Migration-Proof Tiebout Equilibrium:†
Existence and Asymptotic Efficiency

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April 1998

Revised: December 2000

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† This paper was written while both the authors were visiting Boston University. The authors thank people at Boston University for their hospitality. They also thank an anonymous referee and associate editor of this journal for their advice and suggestions. Of course, we take responsibility of any errors that remain.

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Abstract

Tiebout's basic claim was that when public goods are local, competition between jurisdictions solves the free riding problem in the sense that equilibria exist and are always Pareto efficient. Unfortunately, the literature does not quite support this conjecture. For finite economies, one must choose between notions of Tiebout equilibrium which are Pareto optimal but which may be empty, or which are nonempty but may be inefficient. This paper introduces a new equilibrium notion called *migration-proof Tiebout equilibrium* which we argue is a natural refinement of Nash equilibrium for a multijurisdictional environment. We show for sufficiently large economies with homogeneous consumers, such an equilibrium always exists, is unique, and is asymptotically Pareto efficient.

1. Introduction

In his seminal paper, Tiebout (1956) suggests that if public goods are subject to congestion, the benefits of sharing costs over a large number of agents will eventually be offset by the negative effects of crowding. Balancing the effects of cost-sharing and congestion make it advantageous for agents to be partitioned into a system of disjoint jurisdictions. Tiebout speculates that these jurisdictions would offer competing bundles of local public goods and tax liabilities, and that agents would move to jurisdictions whose membership, public good and tax levels most closely approximated their ideal combinations. Tiebout concludes that if public goods are local, agents will reveal their preferences through their locational choice and the free rider problem will disappear.

In more formal terms, Tiebout hypothesizes that if public goods are provided locally, equilibrium will exist and will be Pareto efficient. His paper is quite informal, however, and no proof is offered for this proposition. Unfortunately, there is reason to doubt that his hypothesis is true in general. One of the most famous of papers that makes this point is Bewley (1981) who provides counterexamples to both existence and efficiency of equilibrium. While this is disturbing, such examples do not in themselves constitute a proof that Tiebout markets necessarily fail. Nevertheless, this strikes at the heart of Tiebout's insight and many contributors have attempted to provide a rebuttal.

A major branch of this literature considers Nash-based equilibrium concepts. Nash equilibrium has a certain natural appeal in this context. The requirement of stability against unilateral deviation captures the idea that it is institutionally forbidden in our society to prevent agents for freely migrating between local jurisdictions. There are a number of papers which consider this sort of "free mobility" equilibrium with and without land, using various voting rules to decide of taxes and public good levels, and allowing for different types of tax instruments. A very incomplete list of such work includes Westhoff (1977), Richter (1982), Greenberg (1983), Epple et al. (1984, 1993), Dunz (1989), Konishi (1996), Nechyba (1997).[1] Depending on the details of the specific model in question, it is usually possible to show that Nash equilibrium exists. Unfortunately, the set of free mobility equilibria can be quite large.[2] It is easy for agents to become trapped in suboptimal states for which coalitional deviations would yield a significant Pareto improvement, but which are stable against Nash's unilateral deviations. Thus, the major failing of these Nash-based approaches is that a first welfare theorem does not hold in general.

In order to guarantee efficiency of equilibrium, a natural approach is to consider notions which allow for coalitional deviations and free entry of new jurisdictions. Early work exploring this type of Tiebout equilibrium includes McGuire (1974), Wooders (1978) and Berglas and Pines (1981). Their equilibrium concepts require that given prices and the tax system, no consumer would want to move to another jurisdiction, either currently existing, or potentially available. The remarkable thing about these equilibria is that they are also immune to coalitional deviation. Following Greenberg and Weber (1986) we shall call this class of equilibrium concepts "strong Tiebout equilibria".[3] Notions like the core, strong Nash equilibrium, and the coalition-proof Nash equilibria are in this spirit. Many authors have continued this program and have considered models with various forms of crowding, different treatments of public goods, and alternative price systems and institutional structures.

[1] See especially these last two papers, and Conley and Wooders (1997) for more complete survey of all the literature mentioned in this introduction.

[2] Possible inefficiency of free mobility equilibrium was first discussed in Buchanan and Goetz (1972) and Flatters, Henderson, and Mieszkowski (1974).

[3] See Guesnerie and Oddou (1981) and Greenberg and Weber (1986) for investigations of the strong restrictions needed to get existence of this sort of equilibria even in an economy *without* congestion effect.

The fundamental difficulty with these approaches is while it is often possible to show that such equilibria are efficient, it generally impossible to prove that they exist. See for example, Pauly (1970), who studies the core of a simple jurisditional game, and Wooders (1978) who explores the market equilibrium of a related economy. Among other things, they find that,unless population happens to be an integer multiple of the optimal coalition size, the core or equilibrium set may me empty. The intuition is that otherwise, there are “left-over” agents who generate instability and a failure of existence unless a way can found to place them in non-optimal jurisdictions which nevertheless preclude the possibility of a beneficial coalitional deviation.

The problem of left-overs is quite fundamental and does not yield to easy solution. One approach is to dismiss this as a purely theoretical issue. Wooders (1980) points out that the fraction of left-overs goes to zero as the economy gets large. This allows her to show that while the exact equilibrium may not exist, a properly defined notion of ϵ-equilibrium does. More recently, Cole and Prescott (1997), Conley and Wooders (1997), and Ellickson *et al.* (1999) have carried this intuition forward to show that exact equilibrium exists for continuum versions of such economies. While this is very encouraging, it still does not provide a satisfactory Tiebout theorem when the population is relatively small.

In short, we face a problem similar to that of Goldilocks. On the one hand, the equilibrium notions which allow for coalitional deviations are too strong. In general, such equilibria may not exist. On the other hand, equilibrium notions which allow for only unilateral deviations are too weak. They permit too many Pareto inefficient equilibrium to exist. What we need is a notion which allows neither too much nor too little defection and which also makes sense in the context of a local public goods economy. In other words, we are looking for something which is just right.

The equilibrium notion we use in this paper is motivated by the following consideration. Agents should be allowed not only to deviate unilaterally but also to collectively take advantage of opportunities to improve their welfare as long as such action can be successfully executed. Thus, the equilibrium must be stable against *credible* coalitional deviations. The key question, of course, is what constitutes a credible deviation in the context of the Tiebout economy.[4] One of the standard stories we tell in local public economics is of the rich going to suburbs in an attempt to get away from the poor. This is limited, however, by the possibility that the poor may choose to follow rich in order to free ride of their high levels of public goods provision.[5] Following this logic, we shall require a coalitional deviation to be "migration-proof". That is, we shall require that agents who are left behind do not choose follow the defecting coalition. We will call a Nash equilibrium that is immune to any migration-proof coalitional deviations a *migration-proof Tiebout equilibrium.*

In this paper, we consider an economy with one public and one private good. The major restriction that we impose on our model is that population consist of a single type of agent with single-peaked preferences. Our main results are that (i) for relatively large but still finite populations, the exact migration-proof equilibrium always exists and is unique, and (ii) that this equilibrium is asymptotically efficient in the sense that the per-capita utility approaches the maximum possible as the economics grows without bound. Thus, we provide a confirmation of Tiebout's hypothesis, although for a limited class of economies. The remainder of the paper is organized as follows. Section two describes the model and provides a definition of our notion of Tiebout equilibrium. Section three gives the results. Second four

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[4] Credible deviations in the sense of coalition-proof Nash equilibrium (see Bernheim, Peleg, and Whinston (1987) for the definition) do not make too much sense in the Tiebout economy. Coalition-proofness requires that a coalitional deviation is immune to further (smaller and nested) credible deviation from that coalition. In the Tiebout economy, if such a beneficial deviation exists, then it would also have existed in the original coalition structure. See also Ray (1989).

[5] It is often argued that fiscal zoning and high property tax rates are in reality instruments used by the rich to prevent the poor from chasing them (see Hamilton 1975). Without such devices, the rich could not escape the poor, and so migration to the suburbs would be pointless.

concludes. All the proofs are collected in appendix.

2. The Model

We consider an economy with one private good denoted $x \geq 0$ and one public good denoted $y \geq 0$. The economy contains I identical consumers, denoted $i \in \{1, \ldots, I\} \equiv \mathcal{I}$. Each consumer $i \in \mathcal{I}$ has endowment $\omega > 0$ of private good, and preferences represented by a utility function $U(x, y, n)$, where n is an element of the positive integers Z_{++}. Thus, agents' welfare is affected by private good consumption, public good consumption and the population of the jurisdiction in which he resides. Public good is produced from private good according to the cost function $C(y)$. Thus, $x = C(y)$ means that x units of private good is needed to produce y units of public good. Since consumers are identical, we shall assume that the cost of public good provision cost is shared equally within the jurisdiction.

Define the optimal public good provision level for a coalition with population n as:

$$y^*(n) \equiv \text{argmax}\, U\left(\omega - \frac{C(y)}{n}, y, n\right) \text{ subject to } y \geq 0,\ \omega \geq \frac{C(y)}{n}.$$

By using $y^*(n)$, we can define an indirect utility function $u(n)$ for $n = 1, \ldots$:

$$u(n) = U\left(\omega - \frac{C(y^*(n))}{n}, y^*(n), n\right).$$

To simplify notation, we will extend the domain of the indirect utility function over the set of coalitions in the natural way. Thus, if $S \subset \mathcal{I}$ is a subset of the grand coalition, we will take $u(S) \equiv u(\parallel S \parallel)$ where $\parallel S \parallel$ is the cardinality of the coalition S. This is a slight abuse of notation, but should not lead to any confusion.

Following a standard assumption in local public good economies, we shall assume that $u(n)$ is *single-peaked* at some finite population n^*:

Single-peakedness There exists some $n^* \in Z_{++}$ such that (i) for all $n', n'' \in Z_{++}$ such that $n' < n'' \leq n^*$ it holds that $u(n') < u(n'')$ and (ii) for all $n', n'' \in Z_{++}$ such that $n^* \leq n' < n''$ it holds that $u(n') > u(n'')$.

Informally this says that a consumer's utility level monotonically increases in jurisdiction size till the size of a jurisdiction reaches n^*, and after that, monotonically decreases. For simplicity, we also assume that consumers' preferences are *strict* in the sense that consumers are not indifferent between any pair of sizes of jurisdictions:

Strict preference For all $n', n'' \in Z_{++}$ such that $n' \neq n''$, $u(n') \neq u(n'')$.

This is a generic case in any event, and this assumption simplifies the analysis. Even if this condition is not satisfied essentially the same analysis goes through, but the uniqueness of migration-proof equilibrium may be lost.

2.1 Equilibrium Notions

Since we are concerned in this paper with a local public goods economy, in equilibrium agents choose to live in one and only one jurisdiction. This means that in equilibrium agents partition themselves into disjoint and exhaustive coalitions. Defecting coalitions of agent also have the option to forming a system of disjoint coalitions in an effort to improve on the allocation offered by the grand coalition. Thus, we will need to define the notion of a partition for any coalition or subcoalition of the population.

Definition 1. For any $T \subseteq \mathcal{I}$, a finite set of coalitions of agents $\pi = (S^1, \ldots S^K)$ is said to be **partition of T** if (i) for all $k = 1, \ldots, K$ $S^k \neq \emptyset$, (ii) $\cup_{k=1}^K S^k = T$, and (iii) for all $k \neq \bar{k}$, $S^k \cap S^{\bar{k}} = \emptyset$. Let $\Pi(T)$ denote the set of all possible partitions of the coalition T.

Note that in particular, $\Pi(\mathcal{I})$ denotes the set of possible partitions of the grand coalition. Now we define a few equilibrium notions by using this notation:

Definition 2. A partition $\pi \in \Pi(\mathcal{I})$ is said to be **a free mobility equilibrium**, iff (i) for all $k = 1, \ldots, K$ and all $i \in S^k$, it holds that $u_i(S^k) \geq u_i(\{i\})$, and (ii) for all $k, \bar{k} = 1, \ldots, K$ such that $k \neq \bar{k}$, and all $i \in S^k$, it holds that $u_i(S^k) \geq u_i(S^{\bar{k}} \cup \{i\})$.

This free mobility equilibrium assumes standard Nash type of behavior. This equilibrium notion is often used in the literature, but it tends to include too many equilibria some of which are may be inefficient. The following example demonstrates this:

Example 1: Free mobility equilibria may be inefficient.

Suppose that there are 1200 consumers in the economy and the optimal population $n^* = 200$. Also suppose that $u(1200) > u(1)$. Notice that all agents coalescing into the grand coalition is a free mobility equilibrium since $u(1200) > u(1)$. We can also construct a several more equilibria described by population partitions: $\{600, 600\}$, $\{400, 400, 400\}$, $\{300, 300, 300, 300\}$, $\{240, 240, 240, 240, 240\}$, and $\{200, 200, 200, 200, 200, 200\}$. Clearly, since consumers' preferences are single-peaked, any agent who moves to another jurisdiction would be made worse off. Notice also that a partition consisting of six coalitions containing 200 agents each is best in the Pareto sense.

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Thus, to support Tiebout's theorem, it is necessary to refine the set of free mobility equilibria in some way. One natural way is to ask an equilibrium to be immune to coalitional deviations as well as unilateral deviations. The story is as follows: From a jurisdiction structure, a land developer initiates a new jurisdiction in a suburb, and by direct mail he can organize a coalitional deviation. If consumers who receive the direct mail can improve their payoffs by following the developer's suggestion, then the coalitional deviation will be self-enforcing. A strong Tiebout equilibrium is immune to such coalitional deviations.

Definition 3. A **coalitional deviation** from $\pi = \Pi(\mathcal{I})$ is a partition $\pi^T = (T^1, \ldots T^L) \in \Pi(T)$ of the nonempty coalition $T \subseteq \mathcal{I}$, such that for all $i \in T$, $u_i(T^\ell) > u_i(S^k)$ where $i \in S^k \in \pi$ and $i \in T^\ell \in \pi^T$.

Definition 4. (Greenberg and Weber (1986), Demange (1994)): A **strong Tiebout equilibrium** is a jurisdiction structure π such that (i) π is a free mobility equilibrium, and (ii) π is immune to any coalitional deviation.

Although the definition of strong Tiebout equilibrium makes sense, this equilibrium notion turns out to be too strong. It is very often the case that strong Tiebout equilibrium is empty in the presence of congestion.

Example 2: Nonexistence of strong Tiebout equilibrium. Suppose that $\mathcal{I} = \{1, 2, 3\}$ and $u(2) > u(3) > u(1)$ holds. Since a strong Tiebout equilibrium is a free mobility equilibrium, the unique candidate for a strong Tiebout equilibrium is the grand coalition. However, from this allocation, any two consumer coalition would find it in their interests to deviate will deviate. Thus, there is no strong Tiebout equilibrium in the economy.

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Thus, it is too restrictive to require that an equilibrium to be immune to any coalitional deviation. Although coalition-proof Nash equilibrium reduces the set of possible coalitional deviations, this equilibrium notion does not help us in the above example. There is a unique free mobility equilibrium in the economy (the grand coalition), but two consumer deviation is a credible coalitional deviation since it is immune to further deviations from the coalitional deviation itself. In the following, we introduce another type of refinement of coalitional deviations.

The idea of "migration-proofness" is that a coalitional deviation is feasible only when no outsiders would chase this deviation. From a jurisdiction structure, a land developer initiates a new jurisdiction in a suburb, and by direct mail, he organizes a coalitional deviation. However, if there is no effective tool to exclude outsiders, then even if a new jurisdiction is attractive to the members initially, it may become congested by further immigration of outsiders. If this happens then the new jurisdiction ends up with being very congested and becomes unattractive. The following restriction on coalitional deviations requires that (i) no outsiders want to join the newly established jurisdiction, and (ii) within the coalitional deviation, nobody wants to move to any other jurisdiction.

Definition 5. A migration-proof coalitional deviation from a partition $\pi = (S^1, \ldots, S^K) \in \Pi(\mathcal{I})$ is a coalitional deviation $\pi^T = (T^1, \ldots, T^L) \in \Pi(T)$ by $T \subseteq \mathcal{I}$ such that (i) for all $k = 1, \ldots, K$ and all agents[6] $i \in S^k \backslash T$ it holds all $\ell = 1, \ldots, L$ that $u_i(T^\ell \cup \{i\}) \leq u_i(S^k \backslash T)$, and (ii) for all $\ell, \bar{\ell} = 1, \ldots, L$ such that $\ell \neq \bar{\ell}$, and all $i \in T^\ell \in \pi^T$ it holds that $u_i(T^\ell) \geq u_i(T^{\bar{\ell}} \cup \{i\})$.

Definition 6. A migration-proof Tiebout equilibrium is a partition $\pi \in \Pi(\mathcal{I})$ such that (i) π is a free mobility equilibrium, and (ii) π is immune to any migration-proof coalitional deviation.

In the above three consumer example, there exists a migration-proof Tiebout equilibrium, which is a grand coalition. A two consumer coalition wants to deviate from the grand coalition but it is not immune to a chase by the left-over consumer. Thus, a two person coalitional deviation is not migration-proof.

Note that another way that deviating jurisdictions might be able to induce agents in remaining population not to follow their migration is by offering side payments. For example, the rich might offer to pay the poor to stay in the city as

[6] To be clear, the coalition $S^k \backslash T$ is the set of agents $j \in S^k$ who are not members of the deviating coalition T.

they leave for the suburbs. This is an interesting idea and worth exploring. It opens up a number of other questions, however. One might wonder what keeps the poor from reneging on their agreement not to follow the rich once they have received their payment, or what forces the rich to follow through on their commitment to make the payments in the first place. This becomes even more problematic if there are several deviating coalitions and several coalitions that stay behind. How would one decide which deviating coalitions are responsible for paying-off which "left-behind" coalitions? How would one overcome potential free-riding problems in this case? In general, addressing these issues would require the introduction of some kind of binding contracts. Since we are focusing on an equilibrium concept that is closely related to Nash, however, we choose only to restrict attention in this paper to self-enforcing contracts. We thank an anonymous referee for pointing out this way to extend our work.

3. Results

We begin by characterizing the set of free mobility equilibria. Since we assume identical consumers, a jurisdiction structure can be described by a list of integers $\{n_1, n_2, \ldots, n_K\}$, where $n^k = \parallel S^k \parallel$ The following lemma shows the one of properties of free mobility equilibria:[7]

Lemma 1. *A free mobility equilibrium has at most two different jurisdiction sizes.*

For proof of all results see the appendix.

[7] Free mobility equilibrium is always nonempty. This can be shown by directly applying a potential function approach. See Rosenthal (1973). For an application to a local public goods economy, see Konishi, Le Breton, and Weber (1998). Also see Henderson (1991), Jehiel and Scotchmer (1996) and Perroni and Scharf (1997) for treatments of free mobility equilibrium in other contexts.

Given Lemma 1, we define two classes of possible free-mobility equilibrium. The first is a class of partitions with optimal and oversized jurisdictions only. The second is a class of partitions that also have a single undersized jurisdiction. In all cases, only two distinct sizes of jurisdictions are permitted.

Definition 7. A partition of the grand coalition $\pi \in \Pi(\mathcal{I})$ with a jurisdiction structure $(n^1, \ldots n^K)$ is called a **surplus partition** if (i) for all $k = 1, \ldots, K$ $n^k \geq n^*$, (ii) for all $k, \bar{k} = 1, \ldots, K$, $\mid n^k - n^{\bar{k}} \mid \ \leq 1$.[8] Denote the set of surplus partitions by Π_+.

Definition 8. A partition of the grand coalition $\pi \in \Pi(\mathcal{I})$ with a jurisdiction structure $(n^1, \ldots n^K)$ is called a **shortage partition** if (i) there exists exactly one[9] $\bar{k} = 1, \ldots, K$ such that $n^{\bar{k}} < n^*$, (ii) for all $k, \hat{k} \neq \bar{k}$ $n^k = n^{\hat{k}} \geq n^*$. Denote the set of shortage partitions by Π_-.

Example 3. Suppose that the population consists of $I = 25$ agents, and $n^* = 4$ then (i)(4, 4, 4, 4, 5), (6, 6, 6, 7), and (12, 13) are examples of surplus partitions, (ii) (1, 24), and (3, 11, 11) are examples of shortage partitions, and (iii) (3, 3, 3, 8, 8), (1, 3, 3, 3, 3, 3, 3, 3, 3), (7, 9, 9) are examples of partitions that satisfy neither set of conditions.

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It will be useful to have a definition of an "optimal" surplus partition.

Definition 9. An **optimal surplus partition** for an economy with I agents is the following

$$\pi_+^* \equiv \{\pi \in \Pi_+ \mid \nexists\, \bar{\pi} \in \Pi_+ \text{ such that } K < \bar{K}$$

[8] The reason that the size difference needs to be one or zero is that a consumer can improve her payoff by moving from a larger jurisdiction to a smaller one, otherwise.

[9] The reason that there is only one short jurisdiction in a shortage partition is that otherwise agents in the first short coalition would improve their welfare by joining the second short coalition. Thus, such a partition could not be a free-mobility equilibrium.

$$\text{where } \pi = (S^1, \ldots, S^K) \text{ and } \bar{\pi} = (\bar{S}^1, \ldots, \bar{S}^{\bar{K}})\}.$$

This simply says that the optimal surplus partition is the one with the largest number of jurisdictions. Such a partition is constructed by creating as many optimally sized (that is n^* sized) jurisdictions as possible and then distributing the left-over agents as evenly as possible over these jurisdictions. The following two lemmas are useful in narrowing down the candidates for the migration-proof Tiebout equilibrium.

Lemma 2. *Consider any $\bar{\pi} \in \Pi_+$ such that $\bar{\pi} \neq \pi^*_+$. It holds that $\bar{\pi}$ is not a migration-proof Tiebout equilibrium.*

Lemma 3. *Suppose that there are two distinct shortage partitions which are free mobility equilibria. Then, one of them Pareto-dominates the other, and the Pareto inferior partition is not a migration-proof Tiebout equilibrium.*

We are now ready to consider the existence of migration-proof Tiebout equilibrium. It turns out that in an economy with small population they may not exist as the following example shows:

Example 4. Let $I = 16$ and

$$u(6) > u(5) > u(7) > u(8) > u(4) > u(3) > u(2) > u(1) > \textit{others}.$$

It is easy to check that the unique free mobility equilibrium jurisdiction structure $(8, 8)$. However, suppose that three agents in each of these coalitions defect to form a six person jurisdiction. The agents who are left behind do not wish to follow because the five person jurisdictions they are left with are better than anything they could create by chasing the defecting agents. Obviously, the defecting agents can't do any better by moving since they are now in an optimal jurisdictions. Thus, this is a coalitional deviation which blocks the only free mobility equilibrium. It follows that the set of migration-proof Tiebout equilibrium is empty.

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We therefore turn our attention to large economies with at least $(n^*)^2$ agents. We will show that equilibria exist and are asymptotically Pareto efficient. It will be useful to know how many optimal jurisdictions can be created as well as the number of agents in the population who are "leftover" in the sense that they form a surplus that cannot be put into optimally sized jurisdictions. Formally,

$$k^o \equiv \{k \in Z_+ \mid kn^* \leq I < (k+1)n^*\},$$

and

$$n^\ell \equiv I - k^o n^*.$$

Note that in such large economies, the optimal surplus partition π_+^* consists of $k^o - n^\ell$ jurisdictions of size n^* and n^ℓ jurisdictions of size $n^* + 1$. Distributing the leftover agents evenly over optimally sized jurisdictions generates one of the two candidate for free mobility equilibrium. The other is constructed by placing all the leftover agents into a single shortage jurisdiction. We call this the optimal shortage partition. Formally:

Definition 10. An **optimal shortage partition** for an economy with I agents is the following:

$$\pi_-^* \equiv (n^1, \ldots, n^{k^o+1}) = (n^*, \ldots, n^*, n^\ell).$$

The first of our main results is that for large economies, migration-proof Tiebout equilibrium exists.

Theorem 1. *Suppose that $I \geq (n^*)^2$. Then there exists a migration-proof Tiebout equilibrium.*

Our second main result is that under the same condition in Theorem 1, migration-proof Tiebout equilibrium is unique.

Theorem 2. *Suppose that $I \geq (n^*)^2$. Then either π_+^* or π_-^* is the unique migration-proof Tiebout equilibrium.*

Unfortunately, the migration-proof Tiebout equilibrium need not be Pareto optimal in a strict sense. Consider the following example.

Example 5. Suppose that $n^* = 10$ and there are 104 agents. Also suppose that

$$u(10) > u(9) > u(8) > u(11) > u(4).$$

It is easy to check that the optimal surplus partition has the following structure:

$$(10, 10, 10, 10, 10, 10, 11, 11, 11, 11)$$

and is the only migration-proof Tiebout equilibrium. However, consider the following partition structure:

$$(10, 10, 10, 10, 10, 10, 9, 9, 9, 9, 8).$$

Given the utility function above, all the agents who were in 11 person jurisdictions are strictly better off, while those in 10 person jurisdictions are just as well off. Thus, this is a Pareto improving structure. It is not a migration-proof Tiebout equilibrium, since agents in the smaller jurisdictions would to join together to from optimally sized jurisdictions. For the same reason, this is not a credible deviation from the optimal surplus partition.

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Note, however, that this is asymptotically efficient in the sense that as the economy gets larger, the fraction of agents who are not in optimal jurisdictions diminishes to zero. Equivalently, we could say that the per capita utility of agents at a migration-proof Tiebout equilibrium converges to the per capita utility they receive in the optimal jurisdiction. To be precise:

Definition 11. Suppose that $\pi_I = (S_I^1, \ldots, S_I^K)$ is the unique migration-proof equilibrium partition for an economy with population I. Then the migration-proof equilibrium is said to be **asymptotically efficient** if

$$lim_{I\to\infty} \frac{\sum_k \mid (S_I^k) \mid u(s_I^k)}{I} = u(n^*).$$

Our last major result shows that migration-proof equilibrium does indeed have this property.

Theorem 3. *The migration-proof Tiebout equilibrium is asymptotically Pareto efficient.*

4. Conclusions

The notion that local provision of public goods solves the problem of free riding is very powerful and has significant policy implications. If true, it suggests that competition between jurisdictions solves the problem of efficiently allocating resources to public uses. This implies that federal control over local policy is unnecessary, and that decentralizing spending authority to local governments is probably beneficial.

Unfortunately, a clear theoretical verification of this hypothesis has proven elusive. We argue that this is because the way that Tiebout equilibrium has been formalized in the literature has either been so restrictive as to lead to a failure of existence, or so lax that the set of equilibria is large and contains many Pareto inefficient allocations.

The contribution of this paper is to define a notion of Tiebout equilibrium which walks the line between these two extremes. We consider a model with endogenously formed jurisdictions in an economy with congestion. We require that equilibrium be stable against coalitional deviations, but that these deviations be credible. By this we mean that it must be the case that the agents the defecting coalition leaves behind would not benefit from following them. For example, there would be no point in moving to a new city to get away from one's mother-in-law if one knew that she would simply buy the house next door in your new location. Such a move gains nothing and is therefore not a credible deviation.

We demonstrate for relatively large but finite economies, that migration-proof Tiebout equilibrium exists, is unique, and is asymptotically Pareto optimal. While this is an improvement over existing results in several respects, it comes at cost. To obtain our theorem we restrict attention to economies with a single type of agent with single-peaked preferences. It is not immediately clear that this result can be generalized to several types of agents or to economies with differentiated crowding. Our proofs are constructive, and become exponentially more difficult as the number of types increases. In any event, if one accepts our notion of equilibrium as a reasonable characterization of rational behavior in a multijurisdictional environment, we have shown that Tiebout's conjecture is true for at least a limited economy. Future research will have to determine the breath of this result.

Appendix

Lemma 1. *A free mobility equilibrium has at most two different jurisdiction sizes.*

Proof/

Suppose instead that there were at least three different sizes of jurisdictions at a free mobility equilibrium. Denote the sizes of these small medium and big coalitions as: $n^s < n^m < n^b$, respectively.

(i) Suppose first that $n^* > n^m$. By single-peakedness, any consumer in a jurisdiction with size n^s would be made better off by joining one with population n^m. Thus, such a partition could not be a free mobility equilibrium. It follows that $n^* \leq n^m$. (ii) Next suppose that $n^s \geq n^*$. By single-peakedness, any consumer in a jurisdiction with size n^b would be made better off by joining one with population n^s. Thus, such a partition could not be a free mobility equilibrium. It follows that $n^s < n^*$.

We conclude that $n^s < n^* \leq n^m < n^b$. By the assumption of strict preference, either $u(n^s) < u(n^b)$ or the reverse. Suppose first that $u(n^s) < u(n^b)$, then by single-peakedness, $u(n^m + 1) \geq u(n^b)$ and so any consumer in a jurisdiction with size n^s would be made better off by joining one with population n^m. Suppose instead that $u(n^s) > u(n^b)$. Then by single-peakedness, $u(n^s + 1) \geq u(n^s)$ and so any consumer in a jurisdiction with size n^b would be made better off by joining one with population n^s. Thus, any partition with three different sizes of jurisdictions can not be a free mobility equilibrium.

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Corollary 1. *If a partition is a free mobility equilibrium, it must be either a surplus or a shortage partition.*

Proof/

From Lemma 1 we know that a free mobility equilibrium has at most two sizes of jurisdictions. Let the size of the big and small jurisdiction be denoted n^b and n^s respectively. (Note that there maybe only one size of jurisdiction in which case ignore the restrictions on n^b below.) Consider the following exhaustive set of possibilities:

(a) $n^b > n^*$ and $n^s \geq n^*$.
(b) $n^b \geq n^*$ and exactly one jurisdiction of size $n^s < n^*$.
(c) $n^b \geq n^*$ and more than one jurisdictions of size $n^s < n^*$.
(d) $n^b < n^*$ and $n^s < n^*$ and both sizes exist in the partition.

Suppose in case (a), that $n^b > n^s + 1$. Then by single-peakedness, a unilateral defection by an agent in a big jurisdiction to a small jurisdiction is improving and thus this would not be a free mobility equilibrium. If $n^b = n^s + 1$ or only one size of jurisdiction exists in the partition, then case (a) describes a surplus partition. Case (b) is a shortage partition. Case (c) cannot be a free mobility equilibrium since any agent in a small jurisdiction would be made better off by moving into another small jurisdiction which brings it closer to n^* and is therefore an improvement by single-peakedness. Case (d) cannot be a free mobility equilibrium for exactly the same reasons. We conclude that only surplus or shortage partitions can be free mobility equilibria.

■

Lemma 2. *Consider any $\bar{\pi} \in \Pi_+$ such that $\bar{\pi} \neq \pi^*_+$. It holds that $\bar{\pi}$ is not a migration-proof Tiebout equilibrium.*

Proof/

We dispense with a trivial case first. Suppose that π_+^* is not a free mobility equilibrium. The only unilateral deviation that could possible be of benefit would be to a single person jurisdiction. This is because the only other possibilities are for agents to join a jurisdiction with n^* or n^*+1 agents. In either case they are at best as well off (when an agent in an n^*+1 jurisdiction joins an n^* jurisdiction making it n^*+1, like the one he came from) and may be worse off (when an agent in an n^*+1 or n^* jurisdiction joins an n^*+1 jurisdiction making it n^*+2). Thus, in this case, $u(1) > u(n^*+1)$. But since all other surplus partitions have jurisdictions that are as large or larger than π_+^*, by single-peakedness agents in these partitions would also find defection to a single person jurisdiction advantageous. This implies *no* surplus partition is a free mobility equilibrium and thus none can be a migration-proof Tiebout equilibrium either.

Suppose instead that π_+^* is a free mobility equilibrium. Let n^s and $\bar{n}^s$ be the population of the smaller sized coalitions in π_+^* and $\bar{\pi}$, respectively. Recall by construction, this means the larger coalitions (if any) in each case have population n^s+1 and $\bar{n}^s+1$, respectively. Let k^s and k^b be the number of coalitions of size n^s and n^s+1, respectively, in π_+^*. Define $\bar{k}^s$ and $\bar{k}^b$ for $\bar{\pi}$ similarly. Note that since π_+^* has the largest number of jurisdictions possible, the smaller coalitions in this partition are as small as possible for any surplus partitions. Thus, $\bar{n}^s \geq n^s$. Given this, we consider three possible cases.

1. Suppose first that $\bar{n}^s > n^s+1$. That is, the alternative partition consists of coalitions which are all strictly larger than in the optimal surplus partition. Then it is immediate that π_+^* is a migration-proof coalitional deviation for the grand coalition. To see this note that all agents are in jurisdictions which are both smaller, and closer to the optimal size, n^* when they deviate. Thus, all agents are better off, and since π_+^* is a free mobility equilibrium, this is a migration-proof deviation. It follows that $\bar{\pi}$ is not a migration-proof equilibrium and is weakly Pareto-dominated by π_+^*.
2. Suppose that $\bar{n}^s = n^s$. Since π_+^* is the surplus partition with the largest possible number of coalitions, $\bar{\pi}$ must have strictly fewer, and thus,

$$\bar{k}^b > k^b, \text{ and } \bar{k}^s < k^s.$$

Note the following: since the population is the same in each partition it must be the case the $(\bar{k}^b - k^b)(\bar{n}^s+1)$ is evenly divisible by n^s. Thus, it is possible to take the agents in $(\bar{k}^b - k^b)$ of the larger coalitions in the partition $\bar{\pi}$ and form a number of coalitions of size n^s while leaving the remaining agents unaffected. We claim this deviation is migration-proof. To see this note, that all agents in the defecting coalition end up in jurisdictions which are both smaller, and closer to the optimal size, n^*. Thus, all agents are better off and since π_+^* is a free mobility equilibrium, this is a migration-proof deviation. It follows that $\bar{\pi}$ is not a migration-proof equilibrium and is weakly Pareto-dominated by π_+^*.

3. Finally, suppose that $\bar{n}^s = n^s + 1$. The argument is similar to (2). We consider two subcases
   i. Suppose that $\bar{k}^s \geq k^b$. (Note that these jurisdictions are the same size in this case.) Again, since the population is the same in each partition it must be the case that $\bar{k}^b(\bar{n}^s + 1) + (\bar{k}^s - k^b)\bar{n}^s$ is evenly divisible by n^s. Collect the agents in all of the big coalitions in $\bar{\pi}$ together with the agents in $(\bar{k}^s - k^b)$ of the smaller coalition in this partition and call this the defecting coalition T. Thus, it is possible to take the agents in T and form a number of coalitions of size n^s while leaving the remaining agents unaffected. As before, $\bar{\pi}$ is weakly Pareto-dominated by π.
   ii. Suppose instead that $\bar{k}^s < k^b$. Once again, since the population is the same in each partition it must be the case the $\bar{k}^b(\bar{n}^s + 1)$ is divisible into $k^b - \bar{k}^s$ jurisdictions of size n^s+1 and k^s jurisdictions of size n^s. We leave to reader to construct the rest of the argument.

■

To prove Lemma 3, the following claim is useful. This claim characterizes a shortage partition free mobility equilibrium.

Claim 1. *Suppose that π is a shortage partition as well as a free mobility equilibrium. Denote the size of the one short coalition as n^s and remaining jurisdictions as n^b. Then, it holds that $u(n^b + 1) < u(n^s) < u(n^s + 1) < u(n^b)$.*

Proof/

Since π is a free mobility equilibrium, no agent in size n^b jurisdictions wants to move to size n^s jurisdiction. Thus, $u(n^s + 1) < u(n^b)$. Also, no agent in the size n^s jurisdiction wants to move to size n^b jurisdiction. Thus, $u(n^b + 1) < u(n^s)$. Finally, since $n^s < n^*$, moving closer to the optimally sized jurisdiction is improving by single-peakedness, we know that $u(n^s) < u(n^s + 1)$.

■

Lemma 3. *Suppose that there are two distinct shortage partitions which are free mobility equilibria. Then, one of them Pareto-dominates the other, and the Pareto inferior partition is not a migration-proof Tiebout equilibrium.*

Proof/

Consider two such shortage partitions, π and $\bar{\pi}$. Let n^b and n^s be the size of the big and small coalitions in π and let $\bar{n}^b$ and $\bar{n}^s$ be similarly defined for $\bar{\pi}$.

1. Suppose that $\bar{n}^s = n^s$. Since by assumption, these partitions are not identical, without loss of generality, suppose that $\bar{n}^b > n^b$. This immediately implies that $\bar{n}^b \geq n^b + 1$. But by single-peakedness, $u(\bar{n}^b) \leq u(n^b + 1)$ and by hypothesis $u(n^s) = u(\bar{n}^s)$. By Claim 1, $u(n^b + 1) < u(n^s)$. Putting this together implies that $u(\bar{n}^b) < u(\bar{n}^s)$, but this contradicts Claim 1. Thus, $\bar{n}^s \neq n^s$.

2. We now assume without loss of generality that $\bar{n}^s < n^s$. Note by single-peakedness $u(n^s) > u(\bar{n}^s)$. We will show that $\bar{n}^b > n^b$. We consider two contrary subcases:
   i. Suppose that $\bar{n}^b < n^b$. This immediately implies that $\bar{n}^b + 1 \leq n^b$. But by single-peakedness, $u(\bar{n}^b + 1) \geq u(n^b)$ and by Claim 1, $u(n^s) < u(n^b)$. Putting this together implies that $u(\bar{n}^s) < u(\bar{n}^b + 1)$, but this contradicts Claim 1. Thus, $\bar{n}^b \geq n^b$.
   ii. Now suppose that $\bar{n}^b = n^b$. But this is impossible given that the population is fixed. Since $\bar{n}^s < n^s < n^* \leq n^b$, decreasing the size of the small coalition to $\bar{n}^s$ does not generate enough free population to produce another big coalition. Thus, $\bar{n}^b \neq n^b$.

   Putting together (i) and (ii), we conclude that $\bar{n}^b > n^b$.
3. From the fact that $\bar{n}^b > n^b$, we conclude that $\bar{n}^b \geq n^b + 1$. But by single-peakedness, $u(\bar{n}^b) \leq u(n^b + 1)$. By Claim 1, we know two things: $u(n^b + 1) < u(n^s) < u(n^b)$ and $u(\bar{n}^s) < u(\bar{n}^b)$. Putting this together implies that

$$u(\bar{n}^s) < u(\bar{n}^b) < u(n^s) < u(n^b).$$

   But then the partition π Pareto-dominates partition $\bar{\pi}$. Thus, since all agents in the grand coalition are strictly better off if they form partition π and, since π is a free-mobility equilibrium, π is a migration-proof coalitional deviation from $\bar{\pi}$. We conclude that $\bar{\pi}$ is not a migration-proof Tiebout equilibrium.

■

Theorem 1. *Suppose that $I \geq (n^*)^2$. Then there exists a migration-proof Tiebout equilibrium.*

Proof/

To prove this theorem, we consider two cases. Note that by the assumption of strict preference, these are exhaustive.

1. Suppose that $u(n^\ell) > u(n^*+1)$. We claim that π^*_- is a migration-proof Tiebout equilibrium. First we must demonstrate that π^*_- is a free mobility equilibrium. The agents in optimally sized jurisdictions can never benefit from any deviation. The only deviations open to members of the short jurisdiction are to form a single person jurisdiction which is not improving by single-peakedness, or to join an optimal jurisdiction which is not improving by the hypothesis. Second we must show that there is no migration-proof coalitional deviation. Since the only agents who could improve their welfare are the n^ℓ agent who are not in optimal jurisdictions, we can restrict attention to this group. However, the only coalitional deviations open to them are to form coalition smaller than $n^\ell (< n^*)$ which is not improving by single-peakedness. Thus, π^*_- is a migration-proof Tiebout equilibrium.

2. Suppose that $u(n^\ell) < u(n^*+1)$. We claim that π^*_+ is a migration-proof Tiebout equilibrium. Again, first we must demonstrate that π^*_+ is a free mobility equilibrium. The only agents who could ever benefit from unilateral deviation are those in over-sized jurisdictions. The three deviations available to them are to form a single person jurisdiction, which is not improving by single-peakedness, joining a jurisdiction with $n^* + 1$ members, which is also not improving by single-peakedness, or joining a jurisdiction with n^* members, which leaves him just as well as before. Therefore, π^*_+ is a free-mobility equilibrium. Second, we must show that there is no migration-proof coalitional deviation. The only agents who could potentially deviate are those in the coalitions of size $n^* + 1$ since all other agents are in optimally sized jurisdictions and could not possibly have their welfare improved. One possible coalitional deviation would be to have the extra n^ℓ agents form a single coalition while leaving all the others in optimal coalitions. But by hypothesis this, would leave the n^ℓ agents worse off. By single-peakedness, the same thing would be true if any fraction of these agent tried to form a coalition of size less than n^ℓ. Clearly, forming deviating coalitions larger than $n^* + 1$ is not improving. This leaves only one possibility: forming a deviation with more than one coalition below size n^*. But this is not migration-proof. It would always be in the interest of any agent in the smallest short jurisdiction to join the largest short jurisdiction. This would put the agent in a jurisdiction which is closer to optimally sized and therefore by single-peakedness would be welfare improving. We conclude there is no migration-proof coalitional deviation and so π^*_+ is a migration-proof Tiebout equilibrium.

■

Theorem 2. *Suppose that $I \geq (n^*)^2$. Then either π^*_+ or π^*_- is the unique migration-proof Tiebout equilibrium.*

Proof/

By Corollary 1, if an equilibrium exists it must be either shortage or surplus partition. By Lemma 2 the only candidate for a surplus migration-proof Tiebout equilibrium is π^*_+. We consider two cases:

1. Suppose that $u(n^\ell) > u(n^* + 1)$. We claim that π^*_- is the only migration-proof Tiebout equilibrium. We know from the proof of Theorem 1 in this case π^*_- is a migration-proof Tiebout equilibrium, but Lemma 3 says that there can be no other shortage partition migration-proof Tiebout equilibrium. Thus, it only remains to show that π^*_+ is not a migration-proof Tiebout equilibrium. But given that $u(n^\ell) > u(n^* + 1)$, it is a migration-proof coalitional deviation for one of the agents in each over-sized jurisdiction to form a short jurisdiction of size n^ℓ. All the deviating agents are better off, and the agents who were left

behind are now in optimally sized jurisdictions and would obviously not choose to follow. Thus π^*_- is the only migration-proof Tiebout equilibrium.

2. Suppose that $u(n^\ell) < u(n^* + 1)$. From the proof of Theorem 1, π^*_+ is a migration-proof Tiebout equilibrium, but Lemma 2 says that there can be no other surplus partition migration-proof Tiebout equilibrium. Thus, it only remains to show that no shortage partition can be a migration-proof mobility equilibrium. We begin by demonstrating that π^*_- is not a free mobility equilibrium. By hypothesis, $u(n^\ell) < u(n^*+1)$, and so agents in the short partition would benefit from the unilateral deviation of joining an optimally sized jurisdiction. Thus, π^*_- is not a free-mobility equilibrium. Suppose that there was another shortage partition $\bar{\pi} \in \Pi_-$ which was a free mobility equilibrium. Given that a shortage partition contains exactly one short and one size for larger than optimal jurisdiction, it is immediate that there is exactly one way to create a shortage partition for each population level of larger than optimal jurisdictions. Thus, since π^*_- is the unique shortage partition with jurisdictions of size n^*, all other shortage partitions must contain jurisdictions of size n^*+1 or larger. Let such a shortage partition $\bar{\pi}$ be composed of one size n^s jurisdiction and multiple size n^b jurisdictions ($n^s < n^* < n^* + 1 \leq n^s$). By Claim 1, if $\bar{\pi}$ can be a free mobility equilibrium, which is a necessary condition to be a migration-proof equilibrium, only if $u(n^b + 1) < u(n^s) < u(n^s + 1) < u(n^b)$. Thus, the similar arguments in Lemmas 2 and 3 prove that $\bar{\pi}$ is not immune to a migration-proof coalitional deviation that leads $\bar{\pi}$ to π^*_+. Thus, there is no shortage partition migration-proof Tiebout equilibrium. Thus, depending on the sign of this inequality, either π^*_+ or π^*_- is the unique migration-proof Tiebout equilibrium.

■

Theorem 3. *The migration-proof Tiebout equilibrium is asymptotically Pareto efficient.*

Proof/

This is almost immediate. Notice that in an optimal surplus partition, at most $(n^* + 1)(n^* - 1)$ agents are in jurisdictions of size $n^* + 1$. Similarly, in an optimal shortage partition, at most $n^* - 1$ agents are in suboptimally sized jurisdictions. Thus, whichever of these is the migration-proof Tiebout equilibrium, the number of agents in non-optimal jurisdictions is strictly bounded while those in optimal jurisdiction increase without bound as the population grow. Therefore, almost all agents are in optimal jurisdictions in the limit and the average utility agents receive in migration-proof Tiebout equilibrium converges to $u(n^*)$.

■

References

Berglas, E. and D. Pines (1981): "Clubs, local public goods and transportation models; A synthesis," *Journal of Public Economics,* **15**: 141-162.

Bernheim, D., B. Peleg, and M. Whinston (1987): "Coalition-proof Nash Equilibria. I. Concepts," *Journal of Economic Theory,* **42**: 1-12.

Bewley, T. (1981): "A critique of Tiebout's theory of local public expenditure," *Econometrica,* **49**: 713-740.

Buchanan, J.M. and C. Goetz (1972): "Efficiency Limits of Fiscal Mobility," *Journal of Public Economics,* **1**: 25-45.

Cole, H. and E. Prescott (1997): "Valuation Equilibria with Clubs," *Journal of Economic Theory,* **74**: 19-39.

Conley, J. and M.H. Wooders (1997): "The Tiebout Hypothesis: On the Existence of Pareto Efficient Competitive Equilibrium," Manuscript.

Demange, G. (1994): "Intermediate Preferences and Stable Coalition Structures," *Journal of Mathematical Economics,* **23**: 45-58.

Dunz, K. (1989): "Some comments on majority rule equilibrium in local public goods economies," *Journal of Economics Theory,* **47**: 228-34.

Ellickson, B., B. Grodal, S. Scotchmer, and W. Zame (1999): "Clubs and the Market," *Econometrica,* **67**: 1185-1218.

Epple, D., R. Filimon and T. Romer (1984): "Equilibrium among local jurisdictions: Toward an integrated treatment of voting and residential choice," *Journal of Public Economics,* **24**: 281-308.

Epple, D., R. Filimon and T. Romer (1993): "Existence of voting and housing equilibrium in a system of communities with property taxes," *Regional Science and Urban Economics,* **23**: 585-610.

Flatters, F., V. Henderson, and P. Mieszkowski (1974): "Public Goods, Efficiency, and Regional Fiscal Equalization," *Journal of Public Economics,* **3**: 99-112.

Greenberg, J. and S. Weber (1986): "Strong Tiebout equilibrium under restricted preference domain," *Journal of Economic Theory,* **38**: 101-11.

Guesnerie, R. and C. Oddou (1981): "Second best taxation as a game," *Journal of Economic Theory,* **25**: 67-91.

Hamilton, B. W. (1975): "Zoning and Property Taxation in a System of Local Governments," *Urban Studies,* **12**: 205-211.

Henderson, J. V. (1991): "Separating Tiebout Equilibrium," *Journal of Urban Economics,* **29**: 128-152.

Jehiel, P. and S. Scotchmer (1996): "On the Right of Exclusion in Jurisdiction Formation," Manuscript.

Konishi, H. (1996): "Voting with ballots and feet: Existence of Equilibrium in a local public good economy," *Journal of Economic Theory,* **68**: 480-509.

Konishi, H., M. Le Breton, and S. Weber (1998): "Equilibria in Finite Local Public Goods Economies," *Journal of Economic Theory,* **79**: 224-244.

McGuire, M. (1974): "Group Segregation and Optimal Jurisdictions," *Journal of Political Economy,* **82**: 112-132.

Nechyba, T. (1997): "Existence of Equilibrium and Stratification in Local and Hierarchical Tiebout Economies with Property Taxes and Voting," *Economic Theory,* **10**: 277-304.

Pauly, M. (1970): "Cores and Clubs," *Public Choice,* **9**: 53-65.

Perroni, C., and K Scharf (1997): "Tiebout and Politics: Capital Tax Competition and Jurisdictional Boundaries," *Review of Economic Studies,* Forthcoming.

Ray, D. (1989): "Credible Coalitions and the Core," *International Journal of Game Theory,* **18**: 185-187.

Rosenthal, R.W. (1973): "A Class of Games Possessing a Pure Strategy Nash Equilibrium," *International Journal of Game Theory,* **2**: 65-67.

Tiebout, C. (1956): "A pure theory of local expenditures," *Journal of Political Economy,* **64**: 416-424.

Westhoff, F. (1977): "Existence of equilibrium in economies with a local public good," *Journal of Economic Theory,* **14**: 84-112.

Wooders M.H. (1978): "Equilibria, the core, and jurisdiction structures in economies with a local public good," *Journal of Economic Theory,* **18**: 328-348.

Wooders, M.H. (1980): "The Tiebout Hypothesis: Near optimality in local public good economies," *Econometrica,* **48**: 1467-1486.