

On Efficient Jurisdiction Structures in a Simple Local Public Goods Economy with Interjurisdictional Trade*

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Abstract

This note shows that if commodities are tradable across jurisdictions, then it may be efficient to have heterogeneously sized jurisdictions, even if (i) consumers are identical, (ii) there is one private good and one public good, (iii) utility and production functions are not affected by population (within the relevant range of sizes of jurisdictions).

1 Introduction

In the literature of local public goods economies with (i) single private good, (ii) single public good, and (iii) identical consumers, many papers discuss the optimal jurisdiction size, and claim that it is efficient to partition the population into identical jurisdictions of optimal size (see, for example, Pauly, 1970). This note shows, by example, that it may not be optimal to have identically sized jurisdictions in such a simple economy, if (1) interjurisdictional trade of goods is allowed, and (2) marginal rate of technical substitution (*MRTS*) between public good and private good is not constant (constant returns to

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scale in public good production with private good as an input). It is because there may be gains from trade across heterogeneous jurisdictions when these two conditions are satisfied.

Since many of local public goods are normally services, it is probably reasonable to assume that they are not tradable across jurisdictions. In such a case, it is easy to see that there is an optimal jurisdiction size exists (although it is not necessarily unique). We simply need to maximize jurisdiction member's utility within a feasible set for a single jurisdiction. Similarly, under a constant *MRTS*, there are no gains from trade, and interjurisdictional trade does not improve welfare even if it is feasible. Thus, we can again concentrate on autarchic allocations, and we can find an optimal jurisdiction size again.¹

However, if both of conditions (1) and (2) are satisfied, there may be gains from trade across jurisdictions. In this note, we provide a simple example in which efficient allocations involve a heterogeneous jurisdiction structure when interjurisdictional trade is feasible. The example satisfies not only (i), (ii), and (iii), but two other restrictive conditions: (iv) utility function is concave, quasi-linear, and is not affected by jurisdiction size up to a quota level, and (v) cost function of public good production (in terms of private good) is convex, and is not affected by jurisdiction size.

2 The Model and an Example

Consider the following simple local public goods economy. There is one private good and one public good, and I consumers with identical preference and endowment. Only the private good is initially endowed, but public good can be produced by using private good with a cost function $C(y)$. This means that to produce y units of public good, $C(y)$ units of private good is needed. The technology is common to every jurisdiction. A consumer's net consumption vector is denoted by (x, y) , where x and y represent net consumption of private and public goods. Her utility function is denoted by $U(x, y, n)$ and assumed to be strictly increasing in x and y , quasi-concave, and

¹Flatters, Henderson and Mieszkowski (1974) and Stiglitz (1977) assume a constant *MRTS* between public good and private good, although jurisdiction size affects production level of these two goods: i.e., $nx + y = F(n)$, where $F(n)$ is production function solely based on size of jurisdiction. Obviously, in such a case, relative price of public good in terms of private good is fixed, and there are no gains from trade. As a result, efficient allocations do not involve heterogeneously sized jurisdictions.

quasi-linear in x , where n denotes size of the population in her jurisdiction. We assume that there is large enough private good endowment for transfer and public good production.

A **jurisdiction structure** is a list of sizes of jurisdictions (n^1, \dots, n^L) , where $\sum_{l=1}^L n^l = I$ and n^l is a positive number that stands the population (size) of jurisdiction l . Consumers who live in the same jurisdiction consume the same amount of local public good. For simplicity, we assume equal allocations of private good within a jurisdiction, that is, consumers in the same jurisdiction consume the same amount of private good.² A **feasible allocation with interjurisdictional trade** is a list $((x^l, y^l, n^l)_{l=1}^L)$ such that (i) $y \geq 1$, and (ii) $\sum_{l=1}^L n^l x^l + \sum_{l=1}^L y^l = 0$, where (x^l, y^l) is a net consumption vector of a consumer in jurisdiction l . A **feasible allocation without interjurisdictional trade** is a list $((x^l, y^l, n^l)_{l=1}^L)$ such that (i) $y \geq 1$, and (ii) $n^l x^l = y^l$ for each $l = 1, \dots, L$. Finally, a consumer's utility is a function of her net consumption and the size of her jurisdiction: i.e., a consumer who lives in jurisdiction l gets utility $U = U(x^l, y^l, n^l)$. An **optimal jurisdiction size without interjurisdictional trade** is $n^* \equiv \arg \max_n U(x, y, n)$ s.t. $nx = y$.

EXAMPLE. Consider the following local public goods economy. In this economy the optimal feasible allocation without interjurisdictional trade is dominated by a feasible allocation with interjurisdictional trade.

1. $I = 4$.
2. $C(y) = y$ if $y \in [0, 1]$, and
 $C(y) = 1 + 100(y - 1)$ if $y \geq 1$.
3. $U(x, y, 1) = U(x, y, 2) = x + 0.8y$ if $y \in [0, 3]$,
 $U(x, y, 1) = U(x, y, 2) = x + 2.4 + 0.01(y - 3)$ if $y \geq 3$, and
 $U(x, y, n) = 0$ for $n \geq 3$.³

To see this, first consider the optimal feasible allocation without interjurisdictional trade. It is easy to see that having a jurisdiction with $n \geq 3$

²We assume a quasi-linear utility function, and we can show that any allocation in any identical jurisdictional structure is inefficient. We assume equal treatment only for expositional purposes.

³We can regard $n = 2$ is the quota (upper limit population) for each jurisdiction. Jurisdictions with sizes $n \geq 3$ is not feasible (for some reason).

does not make sense. Thus, focus on the case of $n = 1$ or $n = 2$. Consider the case with $n = 1$. Since marginal cost of production of public good exceeds marginal rate of substitution, no production is optimal. Thus, the resulting utility is zero. Next consider the case of $n = 2$. Since two people can share the marginal cost, it is optimal to produce $y = 1$. Thus, $n^* = 2$ follows, and the resulting utility level of each consumer is $U(-\frac{1}{2}, 1, 2) = 0.3$. Thus, the optimal feasible allocation without interjurisdictional trade is $((x^l, y^l, n^l)_{l=1}^2) = ((-\frac{1}{2}, 1, 2), (-\frac{1}{2}, 1, 2))$ attains utility level 0.3.

Now, we allow interjurisdictional trade. Consider the following allocation with $L = 3$: $n^1 = 2$ and $(x^1, y^1) = (-1.95, 3)$, and $n^2 = n^3 = 1$ and $(x^2, y^2) = (x^3, y^3) = (0.45, 0)$: i.e., $((x^l, y^l, n^l)_{l=1}^3) = ((-1.95, 3, 2), (0.45, 0, 1), (0.45, 0, 1))$. This allocation is feasible (each jurisdiction produces one unit of public good, yet only two person jurisdiction consumes public good), and the resulting utility levels are: $U(-1.95, 3, 2) = U(0.45, 0, 1) = 0.45$. This is higher than 0.3 attained in the previous case. What is happening is that the one person jurisdiction produces one unit of public good and exports it to the two person jurisdiction. In return, the one person jurisdiction receives 1.45 units of private good.

3 Remarks

1. The latter allocation in the example above (with a heterogeneous jurisdiction structure) is actually a market equilibrium allocation with interjurisdictional trade of two goods. The supporting market equilibrium price, the relative price of public good in terms of private good, is 1.45.
2. Note that for any I that is an integer multiple of four, we have the above result. Thus, even if we replicate the economy many times, we do not get autarchic allocations with every jurisdiction size being two.
3. As the readers can easily see, the decreasing returns to scale technology is truly essential. Although the utility and cost functions in the example involve kinks, we can make them smooth without altering the result.
4. This example is certainly not the first one that has the properties described above. There are many similar/related results in club economies

with multiple private goods.⁴ If we confine our attention to the identical consumer case as is assumed in this note, two examples in Gilles and Scotchmer (1997) and Wooders (2001) are particularly related. Example 1 in Gilles and Scotchmer (1997) shows that if there are multiple private goods and if utility functions are affected by the number of consumers in a jurisdiction, then efficient allocation may involve heterogeneously sized jurisdictions. Wooders (2001) notes that if a club is defined as a group of people who consume identical levels of a specific good, then the ‘gin and tonic’ example in Shapley and Shubik (1966) can be interpreted as showing that if consumers have nonconvex preferences then efficiency may require clubs of different characteristics (consumption vectors). Slightly modifying their example (changing endowment only), she demonstrates that efficient allocations can involve heterogeneously sized jurisdictions. Our result is similar to these examples in club economies with multiple private goods, but we do not need population-sensitive or nonconvex preferences. It is simply because we have a production function to play with.

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⁴See Wooders (1999) for a survey of the literature on economies with clubs or local public goods, including those with multiple private goods.

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