show the nonemptiness and the uniqueness of equilibrium in these two economies (Lemmas 1, 2, and 3). These allocations may not be equilibrium allocations in the original economy (condition (15) may be violated). Given the first welfare theorem, we can show that the equilibrium of economies (I) and (II) which attains the higher utility level is the equilibrium of the original economy. Finally, we show that the utility levels of economies (I) and (II) coincide with each other only once at $t^* \in (0, 1)$ (Lemmas 4 and 5).

Let us start the proof of Proposition 5. Since $t_{23} = \bar{t}$ is fixed, the parameter $t (= t_{12} = t_{13})$ characterizes an economy completely. Let the equilibrium allocations for $t \in (0, \bar{t})$ be $(N(t); p^1(t), p^2(t); x^1(t), x^2(t); \theta(t))$. We can define $\bar{p}(t) \equiv p_1^1(t)$ and $p(t) \equiv p_2^2(t)$ as well.³² Let $e_{nh}(t) \equiv (N_{nh}(t); p_{nh}^1(t), p_{nh}^2(t); x_{nh}^1(t), x_{nh}^2(t); 0)$ and $e_h(t) \equiv (N_h(t); p_h^1(t), p_h^2(t); x_h^1(t), x_h^2(t); 1)$ be the equilibrium allocations in economies (I) and (II), respectively. We also define $\bar{p}_{nh}(t) \equiv p_{nh1}^1(t)$, $\bar{p}_h(t) \equiv p_{h1}^1(t)$, $p_{nh}(t) \equiv p_{nh2}^2(t)$, and $p_h(t) \equiv p_{h2}^2(t)$ as well. Equilibrium conditions in economies (I) and (II) are described by the conditions (19)-(18), yet condition (15) is replaced by $p_{nh3}^2(t) = (1+\bar{t})p_{nh}(t)$ and $p_{h3}^2(t) = (1+t)p_h(t) + (t+s)\bar{p}_h(t)$, respectively. Let $u(t)$, $u_{nh}(t)$, and $u_h(t)$ be the equilibrium utility levels in the original economy, economy (I), and economy (II), respectively.

In the following, we prove the uniqueness of equilibrium in economies (I) and (II) (Lemmas 1-3). For this purpose, it is convenient to describe the equilibrium conditions in these economies by using the expenditure function and the compensated demand functions. It is because consumers' utility levels have to be the same in the equilibrium even if their location choices are different due to free mobility of consumers. First, economy (I) is described by the following three equations with three endogenous variables $u(t)$, $p(t)$, and $N(t)$ (For simplicity, we drop subscript nh from all variables. Note also that $\bar{p}(t)$ is assumed to be constant):

$$Nh_1(\bar{p}, (1+t)p, (1+t)p, u) + (1-N)(1+t)h_1((1+t)\bar{p}, p, (1+\bar{t})p, u) = N \quad (23)$$

$$E(\bar{p}, (1+t)p, (1+t)p, u) = \bar{p}, \quad (24)$$

$$E((1+t)\bar{p}, p, (1+\bar{t})p, u) = p. \quad (25)$$

where $h_i(p^j, u)$ is the compensated demand function of commodity i , and $E(p^j, u)$ is the expenditure function of a consumer lives at location j , where u is the utility level and p^j is the price vector at location j . For notational simplicity, we omitted the argument t from variables. Equation (23) is the equilibrium condition for the labor market at location 1 (set $\theta = 1$ in equation (19)). Note that $x_1^1 = h_1(p^1) = h_1(\bar{p}, (1+t)p, (1+t)p)$, and $x_1^2 = h_1(p^2) = h_1((1+t)\bar{p}, p, (1+\bar{t})p)$. The equilibrium conditions for labor markets at 2 and 3 are eliminated by the symmetry of locations 2 and 3, and the Walras law. Equations (24) and (25) are the budget constraints for the consumers living at locations 1 and 2, respectively. The one for location 3 is eliminated by the symmetry of locations 2 and 3. Totally differentiating the system with respect to u , p , N , t , and \bar{t} , we obtain the following:

$$\begin{pmatrix} Nh_{1u}^1 + (1-N)h_{1u}^2 & (1+t)\{N(h_{12}^1 + h_{13}^1) + (1-N) & h_1^1 - (1+t)h_1^2 - 1 \\ & \times (h_{12}^2 + (1+\bar{t})h_{13}^2)\} & \\ E_u^1 & 2(1+t)h_2^1 & 0 \\ E_u^2 & h_2^2 + (1+\bar{t})h_3^2 - 1 & 0 \end{pmatrix} \begin{pmatrix} du \\ dp \\ dN \end{pmatrix}$$

³²We assume that $\bar{p}(t)$ is constant to avoid the indeterminacy of the price vectors. We use notation $\bar{p}(t)$ only to keep symmetry with $p(t)$.

$$= \begin{pmatrix} N(ph_{12}^1 + ph_{13}^1) + (1 - N)h_1^2 \\ +(1 - N)(1 + t)\bar{p}h_{11}^2 \\ 2h_2^1p \\ h_1^2\bar{p} \end{pmatrix} dt - \begin{pmatrix} (1 - N)(1 + t)ph_{13}^2 \\ 0 \\ h_3^2p \end{pmatrix} d\bar{t}, \quad (26)$$

where $h_i^j \equiv h_i(p^j, u)$, $h_{iu}^j \equiv \frac{\partial h_i(p^j, u)}{\partial u}$, $h_{ik}^j \equiv \frac{\partial h_i(p^j, u)}{\partial p_k^j}$, and $E_u^j = \frac{\partial E(p^j, u)}{\partial u}$. By using these notations, we can omit the arguments of each function and derivative. Let the Jacobian in the *LHS* of equation (26) be D_{nh} . By using linear homogeneity of utility function, and the zero homogeneity of compensated demand functions in prices, and equations (23), (24) and (25), we can simplify D_{nh} :

$$D_{nh} = \begin{pmatrix} \frac{N}{u} & -\frac{\bar{p}}{p}(Nh_{11}^1 + (1 - N)(1 + t)^2h_{11}^2) & h_1^1 - (1 + t)h_1^2 - 1 \\ \frac{\bar{p}}{u} & 2(1 + t)h_2^1 \\ \frac{p}{u} & -(1 + t)\frac{\bar{p}}{p}h_1^2 & 0 \\ & & 0 \end{pmatrix}.$$

We can show the following:

Lemma 1 *The determinant of D_{nh} ($|D_{nh}|$) is positive.*

Proof. We calculate the determinant of D_{nh} :

$$|D_{nh}| = \left(h_1^1 - (1 + t)h_1^2 - 1 \right) \frac{(1 + t)}{u} \left\{ -\frac{(\bar{p})^2}{p}h_1^2 - 2ph_2^1 \right\}.$$

The contents of the brace are obviously negative. Thus, all we have to show is that the contents of the parenthesis are also negative. By the definition of the expenditure function, we have the following:

$$E^1 = \bar{p}h_1^1 + (1 + t)ph_2^1 + (1 + t)ph_3^1.$$

From equation (24), we know $E^1 = \bar{p}$. Thus, $\bar{p}h_1^1 < \bar{p}$, and $h_1^1 < 1$. We have proved the contents of the parenthesis are negative. ■

Next, we describe economy (*II*) by the expenditure function and compensated demand functions. An equilibrium in economy (*II*) is described by the following three equations (Again, we drop subscript h from all variables):

$$Nh_1(\bar{p}, (1 + t)p, (1 + t)p, u) + (1 - N)(1 + t)h_1((1 + t)\bar{p}, p, (1 + t)p + (s + t)\bar{p}, u) \\ + (1 - N)(s + t)h_3((1 + t)\bar{p}, p, (1 + t)p + (s + t)\bar{p}, u) = N \quad (27)$$

$$E(\bar{p}, (1 + t)p, (1 + t)p, u) = \bar{p}, \quad (28)$$

$$E((1 + t)\bar{p}, p, (1 + t)p + (s + t)\bar{p}, u) = p. \quad (29)$$

Again, equation (27) denotes the labor market clearing condition at location 1, and equations (28) and (29) are the budget constraints of consumers living at locations 1 and

2, respectively. Totally differentiating the system of equations with respect to u , p , N , t , and s , we obtain the following:

$$\begin{aligned}
& \begin{pmatrix} \frac{N}{u} & (1+t)\{2Nh_{12}^1 + (1-N)(h_{12}^2 + (1+t)h_{13}^2)\} & h_1^1 - (1+t)h_1^2 \\ & +(1-N)(s+t)(h_{32}^2 + (1+t)h_{33}^2) & -(s+t)h_3^2 - 1 \\ \frac{\bar{p}}{u} & 2(1+t)h_2^1 & 0 \\ \frac{p}{u} & h_2^2 + (1+t)h_3^2 - 1 & 0 \end{pmatrix} \begin{pmatrix} du \\ dp \\ dN \end{pmatrix} \\
&= - \begin{pmatrix} 2Nph_{12}^1 + (1-N)(1+t)\{\bar{p}h_{11}^2 + (\bar{p}+p)h_{13}^2\} \\ +(1-N)(s+t)\{\bar{p}h_{31}^2 + (\bar{p}+p)h_{33}^2\} + (1-N)(h_1^1 + h_3^2) \\ 2h_2^1p \\ h_1^2\bar{p} + h_3^2(p + \bar{p}) \end{pmatrix} dt \\
&\quad - \begin{pmatrix} (1-N)(1+t)\{\bar{p}h_{13}^1 + \bar{p}h_{33}^2 + h_3^2\} \\ 2h_2^1p \\ h_3^2(\bar{p} + p) \end{pmatrix} ds. \tag{30}
\end{aligned}$$

Let us denote the matrix in the *LHS* of equation (30) by D_h .

Lemma 2 *The determinant of D_h ($|D_h|$) is positive.*

Proof. We calculate the determinant of D_h :

$$\begin{aligned}
|D_h| &= \frac{1}{u} \left(h_1^1 - (1+t)h_1^2 - (s+t)h_3^2 - 1 \right) \left\{ \bar{p} \left(h_2^2 + (1+t)h_3^2 - 1 \right) - 2(1+t)ph_2^1 \right\} \\
&= \frac{1}{u\bar{p}} \left(-2(1+t)\bar{p}h_1^2 - 2(s+t)\bar{p}h_3^2 - (1+t)ph_3^2 \right) \left\{ \frac{\bar{p}}{p} \left(-(1+t)ph_1^1 - (s+t)\bar{p}h_3^2 \right) - 2(1+t)ph_2^1 \right\}.
\end{aligned}$$

Thus, $|D_h|$ has a positive value. ■

Using Lemmas 1 and 2, we can prove the uniqueness of equilibrium in economies (I) and (II), respectively.

Lemma 3 *In economies (I) and (II), there exists a unique equilibrium for any $t \in [0, \bar{t}]$. The equilibrium allocations are C^1 differentiable with respect to t .*

Proof. First, we prove the statement for economy (I). Let $t = \bar{t} = 0$. Since it is a classical economy without transportation cost, there exists an equilibrium in this economy. Moreover, the symmetry and the strict quasiconcavity of U implies that the equilibrium consumption vectors are completely symmetric and unique. We now gradually increase $t = t_{12} = t_{13}$ and $\bar{t} = t_{23}$. As we can see below, equilibria in economy (I) for various t and \bar{t} are completely described by the system (26). Lemma 1 guarantees that the determinant of the Jacobian matrix in the *LHS* is always positive. This implies that there is no critical point in our comparative static exercise. Thus, given the fact that a unique equilibrium

exists for $t = \bar{t} = 0$, it is easy to see that the equilibrium exists and unique for any t and \bar{t} (see, say, Mas-Colell, Whinston, and Green [28], pages 619-620). It is easy to see that the equilibrium allocation is C^1 differentiable.

Second, we prove the existence and the uniqueness of equilibrium in economy (II). The argument is exactly the same as the one for economy (I). Let $t = s = 0$. Then, again the equilibrium is unique. Lemma 2 proves that the determinant of the Jacobian matrix in equation (30) is again always positive. ■

Uniqueness of equilibria (Lemma 3) implies that $e_{nh}(t)$ and $e_h(t)$ are functions (single-valued correspondences). Let $\mathcal{E}_{nh} \equiv \{t \in [0, \bar{t}] : (1 + \bar{t})p_{nh}(t) \leq (1 + t)p_{nh}(t) + (t + s)\bar{p}_{nh}(t)\}$ and $\mathcal{E}_h \equiv \{t \in [0, \bar{t}] : (1 + \bar{t})p_h(t) \leq (1 + t)p_h(t) + (t + s)\bar{p}_h(t)\}$, where $p_{nh}(t)$ ($p_h(t)$), and $\bar{p}_{nh}(t)$ ($\bar{p}_h(t)$) are equilibrium prices of commodity 2 at location 2 and commodity 1 of location 1 under the no hub (hub) route. By definition, $e_{nh}(t)$ ($e_h(t)$) is an equilibrium in the original economy if and only if $t \in \mathcal{E}_{nh}$ ($t \in \mathcal{E}_h$), since condition (15) is satisfied.

Lemma 4 *Suppose that $s \leq \bar{t}$ holds. Then, $\mathcal{E}_{nh} \cup \mathcal{E}_h = [0, \bar{t}]$, and $\mathcal{E}_{nh} \cap \mathcal{E}_h$ is nonempty and closed in $(0, \bar{t})$.*

Proof. From Proposition 1, there exists an equilibrium in the original economy for any t . Suppose that $(0, \bar{t}) \setminus (\mathcal{E}_{nh} \cup \mathcal{E}_h) \neq \emptyset$. Then, for any $t \in (0, \bar{t}) \setminus (\mathcal{E}_{nh} \cup \mathcal{E}_h)$, in every equilibrium $\theta(t) \in (0, 1)$ must hold. Proposition 3 says that it cannot be the case. A contradiction. This proves $\mathcal{E}_{nh} \cup \mathcal{E}_h = [0, \bar{t}]$. Since e_{nh} and e_h are continuous, p_{nh} and p_h are continuous as well. This implies that \mathcal{E}_{nh} and \mathcal{E}_h are closed (see the definitions). Thus, $\mathcal{E}_{nh} \cap \mathcal{E}_h$ is closed in $[0, \bar{t}]$. Now, we will show that $0 \in \mathcal{E}_h \setminus \mathcal{E}_{nh}$ and $\bar{t} \in \mathcal{E}_{nh} \setminus \mathcal{E}_h$. This guarantees that $\mathcal{E}_{nh} \cap \mathcal{E}_h$ is nonempty and closed in $(0, \bar{t})$ (note $\mathcal{E}_{nh} \cup \mathcal{E}_h = [0, \bar{t}]$). By the first welfare theorem, $u_{nh}(t) > u_h(t)$ ($u_{nh}(t) < u_h(t)$) implies $t \notin \mathcal{E}_h$ ($t \notin \mathcal{E}_{nh}$), which further implies $t \in \mathcal{E}_{nh}$ ($t \in \mathcal{E}_h$) since $\mathcal{E}_{nh} \cup \mathcal{E}_h = [0, \bar{t}]$. It is easy to see that $u_{nh}(\bar{t}) > u_h(\bar{t})$. Thus, $\bar{t} \in \mathcal{E}_{nh} \setminus \mathcal{E}_h$. To show $0 \in \mathcal{E}_h \setminus \mathcal{E}_{nh}$, we utilize the logics in the proof of Proposition 4. Consider the following two cases: (Case-I) $s > 0$: In this case, $p_h(0) > \bar{p}_h(0)$ holds. This together with $s \leq \bar{t}$ imply $(1 + \bar{t})p_h(0) \geq (1 + s)p_h(0) > p_h(0) + s\bar{p}_h(0)$. This implies $0 \in \mathcal{E}_h$. Similarly, we can show $(1 + \bar{t})p_{nh}(0) > p_{nh}(0) + s\bar{p}_{nh}(0)$. This implies $0 \notin \mathcal{E}_{nh}$. (Case-II) $s = 0$: In this case, $(1 + \bar{t})p_h(0) > p_h(0)$ and $(1 + \bar{t})p_{nh}(0) > p_{nh}(0)$ trivially hold. Thus, $0 \in \mathcal{E}_h \setminus \mathcal{E}_{nh}$. This completes the proof. ■

Lemma 5 *Suppose that $s \leq 1$ holds. Then, the set $\mathcal{E}_{nh} \cap \mathcal{E}_h$ is a singleton.*

Proof. Since $\mathcal{E}_{nh} \cap \mathcal{E}_h$ is closed, it can be partitioned into several closed intervals. Pick up one of the interval, $[a, b]$. (Note that $a = b$ can happen.) By definition, at $t = a$, either $(1 + \bar{t})p_{nh}(a) = (1 + a)p_{nh}(a) + (a + s)\bar{p}_{nh}(a)$ or $(1 + \bar{t})p_h(a) = (1 + a)p_h(a) + (a + s)\bar{p}_h(a)$ holds. Since the argument is the same, we simply assume that the first equality holds. By the argument in the proof of Proposition 3, for $N' = N + \int_0^1 \left(\frac{(s+t)(1-N(\tilde{\theta}))x_{nh3}^2(a)}{-x_{nh1}^1(a) + (1+t)x_{nh1}^2(a) + \theta(s+t)x_{nh3}^2(a) + 1} \right) d\tilde{\theta}$, the

allocation $(N'; p_{nh}^1(a), p_{nh}^2(a); x_{nh}^1(a), x_{nh}^2(a); 1)$ is an equilibrium in economy (II) as well. Since Lemma 3 guarantees the uniqueness of equilibrium in economy (II) , we have $e_h(a) \equiv (N_h(a); p_h^1(a), p_h^2(a); x_h^1(a), x_h^2(a); 1) = (N'; p_{nh}^1(a), p_{nh}^2(a); x_{nh}^1(a), x_{nh}^2(a); 1)$. This proves $(x_h^1(a), x_h^2(a)) = (x_{nh}^1(a), x_{nh}^2(a))$. By the same argument, we can prove $(x_h^1(b), x_h^2(b)) = (x_{nh}^1(b), x_{nh}^2(b))$. Thus, $(x_h^1(t), x_h^2(t)) = (x_{nh}^1(t), x_{nh}^2(t))$ for any $t \in \text{bdry}(E_{nh} \cap E_h)$, where $\text{bdry}(\cdot)$ is a topological boundary of \cdot . Now, we prove that the set $\mathcal{E}_{nh} \cap \mathcal{E}_h$ is a singleton. To do this, we prove $\frac{du_{nh}}{dt} > \frac{du_h}{dt}$ at any $t \in \text{bdry}(\mathcal{E}_{nh} \cap \mathcal{E}_h)$. If this is shown, then continuity properties of u_{nh} and u_h guarantee that there is only one element in $\mathcal{E}_{nh} \cap \mathcal{E}_h$. We utilize the property $(x_h^1(t), x_h^2(t)) = (x_{nh}^1(t), x_{nh}^2(t))$ extensively. From equation (26), we can calculate $\frac{du_{nh}}{dt}$ at $t \in \text{bdry}(\mathcal{E}_{nh} \cap \mathcal{E}_h)$:

$$\begin{aligned} \frac{du_{nh}}{dt} &= - \frac{u \begin{vmatrix} 2h_2^1 p & 2(1+t)h_2^1 \\ h_1^2 \bar{p} & -(1+t)\frac{\bar{p}}{p} h_1^2 \end{vmatrix}}{\begin{vmatrix} \bar{p} & 2(1+t)h_2^1 \\ p & -(1+t)\frac{\bar{p}}{p} h_1^2 \end{vmatrix}} \\ &= \frac{4u(1+t)\bar{p}h_1^2 h_2^1}{(1+t)\left(\frac{(\bar{p})^2}{p} h_1^2 + 2ph_2^1\right)} = \frac{4u\bar{p}h_1^2 h_2^1}{\left(\frac{(\bar{p})^2}{p} h_1^2 + 2ph_2^1\right)}. \end{aligned}$$

From equation (30), we can calculate $\frac{du_h}{dt}$ at $t \in \text{bdry}(\mathcal{E}_{nh} \cap \mathcal{E}_h)$:

$$\begin{aligned} \frac{du_h}{dt} &= - \frac{u \begin{vmatrix} 2h_2^1 p & 2(1+t)h_2^1 \\ h_1^2 \bar{p} + h_3^2(\bar{p} + p) & -(1+t)\frac{\bar{p}}{p} h_1^2 - (t+s)h_3^2 \end{vmatrix}}{\begin{vmatrix} \bar{p} & 2(1+t)h_2^1 \\ p & -(1+t)\frac{\bar{p}}{p} h_1^2 - (t+s)h_3^2 \end{vmatrix}} \\ &= - \frac{u \{4\bar{p}(1+t)h_1^2 h_2^1 + 2(t+s)ph_2^1 h_3^2 + 2(1+t)\bar{p}h_2^1 h_3^2 + 2(1+t)ph_2^1 h_3^2\}}{(1+t)\left(\frac{(\bar{p})^2}{p} h_1^2 + 2ph_2^1\right) + (t+s)\bar{p}h_3^2} \\ &= - \frac{u \left\{ (4\bar{p}h_1^2 h_2^1) + 2\frac{t+s}{1+t} ph_2^1 h_3^2 + 2\bar{p}h_2^1 h_3^2 + 2ph_2^1 h_3^2 \right\}}{\left(\frac{(\bar{p})^2}{p} h_1^2 + 2ph_2^1\right) + \frac{t+s}{1+t} \bar{p}h_3^2}. \end{aligned}$$

Now, noting that the consumption vectors in these two economies are the same, we subtract $\frac{du_h}{dt}$ from $\frac{du_{nh}}{dt}$:

$$\begin{aligned} &\frac{1}{u} \left(\frac{du_{nh}}{dt} - \frac{du_h}{dt} \right) \\ &= \frac{\left\{ 2\frac{t+s}{1+t} ph_2^1 h_3^2 + 2ph_2^1 h_3^2 + 2\bar{p}h_2^1 h_3^2 \right\} \left\{ \frac{(\bar{p})^2}{p} h_1^2 + 2ph_2^1 \right\} - 4\bar{p}h_1^2 h_2^1 \times \frac{t+s}{1+t} \bar{p}h_3^2}{\left\{ \frac{(\bar{p})^2}{p} h_1^2 + 2ph_2^1 + \frac{t+s}{1+t} \bar{p}h_3^2 \right\} \left\{ \frac{(\bar{p})^2}{p} h_1^2 + 2ph_2^1 \right\}} \end{aligned}$$

$$\geq \frac{\left\{4\frac{t+s}{1+t}ph_2^1h_3^2 + 2\bar{p}h_2^1h_3^2\right\} \left\{\frac{(\bar{p})^2}{p}h_1^2 + 2ph_2^1\right\} - 4\frac{t+s}{1+t}(\bar{p})^2h_1^2h_2^1h_3^2}{\left\{\frac{(\bar{p})^2}{p}h_1^2 + 2ph_2^1 + \frac{t+s}{1+t}\bar{p}h_3^2\right\} \left\{\frac{(\bar{p})^2}{p}h_1^2 + 2ph_2^1\right\}} > 0.$$

The first inequality (with equality) follows by the assumption $s \leq 1$. Hence, our claim is proved. ■

Proof. Now, we can finally prove Proposition 5. Let $\{t^*\} = \mathcal{E}_{nh} \cap \mathcal{E}_h$ (Lemma 5). Lemma 4 implies $0 < t^* < \bar{t}$. From the proof of Lemma 5, we know that $u_{nh}(t) = u_h(t)$ holds only at $t = t^*$, and $\frac{du_{nh}(t)}{dt} > \frac{du_h(t)}{dt}$ at $t = t^*$. Thus, we can conclude that $u_{nh}(t) > u_h(t)$ for any $t \in (t^*, \bar{t}]$, and $u_{nh}(t) < u_h(t)$ for any $t \in [0, t^*)$. Proposition 3 says that there is a continuum of equilibria at $t = t^*$. Hence, $\mathcal{E}_{nh} = [t^*, \bar{t}]$, and $\mathcal{E}_h = [0, t^*]$ by the first welfare theorem. This completes the proof of Proposition 5. ■

6.6 Proof of Proposition 6

We first prove the following lemma. We specify the argument t for the equilibrium allocation for $t \in (0, t^*)$ to avoid confusion.

Lemma 6 For any $t \in (0, t^*)$, $N(t) > \frac{1}{3}$ if $x_1^1(t) \geq x_2^2(t)$.

Proof. We refer the labor market equilibrium condition at location 2 (equation (20)). Since $t \in (0, t^*)$, $\theta = 1$ follows from Proposition 5. Then, equation (20) becomes as follows (the equilibrium allocation for t is substituted):

$$(1+t)N(t)x_2^1(t) + \left(\frac{1-N(t)}{2}\right)x_2^2(t) + (1+t)\left(\frac{1-N(t)}{2}\right)x_3^2(t) = \left(\frac{1-N(t)}{2}\right).$$

We want to show $N(t)$ is greater than $\frac{1}{3}$. Let $\Phi(N; t) \equiv (1+t)Nx_2^1(t) + \left(\frac{1-N}{2}\right)x_2^2(t) + (1+t)\left(\frac{1-N}{2}\right)x_3^2(t) - \left(\frac{1-N}{2}\right)$. This is the excess demand function when the population at location 1 is N and the consumption vectors are $x^1(t)$ and $x^2(t)$. Since $\Phi(0; t) < 0$, $\Phi(1; t) > 1$, and the equilibrium is unique for any $t \in (0, t^*)$, it is easy to see that $N(t)$ is greater than $\frac{1}{3}$ if $\Phi\left(\frac{1}{3}; t\right) < 0$. Substituting $N = \frac{1}{3}$ into $\Phi(N; t)$, we obtain,

$$\Phi\left(\frac{1}{3}; t\right) = \frac{1}{3} \left[(1+t)x_2^1(t) + x_2^2(t) + (1+t)x_3^2(t) - 1 \right].$$

From the budget constraint at location 1, we know:

$$x_1^1(t) + 2(1+t)\rho(t)x_2^1(t) = 1.$$

Substituting this into $\Phi\left(\frac{1}{3}; t\right)$, we obtain:

$$\Phi\left(\frac{1}{3}; t\right) = \frac{1}{3} \left[x_2^2(t) - x_1^1(t) + (1+t)x_3^2(t) - (1+t)(2\rho(t) - 1)x_2^1(t) \right]$$

$$< \frac{1+t}{3} (x_3^2(t) - x_2^1(t)).$$

The above inequality holds, since $x_2^2(t) \leq x_1^1(t)$ and $\rho(t) > 1$ (Proposition 4). Thus, to show $\Phi(\frac{1}{3}; t) < 0$, it is sufficient to prove $x_3^2(t) \leq x_2^1(t)$. Since $r^1(t) \equiv (1, (1+t)\rho(t), (1+t)\rho(t))$ and $r^2(t) \equiv (\frac{1+t}{\rho(t)}, 1, 1+t+\frac{t+s}{\rho(t)})$, it suffices to show $h_2(1, (1+t)\rho(t), (1+t)\rho(t), u(t)) \geq h_3(\frac{1+t}{\rho(t)}, 1, 1+t+\frac{t+s}{\rho(t)}, u(t)) = h_2(1, 1+t+\frac{t+s}{\rho(t)}, \frac{1+t}{\rho(t)}, u(t))$ (symmetry). To prove $h_2(1, (1+t)\rho(t), (1+t)\rho(t), u(t)) \geq h_2(1, 1+t+\frac{t+s}{\rho(t)}, \frac{1+t}{\rho(t)}, u(t))$, we make the following argument. Since $\frac{1+t}{\rho(t)} < (1+t)\rho(t)$, we have $1+t+\frac{t+s}{\rho(t)} > (1+t)\rho(t)$. Note that $E(1, (1+t)\rho(t), (1+t)\rho(t), u(t)) = E(\frac{1+t}{\rho(t)}, 1, 1+t+\frac{t+s}{\rho(t)}, u(t)) = E(1, 1+t+\frac{t+s}{\rho(t)}, \frac{1+t}{\rho(t)}, u(t)) = 1$. Let $\alpha \in \mathfrak{R}_{++}$. Define $\beta : \mathfrak{R}_{++} \rightarrow \mathfrak{R}_{++}$ to satisfy $E(1, \alpha, \beta(\alpha), u) = 1$ for any $\alpha \in \mathfrak{R}_{++}$. If we show the following for any $\alpha \in \mathfrak{R}_{++}$, then we are done:

$$\frac{dh_2(1, \alpha, \beta(\alpha), u)}{d\alpha} \leq 0.$$

Since $\beta'(\alpha) < 0$, $\frac{\partial h_2(1, \alpha, \beta(\alpha), u)}{\partial \alpha} = h_{22} + h_{23}\beta'(\alpha) < 0$, which completes the proof. ■

Proof. Now, we can prove Proposition 6. It suffices to prove that $x_2^2(t) = x_1^1(t)$ holds under a CES utility function: i.e., $h_2(\frac{1+t}{\rho(t)}, 1, 1+t+\frac{t+s}{\rho(t)}, u(t)) = h_1(1, (1+t)\rho(t), (1+t)\rho(t), u(t))$ or $h_1(1, \frac{1+t}{\rho(t)}, 1+t+\frac{t+s}{\rho(t)}, u(t)) = h_1(1, (1+t)\rho(t), (1+t)\rho(t), u(t))$ (symmetry). Thus, we only need to show that $h_1(1, \alpha, \beta(\alpha))$ is constant for any $\alpha \in \mathfrak{R}_{++}$ (function β is defined in the proof of Lemma 6). By the definition of β , $E_2 + E_3\beta'(\alpha) = h_2 + h_3\beta'(\alpha) = 0$. Thus, $\beta'(\alpha) = -\frac{h_2}{h_3}$. Since U is a CES function, the expenditure function, the compensated demand functions, and the derivative of the compensated demand functions for $\sigma \neq 0$ are:

$$E(p_1, p_2, p_3, u) = u(p_1^\pi + p_2^\pi + p_3^\pi)^{\frac{1}{\pi}},$$

$$h_i(p_1, p_2, p_3, u) = u(p_1^\pi + p_2^\pi + p_3^\pi)^{\frac{1-\pi}{\pi}} p_i^{\pi-1} \quad \text{for } i = 1, 2, 3,$$

$$h_{ij}(p_1, p_2, p_3, u) = u(p_1^\pi + p_2^\pi + p_3^\pi)^{\frac{1-2\pi}{\pi}} p_i^{\pi-1} p_j^{\pi-1} \quad \text{for } i, j = 1, 2, 3, i \neq j,$$

where $\pi = \frac{\sigma}{\sigma-1}$. Using them, it is easy to see the followings:

$$\frac{dh_1(1, \alpha, \beta(\alpha), u)}{d\alpha} = h_{12} - \frac{h_2}{h_3} h_{13} = \frac{1}{h_3} \left(\frac{h_{21}}{h_2} - \frac{h_{31}}{h_3} \right) = 0.$$

Hence, $h_1(1, \alpha, \beta(\alpha), u)$ is constant for any $\alpha \in \mathfrak{R}_{++}$. What if $\sigma = 0$? In this case, the utility function is a Cobb-Douglas function. It is well-known that the expenditure share for each commodity is constant for any prices under Cobb-Douglas preferences. Actually, since $r_1^1 = r_2^2 = 1$, x_1^1 and x_2^2 are actually also expenditure shares, and $x_1^1(t) = x_2^2(t) = \frac{1}{3}$ holds for any t . The proof is completed. ■

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