Anchor Stores*

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Abstract

Planned shopping malls usually have one or more department (anchor) stores and multiple specialized retailers selling substitutable commodities in each commodity category. If consumers know their taste for the anchor's commodity and its price, but learn about a retailer's commodity only by costly search, collocation may benefit both store types. Intra-mall competition reduces markups, but anchors guarantee a minimum surplus from search. A mall with many retailers makes finding a suitable specialized commodity highly probable. For some parameters, additional consumer search dominates the loss in retail markups, so a profit-maximizing land developer would rent mall space to both store types.

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1 Introduction

An "anchor store" is a store that increases, through its name's reputation, the traffic of shoppers at or near its location. Consumers, attracted by the anchor's name, are likely to visit the location ("the mall"), and thus nearby stores' sales and profits are increased by the presence of the anchor. Planned shopping malls usually have one or more department stores and multiple specialized retail stores in each commodity category. Pashigian and Gould [17] provide empirical evidence that rents for anchor (or department) stores are heavily discounted. They interpret the discounted rents charged to department stores as an attempt by land developers to lure these stores to mall locations, creating a positive externality for the mall's retail stores. The free-riding retail stores' surplus can then be extracted through higher rent for retail space.[1]

In the above story, however, it is not clear why consumers are attracted by anchor stores, nor is it clear why anchor stores generate a positive externality for the specialized retailers. Many shopping malls have specialty retailers of shoes, men's apparel and women's apparel, while also having anchor stores that devote substantial retail floor space to each of these product categories. If brand names are so important to consumers and the department store competes with other mall retailers in every product category, department stores are unlikely to exert a positive externality on retailers. Consumers would make their purchases only at the department stores. Consequently, the above "brand name" theory by itself cannot motivate using department stores as anchor stores. Another reason department stores may increase consumer traffic at a mall location is commuting cost savings. That is, since department stores sell commodities in many different categories, consumers may be encouraged to visit the stores to economize on commuting costs (see Stahl [23]). This effect may explain why department stores are regarded as anchor stores. However, it is also not sufficient since it again does not explain why both department and other retail stores sell substitutes.

In this paper, we provide an explanation for the anchor store phenomenon by reinterpreting the brand name theory. In our approach, the significance of the brand names of the department stores is that consumers know more about the characteristics and prices of those commodities than they do about the commodities sold at the retail shops. Thus, if a consumer visits a department

[1] Pashigian and Gould [17] argue that retail stores are "free-riding" on department stores' brand names.

store to look for casual apparel, for example, she knows that she is very likely to find something that suits her taste at an acceptable price. If, by contrast, she visits one of the smaller clothing retailers in the mall, there is a significant chance that she will not find anything acceptable; the retailer may not stock clothing to her taste. However, there is also a chance that she may find clothing that is well suited to her taste at the smaller retailer. That is, the different characteristics of these two types of stores may generate positive externalities for each other, and both types of stores may benefit from collocation.[2] This provides land developers an incentive to choose a similar composition of tenants when writing their commercial leases.

Our basic model comprises an anchor store (a department store) and multiple specialized retail stores. Each store sells exactly one type of indivisible commodity. The two types of stores are competitors in the sense that each consumer buys at most one unit of the commodity, either at the anchor store or at one of the specialized retailers. A consumer knows beforehand her reservation value (willingness-to-pay) for the commodity sold by the anchor store and the commodity's price. By contrast, she must visit specialized retailers to realize her reservation values for their commodities. That is, her reservation value for the commodity sold at each specialized retail store is a random variable, stochastically independent across retailers. Each consumer correctly infers the price set by the retailers (a symmetric Nash equilibrium), then calculates the expected utility of a visit to the shopping center. Visiting the shopping center is costly to each consumer, and this transportation (search) cost is sunk once incurred. Consumers' transportation costs are heterogeneous, so only those whose expected utility is higher than the transportation cost can be expected to contribute to traffic at the shopping mall.

Given this structure, there may be incentives for the two types of stores to collocate. Suppose that there are only specialized retailers at a shopping center. Each consumer infers the retail price at the shopping center and computes her expected utility from a visit. If her expected value is low relative to her search cost, she will not visit the shopping center. Suppose, alternatively, that the shopping center contains an anchor store. In this case, the consumer is guaranteed a minimal level of net surplus by purchasing the anchor store's commodity. As a result, consumers may choose to visit the shopping mall even

[2] If shoppers repeatedly visit a shopping mall, they may accumulate information about the commodities sold by the retailers. However, if (speciality) retail stores change their commodities more often than the department store, we still have similar effects.

if their search cost is fairly high. This implies that the number of consumers who visit the shopping center (the market size, or "traffic") will be increased by the presence of the anchor store. Of course, the anchor store's (presumably low) price also cuts into the profit margins of the specialized retailers. It follows that if the traffic enhancement effect is larger than that of reduced profit margins, the specialized retailers will realize a positive externality from the presence of the anchor store.

Furthermore, note that the anchor store may receive an external benefit from collocating with the specialized retailers (i.e., the retailers are not "free riders"). Suppose that the anchor store is standing alone at a location. A consumer knows how much surplus she can get by visiting the anchor store, so the anchor store has a steady, but possibly scant, traffic of customers. Now suppose instead that the anchor store collocates with the specialized retailers at a shopping center. A consumer's expected surplus from visiting this shopping center will be substantially higher than that of a visit to the stand-alone anchor if she thinks that one of the specialized retailers is likely to have a commodity very well suited to her tastes. Thus mall traffic can again be increased by collocation of the anchor store and specialized retailers. As above, if the reduction in profit margins due to increased competition is offset by the profit-enhancing effect of increased traffic, the anchor store has an incentive to collocate with the specialized retailers. Hence, if conditions are right, collocation of the two types of stores improves profits in a Pareto fashion.

To provide a foundation to the story told by Pashigian and Gould [17], we need to specify how a land developer decides the composition of her shopping mall. We apply the idea of club theory *à la* Tiebout [26] and Buchanan [5]. We consider a single land developer's profit maximization problem: given that each type of store has an outside option, a land developer maximizes her profit by choosing the characteristics of a club (the composition of tenants of the mall). If there are many potential land developers, then the rent structure is determined in the market: how much rent a retail store will pay for a space in a shopping mall with a certain composition of stores. If the anchor and retail stores complement each other, the characteristics of the mall affect each store's gross profit. Thus, a land developer's choice of the characteristics of the mall and the rent structure work, respectively, as the characteristics of a club and a club-membership fee schedule in the club theory.

Our numerical examples support the above analysis, demonstrating three significant characteristics of the model. First, as long as the number of retailers in a mall configuration is not too large, collocation of retail and anchor stores can benefit both types of stores. If the number of specialized retailers is too large relative to production costs, then the commitment to low prices fostered by competition among the retailers is sufficient to exhaust gains from traffic enhancement. In this case, the presence of the anchor store merely reduces profit margins. This observation justifies the commonality of the composition of stores in planned shopping malls (see West [28]). Furthermore, mall configurations including an anchor store and several retailers can be profit-maximizing for a land developer which extracts rents efficiently.

Second, our numerical examples demonstrate that anchor stores which sell attractive commodities at low prices (i.e., give consumers high net surplus) make collocation less attractive to retailers. In particular, as the anchor store sells more attractive commodities, it exposes the retailers in the mall to fiercer price competition, eroding the retailers' incentive to collocate with the anchor. Third, we show that, as long as the anchor store's commodities are not too attractive, Pashigian and Gould's [17] aforementioned observation can be justified.

The rest of the paper is organized as follows: Section 2 is devoted to a brief literature review. Section 3 describes the model and provides sufficient conditions for the existence of an interior equilibrium. Section 4 presents numerical results that illustrate our point. Section 5 concludes.

2 Literature Review

The body of literature on anchor stores is not very large. The only theoretical papers that investigate inter-store externalities in a shopping mall are Benjamin, Boyle, and Sirmans [3] and Brueckner [4]. These papers consider the land developers' space-allocation problem of internalizing externalities among tenant stores, assuming that the developers have decided on the composition of stores at the malls. Assuming reduced-form revenue functions of sales (without explicitly modeling prices and market size), they derive optimal discriminatory rent schemes in a variety of settings.[3] Empirical analyses include

[3]Although it is not directly related to our question, there is an interesting literature on retail lease contracts. In real life, lease contracts in shopping malls have two components: fixed rent and percentage rent linear in sales. Lee [16] looks this as an optimal linear contract that takes care of both risk sharing and moral hazard problems (see Brueckner [4]

West [28], who uses data on shopping malls in Alberta and finds similarity among planned shopping malls. The inference West draws from this result is that owners of shopping malls are selecting retail stores and their locations in a profit-maximizing way. Pashigian and Gould [17] found that anchor stores (department stores) at shopping malls receive substantial rent subsidies. They infer from this result that anchor stores increase mall traffic, increasing the profits of other stores. Thus, rental contracts at shopping malls internalize externalities among stores. Each of the papers cited above takes the land developer's profit-maximization problem to be the origin of the observed mall composition and rent structure (we do the same). Finally, Rauch [18] has an interesting paper on profit-maximizing rent discrimination over time by land developers. Although its focus is different from the others', the idea is related in that land developers strategically discriminate in rents in order to internalize externalities in the mall.[4]

Stahl [25] has a nice survey paper on urban business location. In one section of his paper, Stahl discusses two incentives for retail stores to agglomerate at a location. One incentive toward agglomeration is as follows. If a consumer needs to shop for commodities in several different categories on a given day, it would be much more convenient for her to visit a concentration of stores, each of which sells goods in one of the categories she needs. Clearly, department stores provide this service by themselves to some degree, and this is an important motivation for a landlord to have department stores at her mall.[5] The other incentive is that, within each category of commodities, having more stores can provide customers more variety of commodities, which increases traffic of customers.[6] In this paper, we only focus on this latter in-

as well). Wheaton [29] constructs a dynamic model in which a percentage contract gives landlords a (correct) incentive not to behave opportunistically. Finally, an application of the argument provided in Saggi and Vettas [19] can also explain the coexistence of fixed and percentage contracts. Saggi and Vettas analyze a strategic role of fees (fixed rent) and royalties (percentage rent) in franchise competition. Royalties control competition between own franchisees (own tenant shops) at the expense of making them passive against rivals (other shopping malls).

[4] See also Henderson [12] for the roles of land developers in the context of local jurisdictions.

[5] Stahl [23] is the first paper that discusses this effect. For a related discussion, see Lal and Matutes [15].

[6] Stahl [24] and Wolinsky [31] are the first papers that discuss collocation incentives for retail stores when consumers have imperfect information on commodities, and they incur search costs. Consumers are attracted by a variety of commodities. However, these models

centive in order to separate these two incentives for agglomeration. Thus, we show that it is not essential for an anchor store's goods to be independent or complementary for the store to serve an "anchoring" function.

Finally, we briefly mention the literature on the traditional local public good theory and club theory. In his pioneering paper on local public goods economy, Tiebout [26] (informally) asserts that if consumers can choose their residential jurisdictions, then jurisdictions compete with each other and only efficient jurisdictions survive (see also Buchanan [5]).[7] Although Tiebout himself does not specify jurisdictions' objective functions, Henderson [11] and Sonstelie and Portney [22] supplement his argument by assuming that jurisdictions are owned by land developers who maximize their profits by choosing local public goods provision levels, and the land markets are complete and competitive. In this paper, we will follow this line of discussion with a partial equilibrium approach.[8] We assume that shopping mall owners choose compositions of tenant stores at their shopping malls in order to maximize their profits and that rent markets for tenants are competitive. This explains empirical evidence for the similarity in the compositions of planned shopping malls (see West [28]).

3 The Model

Suppose there is a shopping mall owned by a land developer. She chooses the number and combination of tenants at her shopping center. There are two

cannot analyze the profit reducing effect of intensified price competition since they assume either no price or a common price everywhere. Dudey [9] focuses on the price effect by using a Cournot oligopoly model. Consumers are uninformed about prices, and they choose the shopping center by inferring which shopping center has the lowest prices. Thus, a shopping center with many retail stores attracts more customers. Konishi [14] presents a model in which competing retail stores face these two concentration incentives. All the papers described above assume that commodities and stores are *ex ante* symmetric. Our paper is new in this regard: it has both a standardized good sold by an anchor store and 'high risk-high return' commodities sold by retail stores, discussing the incentive for the two types of stores to collocate.

[7] Tiebout also asserts that consumers sort based on their characteristics. But our paper is not interested in this point.

[8] In this sense, our treatment is closest to Schweizer [20] (and in spirit to Tiebout [26] and Buchanan [5]). See Wildasin [30] as well. For more "general equilibrium" theoretical treatments of local public goods economy such as core convergence/equivalence theorems, see Wooders [32], [33], Scotchmer [21], and Conley and Smith [6].

types of stores: retail stores and an anchor store, each of which sells a single type of indivisible commodity. A tenant combination of the shopping mall can be described by a pair (k, m), where a nonnegative integer $k \leq \bar{k}$ and $m \in \{0, 1\}$ denote the numbers of retail and anchor stores at the shopping mall, respectively ($\bar{k}$ denotes the upper bound for the number of retail stores). Marginal costs of production by the anchor store are normalized to zero, while the marginal costs of the retailers are constant at the common value $c > 0$. Prices of the commodities are given by p_i for retail stores $i = 1, \ldots, k$ and by p_0 for the anchor store. There is a continuum of consumers who *ex ante* have identical tastes but differ in their search costs. Each consumer's reservation value for the commodity sold by retailer $i = 1, \ldots, k$ is v_i, an i.i.d. (over consumers) random variable distributed uniformly over the closed interval $[0, 1]$. That is, consumers are not informed about how much they like the commodity sold by any of the k retailers, and each consumer's valuations for the k retail commodities are stochastically independent. Consumers have a common, nonstochastic valuation v_0 for the product sold by the anchor store. This asymmetric treatment of these two types of stores reflects our view that an anchor store sells a more standardized commodity so that consumers know how much they like its product, yet they do not know how much they are willing to pay for commodities sold by smaller retail stores. Finally, all consumers have the option to purchase nothing (also known as the "outside good") which yields gross and net surplus of zero. Consumers purchase the commodity that gives them the highest net surplus.[9]

Each consumer's reservation values for the commodities $i = 1, \ldots, k$ are revealed only upon a payment of a search cost $t \geq 0$, known to the consumer. One natural interpretation of the search cost is that it reflects a consumer's location relative to the location of the mall (a transportation cost). For simplicity, we assume that t is distributed uniformly over the closed interval $[0, 1]$, e.g., consumers are uniformly distributed along a road of unit length, with the mall located at 0. Once the search cost is incurred by visiting the mall, it is sunk; however, once at the mall, a consumer may visit any store or stores at zero incremental cost (i.e., once at the mall, consumers may check all shops freely). As a result, only consumers whose t's are less than the gross expected utility from shopping at the shopping mall will commute there. *We assume*

[9] If a consumer decide to visit a shopping mall with an anchor store, she will purchase at least one commodity, since we assume $v_0 - p_0 \geq 0$ (see below). If she visits a mall without an anchor store, then she may exercise the option to purchase nothing.

that the prices of the retail commodities are unknown to the consumers in advance of a visit to the mall, while the anchor store commodity price p_0 is known to consumers. Thus, consumers know that they can at least obtain $v_0 - p_0$ by visiting the shopping mall with an anchor store, while they know neither v_i's nor p_i's for retail stores $(i > 0)$ before they visit the mall.[10] This assumption reflects the difference between the two types of stores. Consumers have a very good idea (price and characteristics) about the commodity sold at the anchor store, but they are not well informed about the commodities sold at other specialized retail stores.[11] Consumers rationally infer the prices of retail store commodities based on the common knowledge distributions of valuation and search costs and the firms' production costs. We assume that $v_0 - p_0 \geq 0$, since otherwise nobody prefers (even weakly) the anchor store commodity. This assumption implies that every consumer can at least obtain $v_0 - p_0$ surplus by visiting the shopping mall.

The land developer chooses a combination of tenants (k, m) from the feasible set $\mathcal{F} = \{(k, m) \in \{0, 1, ..., \bar{k}\} \times \{0, 1\}\}$ in order to maximize her total rent revenue, taking retail and anchor stores' outside opportunities into account. Retail and anchor stores have reservation profits (net of rents) ρ_R and ρ_A, respectively. The land developer needs to guarantee these net profits to her tenants to attract them. For simplicity, costs of building a shopping center are ignored.

The timing of the game is as follows:[12]

1. A land developer chooses some combination of anchor and retail stores at her shopping mall.

[10] Once the consumer visits the mall, incurring the search cost, she learns her v_is and p_is as well, and chooses the commodity with the highest realized net surplus (for her).

[11] This is the key assumption that generates (potential) complementarity between these two types of stores.

[12] While we have chosen, for expositional purposes, to have consumers make their search decisions in advance of the retail pricing decisions, our results rely in no way on this ordering. Note that, since consumers' tastes, prior to the move by Nature, are identical, a given consumer's decision to visit the mall location conveys no information exploitable by the retailers in their pricing decisions. Thus, rational consumers could draw correct inferences regarding the prices which would prevail at the mall location even if the prices were not revealed before the search decision was sunk. Consequently, retail pricing and consumer search may be thought of as simultaneous moves, from which it follows that the order in which we present them is a matter of expositional clarity and notational convenience.

2. Consumers decide (independently) whether or not to visit the shopping mall, based on their individual costs of search and expected net surplus upon search. *Consumers are not informed about the prices at the retail stores.* This stage determines the market size (consumer traffic) of the shopping mall.

3. The retail stores choose their prices simultaneously.

4. For each consumer who decides to search, Nature plays and her reservation value for each commodity $i = 1, \ldots, k$ is realized.

5. Each consumer decides which commodity to buy by observing her reservation price for each commodity.[13]

Our equilibrium concept is that of subgame perfect Nash equilibrium. We solve the problem with backward induction, starting with the consumer's purchase decision (stage 5).

3.1 The Consumer's Purchase Decision (Stage 5)

Beginning with the final stage of the game, we examine the choice among commodities for those consumers who have chosen to visit the mall location (those who do not choose to visit the mall receive zero net surplus). At this stage of the game, therefore, each consumer knows her reservation prices $(v_1, \ldots, v_k)$ and the market prices $(p_1, \ldots, p_k)$, purchasing the commodity that gives her the maximum net surplus.

If there is no anchor store at the mall location, she solves

$$\max \left[0, \max_{i=1,\ldots,k} v_i - p_i \right].$$

That is, the consumer chooses the retail commodity i that gives maximum net surplus or chooses the "outside option" of not purchasing (denoted "$\emptyset$")

[13] Depending on the mall configuration chosen by the land developer, the choice set faced by consumers at the mall may include (in addition to the "outside good," which represents the option to purchase nothing) the anchor store's commodity and/or a variety of retail commodities. The above distributional assumptions guarantee that ties occur with zero probability.

which gives zero net surplus. Hence, in a mall without an anchor store, the probability that commodity i is purchased at retail prices $(p_1, ..., p_k)$ is

$$\begin{aligned}\mathbf{P}_i(p_1, ..., p_k; \emptyset) &= \Pr\left(\{v \in [0,1]^k : v_i - p_i \geq v_j - p_j \ \forall j = 1, ..., k, \text{ and } v_i - p_i \geq 0\}\right), \\ &= \Pr\left(\{v \in [0,1]^k : v_j \leq v_i - p_i + p_j \ \forall j \neq i, \text{ and } v_i \geq p_i\}\right), \\ &= \int_{p_i}^{1} \left(\prod_{j \neq i} \Pr\left(\{v_j \in [0,1] : v_j \leq v_i - p_i + p_j\}\right) \right) dv_i.\end{aligned}$$

The last equality holds by stochastic independence of v_j's.[14]

If there is an anchor store at the mall location, she solves

$$\max\left[v_0 - p_0, \max_{i=1,...,k} v_i - p_i\right],$$

where $v_0 - p_0$ represents the net surplus from purchasing the anchor store commodity.[15] The probability that commodity i is purchased with anchor price p_0 and the retail price vector $(p_1, ..., p_k)$ is

$$\begin{aligned}\mathbf{P}_i(p_1, ..., p_k; p_0) &= \Pr\left(\{v \in [0,1]^k : v_i - p_i \geq v_j - p_j \ \forall j = 1, ..., k, \text{ and } v_i - p_i \geq v_0 - p_0\}\right), \\ &= \Pr\left(\{v \in [0,1]^k : v_j \leq v_i - p_i + p_j \ \forall j \neq i, \text{ and } v_i \geq p_i + v_0 - p_0\}\right), \\ &= \int_{p_i + v_0 - p_0}^{1} \left(\prod_{j \neq i} \Pr\left(\{v_j \in [0,1] : v_j \leq v_i - p_i + p_j\}\right) \right) dv_i.\end{aligned}$$

Evidently, the case in which there is no anchor store can be treated as a special case of the anchor store, in which $v_0 = p_0$. That is,

$$\mathbf{P}_i(p_1, ..., p_k; \emptyset) = \mathbf{P}_i(p_1, ..., p_k; v_0).$$

This observation applies to the rest of analysis as well, so in the rest of this paper *we will focus on the case with an anchor store for notational simplicity.* Since Stage 4 is simply Nature's move, we continue with firms' choices of retail prices.

[14] Demand for commodity i is described as market size (consumer traffic) multiplied by probability of sales $\mathbf{P}_i(p_1, ..., p_k; \emptyset)$. A similar comment applies for the case with an anchor store.

[15] While the consumer implicitly has the option not to purchase (receiving zero net surplus), we assume that the consumer always instead chooses the anchor store commodity when $v_0 - p_0 = 0$.

3.2 Equilibrium Retail Prices (Stage 3)

Consumers are not informed of prices before their search, but their expectations about the retailers' pricing behavior should be rational. In considering the retail profit function, we first note that market size, μ, (or "mall traffic") does not affect the equilibrium retail price in our model. The reason is that the marginal cost of production is constant ($c > 0$), and market size has only a scale effect (see Konishi [14]). Moreover, as noted above, consumers are homogeneous with respect to their valuations before Nature moves.

We naturally focus on symmetric price equilibrium, considering only those in which the retailers set equal prices, i.e., $p_1 = \cdots = p_k$, and call this common value p. Since we are looking for a symmetric solution in retail prices given $v_0 - p_0 \geq 0$, suppose retailer i sets price p_i while all other firms set the common price p. From those consumers who arrive at the mall, **retailer i's profit per unit demand** is given by

$$\Pi(p_i, p; k, p_0) = (p_i - c)\hat{\mathbf{P}}(p_i, p; k, p_0),$$

where $\hat{\mathbf{P}}(p_i, p; k, p_0) = \mathbf{P}_i(p, ..., p, p_i, p, ..., p; p_0)$.[16] Where the number of retail stores is k and the anchor store price is p_0, price p^* is called a **symmetric equilibrium retail price** if and only if $\Pi(p^*, p^*; k, p_0) \geq \Pi(p_i, p^*; k, p_0)$ for any $p_i \geq 0$. We denote the symmetric equilibrium retail price where the number of retail stores is k and the anchor store price is p_0 by $p^*(k, p_0)$. The equilibrium profit per unit demand is defined as $\Pi^*(k, p_0) = \Pi(p^*(k, p_0), p^*(k, p_0); k, p_0)$. We have the following proposition (for a proof, see the Appendix).

Proposition 1. The symmetric equilibrium retail price $p^*(k, p_0)$ is unique and is implicitly defined by the following equation:

$$\frac{1}{k}\left[1 - (v_0 - p_0 + p^*(k, p_0))^k\right] - (p^*(k, p_0) - c) = 0.$$

Moreover, the equilibrium profit per unit demand is written as $\Pi^*(k, p_0) = (p^*(k, p_0) - c)^2$.

Note once again that our analysis so far is about the case with an anchor store charging p_0, but the case without an anchor store is a special case of this case with $p_0 = v_0$. Thus $p^*(k, \emptyset) = p^*(k, v_0)$ and $\Pi^*(k, \emptyset) = \Pi^*(k, v_0)$.

[16] Note that the retailer's profit function is nonstochastic, owing to the law of large numbers. For details, see Judd [13].

3.3 Equilibrium Market Size (Stage 2)

Having correct inferences about the prices set by retailers in Stage 3, we next consider consumers' decisions about whether to visit the mall. Based on our foregoing analysis of Stage 5, a consumer's expected utility (gross of her search cost) from searching at the shopping center can be formalized as

$$\begin{aligned} U(p_1, ..., p_k, p_0) \;\; = \;\; & \sum_{i=1}^{k} \left[\int_{p_i+v_0-p_0}^{1} \left((v_i - p_i) \prod_{j \neq i} \Pr\left(\{v_j \in [0,1] : v_j \leq v_i - p_i + p_j\}\right) \right) dv_i \right] \\ & + (v_0 - p_0) \times \Pr\left(\left\{ \{ v \in [0,1]^k : v_j \leq p_j + v_0 - p_0 \; \forall j = 1, ..., k \} \right\} \right), \end{aligned}$$

where, again, $p_0 = v_0$ represents the special case of a shopping center with no anchor store. The integrand above gives the net surplus from purchasing i, $v_i - p_i$, multiplied by the probability that the net surplus from commodity i is the highest among all k specialized commodities. Integration yields the expected payoff over all realizations v_i. The second term is the net surplus from purchasing the anchor store commodity, $v_0 - p_0$, multiplied by the probability that the anchor store commodity attains the highest surplus (i.e., $v_j - p_j \leq v_0 - p_0$ for any $j = 1, ..., k$).

Using the equilibrium price $p^*(k, p_0)$ computed in Stage 3, consumers determine whether or not to visit the mall given their knowledge of (i) the characteristics of the shopping mall (with or without an anchor store, and the number of retail stores at the shopping mall), and (ii) the anchor store's price p_0 if the mall has an anchor store. A consumer visits the mall if and only if her expected surplus, net of transportation costs, from a visit is nonnegative.[17] So, if the mall has an anchor store, only a consumer whose transportation cost t is less than $U(p^*(k, p_0), ..., p^*(k, p_0), p_0)$ will choose to visit the mall, incurring her search cost.[18] Given that transportation costs increase with distance from the retail location, there will be a consumer with a transportation cost $t^*(k, p_0) \equiv U(p^*(k, p_0), ..., p^*(k, p_0), p_0)$ who is indifferent between visiting the mall and not. Since consumers' transportation costs are distributed uniformly (with unit density), **the market size** (the measure of consumers who visit

[17] Note that $U(p_1, ..., p_k, p_0) < 1$ follows for any nonnegative price vector. That is, some consumers do not choose to visit the shopping mall.

[18] Once again note that $U(p^*(k, \emptyset), ..., p^*(k, \emptyset), \emptyset) = U(p^*(k, v_0), ..., p^*(k, v_0), v_0)$ applies, so an analogous condition holds for consumers visiting malls without anchor stores.

the shopping mall) is given by $\mu(k,p_0) \equiv \int_0^{t^*(k,p_0)} dt = t^*(k,p_0)$. That is,

$$\begin{aligned}
\mu(k,p_0) &= U(p^*(k,p_0),...,p^*(k,p_0),p_0) \\
&= k\int_{p^*(k,p_0)+v_0-p_0}^{1} (v-p^*(k,p_0))v^{k-1}dv + (p^*(k,p_0)+v_0-p_0)^k(v_0-p_0) \\
&= \frac{k}{k+1} - p^*(k,p_0) + \frac{(p^*(k,p_0)+v_0-p_0)^{k+1}}{k+1}.
\end{aligned}$$

3.4 Equilibrium Profits

In preparation for the analysis in Stage 1, we calculate the equilibrium profits. By knowing the equilibrium profits per unit demand and the market size, we can calculate the equilibrium profits for retail and anchor stores:

$$\begin{aligned}
\pi^R(k,p_0) &= \Pi^*(k,p_0)\mu(k,p_0) \\
&= (p^*(k,p_0)-c)^2\left(\frac{k}{k+1} - p^*(k,p_0) + \frac{(p^*(k,p_0)+v_0-p_0)^{k+1}}{k+1}\right),
\end{aligned}$$

and

$$\begin{aligned}
\pi^A(k,p_0) &= p_0(p^*(k,p_0)+v_0-p_0)^k\mu(p^*(k,p_0);k,p_0) \\
&= p_0(p^*(k,p_0)+v_0-p_0)^k\left(\frac{k}{k+1} - p^*(k,p_0) + \frac{(p^*(k,p_0)+v_0-p_0)^{k+1}}{k+1}\right).
\end{aligned}$$

When $k=0$, we can also calculate the anchor store's profit (an independent anchor store), since the market size is v_0-p_0 in such a case:

$$\pi^A(0,p_0) = p_0(v_0-p_0).$$

This case can be included in the general formula of $\pi^A(k,p_0)$ by setting $p^*(0,p_0) = 0$. As usual, equilibrium profits for retail stores without an anchor store are:

$$\pi^R(k,\emptyset) = \pi^R(k,v_0).$$

3.5 The Profit-Maximizing Land Developer (Stage 1)

Finally, we analyze the land developer's decision of tenant composition of her shopping mall. The setting is similar to Schweizer's [20] club economy model. There are a large number of potential retail and anchor store tenants, all of whom have outside profit opportunities. Retail and anchor stores are willing to be tenants of a shopping mall only when their net profits are not less than these reservation profits ρ^A and ρ^R, respectively. A land developer chooses the characteristics of her shopping mall, the number of retail and anchor stores, $k \in \{0, 1, ...\bar{k}\}$ and $m \in \{0, 1\}$, respectively, where $\bar{k}$ is the maximum feasible number of retail stores in a shopping mall (owing to space limitations or increasing construction costs in expanding the size of a mall).[19] The land developer's total rent revenue given the composition of tenants (k, m) is $\pi^D(k, m) \equiv kr^R(k, m) + mr^A(k, m)$, where r^R and r^A are, respectively, the rents charged to retail and anchor tenants. The land developer needs to guarantee that her tenants can obtain at least their reservation earnings. Efficient rent extraction then implies that a retail store and an anchor store pay rents $r^R(k, m) = \pi^R(p_0, p^{**}(k, m)) - \rho^R$, and $r^A(k, m) = \pi^A(p_0, p^{**}(k, m)) - \rho^A$, respectively. Consequently,

$$\pi^D(k, m) = k\left(\pi^R(p_0, p^{**}(k, m)) - \rho^R\right) + m\left(\pi^A(p_0, p^{**}(k, m)) - \rho^A\right),$$

where $p^{**}(k, 1) \equiv p^*(k, p_0)$ and $p^{**}(k, 0) \equiv p^*(k, \emptyset) = p^*(k, v_0)$. For simplicity, we assume that cost of building a shopping mall at a given site is zero as long as k and m are not more than $\bar{k}$ and 1, respectively.[20] The land developer chooses the composition of tenants in the mall from the collection of feasible combinations $\mathcal{F} = \{(k, m) \in \{0, 1, ..., \bar{k}\} \times \{0, 1\}\}$ so as to maximize her rent revenue:

$$(k^*, m^*) \in \arg\max_{(k,m)\in\mathcal{F}} \pi^D(k, m).$$

This composition of tenants (k^*, m^*) is the subgame perfect equilibrium path of our game.

4 Numerical Examples

Both because the land developer's underlying optimization problem is discrete and because the retailers' profit functions are not well-behaved (see Konishi

[19]Given our simplistic model (deterministic reservation price for an anchor good), a land developer has no incentive to have multiple anchors. With multiple anchors, Bertrand competition brings p_0 down to zero.

[20]This is obviously an extreme assumption. However, as long as construction and land costs for each additional store are the same (constant marginal cost of construction), then these costs can be included in ρ^A and ρ^R without affecting our analysis.

[14]), we provide numerical examples to characterize these two features, as well as other important features, of the model. Our examples show the sensitivity of the model to changes in the underlying parameters v_0, p_0 and c. In particular, we show how retail prices, market size (mall traffic), firm profits, developer profits and the desirability of collocation vary with these parameters. Altogether, we will consider four examples, with key parameter values given as follows

	v_0	p_0	c
Case I	0.06	0.05	1/3
Case II	0.06	0.03	1/3
Case III	0.10	0.05	1/3
Case IV	0.06	0.05	1/2

The maximum number of retailers which the developer could site at the mall location is assumed to be $\bar{k} = 10$. For ease of exposition, we divide the set of feasible mall configurations into three qualitatively different sets. The first collection, which we will denote "anchor malls" consists of an anchor store collocated with $k \geq 1$ retail stores. The second, denoted a "retail mall" consists solely of $k \geq 1$ collocated retailers with no anchor. The third (singleton) set consists of only an anchor store, and is denoted "anchor only." As above, the prices p^* denote equilibrium *retail store* prices, since the anchor store price is always pegged at the parameter p_0. Finally, note that an independent anchor store always has market size $v_0 - p_0$.

4.1 Case I ($v_0 = 0.06$, $p_0 = 0.05$, $c = 1/3$)

Anchor Malls

	Anchor Only ($k = 0$)	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
p^*	—	0.66167	0.62911	0.59343	0.55742	0.52460	0.49716
μ	0.01000	0.06390	0.12458	0.18972	0.25435	0.31262	0.36121
π^A	0.00050	0.02840	0.00254	0.00208	0.00132	0.00068	0.00031
π^R	—	0.00689	0.01090	0.01283	0.01277	0.01144	0.00970
π^D	0.00050	0.03529	0.02434	0.04057	0.05240	0.05788	0.05851

Retail Malls

	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$
p^*	0.66667	0.63299	0.59607	0.55893	0.52533	0.49747
μ	0.05556	0.11822	0.18549	0.25198	0.31151	0.36075
π^R	0.00617	0.01062	0.01281	0.01282	0.01148	0.00972
π^D	0.00617	0.02124	0.03842	0.05130	0.05742	0.05832

First note that, for both sorts of malls (anchor and retail), the equilibrium retail price declines as the number of retailers k increases. Moreover, for fixed k, an anchor mall always has a lower equilibrium retail price than does a retail mall. We refer to the fact that anchor mall retail prices are uniformly lower than retail mall retail prices as the price effect of anchor stores on malls, since the anchor store acts as a price competitor to the retailers, lowering their margins. Note that if every consumer found it costless to visit the mall location, then it would never advantage retailers to form malls, with or without anchor stores.

In our model, however, consumers must sink a cost to visit the mall location, so retailers and anchor stores may find it worthwhile to commit to low prices by locating near one another, i.e., forming malls.[21] The benefit of retail and anchor agglomeration into malls can be seen by looking at market size (or mall traffic), μ, in the second row of each table. For both sorts of malls, notice that when the number of retailers increases from $k = 1$ to $k = 2$, the mall traffic roughly doubles (from about 0.063 to 0.123 for anchor malls and 0.056 to 0.118 for retail malls). Note that the market size for anchor malls of a given k is uniformly higher than for retail malls. In part, this is a manifestation of the price effect of anchor competition in committing retailers to low prices. At the same time, the presence of an anchor also makes it more likely that a consumer arriving at the mall location will find a commodity which suits her taste. In fact, for malls with anchors, every consumer is guaranteed net surplus of no less than $v_0 - p_0$ (in this case, 0.01) from a visit to the mall.[22]

[21] This effect on incentives for stores to agglomerate at the same location is first discussed in Dudey [9]. See Fischer and Harrington [10] and Konishi [14] as well. Konishi calls this effect "the price cutting effect" on market size.

[22] Here, "net surplus" refers to surplus net of prices but gross of transportation costs t.

Consequently, the anchor store exerts a direct market size effect due to the fact that it is a known commodity available at a known price.

One clear test of the assertion that anchor stores and retailers exert positive externalities on one another is to suppose there were no land developer and ask whether the two different types of firms would choose to collocate. That question may be answered by examination of the anchor and retail profits for anchor malls, retail malls and stand-alone anchor stores. If each type of firm has higher profits under the anchor mall configuration than it would separately, we infer that positive externalities must be operating in both directions. In a configuration where the anchor store stands alone, it earns profit of 0.0005. The anchor's payoffs π^A indicate that it would choose to collocate with retailers, so long as the retailers numbered no more than five. When $k = 6$, the anchor's payoff of 0.00031 is lower than it would receive alone, since the number of retailers has now increased sufficiently that consumers are very likely to find a retail commodity that suits their taste and which will be available at a low price. For the retailers, the payoffs π^R indicate that collocating with the anchor improves their lot so long as $k \leq 3$. For $k = 4$, the (representative) retailer's payoff of 0.01277 when collocated with the anchor is dominated by its payoff of 0.01282 if the anchor is not present.[23]

Assuming $\rho^R = \rho^A = 0$, the total rent-maximizing tenant combination for the land developer is $(k^*, m^*) = (6, 1)$, achieved by charging rents $r^R = \pi^R(6, p_0) = 0.00970$ and $r^A = \pi^A(6, p_0) = 0.00031$. This rent scheme efficiently extracts retail and anchor surplus, leaving them indifferent between signing the lease and not. The land developer's total rent revenue is $6 \times 0.00970 + 0.00031 = 0.05851$.[24]

Figure 1(a) shows how the optimal mall configuration varies with the reservation values of the anchor and retail stores. A higher reservation value ρ^A for the anchor store leads to malls without anchors. Higher reservation values ρ^R for retailers lead to malls with smaller numbers of retailers. It is because the retained profits by the land developer go down by k dollars as ρ^R increases by

[23]Note that this is a separate question from the optimal size of a retail mall with no anchor. In this example, the fact that $k^* = 4$ is the optimal retail mall size may be verified from the table. For more details on optimal retail configurations, see Konishi [14].

[24]Note that the upper bound $\bar{k} = 10$ is not binding in this example. We can show that $(6, 1)$ is the global optimizer among combinations $(k, m) \in Z_+ \times \{0, 1\}$. After $k = 6$, π^D goes down as k increases. For example, $\pi^D = 0.05848$ for $k = 7$.

one dollar, if the number of tenant retail stores is k.[25] Finally, note that, in Figure 1(a)-(d) regions over which pairs (k^*, m^*) are optimal are box-shaped or bounded by lines. Examination of the developer's profit function reveals why. First, the boundary between all optimizers of the form $(0, k)$ and $(1, k)$ for fixed k will be horizontal, since a comparison of $\pi^D(k, 0)$ and $\pi^D(k, 1)$ cancels all terms involving ρ^R, leaving a trivial function. Similar calculations show that all terms involving ρ^A must cancel from a comparison of $\pi^D(k, m)$ and $\pi^D(k', m)$, and that more general comparisons of $\pi^D(k, m)$ with $\pi^D(k', m')$ must result in linear functions only.

4.2 Case II ($v_0 = 0.06$, $p_0 = 0.03$, $c = 1/3$)

Anchor Malls

	Anchor Only ($k = 0$)	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
p^*	—	0.65167	0.62126	0.58799	0.55421	0.52299	0.49645
μ	0.03000	0.08067	0.13748	0.19847	0.25940	0.31511	0.36229
π^A	0.00090	0.03583	0.00175	0.00141	0.00091	0.00049	0.00023
π^R	—	0.00817	0.01140	0.01287	0.01266	0.01133	0.00964
π^D	0.00090	0.04401	0.02455	0.04002	0.05153	0.05716	0.05807

Retail Malls

	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
p^*	0.66667	0.63299	0.59607	0.55893	0.52533	0.49747
μ	0.05556	0.11822	0.18549	0.25198	0.31151	0.36075
π^R	0.00617	0.01062	0.01281	0.01282	0.01148	0.00972
π^D	0.00617	0.02123	0.03814	0.05130	0.05742	0.05832

[25] Note that there is a discrete jump from the equilibrium $(4, 1)$ to $(1, 1)$ as ρ^R becomes sufficiently large. This reflects the fact, noted earlier, that the profit functions are highly nonlinear in k.

In this case, we decrease the anchor store's price to $p_0 = 0.03 = \frac{v_0}{2}$. Because the retailers' marginal cost of production, c, is unchanged, retail malls (which exclude the anchor store) will have identical prices, traffic and payoffs to those in Case I. The decline in the anchor store's price also does not change the relationship, for fixed k, between retail prices in retail and anchor malls. Note, however, that anchor malls in Case II demonstrate somewhat lower prices than did anchor malls in Case I, owing to increased price pressure from the anchor store. Note also that the incremental effect on anchor mall traffic of increased numbers of retailers k is somewhat more subdued in Case II than it is in Case I because the anchor's low prices alone generate significant traffic.

As in Case I, both anchor stores and retailers exert positive externalities on one another up to a mall of size $k = 3$. Finally, observe that the reduction in the anchor store's price has a dramatic effect on the developer's optimal choice of mall configuration. Figure 1(b) shows that, for sufficiently high retailer reservation values, ρ^R, the optimal mall configuration is one in which the anchor is paired with a single retailer. For all smaller reservation values ρ^R, the optimal mall configuration includes either five or six retailers, but does not include an anchor store. That is, so long as retailers are expensive to include in the mall, the mall developer will include only one.[26] If retailers become sufficiently inexpensive, the mall developer may find it worthwhile to eject the anchor store in favor of a large number of retailers. The developer must include many retailers in the mall to emulate the low prices and substantial mall traffic generated by the low-price anchor.

[26] Of course, as ρ^R becomes very large, an anchor-only mall becomes the most desirable alternative.

4.3 Case III ($v_0 = 0.10$, $p_0 = 0.05$, $c = 1/3$)

Anchor Malls

	Anchor Only ($k = 0$)	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
p^*	—	0.64167	0.61333	0.58237	0.55077	0.52117	0.49560
μ	0.05000	0.09754	0.15063	0.20761	0.26488	0.31795	0.36360
π^A	0.00250	0.04329	0.00331	0.00263	0.00173	0.00097	0.00048
π^R	—	0.00927	0.01181	0.01288	0.01252	0.01122	0.00957
π^D	0.00250	0.05256	0.02693	0.04125	0.05182	0.05706	0.05792

Retail Malls

	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
p^*	0.66667	0.63299	0.59607	0.55893	0.52533	0.49747
μ	0.05556	0.11822	0.18549	0.25198	0.31151	0.36075
π^R	0.00617	0.01062	0.01281	0.01282	0.01148	0.00972
π^D	0.00617	0.02123	0.03814	0.05130	0.05742	0.05832

In this case, we set $v_0 = 0.1$, with other parameter values as in the original case. This modification increases, by a factor of five, consumers' net surplus from purchasing the anchor store commodity. Because retail costs do not increase and consumer surplus from the anchor store's commodity increases, Case III is similar in most respects to Case II. In particular, examination of the equations determining the equilibrium retail price and market size shows that they depend solely on the difference $v_0 - p_0$. Nonetheless, the cases are not identical, since a careful examination of the anchor store's profit function shows that it depends on both the absolute level of the price p_0 as well as on the net surplus from the anchor store's commodity $v_0 - p_0$. As a result, given two pairs (v_0, p_0) and $(\hat{v}_0, \hat{p}_0)$ for which $v_0 - p_0 = \hat{v}_0 - \hat{p}_0$, an anchor store with p_0 will make different profits from the one with $\hat{p}_0$. Consequently, a mall developer has a different incentive to include an anchor with p_0 than she does an anchor with $\hat{p}_0$.

4.4 Case IV ($v_0 = 0.06$, $p_0 = 0.05$, $c = 1/2$)

Anchor Malls

	Anchor Only ($k = 0$)	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
p^*	—	0.74500	0.72781	0.70929	0.68998	0.67076	0.65257
μ	0.01000	0.04001	0.07273	0.10763	0.14363	0.17916	0.21258
π^A	0.00050	0.02251	0.00198	0.00200	0.00172	0.00131	0.00090
π^R	—	0.00240	0.00377	0.00471	0.00518	0.00522	0.00495
π^D	0.00050	0.02491	0.00953	0.01615	0.02238	0.02753	0.03059

Retail Malls

	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
p^*	0.75000	0.73205	0.71268	0.69250	0.67249	0.65367
μ	0.03125	0.06538	0.10182	0.13935	0.17626	0.21076
π^R	0.00195	0.00352	0.00461	0.00516	0.00524	0.00498
π^D	0.00195	0.00704	0.01382	0.02066	0.02622	0.02986

Finally, we set $c = \frac{1}{2}$ instead of $c = \frac{1}{3}$, keeping other parameters at their original values. As before, the pattern of declining prices and increasing mall traffic as k increases is evident, although (as one might expect) the retailers' higher marginal costs lead to higher prices in both anchor mall and retail mall configurations. For sufficiently small k, there is again a mutual desire among retailers and the anchor to collocate. For $k \leq 4$, retailers earn higher profits under an anchor mall configuration than they do under a retail mall configuration. For the anchor, collocation is desirable at every k shown in the table, since even at $k = 6$ its payoff of 0.0009 in an anchor mall configuration dominates its payoff of 0.0005 standing along. Further computations show that the anchor prefers collocation up to and including $k = 7$.

Figure 1(d) again shows how the developer's optimal mall configuration varies with the reservation values of the anchor and retailers. In comparison with Figure 1(a), note that higher retail costs induce the developer to include more retailers in the optimal configuration, yet the range of parameters for which an anchor store is included in the mall is significantly smaller . As an explanation for this phenomenon, note that large retail costs c imply that the retail price p^* will be higher. Higher retail prices both reduce the probability that a consumer will find a buyable commodity at the mall and reduce the net surplus of those consumers who do ultimately buy. Consequently, many retail stores are needed to attract customers. However, once too many retail stores collocate, having an anchor store simply reduces retail profit margins. For the anchor store, the situation is similar. If already a certain number of retail stores are collocated, the anchor store can only get a very small share of consumers given the size of demand. Thus, collocation is also not good for the anchor store if k is large.

We have explored the possibility of even higher retail costs c, although we have omitted the numerical results and figures in the interest of space. As c increases, *ceteris paribus*, mall configurations which include an anchor store and two or more retailers cease to be optimal. For $v_0 = 0.06$ and $p_0 = 0.05$, for example, this event occurs at around $c = 0.62$. For sufficiently high values of c ($c = \frac{2}{3}$, for example, is sufficient for $v_0 = 0.06$ and $p_0 = 0.05$), the mall configuration $(1, 1)$ (the anchor store and a single retailer) becomes the only optimizer over the set of reservation values shown in Figures 1(a)-1(d).

4.5 Summary of Numerical Results

A few more general observations can be drawn from the above numerical examples. First, collocation of retail and anchor stores can be mutually beneficial, so long as the number of retailers is not too large. Moreover, mall configurations including an anchor store with several retailers can maximize the land developer's profits (rent payments). Second, the more attractive to consumers is the anchor store's commodity, the lower is the benefit to retailers from collocation. Third, Pashigian and Gould's [17] observation regarding subsidies to anchor stores can be justified when the anchor store's commodity is not too attractive to consumers.

5 Concluding Remarks

In this paper, we showed that different types of stores (anchor stores and retail stores) may have incentives to collocate even if they sell substitutes instead of complements when search is costly for consumers. We assumed that anchor stores sell standard commodities (riskless, yet low value) and retail stores sell specialized commodities (high variance, yet high expected value). The underlying intuition for collocation incentives is that the presence of each type of retailer enhances consumer traffic at the shopping mall, which benefits the retailer or retailers of the other type. Under some parametric restrictions, the value of this increased traffic more than offsets the loss in markups due to competition from additional sellers at the mall. Depending on the structure of payoffs to the anchor and retailers and on the reservation values of each type of agent, it may be in a land developer's interest to rent retail space in the mall to both types of retailers. We applied a Tiebout-like argument in partial equilibrium analysis to explain the similarity in the composition of stores in planned shopping malls.

Here we will mention a few assumptions implicit in the paper and the likely effect of relaxing them. First, in applying the Tiebout theory, we implicitly assumed that shopping malls are far from each other and that they do not compete for customers. Although this is not a realistic assumption, it greatly simplifies the analysis. If two shopping malls are close to each other, a lot of things can happen in equilibrium. They may have similar compositions of stores and peacefully share customers living between the two malls. Or, one shopping mall may get more than the optimal number of stores in order to make itself very attractive so that it can take over the market. That is, if a shopping center becomes very attractive, then the other shopping center nearby can compete with it only by getting excessively many stores as well. However, in such a situation, profits may become negative. As a result, one shopping mall can effectively prey on the other shopping mall by having many tenants.[27] Such an investment may pay off if population density in the area is high. This informal argument can explain various shopping mall size configurations in cities.

Second, we assumed that consumers know their reservation value for the anchor store's commodity perfectly (constant v_0). This assumption is again made for simplicity. Obviously, it would be more realistic to assume that a consumer has less uncertainty in her reservation value for an anchor store's commodity than for that of the retail store. Moreover, relaxing this assumption would allow multiple anchor stores in the same shopping mall without confronting

[27] Related argument is provided in Konishi [14], although he does not discuss the role played by land developers.

Bertrand's paradox.[28] Unfortunately, however, this modification of the model greatly complicates the analysis. We assumed a constant reservation value for the anchor store's commodity in order to provide a clear explanation for collocation of anchor stores and retail stores in a shopping mall.

Finally, we assumed that anchor stores' prices are exogenously set at p_0. We adopted this assumption for simplicity, but it seems reasonable to say that retail prices in anchor stores are similar since they are chain stores. For a theoretically more complete analysis, we may assume that anchor stores set prices strategically as well given the number of retail stores in the shopping mall. However, we decided not to do so, since again it complicates the analysis quite a bit.[29] In our future research, we may analyze a model that has the above features. However, the analysis will be heavily based on numerical computations.

[28] In the current setting, if there were multiple anchor stores, they would sell homogeneous commodities to consumers. Given this, Bertrand price competition brings the equilibrium price down to marginal cost.

[29] Strategic pricing by anchor stores also contradicts our characterization of anchor stores: consumers pretty much know what they get by visiting an anchor store.

Appendix

Proof of Proposition 1.

We can simplify $\hat{\mathbf{P}}(p_i, p; k, p_0))$ as follows:[30]

$$\begin{aligned}\hat{\mathbf{P}}(p_i, p; k, p_0) &= \int_{p_i+v_0-p_0}^{1} [\Pr\{v \in [0,1] : v \leq v_i - p_i + p\}]^{k-1}\, dv_i \\ &= \left\{ \begin{array}{cc} \int_{p_i+v_0-p_0}^{1} (v_i - p_i + p)^{k-1} dv_i & \text{if } p_i \geq p. \\ \\ \int_{p_i+v_0-p_0}^{1-p+p_i} (v_i - p_i + p)^{k-1} dv_i + \int_{1-p+p_i}^{1} dv_i & \text{if } p_i < p. \end{array} \right\}.\end{aligned}$$

This equation denotes the proportion of consumers who purchase commodity i. From Figure 2 (the case $k = 2$), it is evident that there will be asymmetry of the profit function across $p_i = p$. Combining the above formulas, we obtain firm i's profit function per unit market size:

$$\begin{aligned} & \Pi(p_i, p; k, p_0) \\ = \; & (p_i - c)\hat{\mathbf{P}}(p_i, p; k, p_0) \\ = \; & \left\{ \begin{array}{cc} (p_i - c) \int_{p_i+v_0-p_0}^{1} (v_i - p_i + p)^{k-1} dv_i & \text{if } p_i \geq p. \\ \\ (p_i - c) \int_{p_i+v_0-p_0}^{1-p+p_i} (v_i - p_i + p)^{k-1} dv_i + \int_{1-p+p_i}^{1} dv_i & \text{if } p_i < p. \end{array} \right\}.\end{aligned}$$

Note that this profit function is symmetric and is log concave in p_i. By this, we can show the existence of a symmetric Nash equilibrium price (see Dierker [8] and Konishi [14]). By taking derivatives with respect to p_i, we obtain the following:

$$\lim_{\epsilon \downarrow 0} \frac{\Pi(p+\epsilon, p; k, p_0) - \Pi(p, p; k, p_0)}{\epsilon} = \frac{1}{k}\left[1 - (v_0 - p_0 + p)^k\right] - (p - c)$$

and

$$\lim_{\epsilon \downarrow 0} \frac{\Pi(p-\epsilon, p; k, p_0) - \Pi(p, p; k, p_0)}{\epsilon} = \frac{1}{k}\left[1 - (v_0 - p_0 + p)^k\right] - (p - c).$$

[30] Since the marginal densities of the v_i are unity on $[0, 1]$ and the v_i are stochastically independent, the joint density is also unity on $[0, 1]^k$.

Thus, for p to be a symmetric Nash equilibrium, we need the following:

$$\phi(p; k, p_0) \equiv \frac{1}{k}\left[1 - (v_0 - p_0 + p)^k\right] - (p - c) = 0.$$

We now can show that there is a unique (symmetric) interior equilibrium by assuming $v_0 - p_0 < 1 - c$. This assumption is needed for retail stores to have any profits. Evidently, $\frac{\partial}{\partial p}\phi(p; k, p_0) = -(v_0 - p_0 + p)^{k-1} - 1 < 0$. Furthermore, we have

$$\begin{aligned} \phi(c; k, p_0) &= \frac{1}{k}\left[1 - (v_0 - p_0 + c)^k\right] > 0, \\ \phi(1 - v_0 + p_0; k, p_0) &= -(1 - v_0 + p_0 - c) < 0. \end{aligned}$$

It follows that $p^*(k, p_0) \in (c, 1 - v_0 + p_0)$ for any $p_0 \in (1 - c - v_0, 1)$ and any positive integer k. That is, the unique solution $p^*(k, p_0)$ is given implicitly by the following equation:

$$\frac{1}{k}\left[1 - (v_0 - p_0 + p^*(k, p_0))^k\right] - (p^*(k, p_0) - c) = 0.$$

Since p^* is the unique price that satisfies the first order condition, if there exists a symmetric Nash equilibrium price, it has to be p^*. Indeed, we know that there is a symmetric Nash equilibrium by log-concavity of profit functions. Hence p^* is the unique Nash equilibrium price.

By using these first order conditions, we can calculate each retail store's equilibrium profit **per unit demand**:

$$\begin{aligned} \Pi(p^*(k, p_0), p^*(k, p_0); k, p_0) &= (p^*(k, p_0) - c)\int_{p^*(k,p_0)+v_0-p_0}^{1} (v_i)^{k-1} dv_i \\ &= (p^*(k, p_0) - c)\left(\frac{1 - (v_0 - p_0 + p^*(k, p_0))^k}{k}\right) \\ &= (p^*(k, p_0) - c)^2. \end{aligned}$$

We have completed the proof. ■

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Figure 1: Optimal mall configurations (k^*, m^*)

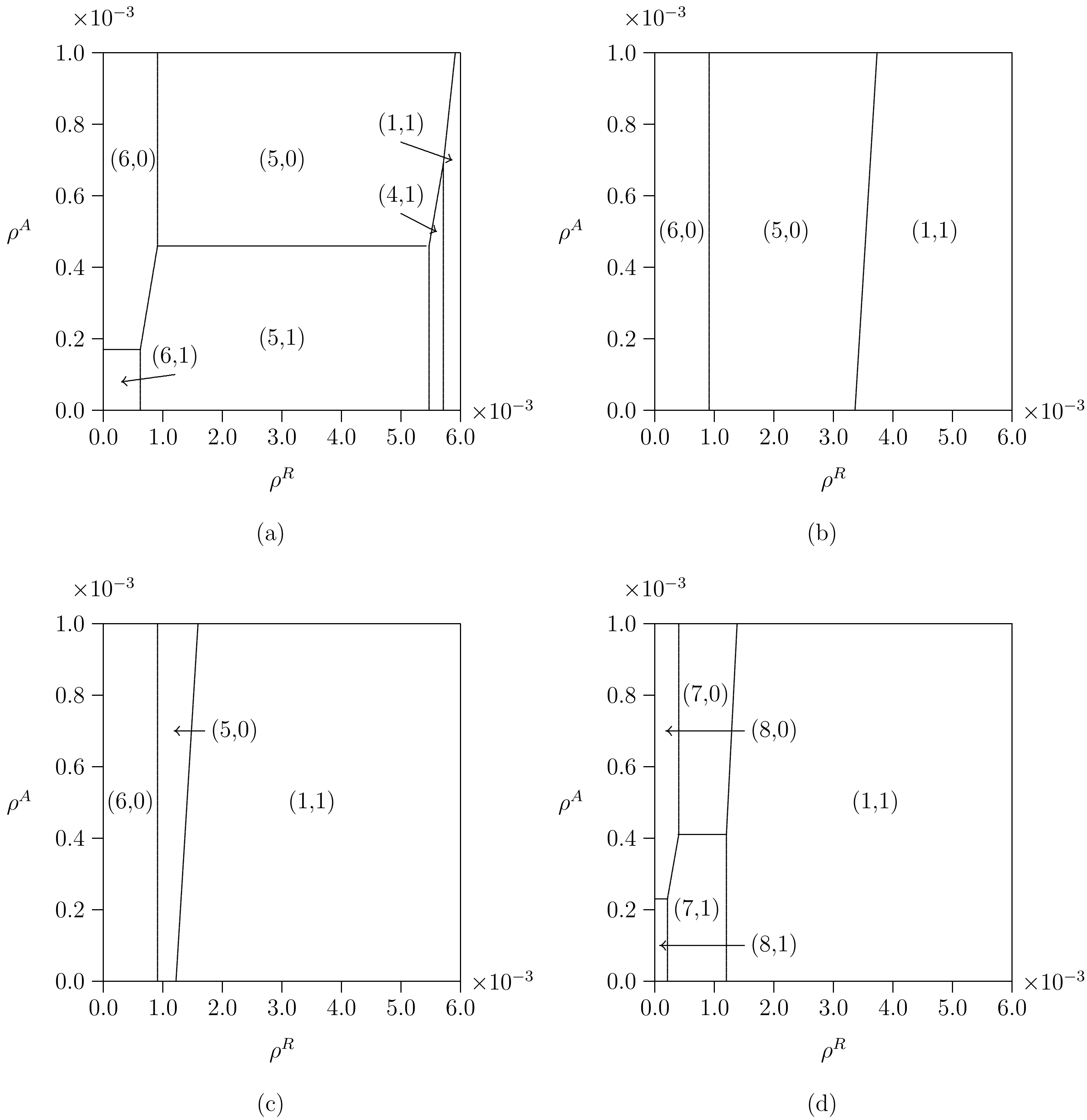
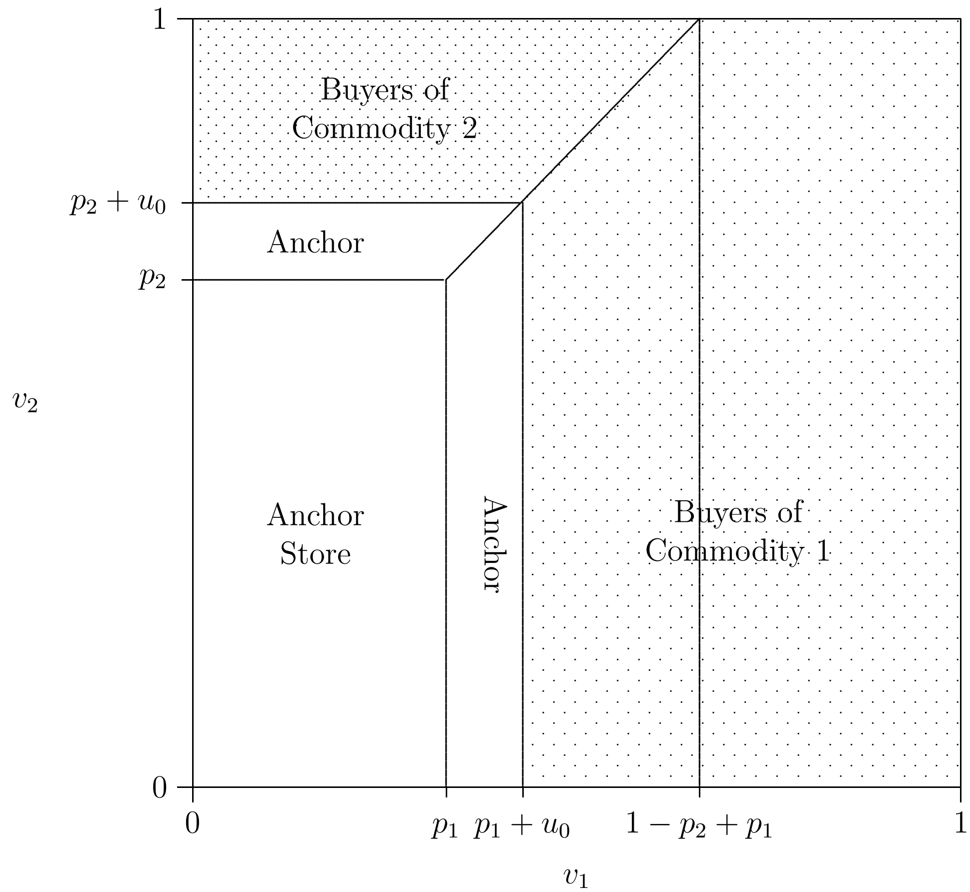


Figure 2: Consumers' choice



Note: $u_0 \equiv v_0 - p_0$