

# Central Bank Learning, Terms of Trade Shocks & Currency Risk: Should Exchange Rate Volatility Matter for Monetary Policy?

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## Abstract

This paper examines the role of interest rate policy in a small open economy subject to terms of trade shocks and time-varying currency risk responding to domestic exchange rate volatility. The private sector makes optimal decisions in an intertemporal non-linear setting with rational, forward-looking expectations. In contrast, the monetary authority practices “least-squares learning” about the evolution of inflation, output growth, and exchange rate depreciation in alternative policy scenarios. Interest rates are set by linear quadratic optimization, with the objectives for inflation, output growth, or depreciation depending on current conditions. The simulation results show that the preferred stance is one which targets inflation and growth, not inflation only nor inflation, growth and depreciation. Including exchange rate changes as targets significantly increases output variability, but marginally reduces inflation variability.

*Key words:* Currency risk, learning, parameterized expectations, policy targets

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# 1 Introduction

A small, open economy is vulnerable to terms of trade shocks, which impinge on the real exchange rate. If exchange rates are also flexible and the financial system is well integrated with the rest of the world, the economy simultaneously faces reactions from international investors through time-varying currency risk. This risk may respond to nominal exchange rate volatility which in turn feedback into further nominal and real exchange rate volatility.

This paper takes up the question of monetary policy in such a setting. Should exchange rate changes be included as policy targets? A central bank committed to low inflation controls neither the terms of trade nor the evolution of currency risk, both of which condition the response of inflation to its policy instruments. In this context, the best the central bank can do is to “learn” the effects indirectly, by frequently “updating” estimates of inflation dynamics and “re-adjusting” its policy rules accordingly.

Of course, central banks, even those with explicit inflation targets, adjust their policy stance from time to time to stimulate growth. Furthermore, when the exchange rate depreciates rapidly, due to adverse external shocks, it should not be surprising that a central banks come under strong pressure to incorporate exchange rate volatility targets in its policy objectives.

Much of the discussion of monetary policy is framed by the well-known Taylor (1993, 1999) rule, whereby interest rates respond to their own lag, as well as to deviations of inflation and output from respective targets. Taylor (1993) points out that this “rule” need not be a mechanical formula, but something which can be operated “informally”, with recognition of the “general instrument responses which underlie the policy rule”. Not surprisingly, the specification of this rule, which reflect the underlying objectives of monetary policy, has been the subject of considerable controversy.<sup>1</sup>

In a closed-economy setting, Christiano and Guest (2000), for example, argue that only the inflation variable should appear as a target. Rotemberg and Woodford (1998) concur, but they argue that a higher average rate of inflation is required for monetary policy to do its job over the medium to long term. They base their argument on the zero lower bound for the nominal interest rate, since at very low inflation rates there is little room for this instrument to manoeuvre.<sup>2</sup>

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<sup>1</sup>Recent technical papers on all aspects of the Taylor rule may be found on the web page, <http://www.stanford.edu/~johntayl/PolRulLink.htm#Technical%20articles>

<sup>2</sup>Erceg, Henderson and Levin (2000) argued that output deviations should also appear in the Taylor rule, but the output measure should be deviations of actual output from the

In an open economy setting, McCallum (2000) takes issue with the Rotemberg and Woodford “policy ineffectiveness” argument under low inflation and zero “lower bounds” for nominal interest rates. McCallum argues that the central bank always has at its disposal a second tool, the exchange rate, so if the economy is stuck at a very low interest rate, there is the option of currency intervention. Chrsitiano (2000) disagrees: McCallum’s argument rests on the assumption that currency depreciation is effective. Furthermore, the Central Bank must be willing to undermine public “confidence” that it stands ready to cut interest rates in the event of major adverse shocks.

For small emerging market economies, Taylor (2000) contends that policy rules that focus on a “smoothed inflation measure and real output” and which do not “try to react too much” to the exchange rate might work well. However, he leaves open the question of some role for the exchange rate.

Bullard and Metra (2001) incorporate “learning” in the Rotemberg-Woodford closed economy framework. In this case, the private sector attempts to learn the specific Taylor rules used by the central bank. They argue for Taylor rules based on *expectations of current* inflation and output deviations from target levels, rather than rules based on lagged values or forecasts further into the future.

However, practically all of these studies are based on *linear* stochastic and dynamic general equilibrium representations, or *linearized* approximations of nonlinear models. The Taylor-type feedback rules are either imposed or derived by linear quadratic optimization. While these approaches may be valid if the shocks impinging on the economy are indeed “small” and “symmetric” deviations from a steady state, they may be inappropriate if the shocks are large, persistent, and asymmetric, as they are in many highly open economies.

Furthermore, few if any of these studies incorporate “learning” on the part of the monetary authority itself. As Sargent reminds us, with “learning”, there must be two models, one used by the agents who are learning, and the “true” one. In contrast to Bullard and Metra (2001), we assume that the private sector uses the true, stochastic dynamic, nonlinear model for formulating its own “laws of motion” for consumption, investment, and trade, with forward-looking rational expectations.

However, unlike the private sector, the monetary policy authority has to learn the “laws of motion” of inflation dynamics from past data, through continuously-updated least squares regression. From the results of these regressions, the monetary authority obtains an optimal interest rate feedback

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level of output generated by a flexible-price economy.

rule based on linear quadratic optimization, using weights in the objective function for inflation which can vary with current conditions. The monetary authority is thus “boundedly rational”, in the sense of Sargent (1999), with “rational” describing the use of least squares, and “bounded” meaning model misspecification.

Our results show that if the central bank decides to incorporate exchange rate dynamics in its learning and policy objectives, the risk of higher output volatility significantly increases. While incorporation of an exchange rate target reduces inflation rate volatility, it does so at high costs.

The policy implication of this paper is that central banks should resist pressures to incorporate exchange rate volatility targets in their objectives for formulating interest rate policy.

The next section describes the theoretical structure of the model for the private sector and the nature of the monetary authority “learning”. The third section discusses the calibration as well as the solution method, while the fourth section analyzes the simulation results of the model. The last section concludes.

## 2 The Model

### 2.1 Consumption

The objective function for the private sector is given by the following utility function:

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \quad (1)$$

where  $C$  is the aggregate consumption index and  $\gamma$  is the coefficient of relative risk aversion. The “household/firm” optimizes the following intertemporal welfare function, with an endogenous discount factor:

$$W = \mathbf{E} \left[ \sum_{t=0}^{\infty} \left( U(C) \cdot \exp^{-\sum_{\tau=0}^{t-1} V(C)} \right) \right] \quad (2)$$

$$V(C) = \beta \ln(1 + C) \quad (3)$$

where  $\beta$  approximates the elasticity of the endogenous discount factor  $V$  with respect to the consumption index.<sup>3</sup>Unless otherwise specified, upper-case variables denote the levels of the variables while lower-case letters

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<sup>3</sup>Mendoza (2000) states the endogenous discounting allows the model to produce well-behaved dynamics and deterministic stationary equilibria in which the rate of time pref-

denote logarithms of the same variables. The exception is the interest rate denoted as  $i$ .

The consumption index is a composite index of non-tradeable goods  $N$  and tradeable goods  $T$  :

$$C = (C^N)^\alpha (C^T)^{1-\alpha} \quad (4)$$

where  $\alpha$  is the proportion of non-traded goods.

Given the aggregate consumption expenditure constraint,

$$PC = P^N C^N + P^T C^T \quad (5)$$

and the definition of the real exchange rate,

$$Z = \frac{P^T}{P^N} \quad (6)$$

the following expressions give the demand for traded and non-traded goods as functions of aggregate expenditure and the real exchange rate  $Z$ :

$$C^N = \alpha_1 Z^{1-\alpha_1} C \quad (7)$$

$$C^T = (1 - \alpha_1) Z^{-\alpha_1} C \quad (8)$$

while the domestic price index may be written as the geometric average of traded and non-traded goods:

$$P = (P^N)^{\alpha_1} (P^T)^{1-\alpha_1} \quad (9)$$

Similarly, consumption of traded goods is a composite index of export goods,  $X$ , and import goods  $F$ :

$$C^T = F^{\alpha_2} X^{1-\alpha_2} \quad (10)$$

where  $\alpha_2$  is the proportion of imported goods. The aggregate expenditure constraint for tradeable goods is given by the following expression:

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erence equals the world real interest rate. Endogenous discounting also allows the model to support equilibria in which credit frictions may remain binding.

$$P^T C^T = EF + EP^x X \quad (11)$$

where  $E$  is the nominal exchange rate, and  $P^x$  is the ratio of foreign export prices to foreign import prices, the terms of trade index (with  $P^m = 1$ )

The demand for export and import goods are functions of the aggregate consumption of traded goods as well as the terms of trade index:

$$F = \alpha_2 (P^x)^{1-\alpha_2} \quad (12)$$

$$X = (1 - \alpha_2) (P^x)^{-\alpha_2} \quad (13)$$

Similarly the price of traded goods may be expressed as a geometric average of the price of imported and export goods:

$$P^T = E (P^x)^{1-\alpha_2} \quad (14)$$

## 2.2 Production

Production of exports and imports is by the Cobb-Douglas technology:

$$Q^x = A^x P^x \epsilon^x (K^x)^{1-\alpha_x} \quad (15)$$

$$Q^f = A^f \epsilon^f (K^f)^{1-\alpha_f} \quad (16)$$

where  $\epsilon^x, \epsilon^f$  represent productivity shocks for export and import-producing firms, while  $(1 - \alpha_x), (1 - \alpha_f)$  are the capital coefficients, and  $A^x, A^f$  the total factor productivity effects.

Total capital is simply the sum of capital in each sector. Hence:

$$K = K^x + K^f \quad (17)$$

The production of non-traded goods is given by the interaction of an exogenous productivity shock with a fixed productive resource,  $\bar{L}$ :

$$Q^n = \epsilon^n \bar{L} \quad (18)$$

## 2.3 Aggregate Budget Constraint

The combined household/firm faces the following budget constraint, in terms of domestic purchasing power:

$$\begin{aligned}
C_t = & \frac{E}{P} \left[ A^f \epsilon^f (K_t^f)^{1-\alpha_f} \right] + \frac{EP^x}{P} \left[ A^x P^x \epsilon^x (K_t^x)^{1-\alpha_x} \right] \\
& + \frac{P^n}{P} \epsilon^n \bar{L} - K_{t+1} + K_t(1 - \delta) - \frac{\phi}{2} [K_{t+1} - K_t]^2 \\
& + \frac{E}{P} L_{t+1}^* - \frac{E}{P} L_t^* [1 + i_t^* + ({}_t e_{t+1} - e_t) + \theta_t - \pi_t] \\
& - D_t - B_{t+1} + B_t(1 + i_t - \pi_t)
\end{aligned} \tag{19}$$

The aggregate resource constraint shows that the firms producing tradeable goods face quadratic adjustment costs when they accumulate capital, with these costs given by the term  $\frac{\phi}{2} [K_{t+1} - K_t]^2$ . For both firms, capital depreciates at a fixed rate  $\delta$ .

Firms and households can borrow internationally at the fixed rate  $i^*$ , but face a cost of borrowing in domestic currency which includes not only the expected rate of depreciation,  $({}_t e_{t+1} - e_t)$  but also a time-varying risk premium  $\theta_t$ , less expected inflation. The variable  $e$  is the logarithm of the nominal exchange rate  $E$ , and  ${}_t e_{t+1}$  the expected logarithmic rate at time  $t$ .

The evolution of currency risk  $\theta_t$  depends on the time-varying volatility of the rate of depreciation, here proxied by the absolute value of the lagged annualized rate of depreciation, as well as on its own lag:

$$\theta_t = \xi_0 + \xi_1 \theta_{t-1} + \xi_2 |e_{t-1} - e_{t-4}| + \eta_t \tag{20}$$

$$\eta \sim N(0, \sigma_\eta^2) \tag{21}$$

The higher the volatility of the rate of depreciation, the higher the level of the risk premium demanded by international lenders. Hence  $\xi_2 > 0$ .

The consolidated household/firm may also lend to the domestic government at interest rate  $i$ . The government runs an exogenous net deficit,  $D_t$ , but finances this deficit by borrowing from the private sector. The government makes its expenditures and collects its lump-sum tax revenue from the non-traded sector.

## 2.4 Euler Equations

The consolidated household and firm solves the following intertemporal welfare optimization problem by choosing the path of ‘‘controls’’  $\{\nu_t\}$ ,

representing consumption  $\{C_t\}$ , aggregate capital  $\{K_{t+1}\}$ , capital in the import-competing industries  $\{K_{t+1}^f\}$ , foreign borrowing  $\{L_{t+1}^*\}$ , and government bonds  $\{B_{t+1}\}$  :

$$\begin{aligned} W_t(\nu_t) &= \max_{\nu_t} U(C_t) + \exp[-V(C_t)]W_{t+1}(\nu_{t+1}) \\ \nu_t &= \{C_t, K_{t+1}, K_{t+1}^f, B_{t+1}, L_{t+1}^*\} \end{aligned} \quad (22)$$

subject to the budget constraint, given in equation (19), as well as the following inequality restrictions:

$$C_t > 0 \quad (23)$$

$$K_t^f > 0 \quad (24)$$

$$K_t^x > 0 \quad (25)$$

The first order conditions are given by the following equations, representing the derivatives of constrained intertemporal optimization with respect to  $K_{t+1}$ ,  $K_{t+1}^f$ ,  $B_{t+1}$ ,  $L_{t+1}^*$ :

$$\lambda_t = C_t^{-\gamma} \quad (26)$$

$$\begin{aligned} \frac{\lambda_t[1 + (\phi K_{t+1} - K_t)]}{\exp[-V(C_t)]\lambda_{t+1}} &= \frac{EP_t^x}{P} \cdot \left[ (1 - \alpha_x) A^x P_{t+1}^x \epsilon_{t+1}^x (K_{t+1} - K_{t+1}^f)^{-\alpha_x} \right] + .. \\ &(1 - \delta) + \phi [K_{t+2} - K_{t+1}] \end{aligned} \quad (27)$$

$$(K_{t+1} - K_{t+1}^f)^{-\alpha_x} = \frac{(1 - \alpha_f) \left[ A^f \epsilon_{t+1}^f (K_{t+1}^f)^{-\alpha_f} \right]}{(1 - \alpha_x) A^x P_{t+1}^x \epsilon_{t+1}^x} \quad (28)$$

$$\frac{\lambda_t}{\exp[-V(C_t)]\lambda_{t+1}} = (1 + i_{t+1} - \pi) \quad (29)$$

$$\frac{\lambda_t}{\exp[-V(C_t)]\lambda_{t+1}} = [1 + i_{t+1}^* + (e_{t+2} - e_{t+1}) + \theta_{t+1} - \pi] \quad (30)$$

The first Euler equation is the familiar condition that the marginal utility of wealth is equal to the marginal utility of income.

The second equation relates to the marginal productivity of capital. Capital should be accumulated until the gross marginal productivity of capital, adjusted for depreciation and transactions costs is equal to the



marginal utility of consumption today divided by the discounted marginal utility tomorrow.

The third equation simply states that the marginal utility of capital in each sector should be equal.

The last two equations tell us that the gross real returns on domestic or foreign assets should also be equal to the marginal utility of consumption today divided by the discounted marginal utility tomorrow.

The first equation may be combined with the fourth equation to solve for current consumption as a function of next period's expected marginal utility:

$$C_t = (\exp[-V(C_t)]\lambda_{t+1} \cdot \{1 + i_{t+1} - \pi\})^{-\frac{1}{\gamma}} \quad (31)$$

The last two equations may be combined to give the interest arbitrage condition:

$$e_{t+1} = (i_{t+1}^* + \theta_{t+1} - i_{t+1}) +_t e_{t+2} \quad (32)$$

Both current consumption and the logarithm of the exchange rate depend on their expected future values.

The solution of the investment equation for aggregate capital and for capital in the two sectors takes place by equating the marginal productivity with the real returns of either domestic or foreign assets:

$$(1 + r_{t+1})[1 + (\phi K_{t+1} - K_t)] = \frac{EP_{t+1}^x}{P} \left[ (1 - \alpha_x) A^x \epsilon_{t+1}^x (K_{t+1} - K_{t+1}^f)^{-\alpha_x} \right] + (1 - \delta) + \phi [K_{t+2} - K_{t+1}] \quad (33)$$

To solve for the capital stock, one first solves for  $K_{t+1}^x$  as a function of the real interest rate and the expected aggregate capital stocks,  $K_{t+1}$  and  $K_{t+2}$ , used to compute the costs of adjustment:

$$K_{t+1}^x = \left( \frac{(1 + R_{t+1})[1 + (\phi K_{t+1} - K_t)] - (1 - \delta) - \phi [K_{t+2} - K_{t+1}]}{\frac{EP_{t+1}^x}{P} (1 - \alpha_x) A^x \epsilon_{t+1}^x} \right)^{-\frac{1}{\alpha_x}} \quad (34)$$

One can solve for  $K_{t+1}^x$  in an iterative manner. In the first, simply set the forward looking variables  $K_{t+1}$  and  $K_{t+2}$  equal to  $K_{t-1}$  and  $K_t$ ,

respectively. Then one may obtain values of  $\{\tilde{K}_t\}$ , whose forward-looking values may be used to recompute  $\{K_t^x\}$  in a second iteration. One can continue in this way until reasonable convergence is obtained. Once  $\{K_t^x\}$  is given, the values of  $\{K_t^f\}$  are obtained by the marginal productivity equality condition.

Aggregate investment is the change in the total capital stock:

$$\Delta K_t = \Delta K^x + \Delta K^f \quad (35)$$

Investment in the capital stock takes place with imported goods,  $F$ .

## 2.5 Macroeconomic Identities and Market Clearing Conditions

From the above equations, the trade balance,  $TB$ , expressed in domestic currency, is simply net exports less net imports, inclusive of goods used for investment:

$$TB_t = \frac{EP^x}{P}[Q_t^x - X_t] - \frac{E}{P}[Q_t^f - \Delta K_t - F_t] \quad (36)$$

while the current account balance,  $CAB$ , is simply the trade balance plus interest on international debt:

$$CAB_t = TB_t - \frac{E}{P}L_t^*[1 + i_t^* + ({}_t e_{t+1} - e_t) + \theta_t - \Pi_t] \quad (37)$$

Under flexible exchange rates, net capital inflows are simply the mirror image of the current account:

$$\frac{E}{P}L_{t+1}^* = \frac{E}{P}L_t^* - CAB_t \quad (38)$$

While the exchange rate is determined by the forward-looking interest parity relation, and the terms of trade are determined exogenously, the price of non-traded goods adjusts in response to demand and supply in this sector. To capture more realistic conditions of “sticky prices” in this sector, this model assumes that the price of non-tradeables follows a partial adjustment process to conditions of excess demand or supply:

$$\ln(P_{t+1}^N) - \ln(P_t^N) = (1 - \psi)[N_t - Q_t^n] \quad (39)$$

where  $\psi$  represents the degree of price stickiness.

## 2.6 The Consolidated Government

### 2.6.1 Fiscal Authority

The government issues bonds to finance budget deficits. It consumes non-traded goods,  $G^N$ , as well as services past government debt:

$$B_{t+1} - B_t = (i - \pi)B_t + D_t \quad (40)$$

where  $G^N$  denotes real government spending on imported and non-traded goods; government tax revenue is assumed to come from the consumption of traded goods. For simplicity, it is assumed that the government does not consume traded-goods. The deficit  $D_t$  affects only the demand for non-traded goods.

The usual no-Ponzi game applies to the evolution of real government debt:

$$t \rightarrow \infty \lim B_t \exp^{-(i-\pi)t} = 0 \quad (41)$$

$$t \rightarrow \infty \lim L_t^* \exp^{-(i^* + \theta + \Delta e_{t+1} - \pi)t} = 0 \quad (42)$$

The fiscal authority will exact lump sum taxes from non-traded goods sector in order to “buy back” domestic debt if it grows above a critical domestic debt/gdp ratio or threshold,  $\tilde{B}$ . Similarly, if the external debt grows above a critical external debt/gdp ratio,  $\tilde{L}^*$ , the fiscal authority will levy taxes in the traded-goods sector in order to reduce or buy-back debt.

### 2.6.2 Monetary Authority

The monetary authority does not know the “correct” model for the evolution of inflation. We assume three different policy scenarios. In the pure inflation target case, the monetary authority estimates the evolution of inflation as a function of its own lag as well as of changes in the interest rate. In the inflation/growth scenario, the central bank estimates the evolution of inflation and growth as functions of their own lags and of changes in the interest rate. Finally, in the inflation/growth/depreciation scenario, the central bank estimates the evolution of all three as functions of their own lags as well as of changes in the interest rate. “Least squares learning” is used to forecast the future values of these “state” variables in each scenario.

$$\Psi_t = \Gamma_{1t}\Psi_{t-1} + \Gamma_{2t}\Delta i_t \quad (43)$$

where  $\Psi_t = [\pi_t]$  in the first scenario,  $\Psi_t = [\pi_t \ \Delta y_t]$  in the second, and  $\Psi_t = [\pi_t, \Delta y_t, \Delta e_t]$  in the third.

Corresponding to each scenario, the government optimizes the following loss function  $\Lambda$ ,

$$\Lambda_1 = \lambda_{1t}(\pi_t - \pi^*)^2 \quad (44)$$

$$\Lambda_2 = \lambda_{1t}(\pi_t - \pi^*)^2 + \lambda_{2t}(\Delta y_t - \psi)^2 \quad (45)$$

$$\Lambda_3 = \lambda_{1t}(\pi_t - \pi^*)^2 + \lambda_{2t}(\Delta y_t - \psi)^2 + \lambda_{3t}(\Delta e_t - \chi)^2 \quad (46)$$

where  $\pi^*$ ,  $\psi$ , and  $\chi$  represent the targets for the inflation, output growth and depreciation rates under alternative policy scenarios.

At time  $t$ , depending on the scenario, the monetary authority specifies the weights on the loss function,  $\lambda_t = \{\lambda_{1t}, \lambda_{2t}, \lambda_{3t}\}$ , and estimates the state-space system. From the parameter set  $\widehat{\Gamma}_t$ , the policy maker sets the systematic part of the interest rate as an optimal feedback function of the state variables  $\Psi_t$ :

$$i_{t+1} = i_t + h(\widehat{\Gamma}_t, \lambda_t)Z_t + \varepsilon_t \quad (47)$$

$$\varepsilon \sim N(0, \sigma_\varepsilon^2) \quad (48)$$

where  $h(\widehat{\Gamma}_t, \lambda_t)$  is the solution of the optimal linear quadratic “regulator” problem, with control variable  $\Delta i$  solved as an feedback response to the state variables, and  $\varepsilon_t$  is a random, non-systematic component of the interest rate at time  $t$ .

In formulating its optimal interest-rate feedback rule, the government acts at time  $t$  as if its estimated model for the evolution of inflation and output growth is true “forever”, and that its relative weights for inflation, or growth or depreciation in the loss function are permanently fixed.

However, as Sargent (1999) points out in a similar model, the monetary authority’s own procedure for re-estimation “falsifies” this pretense as it updates the coefficients  $\{\Gamma_{1t}, \Gamma_{2t}\}$ , and solves the linear quadratic regulator problem for a new optimal response “rule” of the interest rate to the evolution of the state variables  $\Psi_t$ .

The weights for inflation, output growth, and depreciation in the respective loss functions  $\Lambda_i$  depend on the conditions at time  $t$ .

In the pure anti-inflation scenario, if inflation is below the target level  $\pi^*$  then the government does not optimize. The interest rate  $i_t = i_{t-1}$ . This is the “no intervention” case. However, if inflation is above the target rate, the monetary authority puts greater weight on inflation. In this case,  $\lambda_{1t} = 0.9$ . Table I illustrates this scenario.

Table I: Policy Weights	
Inflation Only Target	
$\pi < 0.025$	do nothing
$\pi \geq 0.025$	$\lambda_1 = 0.9$

In the second scenario, if inflation is below the target level  $\pi^*$  and output growth is positive, then the government does not optimize. If inflation is above the target rate, with positive growth, the monetary authority puts greater weight on inflation than on output growth. In this case,  $\lambda_{1t} = 0.9$ . If growth is negative but inflation is above its target, the inflation weight dominates but somewhat more weight is given to output. Finally, if inflation is below its target but output growth is negative, the central bank puts strong weight on the output target. The weights for this policy scenario are summarized in Table II.

Table II: Policy Weights		
Inflation and Growth Targets		
Inflation	Growth	
	$\Delta y < 0$	$\Delta y \geq 0$
$\pi < 0.025$	$\lambda_1 = 0.1$	do
	$\lambda_2 = 0.9$	nothing
$\pi \geq 0.025$	$\lambda_1 = 0.8$	$\lambda_1 = 0.9$
	$\lambda_2 = 0.2$	$\lambda_2 = 0.1$

In the third scenario, targets for the depreciation rate are also taken into account for formulating the policy feedback rule. The relative weights are summarized in Table III.

Table III: Policy Weights			
Inflation, Growth and Exchange Rate Targets			
Inflation	Exchange Rate	Growth	
		$\Delta y \leq 0$	$\Delta y > 0$
$\pi < 0.025$	$\Delta e < 0.025$	$\lambda_1 = 0.1$ $\lambda_2 = 0.8$ $\lambda_3 = 0.1$	do nothing
	$\Delta e \geq 0.025$	$\lambda_1 = 0.1$ $\lambda_2 = 0.7$ $\lambda_3 = 0.2$	$\lambda_1 = 0.1$ $\lambda_2 = 0.1$ $\lambda_3 = 0.8$
$\pi \geq 0.025$	$\Delta e < 0.025$	$\lambda_1 = 0.7$ $\lambda_2 = 0.2$ $\lambda_3 = 0.1$	$\lambda_1 = 0.8$ $\lambda_2 = 0.1$ $\lambda_3 = 0.1$
	$\Delta e \geq 0.025$	$\lambda_1 = 0.5$ $\lambda_2 = 0.3$ $\lambda_3 = 0.2$	$\lambda_1 = 0.7$ $\lambda_2 = 0.1$ $\lambda_3 = 0.2$

### 3 Calibration and Solution

The section discusses the calibration of parameters, initial conditions, and stochastic processes for the exogenous variables of the model as well as the specification of the policy rules and risk premia “reaction function”. Then it briefly summarizes the parameterized expectations algorithm (PEA) for solving the model.

#### 3.1 Parameters and Initial Conditions

The parameter settings for the model appear in Table IV.

Table IV: Parameters	
Consumption	$\gamma = 3.5, \beta = .009$ $\alpha_n = 0.5, \alpha_f = 0.5$
Production	$\alpha_f = 0.7, \alpha_x = 0.5$ $\delta = 0.1, \phi = 0.028$ $A^f = 0.12, A^x = 0.15, A^n = 1.0$
Price Coefficient	$\sigma = 0.7$
Debt Thresholds	$\tilde{b} = 0.5, \tilde{l}^* = 0.5$

Many of the parameter selections follow Mendoza (1995, 2001). The constant relative risk aversion is set at 3.5, somewhat below the value of

5 usually set for developing countries. The shares of non-traded goods in overall consumption is set at 0.5 while the shares of exports and imports in traded goods consumption is 50 percent each. The production function coefficients  $Q^f$  and  $Q^x$ , along with the initial values of capital for each sector, are chosen to ensure that the marginal product of capital in each sector is equal to the real interest plus depreciation, while the level of production meets demand in each sector. In particular the values for  $\alpha_f$  and  $\alpha_x$  reflect the assumption that the production of commodity exports is more capital intensive than manufactured imports.

The initial values of the variables appear in Table V.

Table V: Initial Conditions	
Consumption	$F = 0.2, X = 0.2, N = 0.5$
Production	$Q^f = 0.2, Q^x = 0.2, Q^n = 0.5$
Capital	$K^f = 4, K^x = 3.5$
Prices	$E = P^n = Z = 1$
Interest Rates	$i = 0.04$
Debt	$B = 0.2, L^* = 0.2$

Since the focus of the study is on the effects of time-varying currency risk and terms of trade shocks, the domestic productivity coefficients as well as the foreign interest rate were fixed at unity through the simulations. Table VI gives the values of these fixed variables.

Table VI: Fixed Values	
Foreign Interest rate	$i^* = 0.03,$
productivity shocks	$\epsilon^n = \epsilon^x = \epsilon^f = 1$

### 3.2 Terms of Trade and Currency Risk

The evolution of the terms of trade is specified to mimic the data generating process estimated for several countries.

$$\begin{aligned} \Delta \ln(P_t^x) &= \Delta \ln(P_{t-1}^x) + \eta_t^{P^x} \\ \eta_t^{P^x} &\sim N(0, 0.01) \end{aligned}$$

The parameter values for the evolution of currency risk appear in Table VII.

Constant	$\xi_0 = 0.0$
Lag	$\xi_1 = 0.5$
Change in Exchange Rate	$\xi_2 = 1.0$
Variance of Shock	$\sigma_\eta^2 = 0.0$

These are set to sharpen the focus on the feedback effects of changes in the exchange rate on risk. Thus, there is no long-run constant “currency risk” and no other source of risk, except changes in the exchange rate, so that  $\xi_0 = 0$  and  $\sigma_\eta^2 = 0$ .

### 3.3 Solution Algorithm

Following Marcet (1988, 1993), Den Haan and Marcet (1990, 1994), and Duffy and McNelis (2001), the approach of this study is to “parameterize” the forward-looking expectations in this model, with non-linear functional forms  $\psi^E, \psi^c$ :

$$\mathbf{E}_t(\exp[-V(C_t)]\lambda_{t+1} \cdot \{1 + i_{t+1} - \pi\}) = \psi^c(x_{t-1}; \Omega_\lambda) \quad (49)$$

$$\mathbf{E}_t e_{t+1} = \psi^E(x_{t-1}; \Omega_E) \quad (50)$$

where  $x_t$  represents a vector of observable variables at time  $t$ : consumption of imported and export goods,  $F$  and  $X$ , the marginal utility of consumption  $\lambda$ , the real interest rate  $r$ , and the real exchange rate,  $Z$ ,

$$x_t = \{F, X, \lambda, r, Z\} \quad (51)$$

and  $\Omega_\lambda, \Omega_E$  represent the parameters for the expectation function.

Judd (1996) classifies this approach as a “projection” or a “weighted residual” method for solving functional equations, and notes that the approach was originally developed by Williams and Wright (1982, 1984, 1991). These authors pointed out that the conditional expectation of the future grain price is a “smooth function” of the current state of the market, and that this conditional expectation can be used to characterize equilibrium.

The function forms for  $\psi^E, \psi^c$  are usually second-order polynomial expansions [see, for example, Den Haan and Marcet (1994)]. However, Duffy and McNelis (2001) have shown that neural networks have produced results



with greater accuracy for the same number of parameters, or equal accuracy with fewer parameters, than the second-order polynomial approximation.

The model was simulated for repeated parameter values for  $\{\Omega_\lambda, \Omega_E\}$  until convergence was obtained for the expectational errors. A description of the solution algorithm appears in the appendix.

## 4 Simulation Results

To evaluate the effects of the alternative policy scenarios, 500 simulations of sample length 200 quarters were generated. While we do not explicitly examine welfare in our evaluation of the alternative policy scenarios, we compare the distributions describing the variability of inflation and output generated for these scenarios.

Table VII shows the means and standard deviations of the volatility of inflation, output growth, depreciation, and the change in the interest rate.

Variable	Policy Targets		
	$\pi$	$\pi, \Delta y$	$\pi, \Delta y, \Delta e$
Inflation ( $\pi$ )	0.0194 (0.0034)	0.0137 (0.0035)	0.0082 (0.0018)
Growth ( $\Delta y$ )	0.0532 (0.0060)	0.0378 (0.0067)	0.0517 (0.0140)
Depreciation ( $\Delta e$ )	0.0350 (0.0055)	0.0114 (0.0025)	0.0075 (0.0018)
Interest Changes( $\Delta i$ )	0.0075 (0.0009)	0.0101 (0.0019)	0.0073 (0.0016)

\* in parenthesis

What is most startling about Table VIII is the strong trade-off implied by inflation/growth targeting versus inflation/growth and depreciation targeting. While including depreciation in the objective function does indeed reduce inflation variability, it also increases output variability by more than 30 percent.

The Epanechnikov kernel estimators for the distribution of inflation under the three policy scenarios appears in Figure 1. The Figure shows the reduction in mean (and spread) of volatility as the central bank changes its targets from the narrow pure inflation scenario to the broader inflation/growth/depreciation scenario.

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Figure 1: Kernel Estimates for Inflation Under Alternative Policy Scenarios

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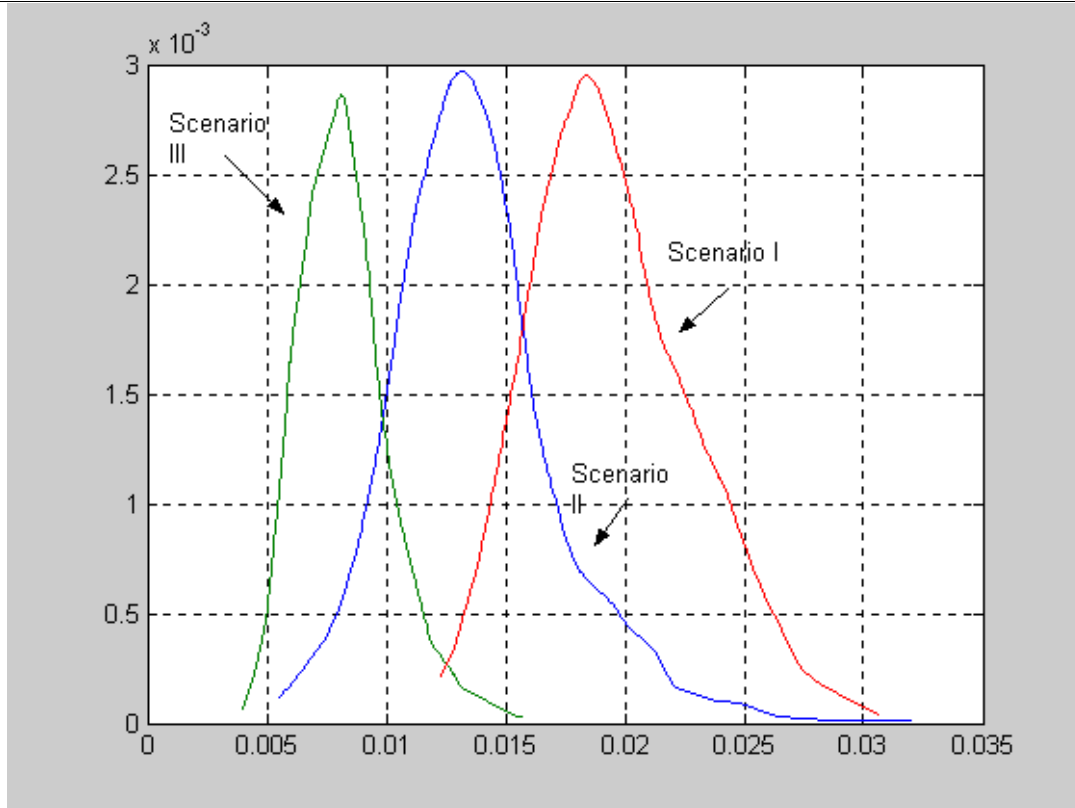


Figure 2 shows the distribution of the volatility estimates for economic growth under the three policy scenarios. As expected, the Figure shows that if GDP growth is ignored by policy makers (as in scenario 1), then its volatility will be larger than when growth is considered (scenario II). However, what is most striking about Figure 2 is that targeting depreciation as well as inflation and growth is likely to lead to a much higher GDP volatility than simply targeting inflation and growth. While incorporating the depreciation rate is likely to reduce inflation volatility, the cost of increased GDP volatility is quite apparent.

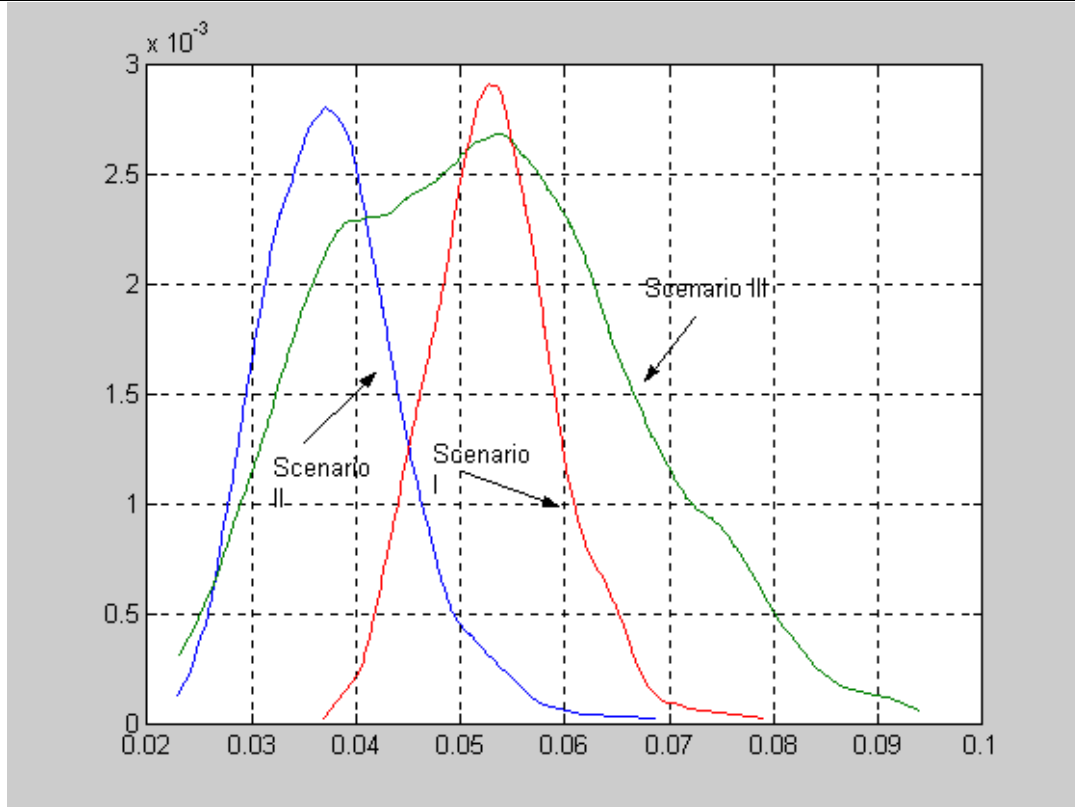
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Figure 2: Kernel Estimates for GDP Growth Under Alternative Policy Scenarios

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## 5 Conclusions

This paper has compared three alternative policy scenarios for a central bank facing terms of trade shocks and time-varying currency risk. Unlike the private sector, the central bank has to learn the laws of motion for its key target variables in order to set the interest rate according to a feedback rule.

The results show that including exchange rate changes in its learning and policy targeting framework will increase output variability significantly, with only small reductions in inflation variability. The policy implication is that central banks which are already targeting inflation and growth should resist pressures to incorporate exchange-rate targets.

Of course, the results of this paper may be conditioned by several key assumptions. One is the learning mechanism. Central banks may indeed have more sophisticated knowledge of underlying inflation dynamics than that which is implied by linear least squares learning. However, linear least squares learning is a good “tracking” mechanism for more complex

dynamic processes and our recursive method serves as an approximation to the Kalman filtering method.

The other strong assumption of this paper is the evolution of currency risk. This variable may well be conditioned by changes in public sector debt, foreign debt as well as inflation and exchange rate changes. How robust our policy results are to the way in which currency risk evolves is an open question. But assuming that currency risk first and foremost responds to past changes in the exchange rate is a sensible first approximation, and would bias the case, if at all, in favor of exchange rate targeting for the central bank.

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# Intertemporal Lagrangean Representation

Below is the equivalent Lagrangean expression of the intertemporal optimization problem of the representative household-firm:

$$\begin{aligned}
 \langle u_t \rangle^{\text{Max}} V = & U(c_t) - \lambda_t \left\{ \frac{E}{P} \left[ A^f \epsilon_t^f (K_t^f)^{1-\alpha_f} \right] + \frac{Ep^x}{P} \left[ A^x p_t^x \epsilon_t^x (K_t^x)^{1-\alpha_x} \right] + \right. \\
 & \frac{P^n}{P} \epsilon_t^n \bar{L} - K_{t+1} + K_t(1 - \delta) - \frac{\phi}{2} [K_{t+1} - K_t]^2 + \\
 & \frac{E}{P} L_{t+1}^* - \frac{E}{P} L_t^* [1 + i_t^* + (te_{t+1} - e_t) + \theta_t - \pi_t] - \\
 & \left. \frac{P^n}{P} def_t - B_{t+1} + B_t(1 + i_t - \pi_t) - c_t \right\} + \\
 & U(c_{t+1}) - \exp(v(c_t)) \lambda_{t+1} \left\{ \frac{E}{P} \left[ A^f \epsilon_{t+1}^f (K_{t+1}^f)^{1-\alpha_f} \right] + \right. \\
 & \frac{Ep_{t+1}^x}{P} \left[ A^x \epsilon_{t+1}^x (K_{t+1}^x)^{1-\alpha_x} \right] + \\
 & \frac{P^n}{P} \epsilon_{t+1}^n \bar{L} - K_{t+2} + K_{t+1}(1 - \delta) - \frac{\phi}{2} [K_{t+2} - K_{t+1}]^2 + \\
 & \frac{E}{P} L_{t+2}^* - \frac{E}{P} L_{t+1}^* [1 + i_{t+1}^* + (te_{t+2} - e_{t+1}) + \theta_{t+1} - \pi_{t+1}] + \\
 & \left. \frac{P^n}{P} def_{t+1} - B_{t+2} + B_{t+1}(1 + i_{t+1} - \pi_{t+1}) - c_{t+1} \right\} \quad (52)
 \end{aligned}$$

$$u_t = \{c_t, K_{t+1}, K_{t+1}^f, B_{t+1}, L_{t+1}^*\} \quad (53)$$

## Solution Algorithm

The solution algorithm for parameterized expectations makes use of neural network specification for the expectations, and a genetic algorithm for the iterative solution method, as well as the quasi-Newton method.

The specification of the functional forms  $\psi^E(x_t; \Omega_E)$  and  $\psi^c(x_t; \Omega_c)$  according to the neural network approximation, is done in the following way:

$$\begin{aligned}
 n_{k,t} &= \sum_{j=1}^{J^*} b_j x_{j,t} \\
 N_{k,t} &= \frac{1}{1 + e^{-n_{k,t}}} \\
 \hat{\psi}_t &= \sum_{k=1}^{K^*} \kappa_k N_{k,t}
 \end{aligned} \quad (54)$$

where  $J^*$  is the number of exogenous or input variables,  $K^*$  is the number of neurons,  $n_t$  is a linear combination of the input variables,  $N_t$  is a logsigmoid



or logistic transformation of  $n_t$ , and  $\hat{\psi}_t$  is the neural network prediction at time  $t$  for either  $(e_{t+1})$  or  $\exp[-V(C_t)]\lambda_{t+1} \cdot \{1 + I_{t+1} - \Pi\}$ .

As seen in this equation, the only difference from ordinary non-linear estimation relating "regressors" to a "regressand" is the use of the hidden nodes or neurons,  $N$ . One forms a neuron by taking a linear combination of the regressors and then transforming this variable by the logistic or logsigmoid function. One then proceeds to thus one or more of these neurons in a linear way to forecast the dependent variable  $\hat{\psi}_t$ .

Judd (1996) notes that the neural networks provide us with an "inherently nonlinear functional form" for approximation, in contrast with methods based on linear combinations of polynomial and trigonometric functions.

Both Judd (1996) and Sargent (1997) have drawn attention to the work of Barron (1993), who found that neural networks do a better job of "approximating" any non-linear function than polynomials, in that sense that a neural network achieves the same degree of in-sample predictive accuracy with fewer parameters, or achieves greater accuracy, using the same number of parameters. For this reason, Judd (1996) concedes that neural networks may be particularly efficient at "multidimensional approximation".

The main choices that one has to make for a neural network is  $J^*$ , the number of regression variables, and  $K^*$ , the number of hidden neurons, for predicting a given variable  $\psi_t$ . Generally, a neural network with only one hidden neuron closely approximates a simple linear model, whereas larger numbers of neurons approximate more complex non-linear relationships. Obviously, with a large number of "regressors"  $x$  and with a large number of neurons  $N$ , one approximates progressively more complex non-linear phenomena, with an increasingly larger parameter set.

The approach of this study is to use relatively simple neural networks, between two and four neurons, in order to show that even relatively simple neural network specifications do well for approximating non-linear relations implied by forward-looking stochastic general equilibrium models.

Since the parameterized expectation solution is a relatively complex non-linear function, the optimization problem is solved with a repeated hybrid approach. First a global search method, genetic algorithm, similar to the one developed by Duffy and McNelis (2001), is used to find the initial parameter set, then a local optimization, the BFGS method, based on the quasi-Newton algorithm, is used to "fine tune" the genetic algorithm solution.

De Falco (1998) applied the genetic algorithm to nonlinear neural network estimation, and found that his results "proved the effectiveness" of

such algorithms for neural network estimation.. The main drawback of the genetic algorithm is that it is slow. For even a reasonable size or dimension of the coefficient vector, the various combinations and permutations of the coefficients which the genetic search may find “optimal” or close to optimal, at various generations, may become very large. This is another example of the well-known “curse of dimensionality” in non-linear optimization. Thus, one needs to let the genetic algorithm “run” over a large number of generations—perhaps several hundred—in order to arrive at results which resemble unique and global minimum points.

Quagliarella and Vicini (1998) point out that hybridization may lead to better solutions than those obtainable using the two methods individually. They argue that it is not necessary to carry out the quasi-Newton optimization until convergence, if one is going to repeat the process several times.. The utility of the quasi-Newton BFGS algorithm is its ability to improve the ”individuals it treats”, so ”its beneficial effects can be obtained just performing a few iterations each time” [Quagliarella and Vicini (1998): 307].