Ex-Post Egalitarianism[*]

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1 Introduction

In any legal system, one finds numerous rules and practices as well as constitutional provisions which are incompatible with utilitarian considerations. These rules and practices often grant benefits to an individual whose well-being is at risk; yet the costs of these benefits to other individuals outweigh the benefits. Thus, for instance, despite the persistent belief of economists that efficiency requires to impose the harshest possible sanctions, legal systems often impose light sanctions and consequently have to bear the high costs of increasing the probability of detection.

It is not merely utilitarianism that fails to explain a diverse range of rules and practices. Other theories that, like utilitarianism, involve ex-ante considerations cannot explain them as well. An ex-ante analysis ranks distributions of ex-ante utilities, before people know the actual outcome they receive.[1] Ex-post analysis, on the other hand, relates to the final distribution of utilities. This analysis too is done before uncertainly related to the distribution of final outcomes is revealed, but relates to the evaluation of distributions of final outcomes.

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[1]Note that when we use the term 'ex-ante' we do not discuss the extreme situation of ex-ante decision process — the situation in which the decision is made behind the 'veil of ignorance,' before the individual is even aware of his personal characteristics.

There are two possible primary explanations for the prevalence of these non-utilitarian rules and practices: Kantian (deontological) explanations and a view we shall label ex-post egalitarianism. Kantian explanations are based on the conviction that there are non-consequentialist obligations. The State is sometimes obliged to act or not to act in certain ways even if acting differently is conducive to utility, or to the ex-ante interests of individuals. Typically those explanations rely on autonomy-based considerations and are based on the belief that respecting the dignity of individuals constrains the promotion of their well-being. For a contemporary sophisticated deontological explanations of various rules and practices, see Kamm [14, vol. II].

Ex-post egalitarianism, on the other hand, requires that the State decides on its action in an egalitarian manner ex-post. It differs from the Kantian approach since it is founded on a comparison between the position of the people who are affected by a certain rule and practice rather than presupposes the existence of moral constraints, which are independent of consequentialist considerations. It differs from utilitarianism since it is concerned with equality, and not only with the maximization of the sum of utilities. Finally, it differs from ex-ante analysis as it considers the well-being of individuals *after* they have the knowledge of the prevailing social outcomes.

The purpose of this paper is to demonstrate that some of the rules and practices, which are traditionally justified in terms of Kantian explanations, can alternatively be explained in terms of ex-post egalitarianism. Two important qualifications should be stated. First, we do not reject Kantian considerations. It is not claimed that the rules and practices examined here cannot be explained on the basis of Kantian considerations. Instead we simply claim that they can also be explained in terms of ex-post egalitarianism. Second, it is not claimed here that the ex-post egalitarianism is an absolute value, which overrides any conflicting values. The rules and practices examined here demonstrate that ex-post egalitarianism competes with other considerations such as ex-ante decision and efficiency.

To demonstrate our approach, consider the issue of sentencing practices. As was argued by Becker [1], sentencing practices pose a great challenge for the utilitarian approach: If criminals react to the expected sanction, then greater deterrence can be achieved by either increasing the probability of detection or by increasing the size of the sanction. However, increasing the probability of detection is much more costly than increasing the size of the sanction. Hence, the most efficient way to deter criminals is to impose the harshest sanction possible and to reduce accordingly the probability of

detection. In reality, however, sanctions do not conform to Becker's recommendations and they are usually proportional to the degree of culpability and wrongfulness of the crimes committed.

We suggest below the following explanation. Sanctions generate inequality because those who are subjected to the sanctions are worse off relative to those who are not. The harsher the sanction is, the larger is the inequality between two classes of individuals, namely those offenders who are subjected to the sanction and those offenders who are not. Harsh sanctions are indeed, as Becker observed, required by consideration of aggregate utilities. Yet, social interests in harsh sanctions must be balanced against social concerns for equality. It may therefore well be the case that what dictates limits on the size of sanctions is ex-post egalitarianism and not retributive justice considerations.

Our approach allows for comparisons among different societies by giving meaning to statements like "Society A is more egalitarian than society B." Furthermore, we show that the more egalitarian societies should also employ less extreme criminal law rules and should be more sensitive to various kinds of injustice, whether it is caused by individual wrongful behavior or by the society's criminal law rules. As we elaborate in Section 4, such societies would use less harsh sentencing practices; furthermore, they would try to avoid wrongful convictions by raising the required burden of proof and, at the same time, they would try not to sacrifice minorities for the sake of the whole society.

In Section 2 we present the social policies and discuss the ex-post and the ex-ante approaches. We define the notion of being an ex-post egalitarian society and provide a way for comparing levels egalitarianism among societies. In Section 3 we present our results and offer some applications in Section 4. Formal analysis is deferred to the Appendix.

2 Social Policies and their Evaluation

Consider a situation where individuals are facing some uncertainty regarding outcomes that are controlled by society, for example road accidents or some health related issues. There are many individuals and we assume that their uncertainties are independent of each other. If society is sufficiently large, we can assume, for practical measures, that the *proportion* of those who receive a certain outcome equals the probability of receiving this outcome.

For example, if each member of society is facing an independent chance of 2% to fall victim to a car theft, then in a large society the probability that the true proportion of victims will differ from 2% by more than ε is negligible. Social policies are often aimed at controlling these proportions, but they rarely try (or are even able) to determine the recipients of each outcome. For example, lower speed limits or wider road shoulders reduce the probability of fatal car accidents, but they do not tell who will be involved in accidents.

There are two extreme ways in which society can view such issues: Ex-ante and ex-post. Suppose that society is facing a choice between two economic policies. One implies an annual increase of 5% in everybody's utility, the other faces each person with an independent risk, where there is a 50% chance of a 10% increase and a 50% chance of no change in his utility. From an ex-ante perspective, both policies are equally attractive, as all expected utility maximizers are indifferent between the two. But the two policies may differ from an ex-post perspective. The first implies a 5% increase in the utility of all. As individual risks are independent, the second policy implies (for a sufficiently large society) a 10% increase in the utility of half of the population and no change in the utility of the rest. If initially everybody's utility was 100, now half of the population will have 110 and half will get only 100. Ex-post, the two policies are not the same and a non-utilitarian society may not be indifferent between them.

Which is the correct analysis, the ex-ante or the ex-post? Consider another situation, where one unit of an indivisible good needs to be given to one of two individuals. Both receive utility 1 from receiving it and zero otherwise. All allocation procedures lead to an uneven allocation of ex-post utility, one person receives 1, the other zero. Diamond [7] suggested randomization over the two individuals as a tool for improving social well being, thus creating ex-ante egalitarianism. There is an extensive discussion in the literature whether social welfare is really improved by such randomizations. Broome [4, 5] argues in favor of ex-post analysis, claiming that equality of *expected* utilities is not a real equality. Harsanyi [11] too, defending his model against Diamond's criticism, argues against attributing ex-ante egalitarianism any significant value. Epstein and Segal [9] on the other hand focus attention on ex-ante analysis, while Karni and Safra [15] and, more explicitly, Ben Porath, Gilboa, and Schmeidler [2] try to combine both ex-post and ex-ante consideration into one evaluation function. This paper offers an ex-post analysis, while assuming that society is not necessarily utilitarian

(that is, evaluations of social policies do not depend solely on the sum of individual utilities). We restrict attention to situations where each social policy leads to a known ex-post distribution of utilities. As argued above, even though we cannot predict individual outcomes, we nevertheless know, for each policy, what proportion of the population will receive each possible utility level. We evaluate such distributions using a social welfare function W which, not being utilitarian, is not necessarily linear.

A utilitarian society is interested in maximizing the sum of individual utilities and does not pay attention to how diverse is the distribution of utilities. Preferences for equality imply that society will be willing to reduce the average utility in order to make the distribution more concentrated around its mean. Such concerns can be represented by quasi concave functions, where the social evaluation of the average of two equally attractive distributions is better than both.[2] Such functions can be monotonic in individual utilities, but are also sensitive to utility differences between individuals, and admit some trade-off between sum of utilities and their disparity. We now present an outline of our model; a more formal analysis appears in the Appendix.

Given two distributions of utility x and y, the mixture $\alpha x + (1 - \alpha)y$ gives each individual α times his allocation under x plus $1 - \alpha$ times his allocation under y. For example, if $x = (3, 1)$ and $y = (1, 5)$, $\frac{1}{2}x + \frac{1}{2}y = (2, 3)$. Our first assumption is that mixtures of equally attractive of ex-post utility distributions improve social welfare.

Quasi Concavity Let x and y be two distributions of utility. If $W(x) = W(y)$, then for all $\alpha \in (0, 1)$, $W(\alpha x + (1 - \alpha)y) \geqslant W(x)$.

Strict quasi concavity requires that for all $x \neq y$, $W(x) = W(y)$ implies $W(\alpha x + (1 - \alpha)y) > W(x)$.[3]

As mentioned above, we are interested here in utility distributions and do not care for the identity of the individual recipients of these utilities. distribution x of utilities can therefore be represented as a cumulative distribution,

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[2]Recall that from an ex-ante perspective such functions imply preferences for randomizations.

[3]Note the relation of this definition to Diamond's [7] argument in favor of ex-ante evaluation — choosing the utility distribution x with probability p and y with probability $1 - p$ yields each individual *expected* utility that is equal to p times his utility under x plus $1 - p$ times his utility under y. Preferences for randomization imply quasi concave preferences over social lotteries. Assuming that all individuals are expected utility maximizers, such preferences imply that the social welfare function is quasi concave in ex-ante utilities (see [9]).

where $F_x(u)$ is the proportion of individuals whose utility given x does not exceed u. Two distributions that lead to the same cumulative distribution should therefore be equally attractive. Formally,

Symmetry If the two utility distributions x and y satisfy $F_x = F_y$, then $W(x) = W(y)$.

The major concept of the paper is that of egalitarian social welfare functions:

Definition 1 *We say that a social welfare function is ex-post egalitarian if it is strictly quasi concave and symmetric.*

For a meaningful comparative statics analysis, we need to be able to compare different societies. For this, we will need the following concept:

Definition 2 *A social welfare function W' is more egalitarian than another social welfare function W if, whenever the society with the social welfare function W is willing to sacrifice the difference between the averages of the two distributions in order to improve equality, so does the society with the social welfare function W'.*

(See the Appendix for a more precise definition). Fig. 1 depicts this concept for the case of two individuals. In this picture, W' is more egalitarian than W and distribution between x and the main diagonal display more equality. In this region, W' is lower than W, hence showing more egalitarianism.

In the literature, considerations of equality that are based on cumulative distributions lead to the concept of aversion to mean preserving spreads. However, by Dekel [6], if W is symmetric and quasi concave, then W must also represent such aversion, hence this concept of equality is related to the concept of quasi concavity discussed above.

3   More Egalitarian Societies

Suppose that society has control over a decision variable $a \in [0, 1]$ that leads to a utility distribution $\gamma(a)$. We assume that $a > b$ implies that there is s such that the distribution of $\gamma(b) + s$ is a mean preserving spread of the distribution of $\gamma(a)$. In other words, as a moves from 0 to 1, $\gamma(a)$ is becoming
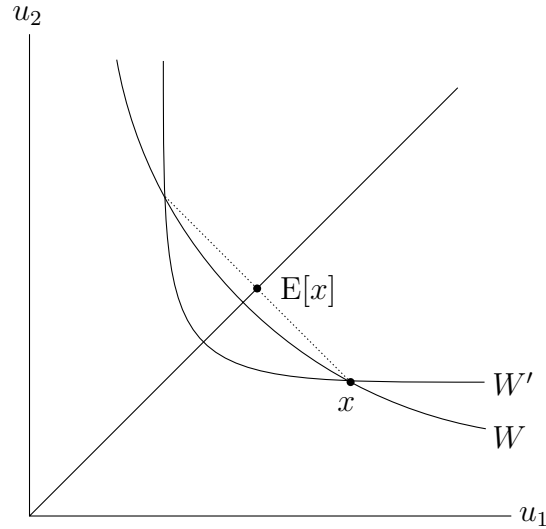
Figure 1: W' is more egalitarian than W

more and more concentrated (even if with respect to a changing average). The curve $\gamma(a)$ is called a *track*. The optimal value of a depends on social preferences, and our aim here is to compare these optimal values for societies with different degrees of ex-post egalitarianism.

The main result of the paper is that the more egalitarian is the society, the further it is willing to sacrifice average utility in order to reduce the variation of the optimal utility distribution. This result is stated in the following theorem.

Theorem 1 *If W' is more egalitarian than W, then along a track γ where there is a substitution between average utility and spread, the optimal point for the more egalitarian function has a weakly lower sum of utilities and more concentration than the less egalitarian function.*

We present a formal statement of this theorem and provide a proof in the Appendix. Moreover, we provide there conditions for a strict result (Theorem 2). Another result, implied by both theorems, is that an egalitarian society chooses a utility distribution that is more concentrated than the one chosen by a utilitarian society. This result is stated in the following corollary (and is proved in the Appendix).

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Corollary 1 *Let W be an ex-post egalitarian social welfare function and let γ be a differentiable track. The optimal utility distribution of W is more concentrated (and has a lower sum of utilities) than the optimal distribution of a utilitarian society.*

4   Applications

In this section we discuss some applications of our main result, namely, that a more egalitarian society will go further in the direction of reducing utility spread, even at the cost of total utility.

4.1   Victims of Crime versus Victims of the Criminal Law System

Under the most fundamental principles of evidence law, facts constitutive of the defendant's guilt have to be proven beyond reasonable doubt.[4] This rule is perceived by practitioners as well as scholars to be grounded in justice-based considerations; it is often described as a right of defendants against the state. The rhetoric of rights and justice used in the justification of the strict standard of proof is often hostile to utilitarian calculations. This is no accident; utilitarianism cannot in general justify the heightened burdens of proof associated with this principle.

To see it, assume that a legal system can adopt either a strict or a lenient rule of proof. Under the strict rule, guilt has to be proven beyond reasonable doubt. Under the lenient rule, guilt has to be proven in a "satisfactory manner." Each one of these alternative rules involves costs and benefits, and there is some tradeoff between convicting the innocent and acquitting the guilty. (See, e.g., Williams [28, p. 188] or Wertheimer [27, pp. 51–52]).

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[4]This is an old principle of Common Law. An important articulation of it can be found in the English landmark case of Woolmington v. DPP [1935] A.C. 462. The Court stated there unambiguously that: "No matter what the charge or where the trial, the principle that the prosecution must prove the guilt of the prisoner is part of the Common Law of England and no attempt to whittle it down can be entertained" (see id at 481). In the US the same principle is regarded as required by the due process clause. The Supreme Court stated that this clause "protects the accused against conviction except upon proof beyond reasonable doubt of every fact necessary to constitute the crime with which he is charged." See In Winship, 397 U.S. 358, 364 (1970). The rule however had much longer and deeper roots which can be traced to the Roman Law (see Williams [28, pp. 186–190]).

It is reasonable to believe that a legal system which adopts the lenient rule fulfills better its task of protecting its citizens from crime than a system which adopts the strict rule and the costs of the strict principle in terms of deterrence may outweigh the benefits the strict principle provides to the innocent people who are better protected under it from wrongful conviction. In that case, utilitarianism dictates a rejection of the strict rule.

In their efforts to justify the strict rule, utilitarians often develop justifications which depend on some restrictive assumptions. Bentham [3], for example, supported the claim that there ought to be a presumption in favor of the accused, writing: "Generally speaking, a too easy acquittal excites regret and uneasiness only among men of reflection; while the condemnation of an accused, who turns out to have been innocent, spreads general dismay; all security appears to be destroyed; no defence can any longer be found, when even innocence is insufficient." Yet moral theorists usually abandon utilitarian justifications for the principle that guilt should be proven beyond reasonable doubt (see Dworkin [8]). Some of their arguments rely on the intuition that wrongful conviction seems to be a direct interference in the lives of an innocent person. In contrast, the failure to prevent a crime is merely an omission on the part of the state.

Ex-post egalitarianism is an in-between approach. Like Bentham and like utilitarianism, it does not ignore the individual costs and benefits of different burdens of proof. However, it also considers social norms and notions of justice, which are represented by the quasi concavity of the social welfare function. Such functions pay more attention to the well being of those members of society who are worse off compared to the rest. To illustrate, consider the following structure.

Society is composed of innocents and criminals. Innocent people face the risk of wrong conviction and the risk of criminal victimhood. Criminals may or may not be caught and convicted (for simplicity, we ignore minor groups such as criminals who are not convicted but are victims of crime). For a given level of burden of proof a, we get an ex-post utility distribution of the form $\gamma(a) = (u_i, p_i)_{i=1}^5$, where u_1 is the utility of convicted innocents, u_2 is the utility of convicted criminals, u_3 is the utility of innocent victims of crime, u_4 is the utility of innocents, and u_5 is the utility of the unconvicted criminals. We assume that crime benefits the criminal, that punishment is more severe than the consequences of the criminal act, and that wrong conviction is worse than just conviction to the bearer of the punishment. Therefore, we assume $u_1 < \cdots < u_5$ (see distribution ①— the continuous line — in Fig. 2). As

explained in Section 2, p_i is the proportion in the population of those who receive utility u_i (and is also the probability of getting this utility level).
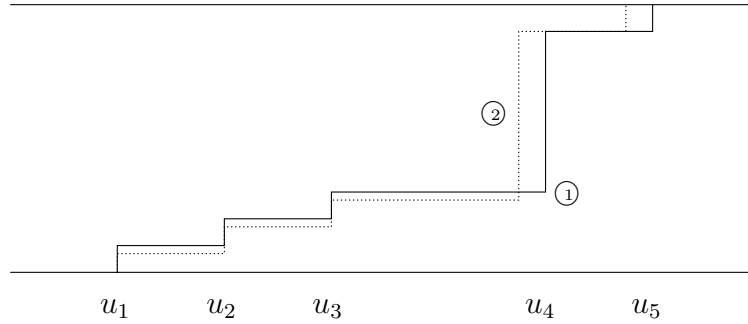

Figure 2: Burden of Proof

Society now raises the burden of proof while simultaneously raising the probability of detection so that the level of crime in society remains untouched. The reason for this simultaneously acts is the desire that the cost of the increase in crime level be shared by the whole society and not only by the additional victims. Criminals are interested in the probability of being punished, which is the product of the probability of detection and the conditional probability of conviction given detection. As crime does not change, this combined probability too does not change. Moreover, assuming one crime per criminal, the number of criminals is not changed. Therefore, the number of those who receive utility levels u_2 and u_5 is not changed. Also, the size of the u_3 part of society is not changed—these being the people who suffer from crime.

Obviously, the combination of higher burden of proof and higher rate of detection that does not change the crime rate will reduce the number of convicted innocents, hence the size of the u_1 level will go down, while the size of the u_4 group will increase.

Better detection is of course not cost free. Suppose that society finances the cost by taxing those who are not convicted and did not suffer from crime, that is, groups 4 and 5.[5] The new distribution is depicted by curve ② (the dotted line). Obviously, the second distribution intersects the first one only

[5]Arguably, one should tax group 3 as well (recall that groups 1 and 2 cannot be taxed as they are serving jail terms). However, we will ignore this tax in order to satisfy the

once, and from below. If the first distribution is optimal, then the new distribution must have a lower expected value, otherwise, as it is less spread than before, a higher expectation would have prevented distribution ① from being optimal even for a utilitarian society, and certainly for an egalitarian one. We strengthen this statement by assuming that the expected value is actually monotonically decreasing as we move from the first to the second distribution.

The conditions of Theorem 1 are thus satisfied (Observe that the set of distributions society can choose from is a track (Section 3). Even when the second distribution is shifted to the right by the whole increased cost of detection, there will be only one crossing point, and the expected value of the shifted distribution will be more than the expected value of distribution ①). We therefore find out that the more egalitarian society will seek a higher level of burden of proof.

Our analysis does not imply that the optimal social policy is to set the burden of proof at its highest possible level ("beyond any reasonable doubt"). We recognize the validity of two claims on the social ruling. The obligation to increase social well-being, but also the obligation to equality. Unlike utilitarianism, and unlike some of the above justifications for the strict rule, we refuse to give one claim lexicographic dominance of the other. Ex-post egalitarian social welfare functions do indeed take both factors into consideration.

4.2   Torture and Other Forms of Cruel Punishment

One of the most puzzling concerns of the legal economist is the ceiling on the size of the criminal sanction. Since Becker [1], legal economists tried to explain the reasons for the widely held intuition that the criminal law system should not impose the maximally possible sanctions.

Becker's argument is simple and yet compelling. An expected punishment of $1000 can be imposed by different combinations of fines and probabilities of apprehension. If the costs of collecting fines are assumed to be zero regardless of the size of the fine, but the costs of apprehending and convicting criminals rise with the probability of apprehension,[6] then the most efficient

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conditions of Theorem 1. For infinitesimal changes this tax is indeed negligible, as the number of those who suffer from (serious) crime is small compared to the number of those who are not affected by it, and the tax is small compared to the sanction against criminals.

[6]These costs are increasing with the probability because higher probabilities imply more police, prosecutors, judges, defense attorneys, etc.

combination is a probability close to zero and a fine arbitrarily close to infinity. (See Becker [1]. See also Posner [20, pp. 219–227]). Under this argument, the only optimal sanction for every crime is the most extreme—possibly torture and death. Such a sanction enables one to obtain that even very low probability of apprehension and conviction would be sufficient to guarantee a sufficiently large expected sanction. Yet, our legal system does not conform to the Becker model. Instead, our system imposes a ceiling on criminal sanctions. Torture is prohibited altogether in modern legal systems and most modern legal systems do not impose capital punishment.

There are several primary explanations for the ceilings on criminal sanctions. Some legal economists dispute some of the presuppositions of Becker's analysis. They point out that criminals may be risk averse or risk preferring and if criminals are not risk neutral Becker's analysis does not yield the same outcome (see Polinsky and Shavell [19]). Moreover, economists point out the importance of marginal deterrence, that is, the incentive to substitute less for more serious crimes as a reason for differentiating between the sanctions imposed for different crimes (see Shavell [25, pp. 1245–1246] and Posner [20, p. 222]). Traditional criminal law provides of course Kantian justification to the ceiling on sanctions. The wrongfulness and the culpability of criminals dictate what they deserve and criminal law should not impose any sanctions which exceed what is being deserved. For a use of this Kantian insight see Nozick [18, pp. 363-397] and Fletcher [10].

Ex-post egalitarian considerations could however explain the very same phenomenon. Criminal sanctions yield benefits that are provided to the public at large at the cost of the particular individual upon whom they are imposed. The imposition of criminal sanctions contributes to the well-being of others by producing deterrence. Yet, if too large sanctions are imposed on a small group of individuals, the disparity between the well-being of those who bear the costs of deterrence and those who benefit from it is too large. The ceiling on the criminal sanction is aimed at constraining this disparity.

As before, society is composed of innocents and criminals. The first group consists of victims of crime (with the utility u_2) and of non-victims (with the utility u_3), while the second group consists of those who are punished (u_1) and of those who are not (u_4).[7] Here too we assume that crime benefits the criminals and that punishment is more severe than the consequences of the criminal act. Therefore, we assume $u_1 < \cdots < u_4$, see distribution ① in

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[7]We ignore here the possible existence of innocent people who are wrongly convicted.
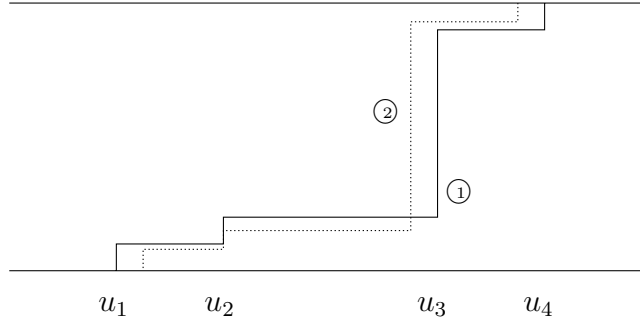
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Fig. 3.

Figure 3: Cruel Punishment

Society now reduces the severance of punishment while simultaneously raising the probability of detection so that the number of convicted criminals is not higher than before. It follows that the number of criminals is thus going down (as before, we assume one crime and one victim per criminal). This of course is not cost free (otherwise, society would not have been at an optimal point), and we assume, as before, that the financial burden falls on the shoulders of those who are not convicted and did not suffer from crime (see fnt. 5).The new distribution is depicted by curve ②. In this distribution, the utility of convicted criminals is higher than before (as the punishment is less severe), and there are not more of them than before. The utility of unconvicted criminals is less than before (they have to pay for better detection), and since the overall crime rate is down and the probability of detection is higher, the size of this group diminishes. As there are fewer crimes, there are less victims of crime, but their utility is the same as before. Finally, less crime means more people who are neither criminals nor victims (the old u_3 group), but since they have to pay for better detection, their utility goes down. Obviously, the new distribution intersects the first one only once, and from below.

If the first distribution is optimal, than the new distribution must have a lower expected value, otherwise, as it is less spread than before, a higher expectation would have prevented distribution ① from being optimal even for a utilitarian society. We thus assume that the expected value is monotonically decreasing as we move from the first to the second distribution.

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It is reasonable to assume that crime benefits criminals less than the harm it imposes on its victims (otherwise, it would have been socially optimal to have more crime, at least for a utilitarian society). In other words, $u_4 - u_3 < u_3 - u_2$. Moreover, since the number of convicted criminals is reduced, the change in the number of victims must be higher than the change in the number of unpunished criminals. In other words, when the new distribution ① is shifted to the right by the full amount of the increased cost of detection, its expected value will be higher than the original distribution, hence the social choice set is a track. The conditions of Theorem 1 are thus satisfied, and we therefore find out that the more egalitarian society will seek less severe punishment than the less egalitarian one.

4.3   Sacrificing Some for the Rest

The problem of whether or not society should sacrifice some (hopefully few) individuals to save the rest is at least 2500 years old. In *Iphigeneia at Aulis*, Euripides describes the attempted scarification of Iphigeneia to appease Artemis' wrath, thus obtaining fair wind to carry the Greek navy out of the bay of Aulis to sail for Troy (see [12]). Similarly, the Rabbinic Law deals with the situation of a group of people who are surrounded by enemies and are asked to surrender one of them to be killed. The law is that they should all be killed, but should not do it, unless the enemy specified the one they want (Tosefta [26] Terumot 7:20). In both sources there is an unambiguous reluctance to accept the idea that social goals may be achieved by sharply reducing the well being of some members of society.

In this subsection we discuss a similar problem, where society can reduce the utility of a minority to improve the position of the majority. We assume a homogeneous society, where initially everyone has utility level u_1. By reducing the utility of some members of society it is possible to increase the utility of the rest—two such possibilities are depicted in Fig. 4 (distributions ② and ③. Distribution ① represents the fully egalitarian situation). Such redistributions of utility can be obtained, for example, by a military draft, jury duty, and other such obligations.

If the expected value of the third distribution is less than that of the second one, then there is no point in switching from ② to ③ even in a utilitarian society. We therefore assume that the expected value in monotonically decreasing as society is moving from distribution ③ to ②. Shifting distribution
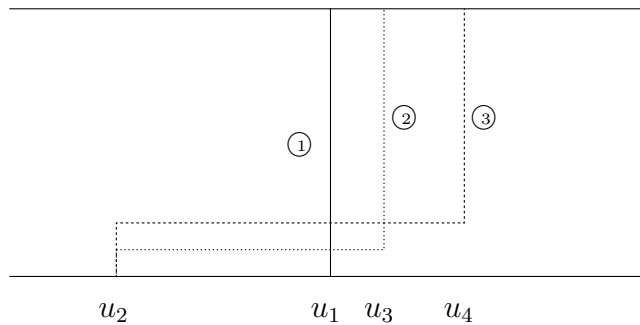
Figure 4: Sacrifying some for the rest

③ to the left to get the same expected value as that of distribution ② will maintain the single crossing property of the two distributions, hence Theorem 1 may be applied. We obtain that the more egalitarian society will opt for less benefits for the privileged at the cost of the underprivileged.

We would like to emphasize that our analysis does not imply that society should not recruit only part of the citizenry, as it may well be that the fully egalitarian distribution is suboptimal, for example, when a relatively modest reduction in utility for a small number of individuals can significantly increase the utility of the rest. But the ex-post egalitarian society is certainly more sensitive than the utilitarian society to issues of utility distribution, and will therefore ask for a higher benefits for the privileged than the utilitarian society before it agrees to further sacrifices by the under-privileged.

5   Summary

Our most fundamental rules and practices constrain the pursuit of maximization of utility. It is often natural to explain these constraints as grounded in deontological considerations founded on principles of dignity and inviolability of persons. This indeed has been the explanation traditionally given to many of the rules and principles constraining the pursuit of utility.

This article provides an additional explanation — one that relies on principles of ex-post egalitarianism. More specifically, it is argued that ex-post egalitarianism can explain a broad array of rules and practices including the requirement that guilt be proven beyond reasonable doubt, the prohibition

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on torture and cruel and unjust punishment and the prohibition of sacrific-
ing some for the sake of saving others. These rules and practices constrain
the pursuit of utility. At the same time, they reallocate utility in a fash-
ion that promotes ex-post egalitarianism. More specifically, they reduce the
costs which otherwise would be imposed on people whose well-being turns
out ex-post to be lower at the expense of those whose well-being turns out
ex-post to be higher.

Unlike ex-ante egalitarianism which is often achieved through randomiza-
tions over social members, ex-post egalitarianism involves a real transfer of
goods from some individuals to others. It is therefore important that these
goods will be divisible, as indeed are the goods in all of our examples.

Appendix

Let $\mathcal{X}$ be the set of real bounded random variables on the probability space
(S, Σ, P) and let F_x denote the cumulative distribution function of $x \in \mathcal{X}$,
where $\mathcal{X}$ is the set of all utility distributions. Generic elements of $\mathcal{X}$ are
denoted x, y, z while degenerate random variable are denoted r, s, t. Scalars
are denoted a, b, c. A preference relation $\succeq$ is a binary relation on $\mathcal{X}$ that is
complete, transitive, continuous,[8] and monotone with respect to the relation
of first-order stochastic dominance (as usual, $\sim$ denotes indifference and $\succ$
denotes strict preference). The preference relation $\succeq$ is symmetric if for all
$x, y \in \mathcal{X}$, $F_x = F_y \implies x \sim y$. It is quasi concave if for all $x, y \in \mathcal{X}$,
$x \sim y \implies \forall a \in (0, 1)$, $ax + (1 - a)y \succeq x$ (strict quasi concavity is defined
with strict preferences). Note that quasi concavity is defined with respect
to outcomes, and not with respect to probabilities. By construction, quasi
concavity implies preferences for averaging, hence it is related to equity-
seeking behavior. Let $\mathcal{U}$ denote the set of all symmetric and quasi concave
preference relations on $\mathcal{X}$. The social welfare functions W and W' used in
the paper represent preferences in $\mathcal{U}$.

Example 1 Consider the set $\mathcal{L} \subset \mathcal{U}$ of preference relations that satisfy pos-
itive linearity: for all $a > 0$ and for all $r \in \mathcal{X}$

$$x \succeq y \iff ax + r \succeq ay + r$$

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[8]We use the topology of weak convergence with the Lévy metric on it. This metric is
defined by $d(F, G) = \inf\{\varepsilon > 0 : \text{For all } x,\ G(x - \varepsilon) - \varepsilon \leqslant F(x) \leqslant G(x + \varepsilon) + \varepsilon\}$; see
Huber [13].

Let $\succeq \in \mathcal{L}$ and pick a non-degenerate utility distribution x. The preference relation $\succeq$ can be fully reconstructed from the set $\{y \in \mathcal{X} : y \sim x$ and $\mathrm{E}[y] = \mathrm{E}[x]\}$ and from the unique degenerate distribution a that satisfy $a \sim x$. The set $\mathcal{L}$ was characterized in Safra and Segal [23]

Definition 3 *Let $\succeq, \succeq' \in \mathcal{U}$. The preference relation $\succeq'$ is more egalitarian than $\succeq$ if for all $x \in \mathcal{X}$*

$$\{y \in \mathcal{X} : y \succeq x \text{ and } \mathrm{E}[y] \leqslant \mathrm{E}[x]\} \subset \{y \in \mathcal{X} : y \succeq' x \text{ and } \mathrm{E}[y] \leqslant \mathrm{E}[x]\}$$

The preference relation $\succeq'$ is strictly more egalitarian than $\succeq$ if these inclusions are strict for all non-degenerate x.

Note that the relation 'more egalitarian than' is a partial order on the set of preference relations $\mathcal{U}$.

Fact 1 *Assume $\succeq'$ is more egalitarian than $\succeq$ and that they are both strictly quasi-concave. Then, for all $x \in \mathcal{X}$*

$$\{y \in \mathcal{X} : y \sim' x \text{ and } \mathrm{E}[y] = \mathrm{E}[x]\} = \{y \in \mathcal{X} : y \sim x \text{ and } \mathrm{E}[y] = \mathrm{E}[x]\}$$

That is, both induce the same preferences over subspaces in which the expected values are fixed.

Proof Let $x \in \mathcal{X}$ and denote $I_x = \{y \in \mathcal{X} : y \sim x \text{ and } \mathrm{E}[y] = \mathrm{E}[x]\}$ and $I'_x = \{y \in \mathcal{X} : y \sim' x \text{ and } \mathrm{E}[y] = \mathrm{E}[x]\}$. First consider $y \in I_x$. By Definition 3, $y \succeq' x$. If $y \succ' x$ then

$$x \in \{z \in \mathcal{X} : z \succeq y \text{ and } \mathrm{E}[z] \leqslant \mathrm{E}[y]\} \backslash \{z \in \mathcal{X} : z \succeq' y \text{ and } \mathrm{E}[z] \leqslant \mathrm{E}[y]\};$$

a contradiction to the assumption that $\succeq'$ is more egalitarian than $\succeq$.

Next consider $y \in I'_x \backslash I_x$ and let a denote the degenerate utility distribution in which all utilities are equal to $\mathrm{E}[x]$. By strict quasi concavity, $a \succ x$ and $a \succ y$. If $x \succ y$ then, by continuity, there exist $\lambda \in (0,1)$ such that $z = \lambda y + (1-\lambda)a$ satisfies $z \sim x$. Since $z \in I_x$, it follows by the above argument that $z \in I'_x$, hence $z \sim' y$; a contradiction to the strict quasi concavity of $\succeq'$. Since $y \notin I_x$, it must be the case that $y \succ x$. By the strict quasi concavity of $\succeq'$ and the continuity of $\succeq$, there exists z satisfying $\mathrm{E}[z] = \mathrm{E}[x]$, $x \succ' z$, and $z \succ x$. Hence,

$$z \in \{y \in \mathcal{X} : y \succeq x \text{ and } \mathrm{E}[y] \leqslant \mathrm{E}[x]\} \backslash \{y \in \mathcal{X} : y \succeq' x \text{ and } \mathrm{E}[y] \leqslant \mathrm{E}[x]\};$$

a contradiction to the assumption that $\succeq'$ is more egalitarian than $\succeq$.　■

Let $x \rhd y$ denote that F_y is a mean preserving spread of F_x while $F_x \neq F_y$. The following fact shows that if $\succeq'$ is (strictly) more egalitarian than $\succeq$ then, whenever the addition of $r > 0$ suffices to compensate the preference relation $\succeq'$ for the lack of equity manifested by y, then r (more than) suffices for the preference relation $\succeq$. This result demonstrates the close connection between our definition and the strong measure of risk aversion suggested by Ross [22]. Note that the analysis of Ross is restricted to a subset of preferences in $\mathcal{U}$ — preferences that correspond to expected utility preferences in the literature on risk.

Fact 2 *If $\succeq'$ is more egalitarian than $\succeq$ then, for all $x, y, r \in \mathcal{X}$ such that $x \rhd y$,*

$$y + r \succeq' x \Longrightarrow y + r \succeq x$$

Similarly, if $\succeq'$ is strictly more equitable than $\succeq$ then

$$y + r \succeq' x \Longrightarrow y + r \succ x$$

Proof Let x, y and r satisfy $x \rhd y$, $y+r \succeq' x$ and $x \succ y+r$. By monotonicity, there exists $z = y+r'$ $(r' > r)$ satisfying $z \sim x$ and $z \sim' x+r'' \succ' x$ $(r'' > 0)$. Hence x belongs to $\{w \in \mathcal{X} : w \succeq z \text{ and } \mathrm{E}[w] \leqslant \mathrm{E}[z]\}$ while it does not belong to $\{w \in \mathcal{X} : w \succeq' z \text{ and } \mathrm{E}[w] \leqslant \mathrm{E}[z]\}$; a contradiction to the assumption that $\succeq'$ is more egalitarian than $\succeq$.　■

Definition 4 *A track is a function $\gamma : [0,1] \to \mathcal{X}$ such that*

$$a > b \Longrightarrow \gamma(a) \rhd \gamma(b) + s$$

for some $s \in \mathcal{X}$.[9]

Note that s can be either a positive or a nonpositive distribution. Also note that if $\gamma(a)$ is a degenerate distribution, than $a = 1$. Otherwise, if $\gamma(a)$ is a mean preserving spread of $\gamma(1) + s$, then $\gamma(a) = \gamma(1) + s$, while the definition of $\rhd$ requires the two to be different from each other.

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[9]s is a function of a and b.

Theorem 1 *Let $\succeq, \succeq' \in \mathcal{U}$ and assume that $\succeq'$ is more egalitarian than $\succeq$. Let γ be a track and let $\gamma(a)$ and $\gamma(a')$ be the unique optimal distribution of $\succeq$ and $\succeq'$ along γ, respectively. Then $a' \geqslant a$.*

Proof Suppose that $a > a'$ and let s satisfy $\gamma(a) \rhd \gamma(a') + s$. Consider first the case $s \geqslant 0$. The quasi concavity of $\succeq'$ implies that $\gamma(a) \succeq' \gamma(a') + s$. By monotonicity, $\gamma(a') + s \succeq' \gamma(a')$, hence $\gamma(a) \succeq' \gamma(a')$, a violation of the assumption that a' is the unique optimal point of $\succeq'$ along γ.

Suppose now that $s < 0$. By assumption, $\gamma(a') \succ' \gamma(a)$. Since $\succeq'$ is more egalitarian than $\succeq$, it follows Fact 2 and from that fact that $\gamma(a) \rhd \gamma(a') + s$ that

$$\gamma(a') = [\gamma(a') + s] + (-s) \succ' \gamma(a) \implies$$
$$\gamma(a') \succ \gamma(a)$$

A contradiction to the assumption that a is the unique optimal point of $\succeq$ along γ. It thus follows that $a' \geqslant a$. ∎

Next we want to establish conditions under which a' is strictly greater than a. Define the positive tangent $\gamma'_+(a)$ to satisfy

$$\gamma'_+(a) = \lim_{a_n \downarrow a} \frac{\gamma(a_n) - \gamma(a)}{\|\gamma(a_n) - \gamma(a)\|}$$

where $\| \cdot \|$ is with respect to the Lévy metric (see ftn. 8).

Definition 5 *The track $\gamma : [0,1] \to \mathcal{X}$ is equitably-differentiable if for all a the tangent $\gamma'_+(a)$ is well defined and there exists $\bar{c} > 0$ such that for all $0 < c < \bar{c}$,*

$$\gamma(a) + c\gamma'_+(a) \rhd \gamma(a) + s \tag{1}$$

for some $s = s(a, c) \in \mathcal{X}$.

The ray $\gamma(a) + c\gamma'_+(a)$ is the linear continuation of the curve γ from the point $\gamma(a)$. The condition of eq. (1) says that small movements in this direction increase the level of equity.

Definition 6 *The preference relation $\succeq \in \mathcal{U}$ is smooth if for all non-degenerate $x \in \mathcal{X}$ there exists a unique linear function $g_x : \mathcal{X} \to \Re$ such that $y \sim x$ implies $g_x(y) \geqslant g_x(x)$, and moreover, g_x is continuous in x.*

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(Recall that all preferences in $\mathcal{U}$ are quasi concave).

Since a hyperplane in $\mathcal{X}$ is an indifference set of a linear function, it follows that if a preference relation $\succeq \in \mathcal{U}$ is smooth then for all $x \in \mathcal{X}$ there is a unique hyperplane H_x supporting the indifference set at that point. Note that Fréchet differentiability may not imply this property (see Machina [16] and Safra and Segal [24]).

Let $\succeq, \succeq' \in \mathcal{U}$ be strictly quasi concave and smooth and assume that $\succeq'$ is strictly more egalitarian than $\succeq$. Let H_x (similarly, H_x') be the supporting hyperplane to the indifference set of $\succeq$ ($\succeq'$) through the non-degenerate distribution x. Let D denote the line generated by all degenerate distributions, $D = \{r : r \in \mathcal{X}\}$, and consider the two-dimensional plane $T = \mathrm{Span}\{D, x\}$. By monotonicity, D is not contained in H_x, hence $H_x \cap T$ is a line. By monotonicity and quasi concavity there exists t satisfying $\{t\} = (H_x \cap T) \cap D$ and, by symmetry and strict quasi concavity, $t < \mathrm{E}[x]$ (see Fig. 5). By construction, $H_x \cap T = \{x\} + \mathrm{Span}\{x - t\}$. Let J_x (J_x') be the indifference curves of $\succeq$ ($\succeq'$) in T through x. Let $G_x = \{y \in \mathcal{X} : \mathrm{E}[y] = \mathrm{E}[x]\}$. Clearly, $H_x = \mathrm{Span}\{H_x \cap G_x, x - t\}$ and, by Fact 1, $H_x' \cap G_x = H_x \cap G_x$.

By definition, the curves J_x and J_x' intersect each other at x such that the following holds: J_x lies above J_x' between x and D and J_x lies below J_x' otherwise. This, however, is not sufficient to imply that $x + (x - t)$ does not belong to H_x'. To have this, we assume the following generic assumption:

($\ast$) For all non-degenerate x, $H(x) \neq H'(x)$.

Note that the set of points $\{x : H_x \cap T = H_x' \cap T\}$ is the complement of an open and dense set (by comparability, $H_x \cap T = H_x' \cap T$ iff $H_x = H_x'$; by smoothness, the set of points where $H_x \neq H_x'$ is open; and the fact that $\succeq'$ is more strictly equitable than $\succeq$ implies that there is no open set on which $H_x = H_x'$).

Theorem 2 *Let $\succeq, \succeq' \in \mathcal{U}$ be strictly quasi concave and smooth. Assume that $\succeq'$ is strictly more egalitarian than $\succeq$, and that condition $(\ast)$ is satisfied. Let γ be an equitably-differentiable track and let $\gamma(a)$ and $\gamma(a')$ be the unique optimal points of $\succeq$ and $\succeq'$ along γ, respectively. If $a < 1$, then $a' > a$.*

Proof Since $a < 1$, $\gamma(a)$ must be a non-degenerate distribution (see the discussion after Definition 4 above). By Theorem 1, it is sufficient to show that $a' \neq a$.
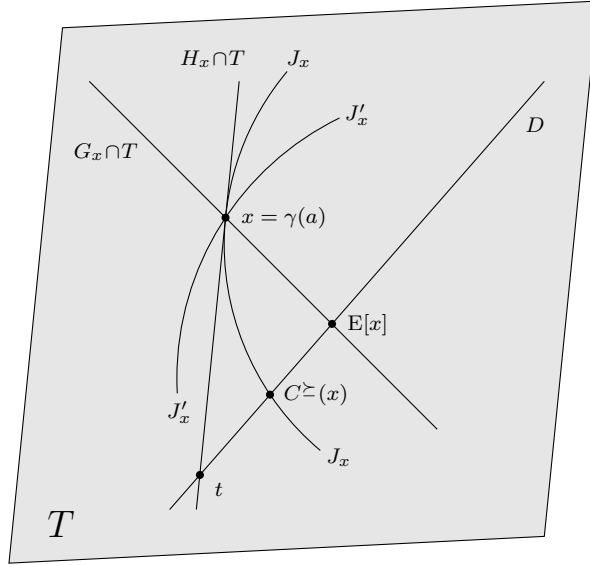
Figure 5: Proof of Theorem 2

Consider the tangent $\gamma'_+(a)$. By the optimality of x, $x + \gamma'_+(a) \in H_x$. Therefore, $x + \gamma'_+(a) = z + d(x - t)$ for some d, where $z \in H_x \cap G_x$ ($= H'_x \cap G_x$). We show that $x + \gamma'_+(a)$ does not belong to $H_x \cap G_x$. Assume the contrary. Since γ is equitably-differentiable it follows that for sufficiently small $c > 0$, $\gamma(a) + c\gamma'_+(a) \triangleright \gamma(a)$ (note that the s in the definition of an equitably-differentiable track satisfies $s = 0$). The smoothness of $\succeq$ implies that for sufficiently small $c > 0$, $\gamma(a + c) \succ \gamma(a)$; a contradiction. Therefore $d \neq 0$.

It follows by condition $(*)$ that $x + (x - t)$ does not belong to H'_x. Now $d \neq 0$ implies that $x + \gamma'_+(a)$ does not belong to H'_x. Therefore, the optimality of $\gamma(a')$ for $\succeq'$ shows that $\gamma(a') \neq x$ and $a' \neq a$. ∎

Corollary 2 *Let $\succeq, \succeq' \in \mathcal{U}$ such that $\succeq'$ is strictly quasi concave and smooth and $\succeq$ is a utilitarian preference relation. Let γ be an equitably-differentiable track and let $\gamma(a)$ and $\gamma(a')$ be the unique optimal points of $\succeq$ and $\succeq'$, respectively. If $a < 1$, then $a' > a$.*

Proof We will use the notations of the previous proof. By Theorem 1, $a' \geqslant a$. The condition that $\succeq$ is a utilitarian preference relation implies

that the supporting hyperplane to the indifference set of $\succeq$ through the non-degenerate distribution x satisfies $H_x = G_x$ (note that $t = \mathrm{E}[x]$) and the tangent vector $x + \gamma'_+(a)$ belongs to $H_x \cap G_x$. As in the former proof, the facts that γ is equitable-differentiable and that $\succeq'$ is smooth, symmetric, and strictly quasi concave imply that for sufficiently small $c > 0$, $\gamma(a+c) \succ' \gamma(a)$, hence $a' > a$. ∎

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