

Productivity Shocks and Consumption Smoothing in the International Economy

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Abstract

We develop a two-country, dynamic general equilibrium model that links cross-country differences in net foreign asset and consumption dynamics to differences in discount factors and steady-state levels of productivity. We compare the results of the model to those of VARs for the G3 economies. We identify country-specific productivity shocks by assuming that productivity does not respond contemporaneously to other variables in these VARs. We identify global productivity shocks by estimating the VARs in common trend representation after testing for and imposing model-based, long-run cointegration restrictions. We then compare the model's predictions for net foreign asset and consumption dynamics in response to productivity shocks with the estimated VAR impulse responses. We find that the two sources of heterogeneity we consider go some way toward reconciling the consumption smoothing hypothesis with the data and explaining variations in net foreign asset and consumption dynamics across countries.

JEL Classification: F41, C33

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1 Introduction

An important finding of the recent empirical open economy literature is that current account dynamics exhibit considerable variation across countries (or heterogeneity, as we shall say interchangeably). The data have also tended to reject optimizing models of the current account that formalize the consumption smoothing hypothesis for open economies. For instance, the present value tests of the current account seem to do a good job in explaining French data (Agénor et al., 1999), but the same model performs poorly for, say, Canada (Sheffrin and Woo, 1991; İřcan, 2002). Contrary to the theory’s predictions, global productivity shocks seem to matter for the current account (Glick and Rogoff, 1995). Moreover, current account responses to country-specific and global productivity shocks differ markedly across countries in ways that are difficult to reconcile with familiar optimizing models of consumption smoothing (Elliott and Fatás, 1996; İřcan, 2000).

One reason why formal models of international consumption smoothing score badly against the data might be that the baseline theoretical framework (as reviewed in Obstfeld and Rogoff, 1996) is too stylized. On the one hand, many empirical analyses rely on small open economy models even though they use data from the largest economies in the world for estimation. On the other hand, most studies based on multi-country models in which countries can have similar economic size assume that net foreign assets are zero in the long-run. Such models of *bilateral interdependence* typically do not allow for any differences in structural parameters across countries and assume symmetric steady state with zero net foreign assets. However, Lane and Milesi-Ferretti (2001, 2002*a, b*) have provided strong evidence of *non-zero* long-run holdings of assets across countries, with important theoretical and empirical implications.

In this paper, we develop a two-country, dynamic general equilibrium model of consumption smoothing and net foreign asset accumulation that allows for structural heterogeneity across countries, leading to cross-country differences in long-run net foreign asset positions. We then take a fresh look at data for the G3 (Germany, Japan, and the U.S.). Overall, we find that our theoretical and empirical results provide plausible explanations for why countries may respond differently to global productivity shocks. Hence, our findings go some way toward reconciling the consumption smoothing hypothesis with the data, and uncovering important sources of heterogeneity in net foreign asset and consumption dynamics across countries.

While a range of structural factors may be responsible for non-zero steady-state net foreign asset holdings (including differences in preferences, technology, demographics, creditworthiness, etc.), we consider two specific potential sources: (i) different subjective discount factors, and (ii) different long-run productivity levels. The departure from the analytically convenient assumption of identical discount factors is crucial. Indeed, our analysis shows that even very small cross-country differences in discount factors lead to quantitatively different net foreign asset positions in the long-run and qualitatively different dynamics in the short-run. The second source of heterogeneity is motivated primarily

by the data, as we document in our empirical analysis. As we demonstrate, however, productivity differences affect the steady-state asset position only when discount factors are different. If the discount factors are identical across countries, foreign assets are zero in the steady-state regardless of the long-run productivity levels.

We incorporate different discount factors and long-run productivity levels into the overlapping-generations model of Ghironi (2000), which allows us to determine the steady-state cross-country distribution of asset holdings endogenously. We identify conditions under which a country is a debtor or creditor in the long-run and analyze both the steady-state and the short-run dynamics. Our analysis shows the importance of relative prices—that of home versus foreign output (the terms of trade) and that of consumption today versus consumption tomorrow (the interest rate)—in the international transmission of productivity shocks in a way that is overlooked in two-country models with zero steady-state foreign asset holdings.

In particular, steady-state assets that differ from zero imply that the interest burden on previously accumulated debt (or income from asset holdings) is important for the dynamics of the log-linearized model. The world interest rate becomes an additional state variable in the solution of the model, and asymmetry of the steady-state yields asymmetric effects of terms of trade variation. As a result, global productivity shocks lead to asymmetric consumption and net foreign asset responses across debtor or creditor countries. By contrast, in the standard model with zero steady-state net foreign assets, such asymmetric responses and dynamics are altogether absent, partly because the interest burden on previous debt drops out of the log-linear law of motion for net asset holdings. Our model also generates interesting asymmetries in the responses of debtor and creditor countries to country-specific shocks. But, these differences are mostly quantitative, and are more difficult to isolate in the data.

In our empirical work, we focus on the G3 countries, treating the rest of the G7 as the rest of the world for each of them. We first identify country-specific and global productivity shocks from vector autoregressions (VARs) consistent with our model as well as a broad set of others. We identify country specific-shocks by assuming that productivity does not respond contemporaneously to other variables in the VAR. We identify global productivity shocks in a manner that is novel in this literature—by estimating these VARs in common trend representation after testing for and imposing model-consistent, long-run, cointegration restrictions. We then compare the model’s predictions for consumption and net foreign asset dynamics following global and country specific shocks with the estimated impulse responses for each of the G3 countries.

We find that structural differences leading to non-zero, long-run net foreign asset positions across countries help account for and interpret consumption and asset accumulation dynamics. In particular, the estimated impulse responses to permanent global productivity shocks differ across countries in a way that is consistent with our theoretical framework: A positive, permanent global shock increases the foreign indebtedness of the U.S., while Japan accumulates net foreign assets. Thus, while the U.S. exhibits the be-

havior of a less patient, more productive economy, Japan emerges as a patient, but less productive economy. Consumption and interest rate responses are also broadly consistent with the predictions of our theoretical model. We are however somewhat less successful in accounting for the empirical responses to country-specific shocks.

In addition to the work of Lane and Milesi-Ferretti (2001, 2002*a, b*), a few other studies have pursued lines of research that share some features with ours. Masson et al. (1994), in particular, look at heterogeneous demographic factors and fiscal policies to explain net foreign asset dynamics of Germany, Japan, and the U.S., with some success in explaining significant variation across countries. These authors, however, do not emphasize macroeconomic interdependence as we do. Henriksen (2002) calibrates a model with heterogeneous demographics to the U.S. and Japan and finds that the predicted paths of U.S. and Japanese current accounts are consistent with the data. We rely on estimation rather than calibration when confronting our model with the data. Kraay and Ventura (2000) study the differences in the responses of the current accounts of debtor and creditor countries to transitory changes in income, but they do not link the determination of initial asset positions and the resulting heterogeneity in dynamics to specific structural features of the economies.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 discusses the model solution and some of its properties. Section 4 presents impulse responses based on a plausible parameterization of the model. Section 5 describes the econometric framework and reports the empirical findings. Section 6 concludes. Two appendices contain technical details of the theoretical analysis and data description.

2 The Theoretical Model

The microfoundations of our model are as in Ghironi (2000), but here we allow for asymmetry in household discount factors and steady-state productivity levels across countries. Readers who are familiar with Ghironi (2000) may wish to review the main assumptions below and move directly to Section 3.

2.1 The Main Assumptions

Demographics and Household Behavior—The world consists of two countries, home and foreign. In each period t , the world economy is populated by a continuum of infinitely lived households between 0 and N_t^W . (A superscript W denotes world variables.) Each household consumes, supplies labor, and holds financial assets. As in Weil (1989*a, b*), households are born on different dates *owning no assets*, but they own the present discounted value of their labor income. The number of households in the home economy, N_t , grows over time at the exogenous rate n , *i.e.*, $N_{t+1} = (1 + n)N_t$. We normalize the size of a household to 1, so that the number of households alive at each point in time is the economy's population. Foreign population grows at the same rate as home population.

We assume that the world economy has existed since the infinite past and normalize world population at time 0 so that $N_0^W = 1$.

Households at home and abroad have perfect foresight, though they can be surprised by initial, unexpected shocks. Households maximize intertemporal utility functions. The period utility function in both countries is logarithmic in consumption of a CES world consumption basket and in the amount of labor effort supplied by the household. Domestic households have discount factor β , $0 < \beta < 1$. Foreign households have discount factor $\alpha\beta$, $0 < \alpha \leq 1$. When $\alpha < 1$, foreign households are more impatient than domestic households.

Goods Market and Production—A continuum of goods $z \in [0, 1]$ are produced in the world by monopolistically competitive, infinitely lived firms, each producing a single differentiated good. Firms produce output using labor as the only factor of production according to a linear technology that is subject to multiplicative, country-wide productivity shocks. We allow steady-state productivity levels to differ across countries. At time 0, the number of goods that are supplied in the world economy is equal to the number of households. The number of households grows over time, but the commodity space remains unchanged. Thus, as time goes, the ownership of firms spreads across a larger number of households. Profits are distributed to consumers via dividends, and the structure of the market for each good is taken as given. The domestic economy produces goods in the interval $[0, a]$, which is also the size of the home population at time 0, whereas the foreign economy produces goods in the range $(a, 1]$.

Asset Markets—The asset menu includes a riskless real bond denominated in units of the world consumption basket and shares in firms. Private agents in both countries trade the real bond domestically and internationally. Shares in home (foreign) firms are held only by home (foreign) residents to ensure diversity of asset portfolios across agents born in the same period in different countries.

2.2 Households

Consumers have identical preferences over a real consumption index (C) and leisure ($1 - L$, where L is labor effort supplied in a competitive labor market, and we normalize the endowment of time in each period to 1). At any time t_0 , the representative home consumer j born in period $v \in [-\infty, t_0]$ maximizes the intertemporal utility function:

$$U_{t_0}^{v,j} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\rho \log C_t^{v,j} + (1 - \rho) \log (1 - L_t^{v,j}) \right], \quad (1)$$

with $0 < \rho < 1$.

The consumption index for the representative domestic consumer is:¹

$$C_t^{v^j} = \left[a^{\frac{1}{\omega}} \left(C_{Ht}^{v^j} \right)^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} \left(C_{Ft}^{v^j} \right)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}},$$

where $\omega > 0$ is the intratemporal elasticity of substitution between domestic and foreign goods. The consumption sub-indexes that aggregate individual domestic and foreign goods are, respectively:

$$C_{Ht}^{v^j} = \left[\left(\frac{1}{a} \right)^{\frac{1}{\theta}} \int_0^a \left(c_t^{v^j}(i) \right)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad C_{Ft}^{v^j} = \left[\left(\frac{1}{1-a} \right)^{\frac{1}{\theta}} \int_a^1 \left(c_{*t}^{v^j}(i) \right)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}},$$

where $c_t^{v^j}(i)$ ($c_{*t}^{v^j}(i)$) denotes time t consumption of good i produced in the home (foreign) country, and $\theta > 1$ is the elasticity of substitution between goods produced in each country.

Foreign agents consume an identical basket of goods. Trade in goods is free. There are no transportation and transaction costs, and each individual good has an identical real price in the two economies.

The representative home consumer enters a period holding bonds and shares purchased in the previous period. He or she receives interests and dividends on these assets, may earn capital gains or incur losses on shares, earns labor income, and consumes.

Denote the date t price (in units of the world consumption basket) of a claim to the representative domestic firm i 's entire future profits (starting on date $t+1$) by V_t^i , and let $x_{t+1}^{v^j i}$ be the share of the representative domestic firm i owned by the representative domestic consumer j born in period v at the end of period t . d_t^i denotes the real dividends that firm i pays on date t (in units of consumption). Then, letting $B_{t+1}^{v^j}$ be the representative home consumer's holdings of bonds entering $t+1$, the period budget constraint is:

$$\begin{aligned} & B_{t+1}^{v^j} + \int_0^a \left(V_t^i x_{t+1}^{v^j i} - V_{t-1}^i x_t^{v^j i} \right) di \\ &= (1+r_t) B_t^{v^j} + \int_0^a d_t^i x_t^{v^j i} di + \int_0^a (V_t^i - V_{t-1}^i) x_t^{v^j i} di + w_t L_t^{v^j} - C_t^{v^j}, \end{aligned} \quad (2)$$

where r_t is the risk-free world real interest rate between $t-1$ and t , and w_t is the real wage, both in units of the consumption basket.²

¹For the sake of simplicity, we will often refer to the representative member of generation v as the "representative consumer" below. Strictly speaking, though, the model we set up is not a representative consumer one, as representative agents of different generations may behave differently.

²Given that individuals are born owning no financial wealth, because they are not linked by altruism to individuals born in previous periods, $B_v^{v^j} = x_v^{v^j i} = 0$. As noted before, however, individuals are born owning the present discounted value of their labor income.

The representative domestic consumer born in period v maximizes the intertemporal utility function (1) subject to the constraint (2). Dropping the j superscript (because symmetric agents make identical choices in equilibrium), optimal labor supply is given by:

$$L_t^v = 1 - \frac{1 - \rho}{\rho} \frac{C_t^v}{w_t}, \quad (3)$$

which equates the marginal cost of supplying labor to the marginal utility of consumption generated by the corresponding increase in labor income.

The first-order condition for optimal holdings of bonds yields the Euler equation:

$$C_t^v = \frac{1}{\beta(1 + r_{t+1})} C_{t+1}^v \quad (4)$$

for all $v \leq t$.

Absence of arbitrage opportunities between bonds and shares requires:

$$1 + r_{t+1} = \frac{d_{t+1}^i + V_{t+1}^i}{V_t^i}. \quad (5)$$

As usual, first-order conditions and the period budget constraint must be combined with appropriate transversality conditions to ensure optimality.

Foreign consumers maximize a similar intertemporal utility function and are subject to an analogous budget constraint as home consumers. The only difference is that the discount factor of foreign households is $\alpha\beta$. Otherwise, a similar labor-leisure tradeoff, Euler equation, no-arbitrage, and transversality conditions hold for foreign households.

2.3 Firms

Output supplied at time t by the representative domestic firm i is a linear function of labor demanded by the firm:³

$$Y_t^{Si} = Z_t L_t^i, \quad (6)$$

where Z_t is exogenous, economy-wide productivity. Production by the representative foreign firm is a linear function of L_t^{i*} , with productivity Z_t^* .

Output demand comes from domestic and foreign consumers. The demand of home good i by the representative domestic household born in period v is:

$$c_t^v(i) = (RP_t^i)^{-\theta} (RP_t)^{\theta-\omega} C_t^v$$

obtained by maximizing C subject to a spending constraint. We denote with RP_t^i the price of good i in units of the composite consumption basket and with RP_t the price of the

³Because all firms in the world economy are born at $t = -\infty$, after which no new goods appear, it is not necessary to index output and labor demands by the firms' date of birth.

sub-basket of home goods in units of consumption. Aggregating across home households alive at time t , *total demand* for home good i coming from domestic consumers is:

$$c_t(i) = (RP_t^i)^{-\theta} (RP_t)^{\theta-\omega} a(1+n)^t c_t,$$

where

$$c_t \equiv \frac{a \left[\dots \frac{n}{(1+n)^{t+1}} C_t^{-t} + \dots + \frac{n}{(1+n)^2} C_t^{-1} + \frac{n}{1+n} C_t^0 \right]}{a(1+n)^t}$$

is *aggregate per capita* home consumption of the composite consumption basket.

Given identity of preferences across countries, total demand for home good i by foreign consumers is

$$c_t^*(i) = (RP_t^i)^{-\theta} (RP_t)^{\theta-\omega} (1-a)(1+n)^t c_t^*,$$

where c_t^* is aggregate per capita foreign consumption, the definition of which is similar to that of c_t .

Total demand for good i produced in the home country is obtained by adding the demands for that good originating in the two countries. Using the results above one can show that:

$$Y_t^{Di} = (RP_t^i)^{-\theta} (RP_t)^{\theta-\omega} \hat{c}_t^W, \quad (7)$$

where \hat{c}_t^W is *aggregate* (as opposed to aggregate per capita) world demand of the composite good: $\hat{c}_t^W \equiv N_t c_t + N_t^* c_t^*$.⁴

Firm i chooses the real price of its product and the amount of labor demanded to maximize the present discounted value of its current and future profits subject to the constraints (6), (7), and the market clearing condition $Y_t^{Si} = Y_t^{Di} (= Y_t^i)$. Given the no-arbitrage condition between bonds and shares (5) and a no-speculative bubble condition, the real price of firm i 's shares at time t_0 is equal to the present discounted value of the real dividends paid by the firm from $t_0 + 1$ on: $V_{t_0}^i = \sum_{s=t_0+1}^{\infty} R_{t_0,s} d_s^i$, where $R_{t_0,s} \equiv$

$\left[\prod_{u=t_0+1}^s (1+r_u) \right]^{-1}$, $R_{t_0,t_0} = 1$. At time t_0 , firm i maximizes the present discounted value of

dividends to be paid from t_0 on: $V_{t_0}^i + d_{t_0}^i = \sum_{s=t_0}^{\infty} R_{t_0,s} d_s^i$. At each point in time, dividends

are given by after-tax real revenues— $(1-\tau)RP_t^i Y_t^i$ —plus a lump-sum transfer (or tax) from the government— T_t^i —minus costs— $w_t L_t^i$. Firm i takes the real price of the sub-basket of home goods, the wage rate, Z , the rate of taxation of revenues (τ), the transfer received from the government, and world demand of the composite good as given.

⁴Where necessary for clarity, we use a “hat” to differentiate the aggregate level of a variable from the aggregate per capita level.

Let λ_t^i denote the Lagrange multiplier on the constraint $Y_t^{Si} = Y_t^{Di}$. The multiplier λ_t^i is the shadow price of an extra unit of output in period t . The first-order condition for the optimal choice of L_t^i yields:

$$\lambda_t = \frac{w_t}{Z_t}. \quad (8)$$

At an optimum, the shadow value of output must equal marginal cost. We drop the i superscript because symmetric firms make identical choices in equilibrium.

The first-order condition with respect to RP_t^i yields the pricing equation:

$$RP_t^i = RP_t = \frac{\theta}{(\theta - 1)(1 - \tau)} \lambda_t, \quad (9)$$

which equates the price charged by firm i to a markup over marginal cost. Identical equilibrium choices by symmetric firms imply that the real price of good i equals the real price of the sub-basket of home goods in equilibrium.

Using the market clearing conditions $Y_t^{Si} = Y_t^{Di}$ and $\hat{c}_t^W = \hat{Y}_t^{SW} = \hat{Y}_t^{DW} (= \hat{Y}_t^W)$, the expressions for supply and demand of good i , and recalling that symmetric firms make identical equilibrium choices, labor demand can be written as:

$$L_t^i = RP_t^{-\omega} \frac{\hat{Y}_t^W}{Z_t}. \quad (10)$$

Ceteris paribus, firm i 's labor demand is a decreasing function of real output price and productivity. It is an increasing function of world consumption demand.⁵

2.4 The Government

We assume that governments run balanced budgets. Governments tax firm revenues and rebate tax income to firms via lump-sum transfers. We assume that governments tax revenues at a rate that compensates for monopoly power and removes the markup charged by firms over marginal costs in equilibrium.⁶ The tax rate is determined by $1 - \tau = \frac{\theta}{\theta - 1}$, which yields $\tau = -\frac{1}{\theta - 1}$. Because the tax rate is negative, firms receive a subsidy on their revenues and pay lump-sum taxes determined by:

$$T_t^i = \tau RP_t^i Y_t^i. \quad (11)$$

⁵Although all domestic firms demand the same amount of labor in equilibrium, we leave the i superscript on labor demand to differentiate labor employed by an individual firm from aggregate per capita employment, which will be denoted by dropping the superscript. Optimality conditions for foreign firms are similar.

⁶This greatly simplifies the solution of the model.

2.5 Aggregation

2.5.1 Households

Aggregate per capita labor supply equations are obtained by aggregating labor-leisure tradeoff equations across generations and dividing by total population at each point in time. The aggregate per capita labor-leisure tradeoffs in the two economies are:

$$L_t = 1 - \frac{1 - \rho}{\rho} \frac{c_t}{w_t}, \quad L_t^* = 1 - \frac{1 - \rho}{\rho} \frac{c_t^*}{w_t^*}. \quad (12)$$

Aggregate labor supply rises with the real wage and decreases with consumption.

Consumption Euler equations in aggregate per capita terms contain an adjustment for consumption by the newborn generation at time $t + 1$:

$$c_t = \frac{1 + n}{\beta(1 + r_{t+1})} \left(c_{t+1} - \frac{n}{1 + n} C_{t+1}^{t+1} \right), \quad c_t^* = \frac{1 + n}{\alpha\beta(1 + r_{t+1})} \left(c_{t+1}^* - \frac{n}{1 + n} C_{t+1}^{t+1*} \right). \quad (13)$$

In addition to ensuring steady-state determinacy, these adjustments for consumption of newborn generations at $t + 1$ in the Euler equations for aggregate per capita consumption provide the degree of freedom necessary for existence of a well defined, *non-degenerate* steady state when discount factors differ across countries.

Newborn households hold no assets, but they own the present discounted value of their labor income. Using the Euler equation (4) and a newborn household's intertemporal budget constraint, it is possible to show that the household's consumption in the first period of its life is a fraction of its human wealth, h :

$$C_t^t = \rho(1 - \beta)h_t, \quad C_t^{t*} = \rho(1 - \alpha\beta)h_t^*. \quad (14)$$

h and h^* are defined as the present discounted values of the households' lifetime endowments of time in terms of the real wages:

$$h_t \equiv \sum_{s=t}^{\infty} R_{t,s} w_s, \quad h_t^* \equiv \sum_{s=t}^{\infty} R_{t,s} w_s^*. \quad (15)$$

The dynamics of h and h^* are described by the following forward-looking difference equations:

$$h_t = \frac{h_{t+1}}{1 + r_{t+1}} + w_t, \quad h_t^* = \frac{h_{t+1}^*}{1 + r_{t+1}} + w_t^*. \quad (16)$$

The law of motion of aggregate per capita assets *held by domestic consumers* is obtained by aggregating the budget constraint (2) across generations alive at each point in time. Using the no-arbitrage condition (5) and recalling that newborn agents hold no assets, aggregate per capita assets of domestic and foreign consumers obey, respectively:

$$\begin{aligned} (1 + n)(B_{t+1} + V_t) &= (1 + r_t)(B_t + V_{t-1}) + w_t L_t - c_t, \\ (1 + n)(B_{t+1}^* + V_t^*) &= (1 + r_t)(B_t^* + V_{t-1}^*) + w_t^* L_t^* - c_t^*, \end{aligned} \quad (17)$$

where V_t and V_t^* denote the aggregate per capita equity values of the home and foreign economies entering period $t + 1$, respectively ($V_t \equiv \frac{aV_t^i}{N_{t+1}}$, $V_t^* \equiv \frac{aV_t^{*i}}{N_{t+1}^*}$).⁷

2.5.2 Firms

Aggregate per capita output in each economy is obtained by expressing production of each differentiated good in units of the composite basket, multiplying by the number of firms, and dividing by population. It is:

$$y_t = RP_t Z_t L_t, \quad y_t^* = RP_t^* Z_t^* L_t^*. \quad (18)$$

For given employment and productivity, each country's real GDP rises with the relative price of the representative good produced in that country, as this is worth more units of the consumption basket.

Aggregate per capita labor demand is:

$$L_t = RP_t^{-\omega} \frac{y_t^W}{Z_t}, \quad L_t^* = RP_t^{*-\omega} \frac{y_t^W}{Z_t^*}, \quad (19)$$

where y_t^W is aggregate per capita world production of the composite good, equal to aggregate per capita world consumption, c_t^W . It is $y_t^W = ay_t + (1 - a)y_t^*$ and $c_t^W = ac_t + (1 - a)c_t^*$, $y_t^W = c_t^W$ to ensure market clearing.

Domestic and foreign relative prices are equal to marginal costs, because government subsidies remove the effect of the monopolistic distortion on pricing in equilibrium:

$$RP_t = \frac{w_t}{Z_t}, \quad RP_t^* = \frac{w_t^*}{Z_t^*} \quad (20)$$

In the absence of arbitrage opportunities between bonds and shares, the aggregate per capita equity value of the home and foreign economies entering period $t + 1$ must evolve according to:

$$V_t = \frac{1 + n}{1 + r_{t+1}} V_{t+1} + \frac{d_{t+1}}{1 + r_{t+1}}, \quad V_t^* = \frac{1 + n}{1 + r_{t+1}^*} V_{t+1}^* + \frac{d_{t+1}^*}{1 + r_{t+1}^*}. \quad (21)$$

where d_t and d_t^* denote aggregate per capita dividends, equal to $(1 - \tau)y_t + T_t - w_t L_t$ and $(1 - \tau^*)y_t^* + T_t^* - w_t^* L_t^*$, respectively. In equilibrium, $\tau = \tau^* = -\frac{1}{\theta - 1}$ implies $d_t = d_t^* = 0$ and $V_t = V_t^* = 0 \forall t$ in the absence of speculative bubbles.

2.5.3 The Government

The government budget constraint in aggregate per capita terms is:

$$T_t = \tau y_t, \quad T_t^* = \tau^* y_t^*. \quad (22)$$

⁷These equations hold in all periods following the initial one. The no arbitrage condition may be violated between time $t_0 - 1$ and t_0 if an unexpected shock surprises agents at the beginning of period t_0 . See Ghironi (2000) for details.

2.5.4 Net Foreign Asset Accumulation

Each country's accumulation of net foreign assets is described by an equation that combines the budget constraints of households, the fact that shares are liabilities of firms toward consumers in the respective economies, and the government budget constraint. In aggregate per capita terms, it is:

$$(1+n)B_{t+1} = (1+r_t)B_t + y_t - c_t, \quad (1+n)B_{t+1}^* = (1+r_t)B_t^* + y_t^* - c_t^*. \quad (23)$$

For asset markets to be in equilibrium, aggregate home assets (liabilities) must equal aggregate foreign liabilities (assets), *i.e.*, it must be $\widehat{B}_t + \widehat{B}_t^* = 0$. In aggregate per capita terms, it must be:

$$aB_t + (1-a)B_t^* = 0. \quad (24)$$

Using (24), the expressions in (23) reduce to $y_t^W = c_t^W$: Consistent with Walras' Law, asset market equilibrium implies goods market equilibrium, and vice versa.

3 Solution

3.1 The Steady State

Appendix A.1 contains the details of the solution for the steady state of our model. Here, we summarize its main characteristics. In what follows, we denote steady-state levels of variables with overbars.

It is known, at least since Becker (1980), that a standard representative agent model with identical discount factors across agents (*i.e.*, $n = 0$, $\alpha = 1$) results in indeterminacy of the steady-state distribution of net foreign assets. If discount factors differ across agents with no other modification to the standard model ($n = 0$, $\alpha < 1$), the distribution of wealth across agents ends up collapsing into one in which the most patient household owns all the wealth. Buiter (1981) and Weil (1989*b*) demonstrated that models with overlapping generations in which households are not linked by intergenerational altruism can deliver a non-degenerate distribution of asset holdings across countries. Our model achieves precisely the same goal by assuming $n > 0$ and absence of intergenerational linkages in the form of altruism or government transfers. Ghironi (2000) shows that, when $\alpha = 1$, this delivers a determinate steady state and stationary dynamics of prices and aggregate per capita quantities following non-permanent shocks.

To demonstrate the influence of structural asymmetry— $\alpha \leq 1$ and $\frac{\bar{Z}}{\bar{Z}^*}$ possibly different from 1—on our analysis, we start with the special case in which all preference parameters are identical across domestic and foreign households. When $\alpha = 1$, steady-state levels of labor effort are identical across countries ($\frac{\bar{L}}{\bar{L}^*} = 1$), and net foreign assets are zero ($\bar{B} = \bar{B}^* = 0$), *regardless of* relative productivity ($\frac{\bar{Z}}{\bar{Z}^*}$). This happens because, when consumers' intertemporal preferences are identical at home and abroad, given a common

world interest rate, households in the two countries have identical incentives to borrow or lend. (The desired slope of the consumption profile is the same for each domestic and foreign household.) In this case, the only possible steady-state equilibrium in the setup of this paper is one in which $\bar{r} = \frac{1-\beta}{\beta}$ and net foreign assets are zero even if $\frac{\bar{Z}}{\bar{Z}^*} \neq 1$. Domestic and foreign GDPs in units of consumption differ ($\bar{y} \neq \bar{y}^*$), and so do consumption levels ($\bar{c} \neq \bar{c}^*$). But consumption equals GDP in each country, so that net foreign assets are zero. Since $\bar{y} = \bar{w}\bar{L}$ and $\bar{y}^* = \bar{w}^*\bar{L}^*$ in equilibrium (because revenue subsidies offset monopoly power in pricing), $\bar{L} = \bar{L}^*$ when $\alpha = 1$ implies that the different GDP levels generated by different productivity levels translate into different real wages and labor incomes across countries.⁸ The more productive country has a higher steady-state real wage and consumption and a lower relative price for the same labor effort as the less productive country.⁹

In the general case $\alpha \leq 1$, Appendix A.1 proves that we can write the solution for \bar{r} , \bar{B} , and cross-country ratios of any pair of other endogenous variables \bar{x} and \bar{x}^* as functions of the steady-state productivity ratio $\frac{\bar{Z}}{\bar{Z}^*}$. The characteristics of these functions depend on the values of structural parameters, and the steady-state levels of \bar{r} , \bar{B} , and other endogenous variables can be obtained numerically given assumptions on \bar{Z} and \bar{Z}^* .¹⁰ Consider the following examples:

1. If $\alpha < 1$ and $\bar{Z} = \bar{Z}^* = 1$, plausible parameter values yield $\bar{B} > 0$ ($\bar{B}^* < 0$), $\bar{c} > \bar{c}^*$, $\bar{L} < \bar{L}^*$, $\bar{w} > \bar{w}^*$, $\bar{RP} > \bar{RP}^*$, $\bar{y} < \bar{y}^*$.¹¹ If domestic agents are more patient than foreign, they accumulate steady-state assets, which make it possible to sustain relatively higher consumption with a smaller labor effort. Lower labor supply generates a higher equilibrium real wage and relative price. The labor effort differential prevails on the relative price differential in generating lower GDP at home than abroad, where higher GDP is required to pay interest on the accumulated debt.

2. If $\alpha < 1$ and $\bar{Z} = 1 < \bar{Z}^*$, plausible parameter values yield $\bar{B} > 0$ ($\bar{B}^* < 0$), $\bar{c} < \bar{c}^*$, $\bar{L} < \bar{L}^*$, $\bar{w} < \bar{w}^*$, $\bar{RP} > \bar{RP}^*$, $\bar{y} < \bar{y}^*$.¹² Sufficiently higher productivity in the more impatient country causes the steady-state real wage differential to switch sign, so that the real wage is now higher in the foreign economy. This induces foreign agents to consume

⁸We assume that labor does not move across countries. Given a steady-state real wage differential, we motivate absence of long-run labor flows by appealing to the presence of unspecified costs of relocating abroad that more than offset the welfare differential implied by differences in real wages.

⁹If $\alpha = 1$ and $\bar{Z} = \bar{Z}^* = 1$, the steady state is symmetric in all respects: $\bar{r} = \frac{1-\beta}{\beta}$, $\bar{B} = \bar{B}^* = 0$, $\bar{c} = \bar{c}^* = \bar{L} = \bar{L}^* = \bar{y} = \bar{y}^* = \rho$, $\bar{w} = \bar{w}^* = \bar{RP} = \bar{RP}^* = 1$. See Ghironi (2000) for the details of the solution in this case.

¹⁰For the reasons discussed above, the functions defined in the appendix are such that, if $\alpha = 1$, it is $\bar{B} = 0$, $\frac{\bar{L}}{\bar{L}^*} = 1$, and $\frac{\bar{c}}{\bar{c}^*} \geq 1$ if $\bar{Z} \geq \bar{Z}^*$.

¹¹These results arise with the benchmark parameterization we discuss below ($\beta = .99$, $\omega = 3$, $a = .5$, $\rho = .33$, $\alpha = .9999$, $n = .01$) as well as under a number of other plausible parameterizations.

¹²The same parameter values as in the previous example and $\bar{Z} = 1$, $\bar{Z}^* = 1.29$ yield these results. (See below on the choice of \bar{Z}^* .)

more, and their consumption rises above that of domestic agents, with an increase in the size of the foreign economy's debt.

When $\alpha < 1$, $\beta(1 + \bar{r}) > 1$ and $\alpha\beta(1 + \bar{r}) < 1$. In conjunction with the Euler equation (4) and its foreign counterpart, this implies that the steady-state consumption profiles of *individual* home households display an upward tilt, whereas there is a downward tilt in the steady-state consumption profiles of foreign households. The Euler equation and the labor-leisure tradeoff for an individual household make it possible to verify that the steady state is also characterized by a downward (upward) tilt in the labor effort of individual home (foreign) households. Even if consumption is increasing and labor effort is decreasing relative to the previous period for each individual home household in steady state, and opposite tilts characterize foreign households, the entry of new households with no assets in each period ensures that aggregate per capita consumption and labor effort are constant. Appendix A.1 shows that the tilt of individual consumption profiles determines whether a country is a steady-state creditor or debtor. Given a constant real wage, the only way for home households to sustain an increasing consumption profile with decreasing labor effort is by accumulating assets. Since there is no home household with negative financial assets in steady state, home aggregate per capita net foreign assets must be positive. As we shall see, the tilt of steady-state individual consumption profiles has important consequences for the dynamics of the economy in response to shocks.

3.2 The Log-Linear Solution

The aggregate model of Section 2.2 can be safely log-linearized around the steady state. The assumptions that $n > 0$ and newborn households enter the economy with no assets generate stationary model dynamics following non-permanent shocks because the steady state is uniquely determined.¹³ We present the log-linear equations in Appendix A.2. The log-linear model can then be solved with the method of undetermined coefficients following Campbell (1994). In what follows we use sans serif fonts to denote percentage deviations from the steady state and focus on the model solution in terms of the minimum state vector, which at time t consists of the predetermined levels of net foreign assets and the (*gross*) risk-free real interest rate (the endogenous states) and the current levels of domestic and foreign productivity (the exogenous states), *i.e.*, $[\mathbf{B}_t, \mathbf{r}_t, \mathbf{Z}_t, \mathbf{Z}_t^*]'$.¹⁴ The

¹³In the representative agent model with $n = 0$, the consumption differential across countries is a random walk. All shocks have permanent consequences via wealth redistribution regardless of their nature. In a stochastic setting, the unconditional variance of endogenous variables is infinite, even if exogenous shocks are bounded. Log-linearization is not a reliable solution technique in this case.

¹⁴Ghironi (2000) shows that the log-linear model has a unique solution when $\alpha = 1$ and steady-state productivities are equal across countries. (In that case, percentage deviations of net foreign assets are defined around the steady-state level of consumption, and the log-linear international asset market equilibrium condition is $a\mathbf{B}_t + (1 - a)\mathbf{B}_t^* = 0$.) While we cannot verify determinacy analytically when the steady state is asymmetric, we conjecture that determinacy of the solution is preserved for α close to 1 and steady-state productivities that are not too far from each other. Indeed, we do not find an excessive

solution of the model can then be written as:

$$\begin{aligned}
\mathbf{B}_{t+1} &= \eta_{BB}\mathbf{B}_t + \eta_{Br}r_t + \eta_{BZ}\mathbf{Z}_t + \eta_{BZ^*}\mathbf{Z}_t^*, \\
r_{t+1} &= \eta_{rB}\mathbf{B}_t + \eta_{rr}r_t + \eta_{rZ}\mathbf{Z}_t + \eta_{rZ^*}\mathbf{Z}_t^*, \\
\mathbf{x}_t &= \eta_{xB}\mathbf{B}_t + \eta_{xr}r_t + \eta_{xZ}\mathbf{Z}_t + \eta_{xZ^*}\mathbf{Z}_t^*, \\
\mathbf{x}_t^* &= \eta_{x^*B}\mathbf{B}_t + \eta_{x^*r}r_t + \eta_{x^*Z}\mathbf{Z}_t + \eta_{x^*Z^*}\mathbf{Z}_t^*,
\end{aligned} \tag{25}$$

where \mathbf{x}_t and \mathbf{x}_t^* are any pair of endogenous variables other than net foreign assets and the interest rate, and the η 's are elasticities of endogenous variables to the endogenous and exogenous components of the state vector. We assume that productivity levels at home and abroad obey the following processes in all periods after the time of an initial impulse ($t = 0$ in the impulse responses below):

$$\mathbf{Z}_t = \phi\mathbf{Z}_{t-1}, \quad \mathbf{Z}_t^* = \phi\mathbf{Z}_{t-1}^*, \quad 0 \leq \phi \leq 1.$$

If $\phi = 1$, impulses to productivity cause the economy to eventually settle at a new steady state that differs from the initial one.

Two important implications emerge from our model. First, non-zero steady-state net foreign assets introduce an additional channel through which the past history of the economy matters for current dynamics relative to the model with zero steady-state assets. The predetermined, risk-free interest rate is an additional state variable in the solution. The intuition is simple. If steady-state net foreign assets are zero (if $\alpha = 1$), the effect of the interest burden on previously accumulated debt is lost in the log-linearization of the laws of motion for domestic and foreign net foreign assets in (23). This is no longer the case when steady-state assets differ from zero, as forcefully argued by Lane and Milesi-Ferretti (2001, 2002*a, b*). This implies that the effect of net foreign asset accumulation on cross-country differences in the levels of other endogenous variables is amplified relative to a model with zero steady-state net foreign assets.

Second, the deviation of net foreign assets from the steady-state in equation (25) can no longer be written as a function of the cross-country productivity differential. As a consequence, worldwide productivity shocks, which have no impact on the current account in the symmetric version of the model, affect net foreign asset accumulation, both through their impact on the world interest rate and the interest rate burden (or income) on previously accumulated assets and through terms of trade effects (see below on this). Several tests of the intertemporal model of the current account are based on the premise that global shocks should have no impact on the current account of an open economy (e.g., Glick and Rogoff, 1995). However, this hypothesis is frequently rejected by the data, resulting in what is viewed as a ‘‘puzzle.’’ Our analysis suggests that variable interest rate effects on outstanding debt and terms of trade dynamics may at least partly explain these findings.

number of stable roots when solving the model numerically.

These novel mechanisms demonstrate the advantages of our theoretical framework and its empirical relevance. The ultimate causes of differences in net foreign asset positions have implications for how we interpret potentially different responses of consumption levels of debtor and creditor countries to disturbances in the data. For instance, we shall argue that empirical net foreign asset and consumption responses to world productivity shocks can be reconciled with the theory in our model.

As we discussed, our analysis highlights a number of issues that are overlooked by small open economy models that do not incorporate a theory of determination of the economy's long-run position to be used in the interpretation of heterogeneous dynamics. Indeed, one could argue that heterogeneity is an inherent feature of small open economy models. However, these models are also frequently used to estimate the familiar present value models of the current account for large economies such as Germany, Japan, and the U.S., and they should be best viewed as “short-cuts” in relation to a more general model.

4 Impulse Responses

In this section, we discuss heterogeneity in responses to shocks using the impulse responses implied by a plausible parameterization of the model. This substantiates the discussion in Section 3.2 and helps us build intuition to interpret the empirical counterparts of these responses in Section 5.

We interpret periods as quarters and choose the following benchmark parameter values: $\beta = .99$ (a standard choice), $\alpha = .9999$ (so that the foreign discount factor is .9899), $\omega = 3$, $\rho = .33$, $a = .5$ (countries have equal size), $n = .01$, $\bar{Z} = 1$, and $\bar{Z}^* = 1.29$.

We choose α very close to 1 because even small differences between the foreign and home discount factors result in very large steady-state net foreign asset positions in the model of this paper. To avoid overstating the effect of interest rate changes, we choose a value of α such that the long-run ratio of debt to quarterly GDP for the foreign economy is 137 percent, or approximately 35 percent on an annualized GDP basis. We discuss the consequences of lower values of α below. The value of ω is in (the lower portion of) the range of estimation results from the trade literature on the U.S. and OECD countries (Feenstra, 1994; Harrigan, 1993; Shiells, Stern, and Deardorff, 1986; Trefler and Lai, 1999).¹⁵ The choice of ρ implies that households in both countries spend one third of their time working in the symmetric steady-state world. $\alpha < 1$ yields a steady-state employment differential. The choice of n is higher than realistic, at least if one has developed economies in mind and n is interpreted strictly as the rate of growth of population.¹⁶ Extending the model to incorporate probability of death as in Blanchard

¹⁵Ghironi (2000) shows that lower (higher but finite) values of ω reduce (amplify) the elasticities of cross country differentials to net foreign asset accumulation in the symmetric version of the model. Consistent with Cole and Obstfeld (1991) and Corsetti and Pesenti (2001), there is no role for asset accumulation if $\omega = 1$ and steady-state assets are zero.

¹⁶The average rate of quarterly population growth for the U.S. between 1973:1 and 2000:3 has been

(1985) would make it possible to reproduce the dynamics generated by $n = .01$ with a lower rate of entry of new households by choosing the proper value of the probability of death. The choice of $n = .01$ thus mimics the behavior of a more complicated, yet largely isomorphic setup.

Our choice of parameter values is plausible if we think of the more impatient economy as the U.S., consistent with the evidence in favor of a lower propensity to save for U.S. households relative to European and Asian ones. As for the steady-state productivity differential, our data suggest that, on average, U.S. productivity has been 29 percent higher than in the rest of the G7.

The parameter values above result in the steady-state configuration of Example 2 above: $\bar{B} > 0$ ($\bar{B}^* < 0$), $\bar{c} < \bar{c}^*$, $\bar{L} < \bar{L}^*$, $\bar{w} < \bar{w}^*$, $\overline{RP} > \overline{RP}^*$, $\bar{y} < \bar{y}^*$. Relative consumer impatience causes the model-U.S. economy to accumulate a steady-state debt against the rest of the world. Nevertheless, higher productivity results in higher real wage, GDP, consumption, and labor effort (the latter is higher than abroad for the need to pay interest on the accumulated debt). Larger U.S. GDP comes with a lower price of U.S. goods relative to the patient economy (home). Numerical values for the steady-state levels of variables are in Table 1, which also displays the values of the elasticities of endogenous variables to the state vector in the model solution.

We consider the following shocks below: country-specific shocks with persistence $\phi = .9$ and a permanent, aggregate shock to world productivity. In all cases, we consider one percent initial impulses. In the case of country-specific shocks, persistence .9 is at the lower end of the range that is usually considered by the international real business cycle literature (e.g., Baxter and Crucini, 1995). (In fact, all our country labor productivity series have auto-regressive parameters between .9 and .97 over the sample period of estimation.) We focus on a permanent world productivity shock for consistency with our empirical work.

4.1 A Productivity Shock in the Creditor Country

Figure 1 shows the impulse responses to a 1 percent increase in productivity in the more patient, less productive country (home). (For ease of interpretation of the empirical results in Section 5, one can think of the more patient, less productive country as Japan.)

Appendix A.3 derives the fundamental current account equation for the home economy in our model along the lines of Obstfeld and Rogoff (1996). It is:

$$CA_t = (r_t - \tilde{r}_t) B_t + w_t - \tilde{w}_t + \frac{\tilde{\Gamma}_t - 1}{\tilde{\Gamma}_t} (\tilde{r}_t B_t + \tilde{w}_t), \quad (26)$$

where $\tilde{x}_t \equiv \frac{\sum_{s=t}^{\infty} R_{t,s} x_s}{\sum_{s=t}^{\infty} R_{t,s}}$ is the permanent, or annuity, level of the variable x_t , and $\tilde{\Gamma}_t \equiv \frac{1}{(1-\beta) \sum_{s=t}^{\infty} R_{t,s}}$. Net foreign asset accumulation is driven by a pure consumption *smoothing*

.0025.

motive and a consumption *tilting* motive generated by the discrepancy between subjective and market discounting. When $B_t > 0$, an increase in the interest rate above its permanent level causes a current account surplus as agents smooth the effect of higher asset income on consumption. Similarly, a real wage above its permanent level induces a surplus through smoothing. The term $\frac{\tilde{\Gamma}_t - 1}{\tilde{\Gamma}_t} (\tilde{r}_t B_t + \tilde{w}_t)$ is due to consumption tilting. Since the home economy is relatively patient, tilting pushes the current account into a steady-state surplus. As long as $\tilde{\Gamma}_t > 1$, *ceteris paribus*, consumption tilting contributes to increases in the current account surplus. A similar equation holds for the foreign economy, and international equilibrium requires $aCA_t + (1 - a)CA_t^* = 0$.

Appendix A.4 shows that, if α is close to 1 and steady-state home labor effort is close to foreign (a condition that is satisfied in our example), the solution for the risk-free, world interest rate in (25) can also be written approximately as:

$$\begin{aligned} r_{t+1} &\approx a \frac{\bar{w}}{\bar{w}W} (Z_{t+1} - Z_t) + (1 - a) \frac{\bar{w}^*}{\bar{w}^*W} (Z_{t+1}^* - Z_t^*) \\ &= -a \frac{\bar{w}}{\bar{w}W} (1 - \phi) Z_t + (1 - a) \frac{\bar{w}^*}{\bar{w}^*W} (1 - \phi) Z_t^*, \end{aligned} \quad (27)$$

which reduces to $r_{t+1} = -a \frac{\bar{w}}{\bar{w}W} (1 - \phi) Z_t$ in the case of a home shock.¹⁷ Since home productivity is expected to decrease and return to the steady state over time after the initial shock, the real interest rate falls on impact and returns to the steady state monotonically.

The shock causes home agents to accumulate more assets. In equation (26), smoothing the consequences of a temporarily higher wage and favorable wage effects through the tilting term holding $\tilde{\Gamma}$ at its steady-state level prevail on an initial drop in interest income and the unfavorable impact of a lower interest rate on the tilting term. Symmetrically, the foreign country's debt increases as foreign agents borrow to share the beneficial effect of higher home productivity and sustain higher consumption with unchanged foreign productivity.¹⁸ Net foreign asset accumulation peaks approximately 6 years after the shock. After that time, net assets return to the steady state gradually.

Increased supply of home goods at any given level of labor effort results in a lower (higher) relative price of home (foreign) goods, *i.e.*, a deterioration of home's terms of trade ($\frac{RP_t}{RP_t^*}$), on impact. This expands (lowers) the demand for home (foreign) labor and causes the home real wage to increase. The foreign real wage increases as a consequence of optimal pricing by foreign firms: In the absence of changes in foreign productivity, the relative price of foreign goods and the foreign real wage are tied to each other.

As the shock dies out, dynamics at home and abroad are driven by net foreign assets. Consistent with empirical evidence on the "transfer problem" in Lane and Milesi-Ferretti

¹⁷The approximation is accurate to the fifth decimal point for the parameterization in our example.

¹⁸International equilibrium implies $B_t^* = B_t$, where B_t^* is the percentage change in the foreign country's debt. Hence, we omit the response of B_t^* from the figures. Figures 1, 2, and 4 show net foreign assets *at the end* of the corresponding period. For this reason, home net foreign assets are denoted with $B1$ in the figures. Appendix A.3 contains details on the derivation of the results in this paragraph.

(2000), *ceteris paribus*, the effect of asset accumulation is to appreciate the terms of trade of the home economy. The intuition is as follows. The effect of wealth accumulation in the home economy is that agents can sustain a higher level of consumption with lower labor effort than in steady state. (The effect of a larger stock of assets prevails on that of a lower interest rate on those assets.) For this reason, labor effort at home falls below the steady state approximately 6 and a half years after the shock in Figure 1. Eventually, lower labor supply translates into less supply of home goods and an increase of their relative price above the steady state. The home labor effort (relative price) returns to the steady state from below (above) as net foreign assets return to their long-run level.

The opposite dynamics take place in the foreign economy: More debt eventually forces foreign agents to increase their supply of labor above the steady state in order to sustain consumption and pay interest. In turn, this lowers the relative price of foreign goods (and the foreign real wage) below the steady state. The foreign labor effort (relative price and real wage) then return to the steady state from above (below). A higher relative price of home goods combined with a lower relative price of foreign goods amounts to appreciation of home's terms of trade.

Home GDP rises above the steady state on impact: Higher productivity and labor effort more than offset the effect of a lower relative price on the consumption-value of home production. Foreign GDP falls, because labor supply falls by more than the increase in the relative price of foreign goods. Eventually, the wealth effects described before cause home (foreign) GDP to fall (rise) below (above) the steady state, from where it returns to its long-run level.

Consumption rises on impact in both countries—though, of course, it rises by more in the home economy. Even if home GDP returns to the steady state from below, home consumption does not fall below the steady state during the transition dynamics. The reason is that increased net foreign assets allow home agents to sustain higher consumption directly and indirectly (by keeping the real wage above the steady state for the length of the transition through lower supply of home labor) even if the consumption value of home output is below the steady state. Instead, foreign consumption falls below the steady state approximately 8 years after the initial shock and returns to the steady state from below. A larger debt eventually causes lower consumption and more supply of labor in the foreign economy until the steady state is reached. The consumption value of foreign output rises as foreign agents supply more effort to smooth the decrease in consumption caused by the debt burden and to drive the latter back to the steady state.

4.2 A Productivity Shock in the Debtor Country

Figure 2 shows the impulse responses to a 1 percent increase in productivity in the more impatient, more productive country (foreign).

Foreign debt decreases as foreign agents smooth the effect of a temporarily higher wage and lower interest rate on consumption. (In addition to these, there is a favorable

current account effect through a decrease in the tilt coefficient $\frac{\tilde{\Gamma}_t^* - 1}{\Gamma_t^*}$ for given interest burden of steady-state debt in the fundamental equation for the foreign current account.) The real interest rate falls and returns to the steady state as the shock dies out. The relative price of foreign goods falls, which leads to more demand of labor effort in the foreign economy and a higher real wage. Dynamics in the home economy mirrors those of foreign variables: The relative price of home goods rises, with a contractionary effect on labor demand. (The real wage increases at home as it is tied to the home relative price.) Foreign GDP increases, whereas home GDP falls, and consumption rises above the steady state in both countries, but does so by more in the foreign country.

Wealth effects dominate the dynamics as the shock dies out. Foreign labor supply decreases below the steady state, because debt is smaller. This results in a higher relative price of foreign goods, combined with a lower consumption value of foreign production as the economy returns to the steady state. A smaller debt implies that foreign consumption returns to the steady state from above. A smaller asset stock is the source of mirroring dynamics at home, where labor supply increases above the steady state, resulting in a lower relative price of home goods but a higher consumption value of production, with consumption that returns to the steady state from above.

4.3 Creditor vs. Debtor Responses

A result that emerges from figures 1 and 2 is that the dynamics after productivity shocks in creditor and debtor countries are not different on *qualitative* grounds. The main difference is *quantitative*. As Table 1 and figures 1 and 2 show, a 1 percent productivity shock in the creditor country (home) causes home assets (foreign debt) to increase by more than a 1 percent productivity shock in the debtor country (foreign) causes foreign debt (home assets) to decrease. Intuitively, foreign agents are more impatient. Hence, they have a smaller incentive to save a portion of the increase in income in the form of lower debt. For the same reason (a stronger incentive to save in the home economy), a home shock causes home consumption to increase by less than a foreign shock causes foreign consumption to increase, and the home shock causes foreign consumption to increase by more than the foreign shock causes home consumption to increase. These differences in elasticities would not exist if it were $\alpha = 1$ and $\bar{Z}^* = \bar{Z}$. In response to shocks of identical relative magnitudes, the home shock would cause home (foreign) consumption to increase by the same amount as the foreign shock causes the foreign (home) consumption to increase. Heterogeneity in the structural characteristics of the two economies that results in an asymmetric steady state also implies quantitative, if not qualitative, heterogeneity in the short-run responses of countries to temporary, country-specific shocks.

Asymmetry of the steady state is responsible for the small quantitative difference in the response of the world interest rate to domestic and foreign productivity shocks (recall equation (27)). Heterogeneity in consumption responses translates into heterogeneity in relative price, employment, and GDP dynamics. In particular, asymmetric demand effects

of shocks generate heterogeneity in own and foreign relative price responses: A home productivity shock causes the home relative price to decrease by more than a foreign shock does to the foreign price. The home shock causes the foreign price to increase by less than the foreign shock does to the home price. The reason is that the foreign shock causes world consumption to increase by more than the home shock does. In the foreign economy, this implies less pressure to lower prices in response to higher productivity. In the home economy, the foreign shock results in a stronger incentive to raise prices in response to higher demand.¹⁹ As a consequence of asymmetric relative price effects, a home shock expands home employment and GDP by more than a foreign shock does for foreign employment and GDP, and the home shock lowers foreign employment and GDP by less than the foreign shock lowers home employment and GDP.

Kraay and Ventura (2000) argue that a favorable productivity shock should cause surplus (deficit) in a creditor (debtor) country. We do not reach the same conclusion in the case of country-specific shocks. Regardless of whether the shock takes place in the creditor or in the debtor country, the country that experiences higher productivity responds by improving its foreign asset position (either by accumulating more assets or by reducing its debt) to smooth the effect of the shock on consumption.²⁰

4.4 A Permanent World Productivity Shock

In the familiar case of a symmetric steady state with zero net foreign assets and equal productivity levels at home and abroad, a permanent increase in world productivity results in no movement in net foreign assets. GDP, the real wage, and consumption in both countries increase immediately by the full amount of the shock. There are no changes in labor effort and relative prices. Anticipating the permanent consequences of the shock, agents in both countries simply find it optimal to consume the entire consumption value of the increase in productivity in all periods without adjusting their labor effort. Symmetry of the shock across countries results in no movement in the terms of trade.²¹

In contrast, asymmetry of the steady state results in interesting dynamics following a permanent shock to world productivity. This is illustrated in Figure 3, which shows

¹⁹In the symmetric steady-state case, the elasticity of RP to Z is identical to the elasticity of RP* to Z*, and the same is true of the elasticity of RP* to Z and RP to Z*. Not only, in the symmetric case, all these elasticities have the same absolute value: A home (foreign) productivity shock lowers the home (foreign) relative price exactly by the same amount as it increases the foreign (home) price.

²⁰We attribute this difference to the absence of physical capital accumulation from our model. Including capital in the production function would make it possible to generate current account deficits in response to favorable productivity shocks through the resulting increase in investment.

²¹In the case of a permanent asymmetric shock—say, to home productivity—net foreign assets do not move, as home agents still find it optimal to consume the entire value of the shock in all periods without changing their labor effort. However, consumption and GDP increase by less than the shock, because the terms of trade of the home economy deteriorate due to the relative increase in the supply of home goods. See Ghironi (2000) for details.

the impulse responses to a 1 percent, permanent increase in productivity at home and abroad.

The home economy accumulates assets over time in response to the shock, the foreign economy accumulates debt. Eventually, the increase in home assets (foreign debt) converges to an amount equal to the increase in world productivity. A permanent productivity shock has no effect on the risk-free interest rate (equation (27)). Therefore, the dynamics in Figure 3 do not originate in the effect of changes in the interest rate on the burden of (income from) the initial steady-state debt (assets). Home and foreign households find it beneficial to engage in further asset trade without changes in the interest rate due to asymmetric income effects that stem from different discounting of future income across countries and the implied tilt in the consumption profiles of individual households.

Appendix A.3 shows that the home current account after a permanent productivity shock that has no effect on the world interest rate is determined by:

$$CA_t = w_t - \tilde{w}_t + \frac{\beta(1 + \bar{r}) - 1}{1 + \bar{r}} W_t, \quad (28)$$

where \tilde{w}_t is the annuity value of the real wage at the steady-state interest rate \bar{r} ($\tilde{w}_t \equiv \frac{\bar{r}}{1 + \bar{r}} \sum_{s=t}^{\infty} \left(\frac{1}{1 + \bar{r}}\right)^{s-t} w_s$), and W_t is beginning-of-period aggregate per capita wealth at the interest rate \bar{r} ($W_t \equiv (1 + \bar{r}) B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1 + \bar{r}}\right)^{s-t} w_s$). The foreign current account obeys a similar equation, with $\alpha\beta$ replacing β , and it satisfies the constraint $aCA_t + (1 - a)CA_t^* = 0$.

As with equation (26), if the real wage is above its permanent level and is expected to decline, consumption smoothing pushes the current account into surplus. Relative patience of home households implies an upward tilt in individual household consumption profiles since $\beta(1 + \bar{r}) > 1$. Therefore, *ceteris paribus*, consumption tilting contributes to home current account surplus. Conversely, the downward tilt of foreign household consumption profiles implied by $\alpha\beta(1 + \bar{r}) < 1$ pushes the foreign current account in the direction of deficit.

At time 0, when the shock happens, home wealth W unambiguously increases, because the real wage w increases in all periods. Given $\beta(1 + \bar{r}) > 1$, this tends to increase the home current account through the consumption tilting channel. However, the path of the home real wage in Figure 3 is *increasing* over time. Therefore, the consumption smoothing channel in (28) would dictate that the home current account should worsen. What we observe is an improvement in home's net foreign asset position, *i.e.*, an increase in home's current account above the steady state. Based on equation (28), this is driven by the fact that the consumption tilting channel prevails on the pure smoothing one. The steady-state incentive of home households to postpone consumption implicit in the upward tilt of individual home consumption profiles results in home households lending more than in the initial steady state. Conversely, the relative impatience of foreign consumers induces them to anticipate consumption and borrow more against their permanently higher human wealth.

The relative price of home goods falls, because the home real wage does not increase as much as productivity on impact; the relative price of foreign goods rises, yielding a deterioration of home's terms of trade. Relative prices return to the steady state over time. There is no long-run effect of the permanent change in asset positions on relative prices because the permanent, worldwide productivity shock eventually results in GDP and consumption increases of the same size as the shock at home and abroad.

The difference in short-run relative price movements across countries originates in the different consumption responses to the shock generated by relative patience versus impatience and their general equilibrium consequences for labor effort and real wages. At time 0, consumption increases in both countries, but it does so by less than the full amount of the productivity shock at home and by more in the foreign economy. Consumption then increases over time in the home economy and decreases abroad, as foreign households must pay interest on an increasing debt. In the long run, the consumption increase in both the home and foreign countries reflects the full amount of the world shock.

Equilibrium labor effort increases at home (decreases abroad), reflecting the expansionary (contractionary) effect of a lower (higher) relative price on labor demand. After the initial jump, labor effort slowly returns to the original steady state in both countries. The real wage increases at home and abroad. It increases by more in the foreign economy, which explains the increase in the foreign relative price, and the decrease in equilibrium labor effort in the foreign country. Both the domestic and the foreign real wages converge over time to a higher steady-state level that reflects the full amount of the world productivity shock. As consumption, the domestic real wage increases over time, the foreign real wage decreases.

GDP also increases in both countries. As for consumption, in the long run, the increase reflects the full amount of the world shock (both relative prices and labor effort return to their original levels). In the short run, GDP increases by more in the patient, less productive country (home) and then decreases toward the new steady state level. Foreign GDP increases over time. Therefore, changes in labor effort prevail on relative price movements in determining the direction of GDP changes.

The key for the dynamics in Figure 3 is the difference in consumption responses implied by patience versus impatience, the lending and borrowing that this generates, and the adjustment of relative prices and the terms of trade that takes place as a consequence. When households in different countries capitalize wealth effects differently due to heterogeneity in subjective discount factors, long-run consumption differs from long-run labor income in each country, and even symmetric, permanent productivity shocks end up redistributing demand across countries in a way that induces agents to adjust their labor effort over time rather than keeping it unchanged. Consumption tilting then results in accumulation of assets (or debt) during the transition dynamics. In the long run, the foreign economy has a permanently larger debt—and its new long-run consumption and GDP levels remain higher than those at home, as in the initial steady state.

Unlike for country specific shocks, we do replicate Kraay and Ventura's (2000) pattern

in the case of a permanent increase in world productivity: As Figure 3 shows, optimal consumption behavior leads the creditor country to respond by accumulating assets, whereas the debtor country responds by running an increasing debt. Thus, a consumption-driven, intertemporal approach to the current account that explicitly accounts for structural, cross-country heterogeneity can explain the regularity documented by Kraay and Ventura at least for the case of permanent world-wide shocks.

4.5 Relative Steady-State Productivity vs. Impatience

How are the results above affected by changes in relative steady-state productivity and/or in the relative degree of impatience in the foreign economy? For example, given $\alpha < 1$, does it matter for the responses to productivity shocks whether $\frac{\bar{Z}}{\bar{Z}^*}$ is equal to or different from 1? As we observed above, whether $\bar{Z} = \bar{Z}^*$ or $\bar{Z} < \bar{Z}^*$ matters for the sign of some cross-country steady-state differentials. (Recall examples 1 and 2 in our discussion of the steady state.) If $\bar{Z} = \bar{Z}^*$, it is no longer the case that $\bar{c} < \bar{c}^*$ and $\bar{w} < \bar{w}^*$. Steady-state home consumption and real wage are higher than abroad if the two countries are equally productive. To investigate the effect of this change on impulse responses, we re-calculated the responses under the assumption $\bar{Z} = \bar{Z}^* = 1$, keeping the values of the structural parameters unchanged. Although the exercise resulted in some quantitative differences in the responses to productivity shocks in the creditor or debtor country, no qualitative difference emerged. The responses were similar to those in figures 1 and 2.²²

What about the effect of a lower value of α ? We know that this will increase the size of foreign steady-state debt. To verify the effect of this change, we re-calculated the responses to country-specific shocks under the assumption $\bar{Z} = 1 < \bar{Z}^* = 1.29$ and the same parameter values as above, but with $\alpha = .999$. In the case of a productivity shock in the debtor country, no qualitative change in the impulse responses is observed and the same intuitions as for Figure 2 apply. The main difference relative to Figure 2 is that the foreign relative price rises above the steady state and the foreign labor effort and GDP fall below earlier than in the benchmark case. (Similarly, the home relative price falls and home labor effort and GDP rise above the steady state earlier than in Figure 2.) The consumption differential is somewhat amplified.

The case of a productivity shock in the creditor country is more interesting and is shown in Figure 4. The following main differences emerge relative to Figure 1. Initially, the home economy accumulates assets to smooth the effect of the shock on consumption, as in Figure 1. However, in Figure 4, home assets fall below the steady state less than two years after time 0 and return to the steady state from below. RP now converges to the steady state from below, L and y from above. Similarly, RP* now converges to the steady state from above, L* and y* from below. The foreign real wage returns to the steady state from above. Foreign consumption now rises above home consumption on impact, and

²²The responses for $\bar{Z} = \bar{Z}^* = 1$ and others not included in the paper are available on request.

both c and c^* return to the long-run level from above.

The intuition for the differences between figures 1 and 4 is as follows. Other things given, y^* below and c^* above the steady state would cause foreign debt to increase. However, the effect of deviations of GDP and consumption from the steady state on debt dynamics must be weighed by the ratios $\frac{\bar{y}^*}{\bar{B}^*}$ and $\frac{\bar{c}^*}{\bar{B}^*}$, respectively.²³ When α is .999 rather than .9999, the implied increase in steady-state foreign debt (\bar{B}^*) causes both of these ratios to be extremely small. As a consequence, debt dynamics after time 0 end up mirroring the dynamics of the real interest rate. Put differently, when steady-state debt is very large, the interest burden on previously accumulated debt becomes the main determinant of debt dynamics. As the interest rate falls and is below the steady state throughout the transition to the long run, foreign debt decreases after the initial increase, and it returns to the steady state from below. A lower interest rate burden allows more impatient foreign households (who anticipate that both the interest rate and debt will be below the steady state for the longer portion of the transition) to increase their consumption above that of home households by borrowing more in the initial periods. In terms of the fundamental current account equation for the foreign economy, the steady-state downward tilt in the consumption profiles of foreign households and a temporarily stronger tilt incentive to consume out of given wage due to lower interest rate are responsible for increased borrowing in the initial periods. Smoothing the consequences of higher wage and lower interest burden on debt improves the current account relative to the steady state in later periods. In turn, foreign debt dynamics are responsible for the dynamics of other endogenous variables once the productivity shock has died out.²⁴

The analysis of this subsection leads us to conclude that the discount factor differential is a more important determinant of model dynamics than relative steady-state productivity. This is so because changes in the degree of relative impatience have a large impact on steady-state net foreign assets, which can amplify the role of the interest burden on previously accumulated debt in the determination of future asset holdings. When α is as “low” as .999, the importance of interest payments becomes paramount, leading to the dynamics in Figure 4.

5 Empirical Analysis

In this section we first set up an empirical framework to analyze country-specific and world productivity shocks. We then use this framework to estimate the empirical impulse responses of endogenous variables to these shocks and compare them with those based on the model. This comparison allows us to assess whether our theoretical model can account for net foreign asset and consumption dynamics in the data.

²³See Appendix A.2. $\frac{\bar{y}^*}{\bar{B}^*}$ and $\frac{\bar{c}^*}{\bar{B}^*}$ are now approximately equal to $-.0764$ and $-.0757$, respectively.

²⁴As for the case $\alpha = .9999$, setting $\bar{Z} = \bar{Z}^* = 1$ does not generate any qualitative change in impulse responses to country-specific shocks relative to the situation in which $\bar{Z} = 1 < \bar{Z}^* = 1.29$.

We should mention at the outset that an exact mapping of our stylized theoretical setup onto the empirical model is not feasible. So, we focus on the qualitative aspects of our empirical and theoretical results. Indeed, the creditor and debtor country distinction in the theoretical analysis has sharper qualitative implications for the global shocks. Thus, our empirical results provide more insight about alternative explanations when we analyze permanent global shocks. The model based responses to temporary country-specific shocks also differ across debtor and creditor countries, but the differences are only quantitative. Consequently, our empirical model has somewhat less power in discriminating between these differences, and across alternative interpretations.

As “home” economies, we focus on the Germany, Japan, and the U.S., the three largest and most advanced economies (G3). To pair each of these home economies with a “foreign” economy representative of the rest of the world, we aggregate data for the group of the seven largest industrial countries (G7) excluding the home economy in question. The empirical analysis is then conducted on each of these ‘country’ pairs and by including the same set of variables for the home and the foreign economy.²⁵

Much empirical literature has focused on the influence of consumption smoothing on current account dynamics, and indeed consumption smoothing is the source of net foreign asset accumulation in our theoretical framework. Consistent with the solution of the log-linear model, we focus on the behavior of home and foreign consumption as determined by the minimum state vector, which consists of four variables: home and foreign productivity, net foreign assets, and the risk-free real interest rate. Therefore, in our empirical analysis, we augment the minimum state vector by home and foreign consumption and estimate the dynamic responses of the augmented, six-variable vector to exogenous productivity shocks. In the empirical analysis, all these variables are in natural logarithms.²⁶

The empirical analysis is based on a quarterly data set we constructed for this purpose. The primary sources are (i) the OECD, *Analytical Database*, which provides comparable data on *business sector* output, consumption, employment and hours worked, and (ii) quarterly net foreign assets constructed by Christopher Baum based on the annual series of Lane and Milesi-Ferretti (2001). Appendix B provides more details on the data sources and variables. The available sample period for all series constructed is 1977:Q1–1997:Q4. Labor productivity is business sector real output per hour worked. Net foreign asset data are vis-à-vis the rest-of-the-world (not the remaining G7) and are re-scaled so that the natural logarithm can be taken for all series. The real interest rate is *ex post* and a country-specific measure. As already noted, labor productivity and consumption series for the rest-of-the-world economy are sums (rather than weighted averages) of the variables of interest converted into U.S. dollars at constant PPP exchange rates.

²⁵It is understood that since we can swap country names in the theoretical model without loss of generality (making home the relatively impatient, more productive economy) each of the of the G3 “home” countries may be interpreted as the home or the foreign country in the current wording of the model.

²⁶Recall that the model solution is in terms of percent deviations from the steady-state.

Consumption and net foreign assets are in per capita terms.

The estimation of the empirical responses to productivity shocks is based on a vector autoregressive (VAR) approach. Standard selection criteria suggested that a lag length of two for all variables was appropriate. To ensure normality, no autocorrelation, and homoscedasticity, we also included a set of seasonal dummy variables and an impulse dummy for the German unification (January 1991).²⁷ Finally, to avoid introducing too many dummy variables in these VARs (especially for the U.S.), we report results estimated on the sub-sample 1980:Q4–1994:Q4. Thus, our sample period stops right before the beginning of the recent period of productivity growth acceleration in the U.S.²⁸

All empirical models that distinguish between country-specific and global productivity shocks have to make important choices to identify and thus measure these shocks, and ours is no exception. One popular strategy is to identify the shocks outside the empirical model based on some independent measurement method. By contrast, we identify country-specific and global shocks by using restrictions that are consistent with our theoretical framework, as well as with a broader set of models. This choice allows our empirical response functions to be more comparable to the theory we developed to interpret the data.

We identify country-specific and global shocks by relying on two different sets of restrictions imposed on the same VARs. To identify country-specific productivity shocks, we only assume that productivity does not respond contemporaneously (*i.e.*, within the same quarter) to other variables in the VAR. To identify global productivity shocks, we assume (i) a long-run, cointegration relation between home and foreign productivity, and (ii) a set of long-run, cointegration relations between the remaining variables and productivity. In both cases of country-specific and world shocks, we do not impose restrictions on the lagged dynamics and leave the remaining (four) shocks unidentified without affecting the interpretation of our results in the VAR models. Thus, while our theoretical analysis focuses on productivity shocks, the empirical framework allows also for other shocks consistent with the approach proposed by Ireland (2003).

When we analyze country-specific shocks, we focus on temporary shocks while, when we analyze global shocks, we focus on permanent shocks, broadly following the existing empirical literature (e.g., Glick and Rogoff, 1995). In particular, in the case of country-specific shocks, the implicit assumption is that the VARs are stationary around a set of deterministic variables; in the case of global shocks, they are stationary around a common stochastic trend for home and foreign productivity.

²⁷The test statistics for these hypotheses at the system level are reported at the bottom of Tables 2–4. Estimated VAR equations are not reported, but are available on request (as all other empirical results not reported).

²⁸Our main data source only goes to 1999 (see Appendix B), so we are unable to cover more recent developments. For the period 1995–98, U.S. labor productivity and net foreign assets exhibit unusual behavior compared to the recent history of those series. This forced us to work on a shorter sample period.

In the rest of this section we describe the identification of productivity shocks in more detail, report the estimated empirical responses and discuss them in light of theoretical responses presented in the previous section. We consider country-specific and global productivity shocks in turn.

5.1 Country-Specific Shocks

5.1.1 Identification

The identification of country-specific productivity shocks is straightforward. The key assumption is that labor productivity does not respond simultaneously to other variables in the system. This is consistent with the assumption of exogenous contemporaneous productivity in our model (as in many others), while allowing for more general, endogenous lagged dynamics. A Cholesky decomposition of the variance-covariance matrix of the reduced form residuals with home and foreign labor productivity entered, respectively, as the first two variables in the system embeds this assumption and does not place additional restrictions on the VAR response to a home productivity shock. In addition, a Cholesky decomposition with productivity variables entered first is consistent with all those frameworks (including ours) in which labor-augmenting technological progress is exogenous.²⁹ We now turn to the discussion of the estimated impulse responses.

5.1.2 Impulse Responses

We have some success in explaining the estimated impulse responses using our theoretical framework, even though some of the impulse responses to country-specific shocks are estimated imprecisely.

Recall that, in the case of country-specific shocks, theoretical impulse responses differ only *quantitatively* across debtor and creditor countries. Both net debtors and net creditors improve their net foreign asset positions in response to a positive, country-specific shock. During our sample period the U.S. was a net debtor, while Japan and Germany were net creditor countries.

Our analysis of the estimated impulses of the endogenous variables for the G3 countries refers to a one-standard deviation shock to the level of home productivity. These impulse responses and their two standard error bands are reported in Figure 5. The impulse responses are in levels of the variables included in the VAR.

In the case of the U.S., in contrast to the theoretical model, foreign debt increases

²⁹We checked the sensitivity of our results by placing foreign productivity first in the VARs, and found the same qualitative results. We also experimented with VARs for detrended variables, filtering all series with a linear trend and a constant. We report results obtained from VARs in levels because they are more comparable with the cointegration-based results for global shocks. Since the sample period is too short to drop additional observations as would be required by band-pass and HP filters, we did not use them.

following the shock and returns to steady state rather quickly. This response, however, is not statistically significant. (In the empirical responses, net foreign assets below zero indicate a worsening of the asset position.) In the case of Japan and Germany, foreign assets increase and deviate persistently from their steady state values, consistent with the basic predictions of the model. But, in the case of Germany, the dynamic responses following the shock appear unstable.³⁰

Consistent with the model’s predictions, Figure 5 shows that, in both Japan and the U.S., domestic consumption responds positively to a country-specific shock, and this effect dies out gradually. Our theory also predicts that *foreign* consumption should respond to a *home* country-specific shock, but the size of this increase should be smaller. The responses of foreign consumption are statistically not different from zero.

The empirical responses of the real interest rate for all three countries exhibit the only clear departure from the predictions of our model. The model implies that, on impact, the risk-free rate *declines*—to induce foreigners to increase their debt or reduce their foreign asset holdings. This is consistent with the temporary increase in home consumption. Our empirical results (fourth row in Figure 5), however, suggest that there may initially be an *increase* in the real interest rate. Of course, this may be partly due to our inability to measure the global risk-free interest rate, but it is also possible that other sources of heterogeneity may be responsible for this result.³¹

5.2 Global Shocks

5.2.1 Identification

To identify permanent global productivity shocks, we follow King et al. (1991) and Maller et al. (1992) and use a common trend representation for the VAR in levels described above, interpreting the innovation to the common trend in the model as a global productivity shock. To proceed in this manner, we need (i) a long-run, cointegration relation between home and foreign productivity, and (ii) a set of cointegration relations between the remaining variables in the system and productivity. More specifically, we proceed as follows.

First, we assume that home and foreign labor productivity are cointegrated, and hence share a common stochastic trend, with cointegration vector given by the long-run relation

³⁰All the empirical results for Germany should be interpreted with caution because the reduced form VAR contains an explosive root (results not reported). A visual inspection of the German series, especially labor productivity data, shows a very marked structural break in January 1991, following the unification. Evidently, our attempts to control for this episode using an impulse dummy have not been entirely satisfactory. Masson et al. (1994) also report econometric difficulties associated with the German data.

³¹It should also be noted that procyclicality of the real interest rate could be a problem for all standard technology-driven models of the business cycle. In these models, as in ours, if technology improves temporarily and consumption jumps up on impact, but is expected to decline afterwards, the real interest rate falls.

observed in the data over the sample period. Thus, in log-levels, we assume:

$$\log \bar{Z} = \gamma_1^{\bar{Z}} + \gamma_2^{\bar{Z}} \log \bar{Z}^*, \quad (29)$$

where $\gamma_1^{\bar{Z}} > 0$ and $\gamma_2^{\bar{Z}} \geq 0$.³²

Second, as discussed in Appendix A.1, the theoretical model delivers three *non-linear* steady-state relations for domestic consumption relative to foreign (\bar{c}/\bar{c}^*), home net foreign assets (\bar{B}), and the risk-free real interest rate (\bar{r}) as functions of the steady-state productivity ratio (\bar{Z}/\bar{Z}^*) (equations (A.21), (A.23), and (A.24)). Since there are no analytic solutions for these functions, we assume that the empirical counterparts of the functions $\bar{r}(\cdot)$ and $\bar{B}(\cdot)$ consist of the following log-linear relations:

$$\log \bar{r} = \gamma_1^{\bar{r}} + \gamma_2^{\bar{r}} \left(\log \bar{Z} - \log \bar{Z}^* \right), \quad (30)$$

$$\log \bar{B} = \gamma_1^{\bar{B}} + \gamma_2^{\bar{B}} \left(\log \bar{Z} - \log \bar{Z}^* \right). \quad (31)$$

Third, as shown in Appendix A.5, steady-state consumption can be rewritten as:

$$\begin{aligned} \log \bar{c} &= \gamma_1^{\bar{c}} + \gamma_2^{\bar{c}} \log \bar{Z} + \gamma_3^{\bar{c}} \log \bar{Z}^*, \\ \log \bar{c}^* &= \gamma_1^{\bar{c}^*} + \gamma_2^{\bar{c}^*} \log \bar{Z} + \gamma_3^{\bar{c}^*} \log \bar{Z}^*. \end{aligned}$$

Finally, by using (29), we obtain:

$$\begin{aligned} \log \bar{Z} &= \gamma_1^{\bar{Z}} + \gamma_2^{\bar{Z}} \log \bar{Z}^*, \\ \log \bar{B} &= \gamma_1^{\bar{B}'} + \gamma_2^{\bar{B}'} \log \bar{Z}, \\ \log \bar{r} &= \gamma_1^{\bar{r}'} + \gamma_2^{\bar{r}'} \log \bar{Z}, \\ \log \bar{c} &= \gamma_1^{\bar{c}'} + \gamma_2^{\bar{c}'} \log \bar{Z}, \\ \log \bar{c}^* &= \gamma_1^{\bar{c}^{*'}} + \gamma_3^{\bar{c}^{*'}} \log \bar{Z}^*. \end{aligned} \quad (32)$$

The system (32) contains five linear relations in six variables. If the six variables considered are I(1) and the five linear combinations are I(0), these represent a set of long-run, cointegration relations and the six variables must share a single common stochastic trend. Hence, we interpret innovations to this common stochastic trend as a *global*, permanent productivity shock.³³

³²Note that, even if this assumption were rejected by our (short) data, this cointegration assumption is economically plausible and consistent with a range of models.

³³Of course, the innovation to the deviation from the cointegration relation between home and foreign productivity could also be interpreted as a global, temporary productivity shock. We do not discuss the responses to this shock to conserve space.

5.2.2 Cointegration Results

Our cointegration results for the U.S. and Japan are broadly consistent with the framework described above, but the evidence for Germany is less favorable. The results also demonstrate striking differences between the U.S. and Japan in terms of the response of foreign assets and consumption to changes in the level of productivity in the long-run, confirming the presence of considerable heterogeneity in the data as found in previous studies.

Tables 2–4 report the results of a cointegration analysis for the G3 countries. They show the results of the application of the Johansen procedure to the VARs in levels specified as discussed above and the results of a test of the (overidentifying) restrictions implicit in the set of five approximate cointegration relations in (32).

Overall, the evidence for the U.S. is supportive of our assumptions. There is only one eigenvalue clearly close to zero in the six-variable VAR (see Table 2), suggesting the presence of five stationary components. As well, if productivity is entered exogenously in the system to form a four-variable VAR system, we do reject the hypothesis that there are less than, or equal to, four stationary components (bottom of Table 2). However, when productivity is entered endogenously in a six-variable system, the Johansen test on the cointegration rank of the VAR rejects the hypothesis that its rank is less than or equal to five. Also, in a bivariate system, the null hypothesis of cointegration between home and foreign productivity is rejected by the data (results not reported), suggesting that cointegration of productivities is the one long-run relation that does not fit the data well among the five considered in (32). Finally, however, if we impose the hypothesis of five stationary components (consistent with (32)) on the six-variable system, the implied over-identifying restrictions are not rejected by the data, and by a wide margin.

Interestingly, the coefficients in the estimated long-run relations have the right signs.³⁴ They suggest that productivity was growing faster in the rest-of-the-world than in the U.S. during our sample period. This is consistent with a catch-up story in which the U.S. is the more productive economy. Note also that the response of net foreign assets to movements in productivity is very large and statistically significant. By contrast, productivity does not seem to affect the real interest rate in the long-run, matching the theoretical model, which predicts that permanent changes in productivity have no effect on the real interest rate. Indeed, this variable may be excluded from the system of equations because of its lack of statistical significance, and this suggests that this specific measure of the real interest rate may be well described by an $I(0)$ process.

The results for Japan are also broadly supportive of our assumptions. When productivity is entered endogenously, there is only one eigenvalue close to zero in the six variable system, and when productivity is entered exogenously, the test on the system cointegration rank rejects the hypothesis of less than, or equal to, four cointegration vectors, albeit only with a five percent significance level. These results suggest that five stationary

³⁴All coefficients in (32) should be positive, except, possibly, γ_2^r .

components is a plausible assumption for Japan too. However, the over-identifying restrictions are marginally rejected in the case of Japan. Nonetheless, estimated coefficients continue to have the expected signs. Note, in particular, that the response of foreign assets to productivity has opposite sign to that of the U.S. system, while the responses of both foreign assets and consumption are considerably smaller than those in the U.S. in absolute value.

Our analysis is less satisfactory for Germany. In this case, the evidence suggests that there are less than five stationary components, even after imposing cointegration between foreign and domestic productivity. In fact, there are at least two eigenvalues very close to zero when productivity is entered into the system endogenously. Moreover, the over-identifying restrictions are rejected at the five percent significance level, although all estimated coefficients continue to have plausible magnitudes and expected signs. But, as already noted, the results for Germany should be interpreted with caution.

In sum, the cointegration analysis suggests that the linear long-run relations consistent with the theoretical model provide an empirically satisfactory specification of the long-run solution of the system for the U.S. and Japan, allowing us to identify global productivity shocks in a model-consistent manner. These results also point to striking qualitative and quantitative cross-country differences in the long-run relation between productivity and foreign assets: In Japan, increased productivity is associated with an accumulation of net foreign assets in the long run, while in the U.S. it is associated with a sharp increase in foreign liabilities. Long-run responses of consumption to changes in productivity are also quantitatively different across countries, with the U.S. consumption response almost twice as large as that of Japan.

5.2.3 Impulse Responses

Figures 6-8 report the point estimates and two standard error bands of the responses of the variables in our VARs to a one standard-deviation innovation to the common stochastic productivity trend for the U.S., Japan, and Germany, respectively. Again, all impulse responses are in levels.³⁵

The empirical responses for U.S. and Japanese net foreign assets in figures 6 and 7 are qualitatively different and strikingly consistent with our theoretical model. Our theoretical impulse responses predict that, on impact, the less patient and more productive economy would reduce its foreign assets (or accumulate foreign debt), while the more patient and less productive economy should accumulate assets (or reduce debt). Given the negative response of its net foreign assets to global productivity shocks and our estimates

³⁵We followed Mallander et al. (1992) for the VARs in common trend representation, and the estimation and simulation were done using the RATS code written by Andrew Warne (available at <http://warne.texlips.org>). In figures 6-8, LZ_h, LC_h, LBT1S_h, and LR1_h1 denote the logs of home productivity, consumption, net foreign assets, and the gross interest rate for the home country, h = U.S., Japan, and Germany, respectively; LZEX_h and LCEX_h denote productivity and consumption in the rest of the G7. See Data Appendix for sources and definitions.

of the long-run relation between domestic and foreign productivity (tables 2 and 3), we view the U.S. as the less patient, more productive country and Japan as the main counterpart of this hypothesis.³⁶

In figures 6 and 7, consumption responses for the U.S. and Japan are also noteworthy. On impact, consumption increases in all countries. Then, in the case of the U.S., own consumption reaches its new steady state from *above*, whereas, in the case of Japan, own consumption reaches the new steady-state from *below*. These are consistent with our theoretical predictions, if again we interpret the U.S. as the less patient and more productive economy, and Japan as the more patient and less productive one. The foreign consumption patterns are also consistent with the predictions of our model: Consumption excluding Japan reaches its new steady state from above, and consumption excluding the U.S. reaches its steady state from below, albeit less clearly than in the other cases.

Our theoretical model also predicts that net foreign asset and consumption dynamics are largely driven by consumption tilting and variations in relative prices, with no dynamic response from the risk-free real interest rate. Although the point estimates indicate a positive initial interest rate response, the standard errors suggest that this is not statistically different from zero.

Our estimates suggest that the German current account response to a global shock is negligible—net foreign assets hardly move in Figure 8. However, as already mentioned, we have less confidence in our results for Germany. A possible interpretation of our finding is that households in Germany and the rest of the G7 face a similar intertemporal tradeoff, perhaps because of a discount rate that is comparable to that of our empirical representation of the rest of the world.

In sum, our empirical findings are consistent with two well-known results reported in previous studies and mentioned in the Introduction: (i) Current accounts do respond to global productivity shocks, and (ii) they exhibit marked heterogeneity across countries in their responses to global productivity shocks. While the small, open economy paradigm provides a short-cut for understanding heterogeneity, such differences arise naturally in our framework. Our analysis, therefore, goes some way toward reconciling the consumption smoothing hypothesis with the data.

6 Conclusions

We proposed a framework to study international consumption smoothing in a two-country model of interdependence with non-zero long-run net foreign asset positions. We considered two sources of structural heterogeneity leading to such non-zero asset holdings: (i) differences in discount factors and (ii) differences in steady-state labor productivity levels. Our simulation results show that even mild and empirically plausible differences in

³⁶Experimenting with a system in which we excluded all other lagged variables from productivity dynamics left all our qualitative findings unaffected.

discount factors can impart considerable heterogeneity in net foreign asset positions and system dynamics following productivity shocks.

Our empirical results suggest that (i) the dynamic responses of net foreign assets and consumption indeed vary considerably across Germany, Japan, and the U.S., and (ii) the empirical responses of U.S. data are consistent with those of the less patient, more productive economy in our model, with Japan emerging as the main counterpart of this story. Thus, our framework goes some way toward reconciling the international consumption smoothing hypothesis and the role of tilts in the consumption profiles of households with empirical evidence. In particular, our analysis accounts for asymmetric responses to global shocks and provides a plausible explanation for the well-documented “puzzle” that current accounts respond to world shocks when standard intertemporal, two-country models suggest they should not.

Several issues remain, however. Our theoretical framework is admittedly stylized. Given the complexity of allowing for *any* cross-country heterogeneity in a general equilibrium model, we confined our analysis to a single *fundamental* source of asymmetry—differences in patience—complemented by differences in average productivity for consistency with our data, and no capital. Clearly, there may be other sources of heterogeneity that can potentially account for empirically relevant consumption smoothing behavior and current account dynamics. We see extending the model to include investment in physical capital and allow for richer asymmetry in production structures across countries as particularly promising. Similarly, it would be useful to incorporate asymmetries in demographics. Nonetheless, we believe that the differences in steady-state and transition dynamics stemming from our model and documented in our empirical analysis are significant, both theoretically and empirically.

Appendix

A Technical Details and Derivations

A.1 The Steady State

The steady state can be obtained as follows. Using steady-state versions of the consumption functions for domestic and foreign newborn households (14) and of the definition of a household’s human wealth (15), steady-state domestic and foreign Euler equations for aggregate per capita consumption at home and abroad are:

$$\bar{c} = \frac{n\rho(1-\beta)(1+\bar{r})}{\bar{r}[1+n-\beta(1+\bar{r})]}\bar{w}, \quad \bar{c}^* = \frac{n\rho(1-\alpha\beta)(1+\bar{r})}{\bar{r}[1+n-\alpha\beta(1+\bar{r})]}\bar{w}^*. \quad (\text{A.1})$$

Steady-state labor-leisure tradeoffs in aggregate per capita terms imply:

$$\bar{L} = 1 - \frac{1-\rho}{\rho} \frac{\bar{c}}{\bar{w}}, \quad \bar{L}^* = 1 - \frac{1-\rho}{\rho} \frac{\bar{c}^*}{\bar{w}^*}. \quad (\text{A.2})$$

Now, international equilibrium requires that world aggregate per capita production of the consumption basket be equal to consumption:

$$a\bar{y} + (1 - a)\bar{y}^* = a\bar{c} + (1 - a)\bar{c}^*. \quad (\text{A.3})$$

Steady-state domestic and foreign GDPs in units of the composite consumption basket are:

$$\bar{y} = \overline{RP} \bar{Z} \bar{L}, \quad \bar{y}^* = \overline{RP}^* \bar{Z}^* \bar{L}^*. \quad (\text{A.4})$$

Optimal price setting is such that $\overline{RP} = \frac{\bar{w}}{\bar{Z}}$, $\overline{RP}^* = \frac{\bar{w}^*}{\bar{Z}^*}$. Hence:

$$\bar{y} = \bar{w} \bar{L}, \quad \bar{y}^* = \bar{w}^* \bar{L}^*. \quad (\text{A.5})$$

Substituting these equations into (A.3) yields:

$$a\bar{w} \bar{L} + (1 - a)\bar{w}^* \bar{L}^* = a\bar{c} + (1 - a)\bar{c}^*. \quad (\text{A.6})$$

Use equations (A.2) to substitute for \bar{L} and \bar{L}^* into (A.6) and rearrange the resulting equation to obtain:

$$\frac{\bar{c}^*}{\rho \bar{w}^*} \left(a \frac{\bar{c}}{\bar{c}^*} + 1 - a \right) = a \frac{\bar{w}}{\bar{w}^*} + 1 - a. \quad (\text{A.7})$$

The steady-state Euler equations in (A.1) imply:

$$\frac{\bar{c}}{\bar{c}^*} = \frac{(1 - \beta) [1 + n - \alpha\beta (1 + \bar{r})] \bar{w}}{(1 - \alpha\beta) [1 + n - \beta (1 + \bar{r})] \bar{w}^*}. \quad (\text{A.8})$$

Also, the equation for foreign consumption implies:

$$\frac{\bar{c}^*}{\bar{w}^*} = \frac{n\rho (1 - \alpha\beta) (1 + \bar{r})}{\bar{r} [1 + n - \alpha\beta (1 + \bar{r})]}. \quad (\text{A.9})$$

Substituting (A.8) and (A.9) into (A.7) yields an equation that relates the world interest rate \bar{r} to the real wage ratio $\frac{\bar{w}}{\bar{w}^*}$:

$$\begin{aligned} & \frac{n(1 - \alpha\beta)(1 + \bar{r})}{\bar{r}[1 + n - \alpha\beta(1 + \bar{r})]} \left\{ a \frac{(1 - \beta)[1 + n - \alpha\beta(1 + \bar{r})] \bar{w}}{(1 - \alpha\beta)[1 + n - \beta(1 + \bar{r})] \bar{w}^*} + 1 - a \right\} \\ &= a \frac{\bar{w}}{\bar{w}^*} + 1 - a. \end{aligned} \quad (\text{A.10})$$

Steady-state labor demand equations, optimal pricing, and the definition of world demand imply:

$$\bar{L} = \left(\frac{\bar{w}}{\bar{Z}} \right)^{-\omega} \frac{a\bar{y} + (1 - a)\bar{y}^*}{\bar{Z}}, \quad \bar{L}^* = \left(\frac{\bar{w}^*}{\bar{Z}^*} \right)^{-\omega} \frac{a\bar{y} + (1 - a)\bar{y}^*}{\bar{Z}^*}. \quad (\text{A.11})$$

Hence,

$$\frac{\bar{L}}{\bar{L}^*} = \left(\frac{\bar{w}}{\bar{w}^*}\right)^{-\omega} \left(\frac{\bar{Z}}{\bar{Z}^*}\right)^{\omega-1}, \text{ or } \frac{\bar{w}}{\bar{w}^*} = \left(\frac{\bar{L}}{\bar{L}^*}\right)^{-\frac{1}{\omega}} \left(\frac{\bar{Z}}{\bar{Z}^*}\right)^{\frac{\omega-1}{\omega}}. \quad (\text{A.12})$$

Substitute for the wage ratio from (A.12) into (A.10) and rearrange:

$$\begin{aligned} & a \left(\frac{\bar{L}}{\bar{L}^*}\right)^{-\frac{1}{\omega}} \left(\frac{\bar{Z}}{\bar{Z}^*}\right)^{\frac{\omega-1}{\omega}} \left\{ \frac{n(1-\beta)(1+\bar{r})}{\bar{r}[1+n-\beta(1+\bar{r})]} - 1 \right\} \\ &= (1-a) \left\{ 1 - \frac{n(1-\alpha\beta)(1+\bar{r})}{\bar{r}[1+n-\alpha\beta(1+\bar{r})]} \right\}. \end{aligned} \quad (\text{A.13})$$

The labor-leisure tradeoffs (A.2) and equation (A.8) imply:

$$1 - \bar{L} = \frac{(1-\beta)[1+n-\alpha\beta(1+\bar{r})]}{(1-\alpha\beta)[1+n-\beta(1+\bar{r})]} (1 - \bar{L}^*). \quad (\text{A.14})$$

Rearrange equation (A.14) as:

$$\frac{\bar{L}}{\bar{L}^*} = \frac{1}{\bar{L}^*} - \frac{(1-\beta)[1+n-\alpha\beta(1+\bar{r})]}{(1-\alpha\beta)[1+n-\beta(1+\bar{r})]} \left(\frac{1}{\bar{L}^*} - 1\right). \quad (\text{A.15})$$

The foreign labor-leisure tradeoff and Euler equation imply:

$$\bar{L}^* = 1 - \frac{1-\rho}{\rho} \frac{n\rho(1-\alpha\beta)(1+\bar{r})}{\bar{r}[1+n-\alpha\beta(1+\bar{r})]}. \quad (\text{A.16})$$

Substituting (A.16) into the right hand side of (A.15) and rearranging yields:

$$\frac{\bar{L}}{\bar{L}^*} = \frac{[1+n-\alpha\beta(1+\bar{r})] \{(\bar{r}-n)[1-\beta(1+\bar{r})] + n\rho(1-\beta)(1+\bar{r})\}}{[1+n-\beta(1+\bar{r})] \{(\bar{r}-n)[1-\alpha\beta(1+\bar{r})] + n\rho(1-\alpha\beta)(1+\bar{r})\}}. \quad (\text{A.17})$$

Equations (A.13) and (A.17) constitute a system of two equations in two unknowns, the steady-state world interest rate \bar{r} and the labor effort ratio $\frac{\bar{L}}{\bar{L}^*}$ as functions of parameters and the productivity ratio $\frac{\bar{Z}}{\bar{Z}^*}$.

If $\alpha = 1$, equation (A.17) yields $\frac{\bar{L}}{\bar{L}^*} = 1$ regardless of $\frac{\bar{Z}}{\bar{Z}^*}$. If agents' intertemporal preferences are identical at home and abroad, the only possible equilibrium is one in which $\bar{r} = \frac{1-\beta}{\beta}$ and net foreign assets are zero even if $\frac{\bar{Z}}{\bar{Z}^*} \neq 1$. To see this, observe that, if $\alpha = 1$, equation (A.13) can be rewritten as:

$$\left\{ \frac{n(1-\beta)(1+\bar{r})}{\bar{r}[1+n-\beta(1+\bar{r})]} - 1 \right\} \left[a \left(\frac{\bar{Z}}{\bar{Z}^*}\right)^{\frac{\omega-1}{\omega}} + 1 - a \right] = 0. \quad (\text{A.18})$$

This equation has solutions $\bar{r} = \frac{1-\beta}{\beta}$ and $\bar{r} = n$, but the latter is not admissible, as it would imply violation of the constraint that both home and foreign real wages must be

strictly positive. To see this, note that the steady-state version of the law of motion for domestic net foreign assets and the equations for home consumption, labor effort, and GDP in (A.1), (A.2), and (A.5), respectively, imply that steady-state net foreign assets are equal to:

$$\bar{B} = \frac{\beta(1 + \bar{r}) - 1}{\bar{r}[1 + n - \beta(1 + \bar{r})]} \bar{w}. \quad (\text{A.19})$$

Similarly,

$$\bar{B}^* = \frac{\alpha\beta(1 + \bar{r}) - 1}{\bar{r}[1 + n - \alpha\beta(1 + \bar{r})]} \bar{w}^*, \quad (\text{A.20})$$

and international equilibrium requires $a\bar{B} + (1 - a)\bar{B}^* = 0$, which is equivalent to the condition used in the text ($\bar{y}^W = \bar{c}^W$) by Walras' Law. If $\alpha = 1$ and $\bar{r} = n$, international equilibrium implies $\bar{w} = -\frac{1-a}{a}\bar{w}^*$, which is ruled out by the constraint that both \bar{w} and \bar{w}^* must be strictly positive. Therefore, if $\alpha = 1$, it must be $\bar{r} = \frac{1-\beta}{\beta}$ and $\bar{B} = \bar{B}^* = 0$ regardless of $\frac{\bar{Z}}{\bar{Z}^*}$.

If $\alpha = 1$ and $\bar{Z} = \bar{Z}^* = 1$, the steady state is the same as in Ghironi (2000): $\bar{B} = 0$, and the steady state is symmetric in all respects: $r = \frac{1-\beta}{\beta}$, $\bar{B} = \bar{B}^* = 0$, $\bar{c} = \bar{c}^* = \bar{L} = \bar{L}^* = \bar{y} = \bar{y}^* = \rho$, $\bar{w} = \bar{w}^* = \bar{R}\bar{P} = \bar{R}\bar{P}^* = 1$.

If $\alpha = 1$ but $\frac{\bar{Z}}{\bar{Z}^*} \neq 1$, domestic and foreign GDPs in units of consumption differ, and so do consumption levels. But consumption equals GDP in each country, so that net foreign assets are zero. Since $\bar{y} = \bar{w}\bar{L}$ and $\bar{y}^* = \bar{w}^*\bar{L}^*$ in equilibrium, $\bar{L} = \bar{L}^*$ when $\alpha = 1$ implies that the different GDP levels generated by different productivity levels translate into different real wages and labor incomes across countries. If $\omega > 1$, the more productive country has a higher steady-state real wage and consumption and a lower relative price for the same labor effort as the less productive country.

In the general case $\alpha \leq 1$, let us write the solution for \bar{r} as a function of $\frac{\bar{Z}}{\bar{Z}^*}$, which we obtain numerically, as:

$$\bar{r} = \bar{r} \left(\frac{\bar{Z}}{\bar{Z}^*} \right). \quad (\text{A.21})$$

Substituting (A.21) into (A.17) yields an equation that can be solved for the steady-state labor effort ratio as a function of relative productivity. We write the solution as:

$$\frac{\bar{L}}{\bar{L}^*} = \bar{L}^R \left(\frac{\bar{Z}}{\bar{Z}^*} \right), \quad (\text{A.22})$$

where $\bar{L}^R \left(\frac{\bar{Z}}{\bar{Z}^*} \right)$ is a function of relative productivity, the characteristics of which depend on the values of structural parameters, and $\bar{L}^R \left(\frac{\bar{Z}}{\bar{Z}^*} \right) = 1$ if $\alpha = 1$.

Given (A.21), we can obtain solutions for steady-state consumption, wage, and GDP

ratios, as well as net foreign assets. In particular:

$$\frac{\bar{c}}{\bar{c}^*} = \bar{c}^R \left(\frac{\bar{Z}}{\bar{Z}^*} \right), \quad (\text{A.23})$$

$$\bar{B} = \bar{B} \left(\frac{\bar{Z}}{\bar{Z}^*} \right). \quad (\text{A.24})$$

As noted above, if $\alpha = 1$, it is $\bar{B} \left(\frac{\bar{Z}}{\bar{Z}^*} \right) = 0$, and $\bar{c}^R \left(\frac{\bar{Z}}{\bar{Z}^*} \right) \geq 1$ if $\bar{Z} \geq \bar{Z}^*$ and $\omega > 1$. If $\alpha < 1$ and $\bar{Z} = \bar{Z}^* = 1$, plausible parameter values yield $\bar{B} > 0$ ($\bar{B}^* < 0$), $\bar{c} > \bar{c}^*$, $\bar{L} < \bar{L}^*$, $\bar{w} > \bar{w}^*$, $\bar{RP} > \bar{RP}^*$, $\bar{y} < \bar{y}^*$. If $\alpha < 1$ and $\bar{Z} = 1 < \bar{Z}^*$, plausible parameter values (and \bar{Z}^* sufficiently above \bar{Z}) yield $\bar{B} > 0$ ($\bar{B}^* < 0$), $\bar{c} < \bar{c}^*$, $\bar{L} < \bar{L}^*$, $\bar{w} < \bar{w}^*$, $\bar{RP} > \bar{RP}^*$, $\bar{y} < \bar{y}^*$. In particular, equations (A.19) and (A.20) show that the tilt of individual household consumption profiles determines whether a country is a net creditor or debtor in steady state.³⁷

A.2 The Log-Linear Model

The main log-linear equations of the model are as follows.

The laws of motion for aggregate per capita home and foreign net foreign assets are:

$$(1+n) \mathbf{B}_{t+1} = (1+\bar{r})(r_t + \mathbf{B}_t) + \frac{\bar{y}}{\bar{B}} y_t - \frac{\bar{c}}{\bar{B}} c_t, \quad (\text{A.25})$$

$$(1+n) \mathbf{B}_{t+1}^* = (1+\bar{r})(r_t + \mathbf{B}_t^*) + \frac{\bar{y}^*}{\bar{B}^*} y_t^* - \frac{\bar{c}^*}{\bar{B}^*} c_t^*. \quad (\text{A.26})$$

International asset markets equilibrium requires that a country's asset accumulation must be mirrored by the other country's debt:

$$\mathbf{B}_t - \mathbf{B}_t^* = 0. \quad (\text{A.27})$$

Home and foreign GDPs are, respectively:

$$y_t = \text{RP}_t + \text{L}_t + \text{Z}_t, \quad (\text{A.28})$$

$$y_t^* = \text{RP}_t^* + \text{L}_t^* + \text{Z}_t^*. \quad (\text{A.29})$$

Labor demand at home and abroad is a function of the relative price of the goods a country produces, of world demand of the consumption basket, and of productivity:

$$\text{L}_t = -\omega \text{RP}_t + y_t^W - \text{Z}_t, \quad (\text{A.30})$$

³⁷This assumes $1+n > \beta(1+\bar{r})$. This condition is necessary for stability of the steady state in the small open economy version of the model with exogenous, constant interest rate and constant wage, and it is satisfied for the parameter values we consider in our model.

$$L_t^* = -\omega RP_t^* + y_t^W - Z_t^*, \quad (\text{A.31})$$

where $\omega > 0$ is the elasticity of substitution between home and foreign goods in consumption and $y_t^W = c_t^W = \frac{a\bar{c}}{a\bar{c}+(1-a)\bar{c}^*}c_t + \frac{(1-a)\bar{c}^*}{a\bar{c}+(1-a)\bar{c}^*}c_t^*$.

Prices are equal to marginal costs:

$$RP_t = w_t - Z_t, \quad (\text{A.32})$$

$$RP_t^* = w_t^* - Z_t^*. \quad (\text{A.33})$$

Labor supply in each country is such that the marginal disutility of an extra unit of labor effort equals the value of the real wage in terms of marginal utility of consumption:

$$L_t = -\frac{1-\rho}{\rho} \frac{\bar{c}}{\bar{w}L} (c_t - w_t), \quad (\text{A.34})$$

$$L_t^* = -\frac{1-\rho}{\rho} \frac{\bar{c}^*}{\bar{w}^*L^*} (c_t^* - w_t^*). \quad (\text{A.35})$$

Euler equations for aggregate per capita domestic and foreign consumption include an additional term that depends on consumption by newborn households at time $t+1$. In turn, newborn households' consumption is a function of the households' human wealth, defined as the present discounted value of the households' infinite lifetime in terms of the real wage. It is:

$$c_t = -r_{t+1} + \frac{1+n}{\beta(1+\bar{r})}c_{t+1} - \frac{n\rho(1-\beta)\bar{h}}{\beta(1+\bar{r})\bar{c}}h_{t+1}, \quad (\text{A.36})$$

$$c_t^* = -r_{t+1} + \frac{1+n}{\alpha\beta(1+\bar{r})}c_{t+1}^* - \frac{n\rho(1-\alpha\beta)\bar{h}^*}{\alpha\beta(1+\bar{r})\bar{c}^*}h_{t+1}^*. \quad (\text{A.37})$$

Human wealth at home and abroad is such that:

$$h_t = \frac{1}{1+\bar{r}}(h_{t+1} - r_{t+1}) + \frac{\bar{w}}{\bar{h}}w_t, \quad (\text{A.38})$$

$$h_t^* = \frac{1}{1+\bar{r}}(h_{t+1}^* - r_{t+1}) + \frac{\bar{w}^*}{\bar{h}^*}w_t^*. \quad (\text{A.39})$$

Finally, to close the model, productivities at home and abroad are described by the processes assumed in the text in all periods after the time of an initial impulse ($t=0$):

$$Z_t = \phi Z_{t-1}, \quad Z_t^* = \phi Z_{t-1}^*, \quad 0 \leq \phi \leq 1. \quad (\text{A.40})$$

A.3 Consumption Smoothing, Tilting, and the Current Account

Focus on the home country, the relative more patient economy that has positive net foreign assets in steady state.

Let A_{t+1}^v denote assets (including shares in firms) held by the representative home household born in period v at the beginning of time $t + 1$. Using the equilibrium, period budget constraint for dates following an initial shock and the no-arbitrage condition between bonds and shares, the household's intertemporal budget constraint is:

$$\sum_{s=t}^{\infty} R_{t,s} C_s^v = (1 + r_t) A_t^v + \sum_{s=t}^{\infty} R_{t,s} w_s L_s^v.$$

Then, using the Euler equation for the household's consumption, the labor-leisure tradeoff, and the definition of human wealth, it is possible to show that the household's consumption of composite good and leisure (evaluated in units of consumption) is a fraction of the household's resources over its remaining lifetime (assets, interest income, and human wealth):

$$C_t^v + w_t (1 - L_t^v) = (1 - \beta) [(1 + r_t) A_t^v + h_t]. \quad (\text{A.41})$$

Aggregating this consumption function across generations yields:

$$c_t + w_t (1 - L_t) = (1 - \beta) [(1 + r_t) A_t + h_t]. \quad (\text{A.42})$$

Now, households' aggregate per capita assets entering period $t + 1$ are the sum of net foreign bond holdings and the equity value of the home economy: $A_{t+1} = B_{t+1} + V_t$. But $V_t = 0 \forall t$ in equilibrium. Hence, equation (A.42) can be rewritten as:

$$c_t + w_t (1 - L_t) = (1 - \beta) [(1 + r_t) B_t + h_t],$$

and taking the home aggregate per capita labor-leisure tradeoff into account yields the aggregate per capita consumption function:

$$c_t = \rho (1 - \beta) [(1 + r_t) B_t + h_t]. \quad (\text{A.43})$$

Foreign aggregate per capita consumption obeys a similar equation, with discount factor $\alpha\beta$. In each period, countries consume a fraction of their asset holdings (principal and interest income) and of their human wealths.

Because dividends are zero, $y_t = w_t L_t$ in equilibrium. Therefore, using the labor-leisure tradeoff and the consumption function (A.43) returns:

$$y_t = w_t - (1 - \beta) (1 - \rho) [(1 + r_t) B_t + h_t]. \quad (\text{A.44})$$

Substituting equations (A.43) and (A.44) into the law of motion for domestic aggregate per capita net foreign assets in (23) yields:

$$B_{t+1} = \frac{\beta (1 + r_t)}{1 + n} B_t + \frac{w_t - (1 - \beta) h_t}{1 + n}. \quad (\text{A.45})$$

A similar equation holds for B_{t+1}^* , replacing β with $\alpha\beta$. International equilibrium then requires $aB_t + (1 - a)B_t^* = 0$.

Define the aggregate per capita current account as:

$$CA_t = (1 + n)B_{t+1} - B_t = r_t B_t + y_t - c_t. \quad (\text{A.46})$$

In steady state, home net foreign assets are constant and positive. For this to happen, the home country must be running a steady-state current account surplus $\overline{CA} = n\overline{B}$ to compensate for the effect of population growth in each period.

Equations (A.45), (A.46), and the definition of h_t imply:

$$CA_t = [\beta(1 + r_t) - 1]B_t + w_t - (1 - \beta) \sum_{s=t}^{\infty} R_{t,s} w_s. \quad (\text{A.47})$$

Define the permanent level of variable x_t as:

$$\tilde{x}_t \equiv \frac{\sum_{s=t}^{\infty} R_{t,s} x_s}{\sum_{s=t}^{\infty} R_{t,s}}.$$

This is the annuity value of x_t at the prevailing interest rate, or the hypothetical constant level of x_t with the same present value as x_t .

Let:

$$\tilde{\Gamma}_t \equiv \frac{1}{(1 - \beta) \sum_{s=t}^{\infty} R_{t,s}}.$$

The following equalities hold:

$$\begin{aligned} \frac{\tilde{\Gamma}_t - 1}{\tilde{\Gamma}_t} &= 1 - (1 - \beta) \sum_{s=t}^{\infty} R_{t,s}, \quad \sum_{s=t+1}^{\infty} R_{t,s} r_s = 1, \\ \frac{1 + r_t}{\sum_{s=t}^{\infty} R_{t,s}} &= \frac{r_t + \sum_{s=t+1}^{\infty} R_{t,s} r_s}{\sum_{s=t}^{\infty} R_{t,s}} = \frac{\sum_{s=t}^{\infty} R_{t,s} r_s}{\sum_{s=t}^{\infty} R_{t,s}} = \tilde{r}_t, \quad \text{or} \quad \tilde{r}_t \sum_{s=t}^{\infty} R_{t,s} = 1 + r_t. \end{aligned}$$

Using these equalities, it is possible to show that:

$$CA_t = (r_t - \tilde{r}_t) B_t + w_t - \tilde{w}_t + \frac{\tilde{\Gamma}_t - 1}{\tilde{\Gamma}_t} (\tilde{r}_t B_t + \tilde{w}_t). \quad (\text{A.48})$$

Equation (A.48) is the fundamental current account equation for the home economy. To understand current account dynamics following transitory shocks that cause changes in the interest rate such as in figures 1, 2, and 4, observe that differentiation of (A.48) yields:

$$dCA_t = (dr_t - d\tilde{r}_t) \overline{B} + dw_t - d\tilde{w}_t + d \left(\frac{\tilde{\Gamma}_t - 1}{\tilde{\Gamma}_t} \right) (\overline{r} \overline{B} + \overline{w}) + \frac{\beta(1 + \overline{r}) - 1}{\overline{r}} (d\tilde{r}_t \overline{B} + \overline{r} dB_t + d\tilde{w}_t). \quad (\text{A.49})$$

It is possible to verify that:

$$\begin{aligned}
d\tilde{r}_t &= \frac{\bar{r}}{1+\bar{r}} \sum_{s=t}^{\infty} \left(\frac{1}{1+\bar{r}} \right)^{s-t} dr_s, \\
d\tilde{w}_t &= \frac{\bar{r}}{1+\bar{r}} \sum_{s=t}^{\infty} \left(\frac{1}{1+\bar{r}} \right)^{s-t} dw_s, \\
d\left(\frac{\tilde{\Gamma}_t - 1}{\tilde{\Gamma}_t} \right) &= (1-\beta) \sum_{s=t}^{\infty} \frac{d \left[\prod_{u=t}^s (1+r_u) \right]}{(1+\bar{r})^{2(s-t)}}.
\end{aligned}$$

Consider the case of a home productivity shock with persistence $\phi = .9$ (Figure 1). The gross interest rate is below the steady state during the transition dynamics. Therefore, $dr_s < 0$ and $d\tilde{r}_t < 0$. Since the path of the interest rate is increasing after time 0, $dr_t - d\tilde{r}_t < 0$. In addition, $d(1+r_u) < 0 \forall u$ implies $d\left(\frac{\tilde{\Gamma}_t-1}{\tilde{\Gamma}_t}\right) < 0$. Finally, $dw_s > 0$ implies $d\tilde{w}_t > 0$, and a decreasing path of the real wage after time 0 ensures $dw_t - d\tilde{w}_t > 0$. Using these results and $\beta(1+\bar{r}) > 1$ in conjunction with equation (A.49) makes it possible to conclude that the terms $dw_t - d\tilde{w}_t$ and $\frac{\beta(1+\bar{r})-1}{\bar{r}}d\tilde{w}_t$ prevail on the other terms in equation (A.49) in determining the reaction of the home current account to the shock. A similar exercise can be done for figures 2 and 4 using the analogous equation and results for the foreign country.

When there is a world productivity shock that leaves the interest rate unchanged at the steady-state level \bar{r} (as in Section 4.4), equation (A.48) implies:

$$CA_t = w_t - \tilde{w}_t + \frac{\beta(1+\bar{r})-1}{\bar{r}}(\bar{r}B_t + \tilde{w}_t). \quad (\text{A.50})$$

Using $\tilde{w}_t = \frac{\bar{r}}{1+\bar{r}} \sum_{s=t}^{\infty} \left(\frac{1}{1+\bar{r}} \right)^{s-t} w_s$ and defining beginning-of-period aggregate per capita wealth, W_t , at constant interest rate \bar{r} as $W_t \equiv (1+\bar{r})B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+\bar{r}} \right)^{s-t} w_s$ yields equation (28). Equations similar to (A.48) and (A.50) hold for the foreign economy, and it must be $aCA_t + (1-a)CA_t^* = 0$ for international equilibrium.

A.4 The World Interest Rate

Equation (A.45) provides a law of motion for home net foreign assets that takes optimal behavior of households and firms into account. A similar equation holds abroad:

$$B_{t+1}^* = \frac{\alpha\beta(1+r_t)}{1+n} B_t^* + \frac{w_t^* - (1-\alpha\beta)h_t^*}{1+n}. \quad (\text{A.51})$$

Taking a weighted average of equations (A.45) and (A.51), imposing international asset market equilibrium, and rearranging yields:

$$w_t^W = (1 - \beta) h_t^W + \beta (1 - \alpha) (1 - a) [(1 + r_t) B_t^* + h_t^*], \quad (\text{A.52})$$

where $w_t^W \equiv aw_t + (1 - a) w_t^*$ and $h_t^W \equiv ah_t + (1 - a) h_t^*$. Finally, using the analog to (A.43) for the foreign economy, we can rewrite equation (A.52) as:

$$w_t^W = (1 - \beta) h_t^W + \frac{\beta (1 - \alpha) (1 - a)}{\rho (1 - \alpha \beta)} c_t^*. \quad (\text{A.53})$$

We assume that α is sufficiently close to 1 that the effect of the second term on the right-hand side of equation (A.53) is negligible. (Formally, we assume that $1 - \alpha$ is of order 2 or higher in the amplitude of the shocks we consider.) It follows that:

$$w_t^W \approx (1 - \beta) h_t^W. \quad (\text{A.54})$$

Now, the equations in (16) imply:

$$h_t^W = \frac{h_{t+1}^W}{1 + r_{t+1}} + w_t^W, \quad (\text{A.55})$$

and combining (A.54) and (A.55) shows that the risk-free, real interest rate is approximately determined by the expected rate of world-wide real wage growth:

$$1 + r_{t+1} \approx \frac{w_{t+1}^W}{\beta w_t^W},$$

where $w_t^W \equiv aw_t + (1 - a) w_t^*$. In log-linear terms:

$$r_{t+1} \approx \mathbf{w}_{t+1}^W - \mathbf{w}_t^W, \quad (\text{A.56})$$

with $\mathbf{w}_t^W = a \frac{\bar{w}}{\bar{w}^W} \mathbf{w}_t + (1 - a) \frac{\bar{w}^*}{\bar{w}^W} \mathbf{w}_t^*$.

Real wages at home and abroad are tied to relative prices and productivity by optimal pricing in the two economies and the markup-offsetting subsidies: $RP_t = \frac{w_t}{Z_t}$ and $RP_t^* = \frac{w_t^*}{Z_t^*}$. Put differently, competitive labor markets and markup-offsetting subsidies imply that workers in the two countries are paid the consumption-value of their marginal products: $w_t = RP_t Z_t$ and $w_t^* = RP_t^* Z_t^*$. Therefore, $w_t^W = a RP_t Z_t + (1 - a) RP_t^* Z_t^*$, or, in log-linear terms:

$$\mathbf{w}_t^W = a \frac{\bar{w}}{\bar{w}^W} (\mathbf{RP}_t + \mathbf{Z}_t) + (1 - a) \frac{\bar{w}^*}{\bar{w}^W} (\mathbf{RP}_t^* + \mathbf{Z}_t^*), \quad (\text{A.57})$$

and

$$\begin{aligned} r_{t+1} \approx & a \frac{\bar{w}}{\bar{w}^W} (\mathbf{RP}_{t+1} - \mathbf{RP}_t + \mathbf{Z}_{t+1} - \mathbf{Z}_t) \\ & + (1 - a) \frac{\bar{w}^*}{\bar{w}^W} (\mathbf{RP}_{t+1}^* - \mathbf{RP}_t^* + \mathbf{Z}_{t+1}^* - \mathbf{Z}_t^*). \end{aligned} \quad (\text{A.58})$$

Now, equation (A.57) can be rewritten as:

$$a \frac{\bar{w}}{\bar{w}^W} \text{RP}_t + (1-a) \frac{\bar{w}^*}{\bar{w}^W} \text{RP}_t^* = \mathbf{w}_t^W - \left[a \frac{\bar{w}}{\bar{w}^W} Z_t + (1-a) \frac{\bar{w}^*}{\bar{w}^W} Z_t^* \right].$$

When the steady state is symmetric, $\bar{w} = \bar{w}^* = \bar{w}^W$, $\mathbf{w}_t^W = Z_t^W \equiv aZ_t + (1-a)Z_t^*$, and $a\text{RP}_t + (1-a)\text{RP}_t^* = 0$. In this case, there is no relative price effect on the real interest rate, which is simply equal to expected world-wide productivity growth: $r_{t+1} = Z_{t+1}^W - Z_t^W$ as in Ghironi (2000).

When the steady state is asymmetric, but steady-state home labor effort is close to foreign, we show below that it is $\mathbf{w}_t^W \approx a \frac{\bar{w}}{\bar{w}^W} Z_t + (1-a) \frac{\bar{w}^*}{\bar{w}^W} Z_t^*$, or $a \frac{\bar{w}}{\bar{w}^W} \text{RP}_t + (1-a) \frac{\bar{w}^*}{\bar{w}^W} \text{RP}_t^* \approx 0$. In this case, relative price effects on the world interest rate are negligible (home and foreign relative price effects cancel each other out), and equation (A.58) is well approximated by:

$$r_{t+1} \approx a \frac{\bar{w}}{\bar{w}^W} (Z_{t+1} - Z_t) + (1-a) \frac{\bar{w}^*}{\bar{w}^W} (Z_{t+1}^* - Z_t^*). \quad (\text{A.59})$$

The interest rate is no longer equal to world productivity growth, but it is approximately equal to a weighted average of productivity growth in the two countries, where the weights are adjusted to reflect the share of a country's wage in the world real wage. (To make this point, it is not sufficient to argue simply that α close to 1 implies that \bar{w} is close to \bar{w}^* —and therefore both are close to \bar{w}^W . Our numerical calculations show that even α very close to 1 can result in sizable differences in steady-state real wages across countries. Due to non-linearity, this happens even if steady-state levels of labor effort are close to each other.)

To prove the result in (A.59), take a weighted average of equations (A.28) and (A.29) with weights $a \frac{\bar{w}}{\bar{w}^W}$ and $(1-a) \frac{\bar{w}^*}{\bar{w}^W}$, respectively. It is:

$$\begin{aligned} & a \frac{\bar{w}}{\bar{w}^W} y_t + (1-a) \frac{\bar{w}^*}{\bar{w}^W} y_t^* \\ = & a \frac{\bar{w}}{\bar{w}^W} \text{RP}_t + (1-a) \frac{\bar{w}^*}{\bar{w}^W} \text{RP}_t^* + a \frac{\bar{w}}{\bar{w}^W} Z_t + (1-a) \frac{\bar{w}^*}{\bar{w}^W} Z_t^* \\ & + a \frac{\bar{w}}{\bar{w}^W} L_t + (1-a) \frac{\bar{w}^*}{\bar{w}^W} L_t^*. \end{aligned} \quad (\text{A.60})$$

Take the same weighted average of (A.30) and (A.31), use the fact that $a \frac{\bar{w}}{\bar{w}^W} + (1-a) \frac{\bar{w}^*}{\bar{w}^W} = 1$, and rearrange:

$$\begin{aligned} y_t^W &= \omega \left[a \frac{\bar{w}}{\bar{w}^W} \text{RP}_t + (1-a) \frac{\bar{w}^*}{\bar{w}^W} \text{RP}_t^* \right] + a \frac{\bar{w}}{\bar{w}^W} Z_t + (1-a) \frac{\bar{w}^*}{\bar{w}^W} Z_t^* \\ &+ a \frac{\bar{w}}{\bar{w}^W} L_t + (1-a) \frac{\bar{w}^*}{\bar{w}^W} L_t^*. \end{aligned} \quad (\text{A.61})$$

Subtract equation (A.60) from (A.61):

$$y_t^W - \left[a \frac{\bar{w}}{\bar{w}^W} y_t + (1-a) \frac{\bar{w}^*}{\bar{w}^W} y_t^* \right] = (\omega - 1) \left[a \frac{\bar{w}}{\bar{w}^W} \text{RP}_t + (1-a) \frac{\bar{w}^*}{\bar{w}^W} \text{RP}_t^* \right]. \quad (\text{A.62})$$

Repeat the exercise of equations (A.60)–(A.62) with weights $a\frac{\bar{y}}{\bar{y}^W}$ and $(1-a)\frac{\bar{y}^*}{\bar{y}^*W}$ (of course, $a\frac{\bar{y}}{\bar{y}^W} + (1-a)\frac{\bar{y}^*}{\bar{y}^*W} = 1$). Because $\mathbf{y}_t^W = a\frac{\bar{y}}{\bar{y}^W}\mathbf{y}_t + (1-a)\frac{\bar{y}^*}{\bar{y}^*W}\mathbf{y}_t^*$, it must be:

$$a\frac{\bar{y}}{\bar{y}^W}\mathbf{RP}_t + (1-a)\frac{\bar{y}^*}{\bar{y}^*W}\mathbf{RP}_t^* = 0. \quad (\text{A.63})$$

Solve (A.63) for \mathbf{RP}_t ($= -\frac{1-a}{a}\frac{\bar{y}^*}{\bar{y}^W}\mathbf{RP}_t^*$) and substitute into (A.62), recalling that $\bar{y} = \bar{w}\bar{L}$ and $\bar{y}^* = \bar{w}^*\bar{L}^*$. It is:

$$\mathbf{y}_t^W - \left[a\frac{\bar{w}}{\bar{w}^W}\mathbf{y}_t + (1-a)\frac{\bar{w}^*}{\bar{w}^*W}\mathbf{y}_t^* \right] = -(\omega-1)(1-a)\frac{\bar{w}^*}{\bar{w}^*W} \left(\frac{\bar{L}^*}{\bar{L}} - 1 \right) \mathbf{RP}_t^*. \quad (\text{A.64})$$

If $\alpha = 1$, $\bar{L} = \bar{L}^*$ regardless of \bar{Z} vs. \bar{Z}^* . In that case, $\mathbf{y}_t^W = [a\frac{\bar{w}}{\bar{w}^W}\mathbf{y}_t + (1-a)\frac{\bar{w}^*}{\bar{w}^*W}\mathbf{y}_t^*]$ and $a\frac{\bar{w}}{\bar{w}^W}\mathbf{RP}_t + (1-a)\frac{\bar{w}^*}{\bar{w}^*W}\mathbf{RP}_t^* = 0$. (In fact, $\frac{\bar{y}}{\bar{y}^W} = \frac{\bar{w}\bar{L}}{a\bar{w}\bar{L} + (1-a)\bar{w}^*\bar{L}^*} = \frac{\bar{w}}{a\bar{w} + (1-a)\bar{w}^*\frac{\bar{L}^*}{\bar{L}}} \neq \frac{\bar{w}}{\bar{w}^W}$, unless $\bar{L} = \bar{L}^*$. Similarly for $\frac{\bar{y}^*}{\bar{y}^*W}$ and $\frac{\bar{w}^*}{\bar{w}^*W}$.) If $\alpha < 1$, equations (A.62) and (A.64) show that $a\frac{\bar{w}}{\bar{w}^W}\mathbf{RP}_t + (1-a)\frac{\bar{w}^*}{\bar{w}^*W}\mathbf{RP}_t^*$ is proportional to the difference between $\frac{\bar{L}^*}{\bar{L}}$ and 1. Therefore, if steady-state home labor effort is close to foreign, as it is the case for the values of α that we consider, $a\frac{\bar{w}}{\bar{w}^W}\mathbf{RP}_t + (1-a)\frac{\bar{w}^*}{\bar{w}^*W}\mathbf{RP}_t^*$ is close to zero, and equation (A.59) ((27) in the main text) approximates the solution for the world interest rate accurately.

A.5 Empirical Long-Run Restrictions

The functions $\bar{r}(\cdot)$, $\bar{L}^R(\cdot)$, $\bar{c}^R(\cdot)$, and $\bar{B}(\cdot)$ in equations (A.21)–(A.24) are non-linear and depend on the structural parameters. To obtain the empirical long-run restrictions in (32), we proceed as follows.

First, we assume that steady-state domestic and foreign productivity levels are such that:

$$\bar{Z} = \Gamma_1^{\bar{Z}} \left(\bar{Z}^* \right)^{\gamma_2^{\bar{Z}}}, \quad (\text{A.65})$$

where $\Gamma_1^{\bar{Z}}$ and $\gamma_2^{\bar{Z}}$ are coefficients such that $\Gamma_1^{\bar{Z}} > 0$ and $\gamma_2^{\bar{Z}} \geq 0$. In logs:

$$\log \bar{Z} = \gamma_1^{\bar{Z}} + \gamma_2^{\bar{Z}} \log \bar{Z}^*,$$

where $\gamma_1^{\bar{Z}} = \log \Gamma_1^{\bar{Z}}$.

Next, we assume that the empirical counterpart to the functions $\bar{r}(\cdot)$, $\bar{c}^R(\cdot)$, and $\bar{B}(\cdot)$ consists of the following log-linear relations:

$$\begin{aligned} \log \bar{r} &= \gamma_1^{\bar{r}} + \gamma_2^{\bar{r}} \left(\log \bar{Z} - \log \bar{Z}^* \right), \\ \log \bar{c} - \log \bar{c}^* &= \gamma_1^{\bar{c}^R} + \gamma_2^{\bar{c}^R} \left(\log \bar{Z} - \log \bar{Z}^* \right), \\ \log \bar{B} &= \gamma_1^{\bar{B}} + \gamma_2^{\bar{B}} \left(\log \bar{Z} - \log \bar{Z}^* \right). \end{aligned}$$

For any pair of individual country variables x_t and x_t^* , we define the variables $x_t^A \equiv (x_t)^a (x_t^*)^{1-a}$ and $x_t^R \equiv \frac{x_t}{x_t^*}$.³⁸ Given these definitions, the following equalities hold:

$$x_t = x_t^A (x_t^R)^{1-a}, \quad x_t^* = x_t^A (x_t^R)^{-a}. \quad (\text{A.66})$$

In logs:

$$\log x_t = \log x_t^A + (1-a) \log x_t^R, \quad \log x_t^* = \log x_t^A - a \log x_t^R, \quad (\text{A.67})$$

with $\log x_t^R = \log x_t - \log x_t^*$. Therefore, in steady state:

$$\log \bar{c} = \log \bar{c}^A + (1-a) (\log \bar{c} - \log \bar{c}^*), \quad \log \bar{c}^* = \log \bar{c}^A - a (\log \bar{c} - \log \bar{c}^*). \quad (\text{A.68})$$

Now, using $\log \bar{c} - \log \bar{c}^* = \gamma_1^{\bar{c}^R} + \gamma_2^{\bar{c}^R} (\log \bar{Z} - \log \bar{Z}^*)$, we have:

$$\log \bar{c} = \log \bar{c}^A + (1-a) \gamma_1^{\bar{c}^R} + (1-a) \gamma_2^{\bar{c}^R} (\log \bar{Z} - \log \bar{Z}^*), \quad (\text{A.69})$$

$$\log \bar{c}^* = \log \bar{c}^A - a \gamma_1^{\bar{c}^R} - a \gamma_2^{\bar{c}^R} (\log \bar{Z} - \log \bar{Z}^*). \quad (\text{A.70})$$

To obtain a solution for \bar{c}^A , we note that \bar{c}^A must be a function of steady-state productivity at home and abroad: $\bar{c}^A = \bar{c}^A(\bar{Z}, \bar{Z}^*)$. We then approximate this function with a log-linear relation, so that:

$$\log \bar{c}^A = \gamma_1^{\bar{c}^A} + \gamma_2^{\bar{c}^A} \log \bar{Z} + \gamma_3^{\bar{c}^A} \log \bar{Z}^*. \quad (\text{A.71})$$

Hence, we can write:

$$\begin{aligned} \log \bar{c} &= \gamma_1^{\bar{c}^A} + (1-a) \gamma_1^{\bar{c}^R} + \gamma_2^{\bar{c}^A} \log \bar{Z} + \gamma_3^{\bar{c}^A} \log \bar{Z}^* + (1-a) \gamma_2^{\bar{c}^R} (\log \bar{Z} - \log \bar{Z}^*) \\ &= \gamma_1^{\bar{c}} + \gamma_2^{\bar{c}^A} \log \bar{Z} + \gamma_3^{\bar{c}^A} \log \bar{Z}^* + (1-a) \gamma_2^{\bar{c}^R} (\log \bar{Z} - \log \bar{Z}^*), \end{aligned} \quad (\text{A.72})$$

$$\begin{aligned} \log \bar{c}^* &= \gamma_1^{\bar{c}^A} - a \gamma_1^{\bar{c}^R} + \gamma_2^{\bar{c}^A} \log \bar{Z} + \gamma_3^{\bar{c}^A} \log \bar{Z}^* - a \gamma_2^{\bar{c}^R} (\log \bar{Z} - \log \bar{Z}^*) \\ &= \gamma_1^{\bar{c}^*} + \gamma_2^{\bar{c}^A} \log \bar{Z} + \gamma_3^{\bar{c}^A} \log \bar{Z}^* - a \gamma_2^{\bar{c}^R} (\log \bar{Z} - \log \bar{Z}^*), \end{aligned} \quad (\text{A.73})$$

where $\gamma_1^{\bar{c}} \equiv \gamma_1^{\bar{c}^A} + (1-a) \gamma_1^{\bar{c}^R}$ and $\gamma_1^{\bar{c}^*} \equiv \gamma_1^{\bar{c}^A} - a \gamma_1^{\bar{c}^R}$.

Equations (A.72) and (A.73) show that steady-state domestic and foreign consumption levels are functions of steady-state productivity in each country and of the cross-country productivity differential. Rearranging terms and defining $\gamma_2^{\bar{c}} \equiv \gamma_2^{\bar{c}^A} + (1-a) \gamma_2^{\bar{c}^R}$, $\gamma_3^{\bar{c}} \equiv \gamma_3^{\bar{c}^A} - (1-a) \gamma_2^{\bar{c}^R}$, $\gamma_2^{\bar{c}^*} \equiv \gamma_2^{\bar{c}^A} - a \gamma_2^{\bar{c}^R}$, and $\gamma_3^{\bar{c}^*} \equiv \gamma_3^{\bar{c}^A} + a \gamma_2^{\bar{c}^R}$ gives the long-run restrictions on the behavior of home and foreign consumption stated in the text.

³⁸The variable x_t^A differs from the definition of world aggregate per capita x_t^W ($\equiv ax_t + (1-a)x_t^*$) used in the text. For this reason, we use a different superscript.

B Data

B.1 Sources and Coverage

We use quarterly data for the G7 countries, to the best of our abilities, comparable across countries. We primarily use three data sources: (i) The OECD *Analytical Database* (AD, retrieved on 18 February 2002), which provides quarterly data on *business sector* output, consumption, and employment; (ii) Quarterly net foreign assets (NFA) data graciously provided by Christopher Baum, who builds on the annual series constructed by Lane and Milesi-Ferretti (2001); and (iii) the IMF *International Financial Statistics* (IFS).

The country abbreviations are as follows: Canada/CAN, France/FRA, Germany/GER, Italy/ITA, Japan/JPN, United Kingdom/UK, United States/US.

While some series go back to 1960, available OECD data for the business sector are mostly limited to the period from 1970:Q1 to 1999:Q4. The sample period for NFA is from 1977:Q1, the first year for which we have NFA data for Japan (the series which constrains our analysis), to 1997:Q4. When needed, we re-scaled data to ensure strictly positive series before taking logs.

B.2 Variables

Output—Gross domestic product (GDP), business sector, volume, factor cost, in millions of local currency units. For Canada and the U.S., the base year is 1997 and 1996, respectively. For the rest of the G7, the base year is 1995. We re-based the Canadian and U.S. business sector GDP so that 1995 is the common base year. We used the GDP business sector deflator for this purpose.

Employment—Employment of the business sector, millions of persons.

Hours worked—Actual hours worked per employee in the business sector. The U.S. series is an index. We back-casted it starting from 1989 by using the annual average hours actually worked obtained from the OECD, *Employment Outlook*, 2001 Edition, Table F.

Exchange rate—Purchasing power parity (PPP), local currency per U.S. dollar, from the AD; annual series interpolated to quarterly by means of a cubic spline.

Consumption—Business sector private final consumption deflated by the (business sector) GDP deflator, except in the case of the UK, for which we lacked the series on GDP deflator and used the private final consumption deflator.

Population—Annual series from AD interpolated by using a cubic spline.

NFA—Deflated with the CPI from the IFS and expressed in per capita terms. These series are then converted into current U.S. dollars by using market exchange rates (from the IFS) and deflated by the U.S. GDP business sector deflator (from the AD).

Real interest rate (r)—Nominal interest rate (i) adjusted for annualized quarterly inflation in the average CPI:

$$1 + r_t = \log \frac{1 + i_t/100}{(\text{CPI}_t/\text{CPI}_{t-1})^4}.$$

The nominal interest rate is a quarterly average of 3-month T-bill rates on an annual basis, except for Japan. For Japan we used the call money market rate because we lacked comparable data.

B.3 Labor Productivity and Per Capita Consumption

To obtain “per unit of labor service,” we first calculated business sector GDP per hour worked in local currency units as GDP per employee hour worked:

$$gdp = GDP / (\text{employment} \times \text{hours worked}).$$

We then converted this variable to a common currency. To do this, we pursued two alternative strategies with very similar results (see Table B1 below).

The first method uses national price deflators. In this case, we deflated GDP per employee hour worked by national business sector GDP deflators (P),

$$Rgdp(t) = gdp(t) / P(t),$$

and then calculated $Rgdp$ per hour worked in US dollars by using 1995 PPP (PPP_{1995}):

$$Z(t) = Rgdp(t) / PPP_{1995}.$$

This is our basic labor productivity measure.

The second method uses the U.S. price deflator for all countries. In this case, we first converted GDP per employee hour worked into US dollars using interpolated PPP series, and then deflated all series by the US business sector GDP deflator (P^{US}):

$$Z'(t) = [gdp(t) / PPP(t)] / P^{US}(t).$$

The only exception is the UK, for which we do not have the GDP business sector deflator data prior to 1987:Q1. In this case, we used the consumption deflator to convert real GDP business sector values into nominal GDP, and then proceeded as above.

We calculated rest-of-the-world labor productivity as follows: We first computed real GDP in national currency units (RGDP), then converted the RGDP of those countries that make up the rest-of-the-world economy into U.S. dollars using PPP_{1995} , we summed over countries, and finally divided by the rest-of-the-world total employee hours.

Per capita consumption is private final consumption expenditure divided by population. Our measure of consumption is real per capita consumption in PPP U.S. dollars. We computed per capita consumption data using the two methodologies discussed above for labor productivity. Similarly, we obtained rest-of-the-world per capita consumption following the same steps as in the case of labor productivity.

Table B1: Correlations Between Alternative Measures

	CAN	FRA	GER	ITA	JPN	UK	US
Z	0.99093	0.99437	0.99963	0.99565	0.99697	0.99329	1.00000
C	0.99337	0.99913	0.97749	0.99644	0.99666	0.99616	1.00000

Notes: Z is labor productivity, and C is per capita consumption.

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Table 1. The Benchmark Solution

Steady-State Levels

$$\begin{aligned} \bar{r} &= .01015554120 & \bar{B} &= .5588806661 \\ \bar{L} &= .3299443011 & \bar{L}^* &= .3300470075 \\ \bar{w} &= 1.045291364 & \bar{w}^* &= 1.238564933 \\ \bar{y} &= .3448879285 & \bar{y}^* &= .4087846498 \\ \bar{c} &= .3449748265 & \bar{c}^* &= .4086977516 \end{aligned}$$

Elasticities

	B_t	r_t	$\phi = .9$		$\phi = 1$	
			Z_t	Z_t^*	Z_t	Z_t^*
B_{t+1}	.9924	.9924	.3737	-.3661	.0039	.0037
r_{t+1}	.0000	.0000	-.0458	-.0542	.0000	.0000
RP_t	.0028	.0028	-.3174	.3146	-.1823	.1794
RP_t^*	-.0024	-.0024	.2678	-.2654	.1538	-.1514
L_t	-.0085	-.0085	.4099	-.4014	.0044	.0041
L_t^*	.0072	.0072	-.3458	.3386	-.0037	-.0035
w_t	.0028	.0028	.6826	.3146	.8177	.1794
w_t^*	-.0024	-.0024	.2678	.7346	.1538	.8486
h_t	.0017	.0017	.4833	.5150	.8200	.1784
h_t^*	-.0013	-.0013	.4362	.5652	.1549	.8465
y_t	-.0057	-.0057	1.0925	-.0868	.8222	.1835
y_t^*	.0048	.0048	-.0780	1.0732	.1501	.8451
c_t	.0070	.0070	.4807	.5122	.8156	.1774
c_t^*	-.0059	-.0059	.4382	.5678	.1556	.8504

Table 2. United States: Cointegration results (1979:Q4-1994:Q4)

Johansen procedure (with productivity entered endogenously):

eigenvalue	loglik	for rank
	1298.406	0
0.51379	1320.040	1
0.42826	1336.812	2
0.34351	1349.438	3
0.26254	1358.574	4
0.11075	1362.095	5
4.2240e-005	1362.096	6
H0:rank<= Trace test [Prob]		
0	127.38	[0.000] **
1	84.114	[0.002] **
2	50.569	[0.026] *
3	25.318	[0.155]
4	7.0449	[0.579]
5	0.0025	[0.960]

Estimated cointegration vectors:

Z_US	1.0000	-1.8527	0.00000	5.6363	0.00000
ZEX_US	-0.6000	0.00000	-1.0000	0.00000	0.00000
C_US	0.00000	1.0000	0.00000	0.00000	0.00000
CEX_US	0.00000	0.00000	1.0000	0.00000	0.00000
NFA_US	0.00000	0.00000	0.00000	1.0000	0.00000
R_US	0.00000	0.00000	0.00000	0.00000	1.0000

LR test of restrictions: $\text{Chi}^2(3) = 2.1357 [0.5447]$

Johansen procedure (with productivity entered exogenously)

I(1) cointegration analysis, 1980 (1) to 1995 (4)

eigenvalue	loglik	for rank
	842.0402	0
0.51864	865.4368	1
0.31170	877.3895	2
0.23755	886.0684	3
0.10407	889.5851	4
H0:rank<= Trace test [Prob]		
0	95.090	[0.000] **
1	48.297	[0.000] **
2	24.391	[0.001] **
3	7.0333	[0.008] **

Specification tests:

Vector AR 1-4 test:	$F(64,119) = 1.2069 [0.1881]$
Vector Normality test:	$\text{Chi}^2(8) = 13.184 [0.1057]$
Vector hetero test:	$F(200,193) = 0.70217 [0.9933]$

Table 3. Japan: Cointegration results (1979:Q4-1994:Q4)

Johansen procedure (with productivity entered endogenously):

eigenvalue	loglik for rank
	1265.169 0
0.57377	1290.753 1
0.52972	1313.385 2
0.24911	1321.980 3
0.15328	1326.972 4
0.13512	1331.327 5
0.017610	1331.860 6
H0:rank<=	Trace test [Prob]
0	133.38 [0.000] **
1	82.214 [0.003] **
2	36.949 [0.354]
3	19.759 [0.450]
4	9.7761 [0.304]
5	1.0660 [0.302]

Estimated cointegration vectors:

Z_JPN	1.0000	-0.94173	0.00000	-1.7095	0.00000
ZEX_JPN	-1.7106	0.00000	-1.0724	0.00000	0.00000
C_JPN	0.00000	1.0000	0.00000	0.00000	0.00000
CEX_JPN	0.00000	0.00000	1.0000	0.00000	0.00000
NFA_JPN	0.00000	0.00000	0.00000	1.0000	0.00000
R_JPN1	0.00000	0.00000	0.00000	0.00000	1.0000

LR test of restrictions: $\chi^2(1) = 4.0522 [0.0441]^*$

Johansen procedure (with productivity entered exogenously)

I(1) cointegration analysis, 1980 (1) to 1995 (4)

eigenvalue	loglik for rank
	804.0371 0
0.45967	823.7355 1
0.28793	834.6019 2
0.23898	843.3410 3
0.073993	845.8009 4
H0:rank<=	Trace test [Prob]
0	83.528 [0.000] **
1	44.131 [0.000] **
2	22.398 [0.003] **
3	4.9199 [0.027] *

Specification tests:

Vector AR 1-4 test:	F(64,119)= 1.1085 [0.3111]
Vector Normality test:	$\chi^2(8) = 47.169 [0.0000]**$
Vector hetero test:	F(200,193)= 0.77486 [0.9629]

Table 4. Germany: Cointegration results(1979Q4-1994:Q4)

Johansen procedure (with productivity entered endogenously):

I(1) cointegration analysis, 1980 (1) to 1994 (4)

eigenvalue	loglik	for rank
	1270.652	0
0.63002	1300.481	1
0.43938	1317.842	2
0.29438	1328.303	3
0.20248	1335.090	4
0.072443	1337.346	5
0.017149	1337.865	6

H0:rank<=	Trace test	[Prob]
0	134.43	[0.000] **
1	74.768	[0.018] *
2	40.046	[0.223]
3	19.125	[0.495]
4	5.5499	[0.749]
5	1.0378	[0.308]

Estimated cointegration vectors:

Z_GER	1.0000	-0.29742	0.00000	-0.94476	0.00000
ZEX_GER	-1.0000	0.00000	-1.3148	0.00000	0.00000
C_GER	0.00000	1.0000	0.00000	0.00000	0.00000
CEX_GER	0.00000	0.00000	1.0000	0.00000	0.00000
NFA_GER	0.00000	0.00000	0.00000	1.0000	0.00000
R_GER1	0.00000	0.00000	0.00000	0.00000	1.0000

LR test of restrictions: $\text{Chi}^2(2) = 6.4482 [0.0398]^*$

Johansen procedure (with productivity entered exogenously)

I(1) cointegration analysis, 1980 (1) to 1995 (4)

eigenvalue	loglik	for rank
	832.0418	0
0.52953	856.1703	1
0.42286	873.7597	2
0.17307	879.8410	3
0.015762	880.3494	4

H0:rank<=	Trace test	[Prob]
0	96.615	[0.000] **
1	48.358	[0.000] **
2	13.179	[0.108]
3	1.0168	[0.313]

Specification tests:

Vector AR 1-4 test: $F(64,119) = 1.4452 [0.0424]^*$
 Vector Normality test: $\text{Chi}^2(8) = 17.885 [0.0221]^*$
 Vector hetero test: $F(200,193) = 0.45137 [1.0000]$

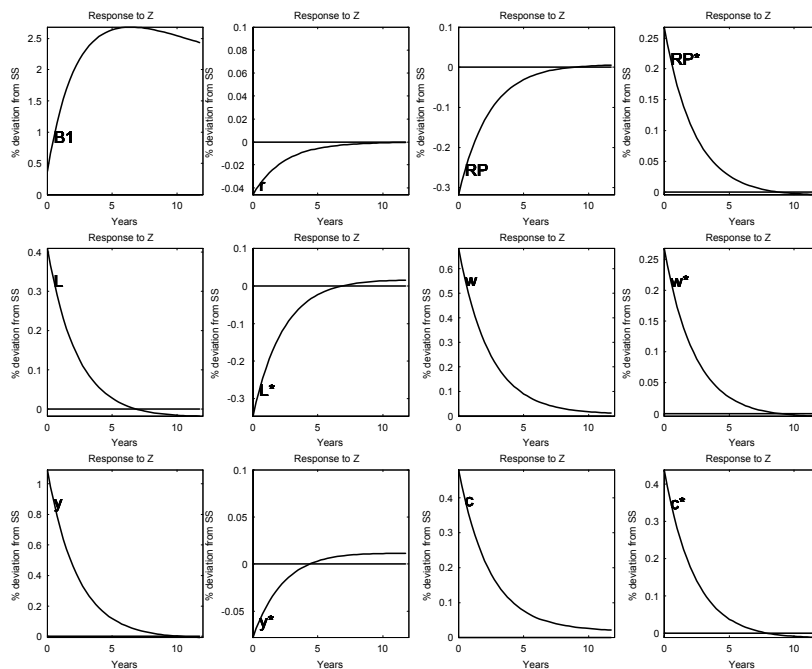


Figure 1: Productivity Shock in the Creditor Country

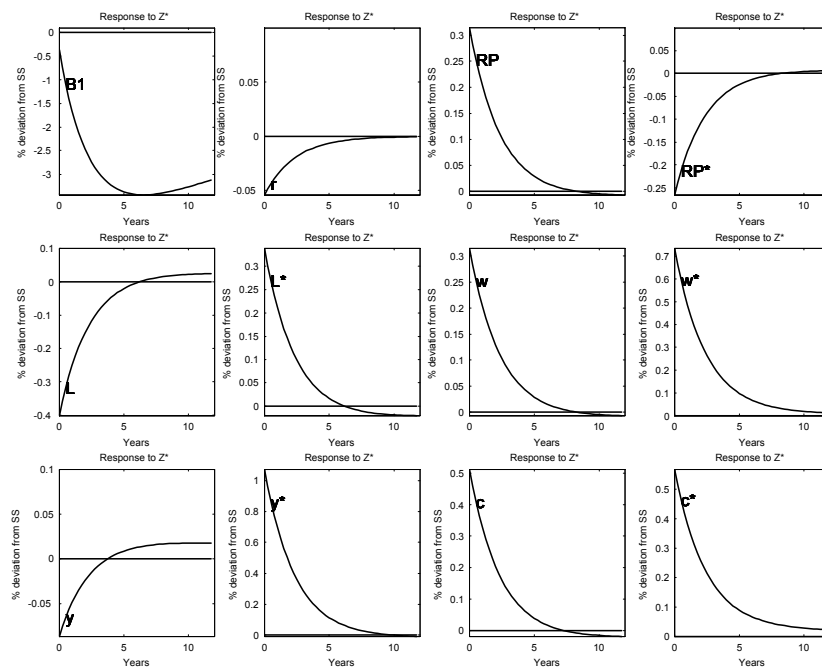


Figure 2: Productivity Shock in the Debtor Country

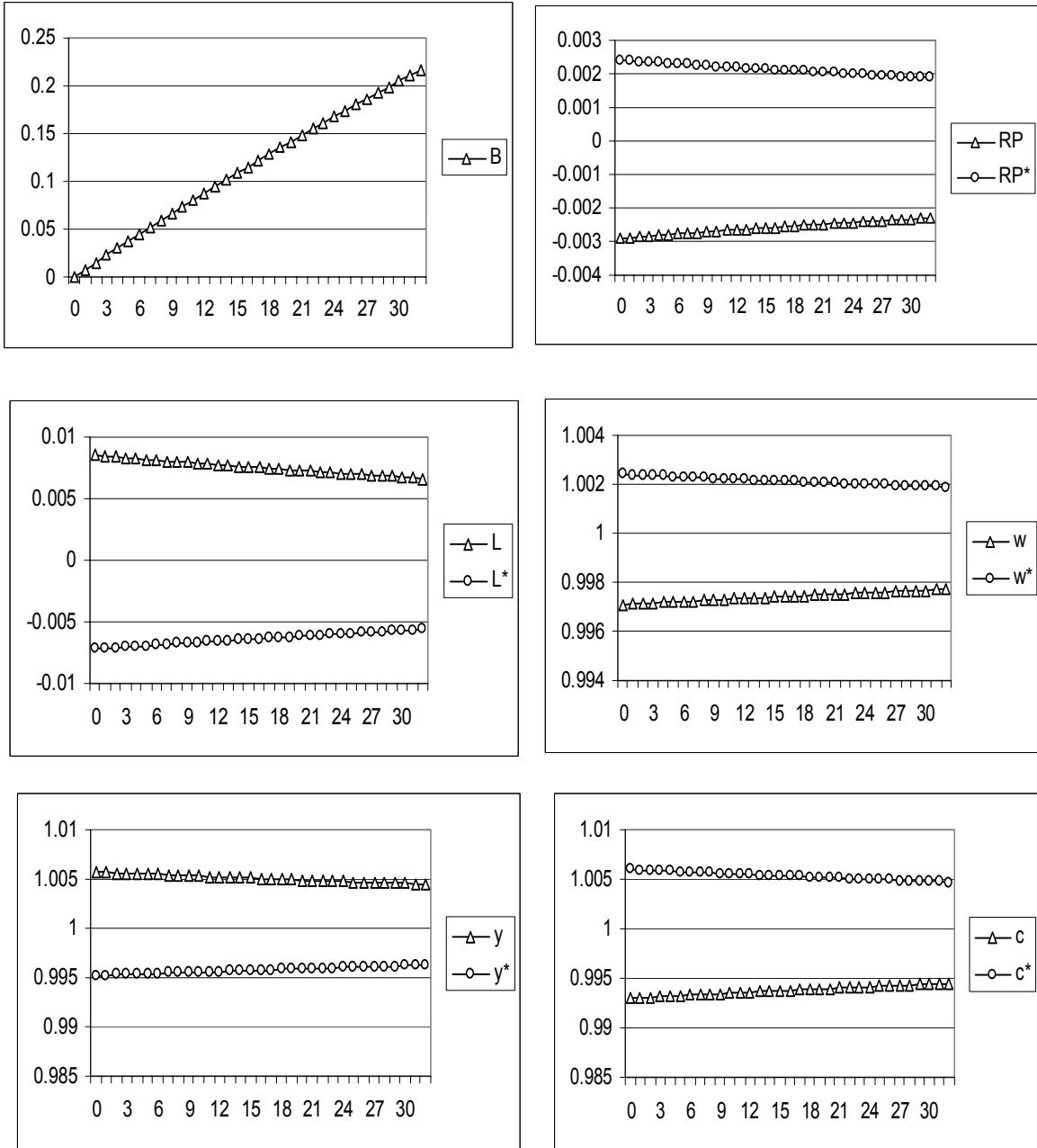


Figure 3. Permanent World Productivity Shock

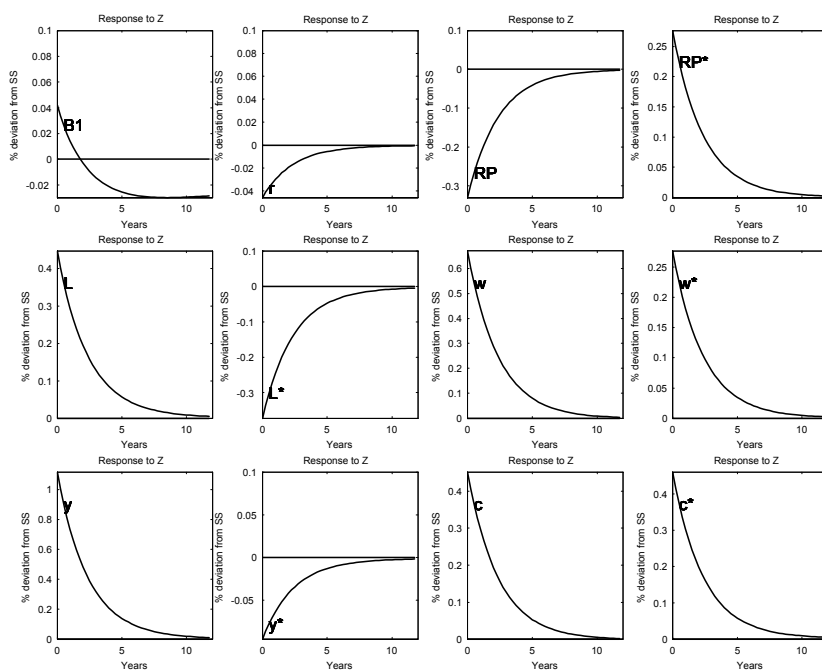
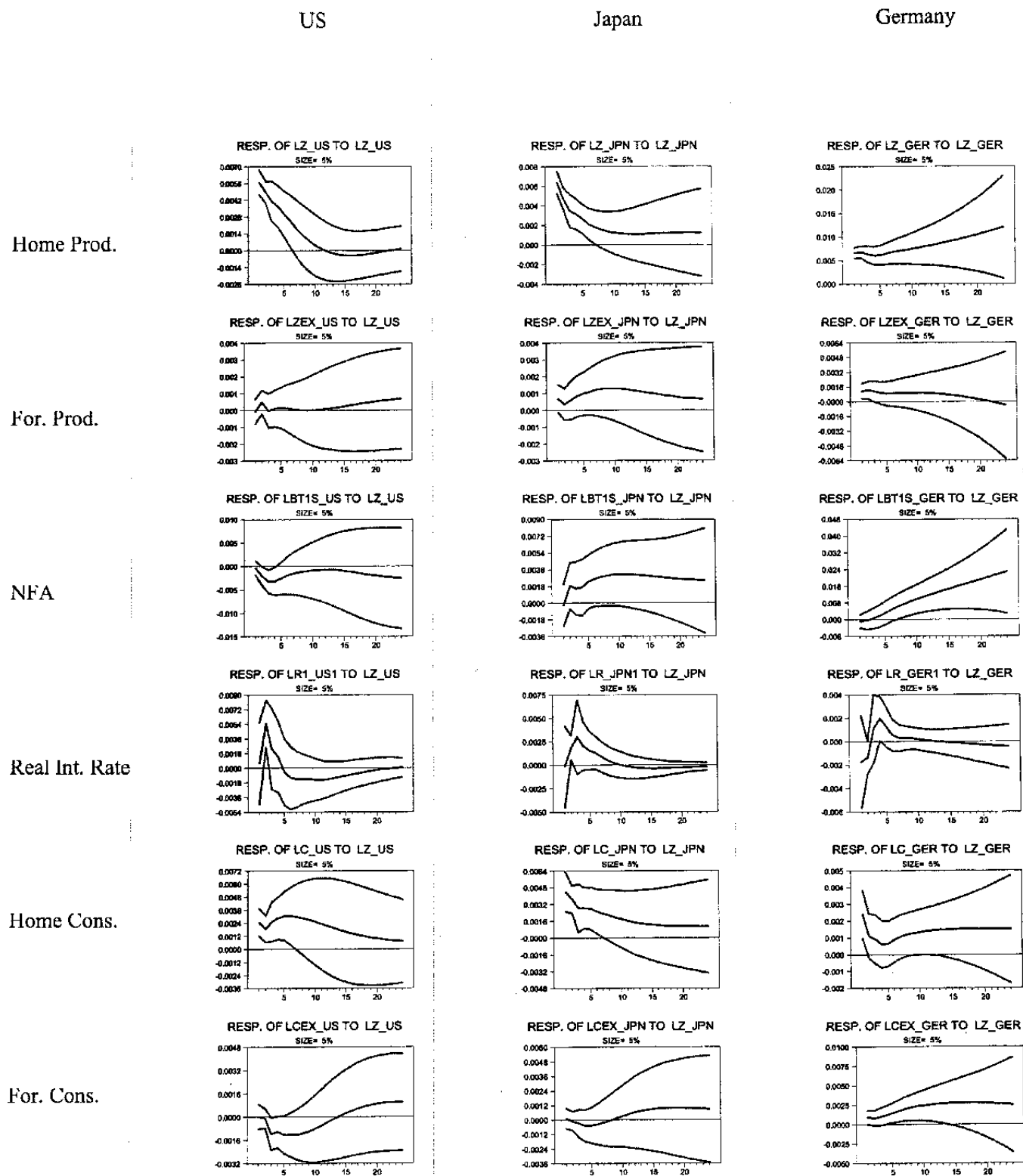


Figure 4: Productivity Shock in the Creditor Country (Lower α)

Figure 5. Response to a Country Specific Shock



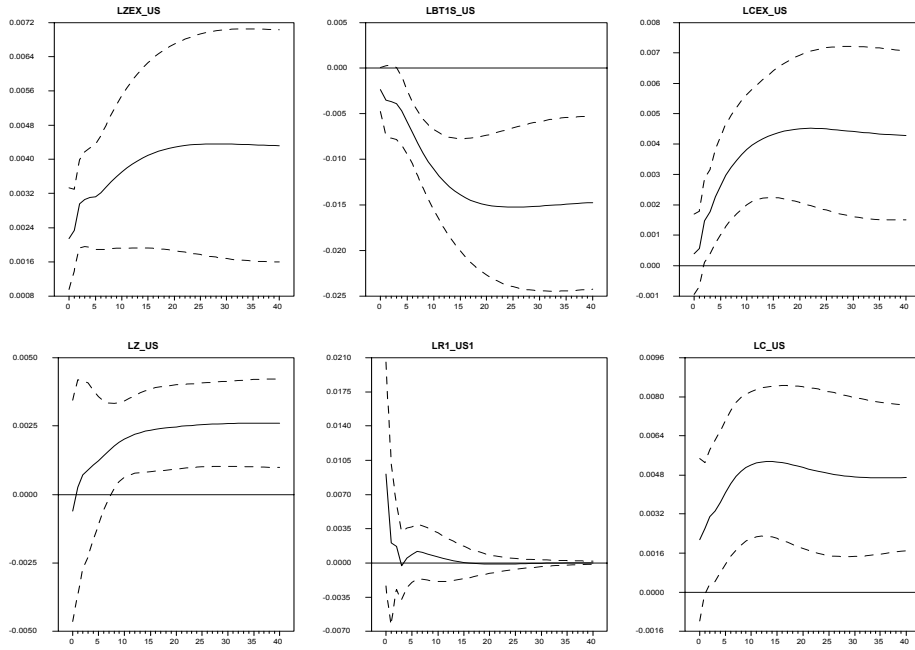


Figure 6: U.S.: Response to a Permanent World Shock

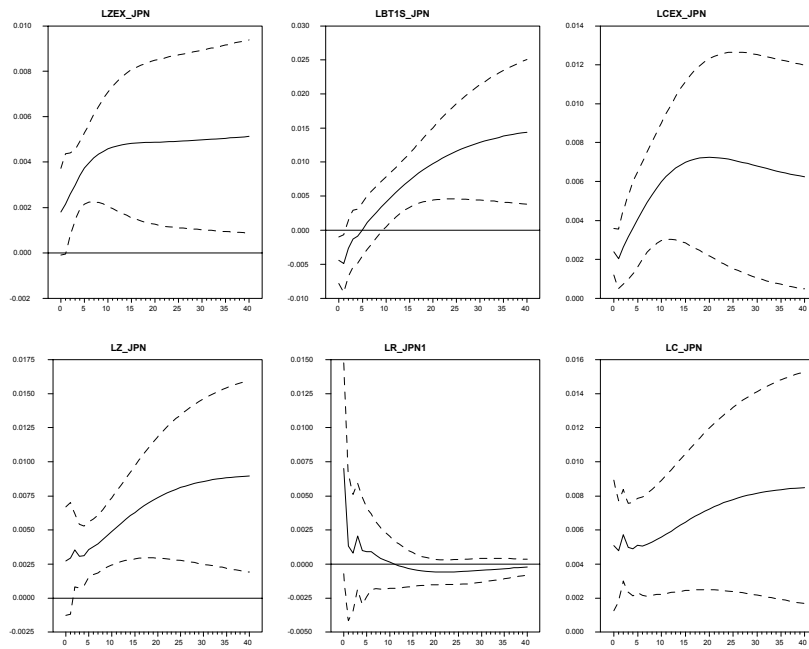


Figure 7: Japan: Response to a Permanent World Shock

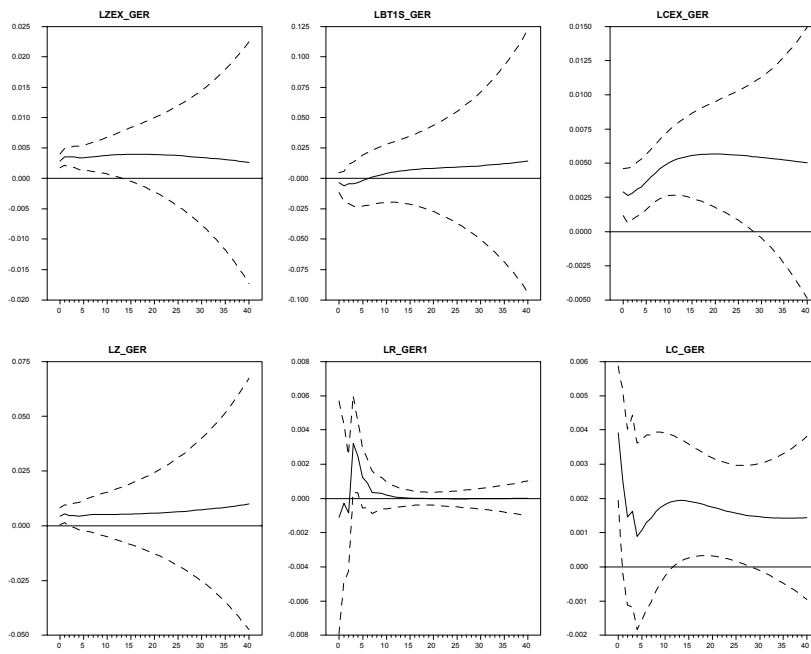


Figure 8: Germany: Response to a Permanent World Shock