Volatility and Sovereign Default

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This version: October 2003

Abstract

Over and over again, history shows that countries default on external debt when their economies experience a downturn. This paper presents a theoretical model of international lending that is consistent with this evidence. Productivity is stochastic and international capital markets are incomplete in two ways: the only internationally traded assets are non-contingent real bonds and borrowers cannot commit to repay loans. Low productivity realizations get carried forward through low investment that lower output and consumption and eventually result in self-fulfilling or solvency debt crises. When lenders are atomistic, self-fulfilling crises may arise for intermediate debt levels that would not trigger a default with a large lender. Alternative reforms to eliminate liquidity crises are analyzed. An international lender of last resort can eliminate liquidity crises provided it implements full bailouts via purchasing debt at its market price.

JEL Classification Code: F3, F34
Keywords: Sovereign debt, Default, Self-fulfilling crises.

1 Introduction

The history of international lending shows that, time and time again, countries that borrow internationally are asked to repay when their economies are in a recession. Countries are asked to cut consumption and investment in order to repay their international loans exactly when they would like to borrow from abroad. In most cases, the borrowing country is a small open economy whose idiosyncratic shocks are unlikely to affect world economic conditions.

*I thank Peter Kenen for useful discussions on this paper.

The recent experience of Argentina is an example but other countries had similar experiences. Mexico defaulted on its external debt in 1983 and in 1995; both default episodes were preceded by economic deterioration and sharp reversal of economic flows. Chile, Brazil and Turkey also defaulted on their external debt in the early 1980s after their economies experienced severe downturns.

Lending models with complete markets do not match this empirical regularity. If markets are complete, i.e. state-contingent borrowing is feasible, and the government can commit to repay its debts, default arises in bad economic times but at very high levels of debt – much higher than those we see in the data. If markets are complete but the government cannot commit to repay its debts and default is punished by subsequent exclusion from international borrowing and lending, the incentive to default is strongest in good economic times. The empirical evidence suggest the opposite.

Before World War II, international lending took mostly the form of bond lending.[1] In the early postwar years, however, bond finance dried up and bank loans became widespread between 1974 and 1982. Loans from relatively few commercial banks represented most of the emerging economies' borrowing in the 1970s and resulted in the debt crisis of the early 1980s. Most of these defaults were solvency crises, characterized by high net external debt to GDP ratio. As a counterpart to that, some U.S. and European banks had exposures exceeding 50 percent in that crisis.

The defaults of the 1980s prompted the international capital markets to partly switch back from bank loans to bonds. An increasing number of emerging economies now borrow from the international capital market by issuing bonds again. For example, bond issues in Latin America grew from less than 1 billion US$ in 1989 to 11.17 billion US$ in 1993 (see Cline [7]). This impressive growth of bond capital flows stands in sharp contrast with the stagnation of new long-term loans from the commercial banks, which fell from 1.92 to 0.53 billion US$ over the same period in Latin America. Figure 1 shows bond capital flows and loans from commercial banks for 8 Latin American countries: Argentina, Brazil, Chile, Colombia, Mexico, Peru, Uruguay and Venezuela. Even ignoring the large jump in 1989, due to the conversion of restructured bank loans into Brady bonds, the trend is evident.

The change in the composition of international capital lending has been accompanied by the occurrence of sudden reversals of capital flows and defaults. The Mexican crisis of 1994-95 is an example. To reduce the cost of financing the deficit and preserve investor confidence, in 1994 the Mexican authorities replaced domestic-interest-based cetes with dollar-indexed tesobonos, all with maturities of less than a year. Even though net external debt for Mexico was only 25% of GDP, the bunching of its maturities and the fact that the outstanding stock of tesobonos exceeded gross international

[1] Lindert and Morton [19] analyze the historical record of bond lending and find that, on the whole, it has given a higher real rate of return than the alternative of lending to domestic governments.

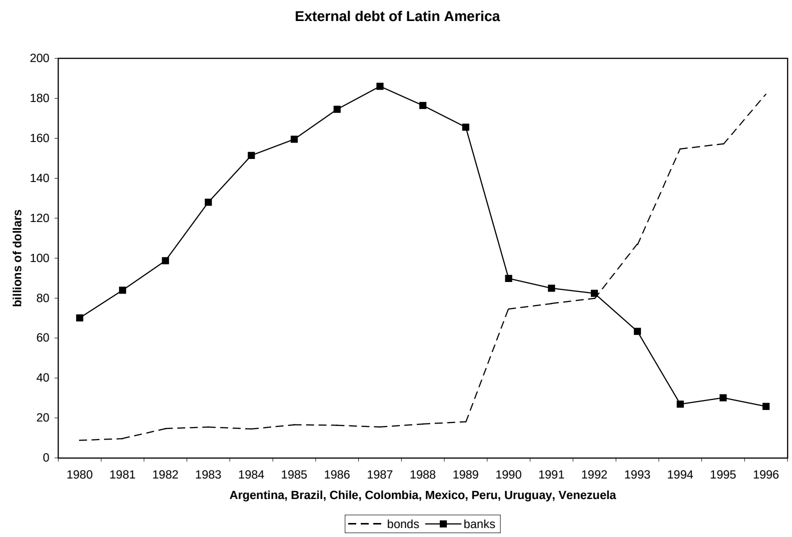

Figure 1: Latin America: composition of external debt as % of GDP

reserves by the end of 1994, led to default.

The problem that arises with bond financing and spread ownership is the debtor's exposure to liquidity crises. In solvency crises, like those of the 1980s, the debtor would not repay its outstanding debt even if it could be rolled over; in liquidity crises, like the Mexican default of 1994-95, the debtor would repay the outstanding debt if new debt could be issued.

This paper provides a theoretical framework that sqaures with the evidence described above. In this model, international capital markets are incomplete because borrowing can only occur via issuing bonds that promise a fixed repayment, independent of what state of the world occurs. Hence, international loan contracts and their repayments cannot be made state-contingent. At the same time, borrowers cannot commit to repay their loans so lenders are reluctant to extend credit beyond what they expect the borrower will have an incentive to repay. And when lenders are atomistic, as it may be the case with bond ownership, foreign lenders may refuse to roll over the debt even at relatively low levels. Most importantly, external credit dries up when the debtor country is experiencing low growth or is in a recession – which is exactly when it would like to borrow more.

The recent financial crises in emerging market economies have been costly for the countries that were at the center of the crises as well as the countries affected by the spillovers of the crises. The question of how to avoid liquidity crises and how to solve a crisis in an orderly manner, once it arises, are at the heart of the proposals to reform the international financial architecture. This paper takes a first step in this direction. Its framework allows studying policies and reforms to eliminate liquidity crises and assess their welfare consequences. I consider one of the proposals that has received most attention so far: the creation of an international lender of last resort. It turns out that an international lender of last resort can eliminate liquidity crises and implement the constrained efficient allocation (given that markets are incomplete) provided it implements full bailouts via purchasing debt at its market price. By full bailout I mean that the international lender of last resort should not extend the minimum amount of credit that barely avoids the crisis, but should lend the amount that the borrower was seeking to borrow from the international capital market. By fair price I mean that the price at which the international lender of last resort should purchase debt must be the market price, which reflects the risk of a solvency crisis. If debt is purchased at a price higher than its market value, a moral hazard problem arises and an inefficient amount of borrowing takes place in equilibrium.

The existing literature on sovereign debt and default is large; an excellent review of this literature is given by Eaton and Fernandez [10]. Eaton and Gersowitz [11] were the first to look at international lending without commitment to repay and find that borrowing is indeed limited. Bulow and Rogoff [4] study the case where a government can safely invest abroad regardless of any past default; they find that international lending must be supported by direct sanctions, as a country's reputation for repayment would not support any borrowing. My work assumes that default is punished by permanent exclusion from borrowing and lending. The history of international lending shows that partial repayments often follow default. Fernandez and Rosenthal [13] study debt renegotiations between a sovereign government and a large creditor. In my setting borrowing occurs by issuing bonds to a large number of atomistic lenders; and the problem with bonds, as pointed out by the literature on the new international financial architecture, is that debt renegotiation may be difficult, if not impossible, when bond ownership is spread.

Atkeson [1] presents a model of international lending with a moral hazard problem in investment. He shows that the borrowing country experiences a capital outflow when the worst output realization takes place. This occurs because of the unobservability of investment. In my paper there is perfect information and the reason why credit is withdrawn during economic downturns is because the incentive to repay is low in such contingencies and debt contracts are not state contingent.

Cole and Kehoe [8] have a model of self-fulfilling debt crises where consumers are risk-averse with respect to public consumption.[2] They characterize the optimal policy response of the government to the threat of a liquidity crisis, which consists in a reduction of the stock of debt when the country is in the zone where a self-fulfilling crisis may arise. My framework is more general than that of Cole and Kehoe. I allow for a production structure with stochastic technology and for risk aversion in private consumption. It is precisely because agents are risk-averse that a bad productivity

[2] Self-fulfilling crises are first studied in Obstfeld [21].

shock motivates borrowing. Moreover, the welfare costs of autarky are well defined in this setting.

The paper is organized as follows. Section 2 presents the basic model; section 3 presents the autarkic equilibrium. Section 4 studies the equilibrium when commitment to repay is not feasible. The emergence of self-fulfilling debt crises is explained in section 4.1 and section 5 discusses the debt overhang effect and the debt Laffer curve. Section 6 studies the welfare costs of liquidity crises and 7 discusses institution-based bailouts. Section 8 concludes by pointing out the directions for future work.

2 The model

Consider a small open economy in discrete time. There is a single good in each period, which can be either consumed or saved as capital. Production utilizes capital and, implicitly, inelastically supplied labor. There are three types of agents in this economy: consumers, foreign lenders and the domestic government.[3]

There is a continuum with measure one of identical infinitely-lived consumers who consume, invest and pay taxes to the government; consumers cannot access the international credit market directly, i.e. they cannot borrow from or lend to foreign agents or institutions; the government, however, can. This assumption captures some of the difficulties inherent with international lending. For example, emerging economies often impose restrictions on private flows of resources in and out of their boundaries; also, it is hard for lenders to monitor how private individuals use the proceeds of the loans, but it is easier to monitor a government. Loan repayments cannot be enforced and internatinal loans are hard to collateralize; in case of default, it is easier to negotiate partial repayments with a government rather than with many small private borrowers. Since the government is benevolent, it borrows and lends so as to maximize welfare of the citizens; unlike private consumers, who behave atomistically, the government is a large agent and behaves strategically, taking into account its decisions' effects on the price of debt and on the stock of capital.

The representative consumer has preferences

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} E_t u(c_s) \tag{1}$$

where E_t means expectations based on the knowledge available at time t, c_s is private consumption in period s, $u(\cdot)$ is assumed to be continuously differentiable, strictly concave, and monotonically increasing, and $\beta < 1$ is the subjective discount factor. In every period, consumers own the outcome of production that, after paying taxes, is

[3]The setup is close to Cole and Kehoe [8], but in a more general framework.

allocated in consumption and investment. The consumer's budget constraint is

$$y_t - \tau_t = c_t + k_{t+1} \tag{2}$$

where y_t is production, k_t is the consumer's individual capital stock and τ_t is the lump-sum tax paid to the government, all this at time t. A unit of capital is created from a unit of the consumption good. This implies that the relative price of capital goods in terms of consumption always equals 1. For simplicity, I assume that capital depreciates completely after its use. Output is produced using capital and the production function is

$$y_t = A_t k_t^\alpha \tag{3}$$

where A_t is a stochastic multiplicative productivity factor. Productivity follows the simple process

$$A_t = A + \epsilon_t, \qquad A > 0, \tag{4}$$

where ϵ_t is a i.i.d. shock with mean zero and variance σ_ϵ, distributed over $[-\epsilon, \epsilon]$ with probability density function $\zeta(\epsilon_t)$. The initial capital stock k_t is given.[4]

There is a domestic government in the economy; the government is benevolent in the sense that its objective is to maximize the utility of consumers, U_t. Unlike the private consumers, the government can access the international credit market and can therefore borrow and lend abroad. Credit markets are incomplete because the government can only borrow or lend via issuing or purchasing bonds that promise a fixed repayment, independent of the productivity realization. Hence, international loan contracts and/or their repayments cannot be made state-contingent. The government levies a lump-sum tax τ_t from consumers at time t; $\tau_t < 0$ is a transfer. In every period, the government issues new debt b_{t+1}, chooses whether to repay its outstanding debt b_t, by setting $z_t = 1$, or default on it by setting $z_t = 0$, and it chooses the tax τ_t, subject to the constraint

$$q_t b_{t+1} = b_t z_t - \tau_t. \tag{5}$$

q_t is the price of a one-period government bond that pays one unit of the consumption good in period $t+1$ if default does not occur. There is no need to impose a constraint on the government on how much it can borrow in order to avoid Ponzi schemes because, if the government tries to sell too much debt, the price q_t goes to zero. The initial stock of debt b_t is given.

It is assumed that, following default, the government loses access to international borrowing and lending and keeps the whole amount owed to the creditors. Refraining from lending abroad is subgame perfect for the defaulting government, as any

[4]The assumption that shocks are i.i.d. is not crucial for the final results; on the other hand, the assumption that capital depreciates fully is important. If capital did not depreciate at all, the country would have a stronger incentive to default with a good productivity shock as autarky would be less costly with a high capital stock.

lending could be seized by the creditors who want to recuperate their loans. Permanent exclusion from the international capital market is not entirely realistic and is not renegotiation-proof; in fact, large (i.e. non-atomistic) lenders are willing to forgive part of the debt in order to recuperate something and the borrower may be willing to repay part of the debt in order to regain access to the credit market (see Fernandez and Rosenthal [13]). But with debt ownership spread among a large number of small lenders, as it is assumed here, coordination problems make renegotiation difficult. These issues will be discussed in detail later.

There is also a continuum with measure one of identical, infinitely lived foreign lenders. The individual lender is risk neutral with utility function

$$J_t = \sum_{s=t}^{\infty} \beta^{s-t} E_t x_s \tag{6}$$

where x_t is the lender's private consumption. Each lender is endowed with $\overline{x}$ units of consumption good in each period, which can be lent or consumed later in the period. The lender's budget constraint is

$$q_t B_{t+1} = B_t z_t + \overline{x} - x_t. \tag{7}$$

When deciding how much new debt to buy, the lender faces the constraint

$$\overline{x} \geq q_t B_{t+1}. \tag{8}$$

I am going to assume that $\overline{x} >> B_{t+1}, \forall t$: this is a small open economy whose borrowing is small relative to the size of the international capital market. Foreign lenders' behavior depends on the realization of a sunspot variable ϕ: if $\phi_t = 1$, which happens with probability μ, foreign lenders are optimistic and each one of them believes that all others will purchase the new debt offering b_{t+1}; if $\phi_t = 0$, which happens with probability $1-\mu$, foreign lenders are pessimistic and believe that the new debt b_{t+1} will not be purchased. In equilibrium, foreign lenders' expectation are fulfilled and the outstanding debt b_t is defaulted when they are pessimistic and foreign debt is high enough.

The market clearing condition for the government's debt is $b_{t+1} = B_{t+1}$; I assume that the foreign lenders behave competitively in making their choice of b_{t+1}; consumers also behave competitively and take next period's prices and the government's actions as given; in equilibrium, all consumers are identical and K_{t+1} is the aggregate capital stock at the beginning of period $t+1$.

The timing of events is as follows: 1) the productivity shock ϵ_t is realized; 2) the government, taking the price schedule q_t as given, offers b_{t+1}; 3) the sunspot variable ϕ_t is realized; 4) the foreign lenders choose B_{t+1}; 5) the government decides whether to default or not ($z_t = 0$ or 1) and chooses the tax τ_t; 6) consumers decide how much to consume and to invest, c_t and k_{t+1}.

The timing of bond issue and repayment is fundamental to allow for self-fulfilling crises. The government issues new debt *before* it repays the old one; if current productivity is low and the stock of debt is high enough, foreign lenders' expectations crucially determine if default on the outstanding debt will take place. Given the fundamentals, optimistic lenders anticipate roll-over and no default on the outstanding debt; hence they purchase the new debt and repayment of the old debt occurs. With exactly the same fundamentals but pessimistic lenders that anticipate no roll-over, the new debt is not purchased and the old debt is defaulted. The probability of a crisis is arbitrary after the productivity shock is realized; ex-ante, the probability of a crisis is higher for lower productivity realizations.

3 The equilibrium with autarky

First, I study the autarkic equilibrium of this economy – namely, the equilibrium with no borrowing and lending from the rest of the world; this is the relevant equilibrium after default. When the economy is barred from international borrowing and lending, the government has no role and taxes are zero. Consumers choose how much to invest k_{t+1} in order to maximize (1) subject to (2); the first order condition is

$$u'(c_t) = \beta E_t \left[u'(c_{t+1}) A_{t+1} \alpha k_{t+1}^{\alpha-1} \right], \tag{9}$$

which shows that the optimal investment decision is such that the marginal disutility of investing an extra-unit of capital and cutting current consumption must be equal to the expected marginal increase in future consumption, which depends on future productivity. The right-hand side of (9) is the expected value of a product and can therefore be rewritten as

$$u'(c_t) = \beta E_t \left[u'(c_{t+1}) \right] E_t[A_{t+1} \alpha k_{t+1}^{\alpha-1}] + Cov(u'(c_{t+1}), A_{t+1} \alpha k_{t+1}^{\alpha-1}). \tag{10}$$

The covariance is negative because, when productivity is high, consumption is high and its marginal utility is low. Investment in domestic capital, which is the only asset available to citizens for smoothing their consumption, does not provide an edge against the fluctuations in productivity; as a result, consumers invest on average less than in an economy where consumption fluctuations can be insured against. Income is procyclical in this economy and, as a result, investment, income and consumption are also procyclical. Since consumers are risk-averse, the volatility of consumption makes them ex-ante worse off in the autarkic equilibrium than in an equilibrium where consumption fluctuations can be insured away. Given k_t, ϵ_t, the expected utility from t on in the autarkic equilibrium is

$$U_t^a(k_t, \epsilon_t) = u(y_t - k_{t+1}^a) + \sum_{s=t+1}^{\infty} \beta^{s-t} E_t u(y_s - k_{s+1}^a). \tag{11}$$

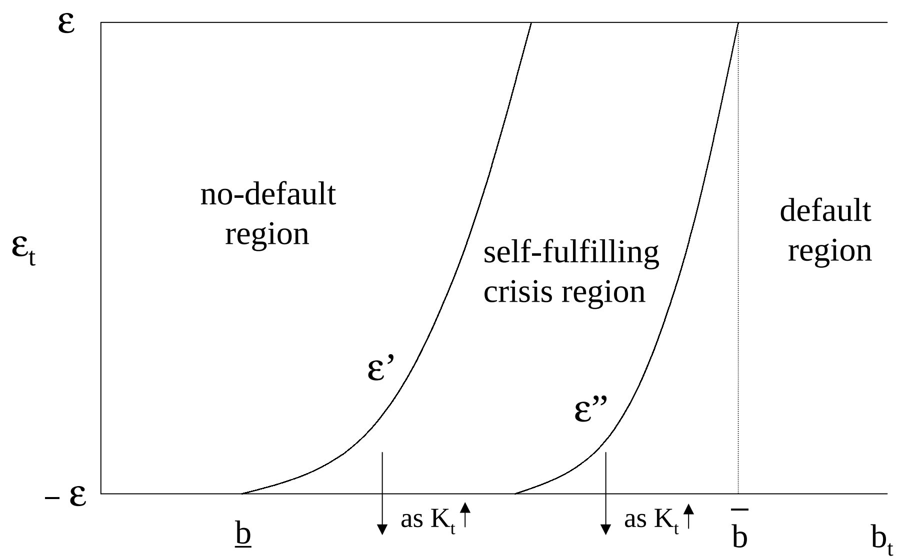

Figure 2: Debt regions

4 The equilibrium without commitment

This section studies the equilibrium when the government cannot commit to repay its debt. Appendix A studies the equilibrium when commitment to repay is feasible. Here, it is assumed that default causes: 1) the lender to lose its principal; 2) the borrower to lose its access to international borrowing and lending forever. I construct a recursive equilibrium in which commitment is not feasible and agents act sequentially and rationally. I consider the maximizing choice of the consumers, the maximizing choice of foreign lenders, and the maximizing choices of the government, which acts twice in period t: first, it decides how much debt to issue b_{t+1}, then it decides whether to repudiate the old debt, $z_t = 0$ or 1 (and residually it decides the tax τ_t).

The definition of equilibrium used here follows the definition of sustainable equilibrium of Chari and Kehoe [5], [6] and the definition of credible equilibrium by Stokey [24]. Appendix B gives the formal definitions.

4.1 Regions with crises: a graphical interpretation

This section gives a graphical intuition of the equilibrium of the model. Figure 2 shows that there are three regions in the (b_t, ϵ_t) space: a no-default region, a self-fulfilling crises region, and a default region. Given b_t, K_t, once the productivity shock ϵ_t is realized, the government knows in which of the three regions it lies and whether it is vulnerable to a crisis. For low levels of outstanding debt, i.e. for $b_t \leq \underline{b}(K_t)$, the probability of default is zero. Even if the productivity realization is low, the government prefers to repay the outstanding stock of debt rather than defaulting and being excluded from the international capital market forever. Since b_t is low, the cost of repaying is lower than the expected gains from being able to borrow in the future. Notice that the no-default region is wider in its upper part: the government is not vulnerable to crises when productivity is high. Since the probability of default is zero, the new debt b_{t+1} is purchased with probability one: the international capital market is "perfect" in this region.

For intermediate levels of debt, i.e. for $\underline{b}(K_t) < b_t \leq \bar{b}(K_t)$, the government is vulnerable to self-fulfilling crises. For debt-productivity realizations in this region, repayment and default are equilibria. If the sunspot variable ϕ_t is equal to 1, which happens with probability μ, foreign lenders purchase the new debt b_{t+1} sold by the government that, with these proceeds, repays the outstanding debt b_t. If the sunspot variable ϕ_t is equal to zero, which happens with probability $1-\mu$, foreign lenders do not purchase the new debt b_{t+1}; unable to roll over the outstanding debt, the government should levy high taxes to repay it and prefers to default. Notice that repayment and default arise with the same fundamentals; the only difference is whether foreign lenders purchase the new debt or not. In the first case, each lender is optimistic and purchases the new debt as she believes that all other lenders will also do so. In the second case, each lender is pessimistic and does not purchase the new debt because she believes that no one else will: if productivity is low enough, failure to sell the new debt triggers a default on the outstanding debt. This is a self-fulfilling debt crisis. At the heart of self-fulfilling crises is the atomistic behavior of foreign lenders. One large lender, or few of them, internalizes the government's need of issuing new debt to repay old debt and will not cut credit.

The left boundary between the no-default and the self-fulfilling crises region is the combination of debt levels and productivity realizations such that the government is indifferent between repaying the outstanding debt b_t and defaulting on it when foreign lenders do *not* purchase new debt b_{t+1}. These combinations are labelled $\epsilon'(b_t, K_t)$ and they will be defined formally in the next section. ϵ' is postively sloped because the goverment needs to levy higher taxes to repay higher leves of b_t without issuing any new debt b_{t+1}; hence, productivity must be higher for the government to be indifferent between repaying and defaulting.

The right boundary between the self-fulfilling crises and the default region is the combinations of debt and productivity realizations such that the government is indifferent between repaying the outstanding debt b_t and defaulting on it when foreign lenders purchase the new debt b_{t+1}. These combinations are labelled $\epsilon''(b_t, K_t)$; this locus is upward sloping in the (b_t, ϵ_t) space for similar reasons as to why ϵ' is upward sloping.

For high debt levels, i.e. for $b_t > \bar{b}(K_t)$, there is a solvency crisis and default occurs with probability one. Here debt is so high and productivity so low that the new debt b_{t+1} won't be purchased because it is worthless as it will be defaulted for sure; hence, the government defaults on b_t with probability one.

Output fluctuations play an important role in sovereign default. Self-fulfilling and solvency crises occur both with bad productivity realizations; the former with relatively low debt levels, the latter with relatively high debt levels.

A higher stock of initial capital K_t makes default less likely by enlarging the no-default region. More precisely, ϵ' and ϵ'' shift downward and to the right. The assumption that capital fully depreciates every period is important for this result. If capital does not depreciate fully, a high stock of capital makes autarky better and raises the incentive to default; this must be balanced against a weaker incentive to default coming from higher output and lower borrowing from the international capital market, i.e. a low b_{t+1}.

Figure 3 shows the lenders' price schedule for a given capital stock. These are the debt price-quantity combinations that foreign lenders are willing to purchase before the sunspot variable is realized. When the government chooses its new debt offering b_{t+1}, it takes this price schedule as given and chooses the price-quantity combination on it that maximizes its utility. Two price schedules are depicted in figure 3. The most outward curve is for $\mu = 1$, namely when foreign lenders are always optimistic; the downward sloping section of the curve corresponds to debt levels associated with positive probabilities that $b_{t+1} > \underline{b}(k_{t+1})$. The most inward curve is for $\mu = 0$, that corresponds to the case when foreign lenders are always pessimistic; notice that the government can borrow less and at a lower price relative to the $\mu = 1$ case. The curve for $\mu \in (0, 1)$ lies in between these two curves.

The price of the newly issued debt is equal to β for debt levels in the no-default region ($b_{t+1} < \bar{b}(K_{t+1})$, which are repaid with probability one. For debt levels between $\bar{b}(K_{t+1})$ and $\underline{b}(K_{t+1})$, foreign lenders anticipate repayment with probability strictly less than one and, to compensate them for the risk of default, the price falls below β.[5] The price schedule is flat at zero for debt levels above $\underline{b}(K_{t+1})$: debts in the default region are worthless and foreign lenders do not purchase them for a positive price.

4.2 Characterization

Now I characterize the policy functions of each agent in the economy. I start with the consumers, who are the last the act in each period. Let $\epsilon'(b_t, K_t)$ be the productivity realization at t such that, if foreign lenders do not purchase any new debt and set $B_{t+1} = 0$, the government is indifferent to repay the oustanding debt b_t or defaulting on it. In the notation of appendix B,[6] ϵ' is the productivity realization, given b_t and

[5]Since $d^2 q_t / db_{t+1}^2 < 0$, the price schedule is convex.

[6]To keep the notation simple, the dependence of ϵ', ϵ'' on b_t, K_t will be dropped whenever it does not create confusion.

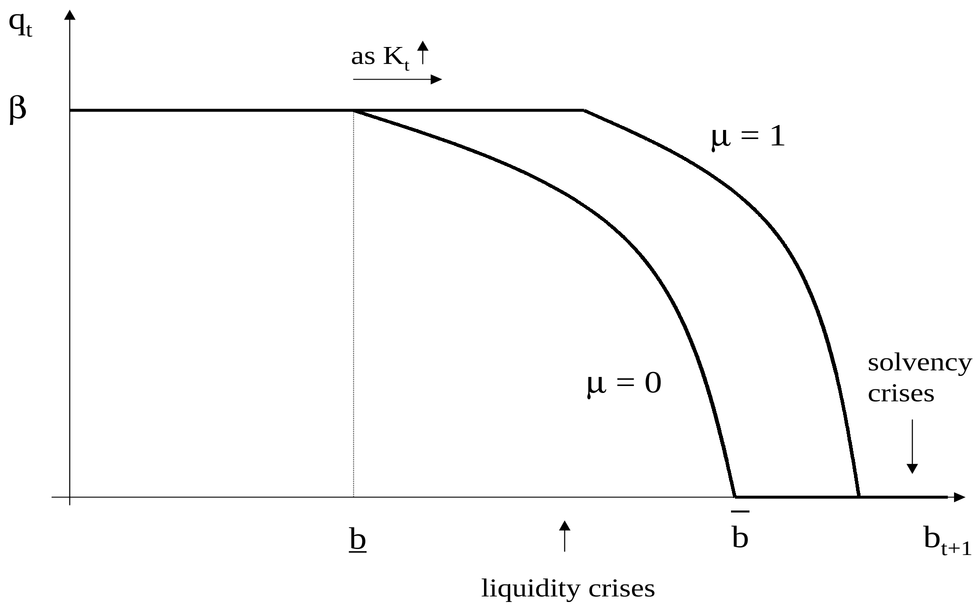

Figure 3: Price schedule

K_t, that defines the aggregate state $s'_t = (B_t, K_t, z_{t-1}, \epsilon'_t)$ such that

$$V^n(s'_t, 0, q_t) = V^d(s'_t, 0, q_t),$$

where d stands for default, n stands for no-default and $V^i, i = n, d$ is the value function in the case of no-default or default, respectively. Intuitively, failure to sell new debt when productivity is low, namely $\epsilon_t < \epsilon'$, means that the government chooses to default rather than repay, thereby fulfilling the lenders' expectations. To honor the outstanding debt without issuing any new lending, the government must raise taxes $\tau_t = b_t$ and reduce current consumption proportionally; if current output is low because of low capital and/or a low productivity realization, default may be a better option.

Similarly, let $\epsilon''(b_t, K_t)$ be the productivity realization at t such that, if foreign lenders purchase the new debt issued by the government and set $B_{t+1} = b_{t+1}$, the government is indifferent to repay the oustanding debt b_t or defaulting on it. In the notation of appendix B, ϵ'' is the productivity realization, given b_t and K_t, that defines the aggregate state $s''_t = (B_t, K_t, z_{t-1}, \epsilon''_t)$ such that

$$V^n(s''_t, b_{t+1}, q_t) = V^d(s''_t, b_{t+1}, q_t).$$

Intuitively, the government's participation constraint is binding at ϵ''; for productivity realizations lower than ϵ'', a solvency crisis occurs and the government defaults even if foreign lenders purchase the new debt. Of course, foreign lenders anticipate the default and, in equilibrium, do not purchase any new debt when $\epsilon_t < \epsilon''$. Appendix D proves $\epsilon' > \epsilon''$; appendix C proves that

$$\frac{d\epsilon'}{db_t} > 0, \qquad \frac{d\epsilon'}{dK_t} < 0, \qquad \frac{d\epsilon''}{db_t} > 0, \qquad \frac{d\epsilon''}{dK_t} < 0.$$

Given the definitions above, the probability at the beginning of period t that the outstanding stock of debt b_t will be repaid later on in the period is given by

$$(1-\mu)\int_{\epsilon'}^{\epsilon} \zeta(\epsilon_t)d\epsilon_t + \mu \int_{\epsilon''}^{\epsilon} \zeta(\epsilon_t)d\epsilon_t. \tag{12}$$

The first term on the right-hand side is the probability that the sunspot variable ϕ_t will take value 0 and the productivity realization will be sufficiently high for the government to be better off repaying the whole outstanding debt even without a roll over; the second term is the probability that ϕ_t will take value 1 and the government participation constraint will not be binding.

If default has not occurred at time $t-1$, the choices of consumption and investment solve the following problem

$$\max_{k_{t+1}} u(c_t) + \beta\left[(1-\mu)\int_{\epsilon'}^{\epsilon} u(c_{t+1}^n)\zeta(\epsilon_{t+1})d\epsilon_{t+1} + \mu\int_{\epsilon''}^{\epsilon} u(c_{t+1}^d)\zeta(\epsilon_{t+1})d\epsilon_{t+1}\right] \tag{13}$$

subject to

$$c_t = y_t - \tau_t - k_{t+1}$$
$$c_{t+1}^n = y_{t+1} - \tau_{t+1} - k_{t+2}^n$$
$$c_{t+1}^d = y_{t+1} - k_{t+2}^d$$

where c_{t+1}^n and c_{t+1}^d are consumption in period $t+1$ contingent on the government not defaulting and defaulting, respectively, in period $t+1$. The first-order condition for the problem is

$$u'(c_t) = \beta\left[(1-\mu)\int_{\epsilon'}^{\epsilon} u'(c_{t+1}^n)(A+\epsilon_{t+1})\alpha k_{t+1}^{\alpha-1}\zeta(\epsilon_{t+1})d\epsilon_{t+1} + \right. \tag{14}$$

$$\mu\int_{\epsilon''}^{\epsilon} u'(c_{t+1}^n)(A+\epsilon_{t+1})\alpha k_{t+1}^{\alpha-1}\zeta(\epsilon_{t+1})d\epsilon_{t+1} + (1-\mu)\int_{-\epsilon}^{\epsilon'} u'(c_{t+1}^d)(A+\epsilon_{t+1})\alpha k_{t+1}^{\alpha-1}\zeta(\epsilon_{t+1})d\epsilon_{t+1}$$

$$\left. +\mu\int_{-\epsilon}^{\epsilon''} u'(c_{t+1}^d)(A+\epsilon_{t+1})\alpha k_{t+1}^{\alpha-1}\zeta(\epsilon_{t+1})d\epsilon_{t+1}\right].$$

If the probability of default is high, investment is mainly determined according to the autarkic solution; if the probability of default is low, investment resembles more the solution with borrowing.

Foreign lenders are atomistic agents that behave competitively but not strategically; more precisely, they do not internalize the effect of their individual actions on the aggregate state. Given the aggregate state s_t and the new debt offering b_{t+1}, the foreign lenders choose whether to purchase it or not. If they decide to purchase the new debt, then $b_{t+1} = B_{t+1}$. The first-order condition for the problem in (B.13) defines the price at which foreign lenders purchase the debt

$$q_t = \beta \left[(1-\mu) \int_{\epsilon'}^{\epsilon} \zeta(\epsilon_{t+1}) d\epsilon_{t+1} + \mu \int_{\epsilon''}^{\epsilon} \zeta(\epsilon_{t+1}) d\epsilon_{t+1} \right]. \tag{15}$$

Foreign lenders are atomistic and each one of them takes the above expression parametrically. They will purchase new debt at the price q_t, which compensates them for the future risk of default; equation (15) implicitly defines the lenders' supply schedule. Notice that competition among foreign lenders drives the price down to q_t.

Consider now the government's decisions. First, the government chooses the new debt offering subject to the lenders' supply schedule (15). At this stage, the government knows the aggregate state s_t, which implies that the probability of default on the outstanding debt b_t is completely exogenous. In other words, given b_t, K_t and ϵ_t, the government is or is not in the self-fulfilling crises region; it defaults on b_t with probability $1-\mu$, i.e. if the sunspot variable $\phi_t = 0$, if it is in the self-fulfilling region; if, given b_t, K_t and ϵ_t, the government is in the default region, its choice of b_{t+1} does not matter because foreign lenders will not purchase it for a positive price. This implies that the choice of z_t is not affected by b_{t+1} (the best the government can do, once it is in the self-fulfilling crises region, is to sell the optimal amount of debt) and we only need to study the choice of b_{t+1} when $z_t = 1$ and b_t is repaid. Formally, the government solves the problem

$$\begin{aligned} V^n(s_t) = \max_{b_{t+1}} u(c_t^n) + \beta \Bigg[&(1-\mu) \int_{\epsilon'}^{\epsilon} V^n(s_{t+1}) \zeta(\epsilon_{t+1}) d\epsilon_{t+1} + \mu \int_{\epsilon''}^{\epsilon} V^n(s_{t+1}) \zeta(\epsilon_{t+1}) d\epsilon_{t+1} \\ &+ (1-\mu) \int_{-\epsilon}^{\epsilon'} V^d(s_{t+1}) \zeta(\epsilon_{t+1}) d\epsilon_{t+1} + \mu \int_{-\epsilon}^{\epsilon''} V^d(s_{t+1}) \zeta(\epsilon_{t+1}) d\epsilon_{t+1} \Bigg]. \end{aligned} \tag{16}$$

subject to

$$c_t = y_t - k_{t+1} + q_t b_{t+1} - b_t$$

where $V^n(s_{t+1})$ is the payoff to the government conditional on not defaulting in period $t+1$ and $V^d(s_{t+1})$ is the payoff to the government conditional on defaulting in period $t+1$ (and therefore equal to $U^a(k_{t+1}, \epsilon_{t+1})$ defined in (11)). The first-order condition for the problem is

$$u'(c_t^n) q_t \left[1 + \eta_{t+1} \right] = \beta \left[(1-\mu) \int_{\epsilon'}^{\epsilon} u'(c_{t+1}^n)(1 + \eta_{t+2}) \zeta(\epsilon_{t+1}) d\epsilon_{t+1} \right. \tag{17}$$

$$+\mu\int_{\epsilon''}^{\epsilon} u'(c^n_{t+1})(1+\eta_{t+2})\zeta(\epsilon_{t+1})d\epsilon_{t+1}\Big]$$

where $\eta_{t+1} \equiv (\partial q_t/\partial b_{t+1})/(b_{t+1}/q_t)$ is the price elasticity of the demand for b_{t+1} that, as shown in figure 3, is negative when b_{t+1} is high enough to make the government vulnerable to self-fulfilling crises.[7] A small increase in b_{t+1} raises current consumption by reducing the current tax by $q_t(1+\eta_{t+1})$; however, a higher b_{t+1}, if repaid, means higher taxes and lower consumption in period $t+1$. Moreover, the higher the debt issued today, the higher the debt issued tomorrow and, therefore, the lower the price that the government will receive for it. The first-order condition (17) has a very simple interpretation when the government is in the no-default region today and will be, with probability one, in the no-default region tomorrow. In this case, $\eta_{t+1} = \eta_{t+2} = 0, q_t = 1, \epsilon' < -\epsilon, \epsilon'' < -\epsilon$. When the government is in the self-fulfilling crises region, it balances two incentives: to smooth consumption over time (raise debt when a bad shock hits) and to reduce debt to exit the self-fulfilling crises region (lower debt means higher price).[8]

Later on in the same period, the government decides whether to repay or default the stock of old debt ($z_t = 1$ or 0) and levies a lump-sum tax on consumers τ_t, whose amount is uniquely defined by the government budget constraint. Given the aggregate state s_t, the amount of new debt purchased by the foreign lenders B_{t+1} and the price at which it was sold q_t, the government repays b_t if its participation constraint is satisfied, namely

$$V^n(s_t, B_{t+1}, q_t) \geq V^d(s_t, B_{t+1}, q_t),$$

where

$$V^n(s_t, B_{t+1}, q_t) = u(y_t - \tau_t - k_{t+1}) + \beta E_t\Big[(1-\mu)\int_{\epsilon'}^{\epsilon} V^n(s_{t+1})\zeta(\epsilon_{t+1})d\epsilon_{t+1} + \qquad (18)$$

$$\mu\int_{\epsilon''}^{\epsilon} V^n(s_{t+1})\zeta(\epsilon_{t+1})d\epsilon_{t+1} + (1-\mu)\int_{-\epsilon}^{\epsilon'} V^d(s_{t+1})\zeta(\epsilon_{t+1})d\epsilon_{t+1} + \mu\int_{-\epsilon}^{\epsilon''} V^d(s_{t+1})\zeta(\epsilon_{t+1})d\epsilon_{t+1}\Big].$$

and $V^d(s_t, B_{t+1}, q_t)$ given by (11). In words, the government honors its outstanding debt only if it has the incentive to do so.

[7]More precisely,

$$\frac{dq_t}{db_{t+1}} = -\beta\left[(1-\mu)\frac{d\epsilon'}{db_{t+1}} + \mu\frac{d\epsilon''}{db_{t+1}}\right] \leq 0.$$

[8]Notice that b_{t+1} affects current investment via its effect on current taxes; thanks to the envelope theorem and the fact that consumers optimally allocate each extra unit of after-tax income between consumption and investment, this effect drops out of (17).

5 Debt overhang and the debt Laffer curve

It is often argued that large external debt is the cause of growth slowdown in a debtor country because its legacy effectively taxes available resources and reduces investment. Many developing countries in the 1980s saw their investment figures fall precipitously as the debt crisis developed and foreign credit almost disappeared. This question can be addressed precisely within the model developed so far. Conditional on $z_{t+1} = 1$, totally differentiating the first-order condition (14), I find that

$$\frac{dk^n_{t+1}}{db_t} = \frac{-u''(c^n_t)}{\Omega} < 0, \tag{19}$$

where $\Omega < 0$.[9] This is the debt *overhang* effect on the debtor's investment: inherited liabilities, if repaid, reduce capital in the debtor country. On the other hand

$$\frac{dk_{t+1}}{db_{t+1}} = \frac{1}{\Omega}\Bigg\{u''(c_t)[q_t + b_{t+1}\frac{dq_t}{db_{t+1}} - b_t\frac{dz_t}{db_{t+1}}] + \mu\int_{\epsilon''}^{\epsilon} u''(c^n_{t+1})\zeta(\epsilon_{t+1})d\epsilon_{t+1}$$
$$+(1-\mu)\int_{\epsilon'}^{\epsilon} u''(c^n_{t+1})\zeta(\epsilon_{t+1})d\epsilon_{t+1}\Bigg\} > 0. \tag{20}$$

This is the *investment* effect of new debt: new credit lowers current taxes and raises current investment.

The current initiative to extend debt relief to heavily indebted poor countries (HIPC) as well as the initiative to partially forgive the debt of some nations in the 1980s are based on the idea that, when debt is too high, the debt overhang problem is so severe that forgiving a portion of the debt raises expected debt repayment. In other words, the value of the debt increases by forgiving some debt. This concept has been labeled the debt Laffer curve, by analogy with the usual tax Laffer curve that shows that tax revenues may fall as the tax is raised. The context within which Krugman [17]

[9] More precisely,

$$\Omega = u''(c_t) + \beta E_t\Bigg[(1-\mu)\int_{\epsilon'}^{\epsilon} u''(c^n_{t+1})(A_{t+1}\alpha k^{\alpha-1}_{t+1})^2 + \mu\int_{\epsilon''}^{\epsilon} u''(c^n_{t+1})(A_{t+1}\alpha k^{\alpha-1}_{t+1})^2 +$$
$$(1-\mu)\int_{\epsilon'}^{\epsilon} u'(c^n_{t+1})A_{t+1}\alpha(\alpha-1)k^{\alpha-2}_{t+1} + \mu\int_{\epsilon''}^{\epsilon} u'(c^n_{t+1})A_{t+1}\alpha(\alpha-1)k^{\alpha-2}_{t+1}\Bigg]$$
$$+\beta\Bigg[(1-\mu)\int_{-\epsilon}^{\epsilon'} u''(c^d_{t+1})(A_{t+1}\alpha k^{\alpha-1}_{t+1})^2 + \mu\int_{-\epsilon}^{\epsilon''} u''(c^d_{t+1})(A_{t+1}\alpha k^{\alpha-1}_{t+1})^2$$
$$+(1-\mu)\int_{-\epsilon}^{\epsilon'} u'(c^d_{t+1})A_{t+1}\alpha(\alpha-1)k^{\alpha-2}_{t+1} + \mu\int_{-\epsilon}^{\epsilon''} u'(c^d_{t+1})A_{t+1}\alpha(\alpha-1)k^{\alpha-2}_{t+1}\Bigg] < 0.$$

and Sachs [22] developed the debt Laffer curve is one where creditors can seize part of the country's output in case of a default.

The concept of debt Laffer curve can be investigated in this setting. The face value of the stock of debt is b_{t+1}; let $W(b_{t+1})$ be the market value of the debt as defined by

$$W(b_{t+1}) = b_{t+1}\left[\mu\int_{\epsilon''}^{\epsilon}\zeta(\epsilon_{t+1})d\epsilon_{t+1} + (1-\mu)\int_{\epsilon'}^{\epsilon}\zeta(\epsilon_{t+1})d\epsilon_{t+1}\right], \tag{21}$$

for debt levels above $\underline{b}(K_{t+1})$, i.e. in the default region, the market value is zero. The market value of the debt $W(b_{t+1})$ is plotted in figure 4 for $\mu > 0$. To understand why it is inverse-U shaped, consider its differentiation with respect to b_{t+1}:

$$\frac{dW}{db_{t+1}} = \mu\int_{\epsilon''}^{\epsilon}\zeta(\epsilon_{t+1})d\epsilon_{t+1} + (1-\mu)\int_{\epsilon'}^{\epsilon}\zeta(\epsilon_{t+1})d\epsilon_{t+1} - b_{t+1}\left[\mu\frac{d\epsilon''}{db_{t+1}} + (1-\mu)\frac{d\epsilon'}{db_{t+1}}\right]. \tag{22}$$

The first two terms on the right-hand side are the probability of repayment; the third term on the right-hand side is the (negative) effect of higher debt on the probability of repayment. For debt levels in the no-default region, the debt is repaid with probability one, the second term is zero[10] and the debt Laffer curve is a straight line out of the origin with slope 1; for debt levels in the self-fulfilling region, a rise in the debt lowers the probability of repayment and investment and the Laffer curve flattens, and eventually it slopes downward.

6 Welfare

Defining aggregate welfare in a model with heterogeneous agents raises some obvious difficulties, especially in this setting where lenders are risk-neutral and debtors are risk-averse. One possible definition consists in the weighted sum of the utilities of all agents in the economy:

$$AW_t = [\omega J_t + (1-\omega)U_t], \quad 0 < \omega < 1 \tag{23}$$

where J_t is the expected utility of the foreign lenders and U_t is the expected utility of the consumers at the beginning of period t. These utilities have been defined in (1) and (6).

A benevolent central planner maximizes (23) under the resource constraint of the lenders and the budget constraint of the consumers. I concentrate on the solution to this problem where both the lenders and the debtors are fully rational (i.e. where lenders do not purchase debt that won't repay at least $1/\beta$ in expected terms) and where there are no ex-post transfers that were not specified in the contracts.

[10]More precisely, ϵ' and $\epsilon'' \notin [-\epsilon, \epsilon]$.

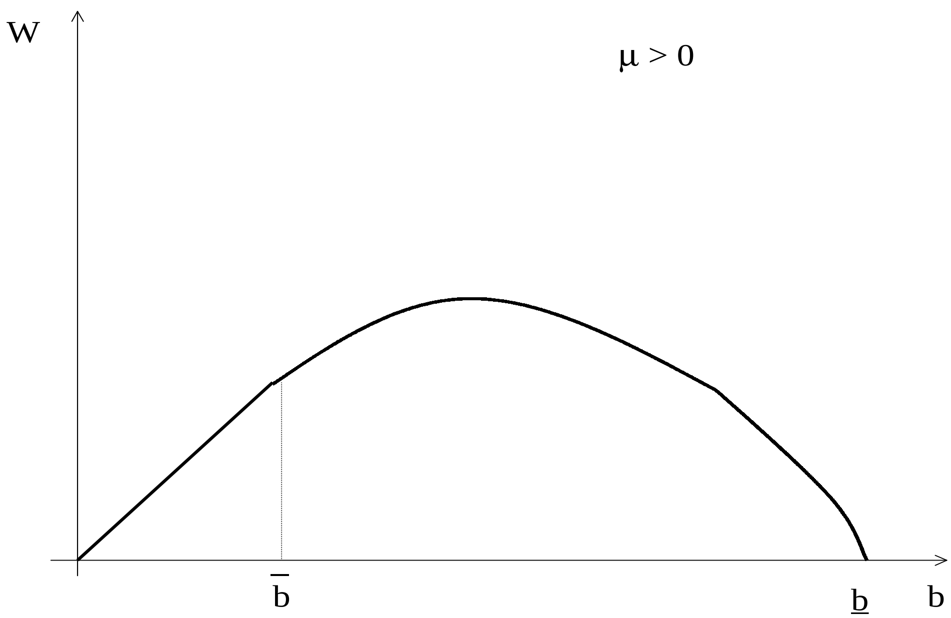

Figure 4: Debt Laffer curve

If the benevolent central planner could provide the government with a commitment device, it would certainly do so because commitment delivers the most efficient allocations. This is equivalent to the benevolent central planner setting up a system of punishments, possibly very large, following default so that the government will not default. Here I consider the case where the benevolent planner cannot provide such a commitment device.

A benevolent central planner solves the coordination failure that is at the heart of a liquidity crisis if it can eliminate the atomistic behavior of lenders. The planner recognizes that, if lenders withdraw their credit when productivity is low, the debtor country may have no option but default; hence, welfare is maximized if any new debt below $\underline{b}(k_t)$ is purchased at the price

$$q_t = \beta \int_{\epsilon''}^{\epsilon} \zeta(\epsilon_{t+1}) d\epsilon_{t+1}.$$

In words, the benevolent central planner implements the decentralized solution when $\mu = 1$.

In this setting, we can easily measure the welfare costs of liquidity crises. Ex-ante and before a crisis occurs, the anticipation of a credit withdrawal lowers the price at which the debtor sells its debt. This is the vertical distance between the price schedules in figure 3: the lower μ, the more pessimistic the lenders and the lower the price q_t. If lenders could be made more optimistic by endowing them with $\mu = 1$, the government borrows more cheaply in bad times and, on average, invests and produces more. Suppose this economy has been in an equilibrium with $\mu < 1$ until period $t-1$ and there has been no default; if this economy switches permanently to $\mu = 1$ at the very beginning of period t, aggregate welfare increases. At t, J_t increases by

$$b_t(1-\mu)\int_{\epsilon''}^{\epsilon'} \zeta(\epsilon_t)d\epsilon_t \geq 0. \tag{24}$$

Repayment of b_t is now expected with higher probability. This is a once-and-for-all effect; the expected utility of lenders after t is unchanged because higher probability of repayment brings higher debt prices. The change in consumers' expected utility is

$$(1-\mu)\int_{\epsilon''}^{\epsilon'} \left[V^n(s_{t+1}) - V^d(s_{t+1})\right]\zeta(\epsilon_{t+1})d\epsilon_{t+1} > 0. \tag{25}$$

Intuitively, consumers' expected utilityimproves because self-fulfilling crises disappear; this raises q_t, which in turn raises current consumption, current investment and future expected consumption.

7 Bailout

Since liquidity crises impose welfare costs, ex-ante as well as ex-post, it is desirable to eliminate them. A number of measures have been proposed lately to reduce the risk of liquidity crises or to solve a liquidity crisis, once it arises, in a more orderly manner. For example, bailouts, the creation of an international lender of last resort and the creation of contingent credit lines by the IMF belong to the first category (see Fischer [14]); debt standstills, debt rollover options and bondholders committees belong to the second category and their use has been recently suggested by Miller and Zhang [20], Buiter and Siber [2] and Eichengreen [12], respectively. Jeanne [18] studies the welfare consequences of these measures within a model where debt repayment is feasible only if the government enacts a fiscal reform.

This section studies the welfare effects of creating an international lender of last resort; in this setting, this is equivalent to a bailout organized by an international institution such as the IMF. Suppose a liquidity crisis occurs at time t. In the setting developed earlier, an international lender of last resort, the IMF henceforth, intervenes in the following way: it gets an endowment T_t by levying a tax on foreign lenders and uses it to purchase all or part of the new debt that the government tried to sell without success. For simplicity, suppose it is costless for the IMF to levy its endowments on the lenders; moreover, the IMF pays an expected rate of return of $1/\beta$ on them. After the new debt is purchased by the IMF, the government repays in full to foreign lenders the amount it initially defaulted on, b_t; then the consumers take their investment decision.

I assume that, while the IMF bailout is taking place, the sovereign government cannot borrow from anyone else than the IMF. This means that the government cannot sell debt to foreign lenders directly until it has repaid in full the IMF. Starting from $t+1$, the IMF is the sole owner of government debt. Once the IMF has been repaid in full, it pays back what it owes to foreign lenders and the debtor government is allowed to borrow again directly from the lenders. More formally, the foreign lenders' resource constraint in the default period t is

$$x_t = \overline{x} - T_t + b_t \tag{26}$$

where T_t is the transfer to the IMF at time t. After t and until the bailout comes to an end, the resource constraint in period s is

$$x_s = \overline{x} - T_s + R_s, \tag{27}$$

where R_s is the IMF repayment to foreign lenders at time s, with $T_s, R_s \geq 0$. The IMF maximizes the sum of the expected repayments by the government, d_s:

$$J_t^{IMF} = \sum_{s=t}^{\infty} \beta^{s-t} E_t d_s z_s \tag{28}$$

subject to the resource constraint at s

$$q_s d_{s+1} + R_s = d_s z_s + T_s. \tag{29}$$

Here I concentrate on two alternative bailout strategies and rank them in terms of aggregate welfare. In the first bailout scenario, labeled *full bailout*, the IMF purchases all the debt that the countried tried to sell at its market price:

$$d_{t+1} = b_{t+1} \qquad \text{and} \qquad q_t = \beta \int_{\epsilon''}^{\epsilon} \zeta(\epsilon_{t+1}) d\epsilon_{t+1}.$$

The IMF purchases the debt at the market price when $\mu = 1$. It is easy to see that the full bailout is efficient and it implements the first-best solution. In fact, the expected rate of return on the debt purchased during the bailout is $1/\beta$; at the same time, the expected rate of return on the transfers to the IMF is also $1/\beta$. Hence, all the transactions carried out in the full bailout can be decentralized. The country's expected utility is the same as in a setting where liquidity crises never arise. Moreover, if foreign lenders anticipate the IMF intervention, they will be indifferent between purchasing the new debt at its market price and withdrawing their credit, cause a liquidity crisis and paying transfers to the IMF: the two equilibria have the same expected rate of return, which is in turn equivalent to the return from consuming their endowment $\bar{x}$.

Therefore, the creation of an international lender of last resort eliminates the occurrence of liquidity crises altogether in this setting.

For the full bailout to be efficient, it is necessary that the IMF purchases the debt at its market price q_t. If d_{t+1} is purchased at a price above q_t, the foreign lenders will on average take a loss from lending to the government. This implies that the IMF enters into the picture only for a liquidity, not a solvency a crisis.

The second scenario is the *partial* bailout: in the period the liquidity crisis arises, say t, the IMF purchases an amount of debt that is less than what the country originally tried to sell. More precisely,

$$d_{t+1} = g_{t+1} \qquad \text{and} \qquad q_t = \beta \int_{\epsilon''}^{\epsilon} \zeta(\epsilon_{t+1}) d\epsilon_{t+1},$$

where g_{t+1} is the level of debt that makes the government indifferent between defaulting and repaying:

$$V^n(s_t, g_{t+1}, q_t) = V^d(s_t, g_{t+1}, q_t).$$

Since the IMF purchases debt at its market price, the expected rate of return on the country's debt is, once again, $1/\beta$; however, the partial bailout reduces the expected utility of the debtor country because $g_{t+1} < b_{t+1}$. Hence, aggregate welfare is lower under partial than full bailout.

To summarize, an international lender of last resort implementing a full bailout at market prices maximizes aggregate welfare in the sense of (23) and eliminates self-fulfilling runs altogether. Solvency crises, on the other hand, are not eliminated and the probability of their occurrence is reflected in the debt price. Notice that the creation of a contingent credit line that the debtor country can use in the event of a liquidity crisis is equivalent to the full bailout provided the interest rate fully reflects the risk of a solvency crisis: $(1 + r_{t+1}) = 1/q_t$.

Many economists have argued that the creation of an international lender of last resort creates a moral hazard problem. This problem can be easily analyzed in this setting. Suppose the IMF bails out the country both under liquidity and solvency crises by levying taxes on foreign lenders. The country's debt is sold at the price $q_t = \beta$ even for debt levels larger than $\underline{b}(K_{t+1})$. In fact, each foreign lender anticipates to be taxed by the IMF in the event of default, irrespectively of whether she is a debt holder or not. Hence, she is better off purchasing the debt because that entitles her to a credit of b_{t+1} in the event of default.

The moral hazard problem of bailing out a country following a solvency crisis is presented in figure 5. If the price schedule remains flat at β for debt levels above $\underline{b}(K_{t+1})$, inefficiently high levels of debt get financed at low interest rates; for debt levels between $\underline{b}(K_{t+1})$ and $\overline{b}(K_{t+1})$, debt should be purchased at a discount, while it is not; for debt leves above $\underline{b}(K_{t+1})$, the debt would not be purchased at a positive price

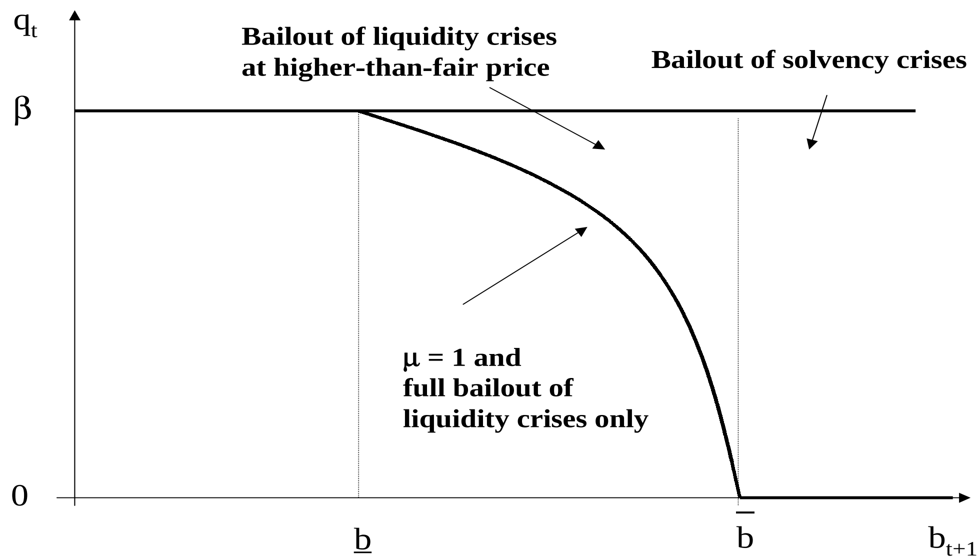

Figure 5: Moral hazard in bailout

in the efficient allocation. Hence, too much borrowing at too high price takes place in this equilibrium.

8 Conclusions

I have studied a general equilibrium model with stochastic productivity and incomplete international lending markets that deliver the following results. First, when foreign lenders are atomistic, self-fulfilling debt crises can arise for intermediate debt levels. Second, both self-fulfilling and solvency crises arise when the borrower suffer an adverse shock. The model appears to be well equipped to explain some features that emerge from the history of international lending.

This work can be extended in several directions. Under the general conditions and functional forms assumed here, I have been able to characterize the optimal policies of the government and consumers but have not solved explicitly for them. In the future, it would be interesting to choose specific functional forms and random processes that allow one to solve analytically for the dynamic behavior of debt and capital.

When productivity is persistent, namely $\rho > 0$, an adverse productivity realization may reduce output for more than one period. I believe that, given the initial stocks of debt and capital, a negative productivity realization with $\rho > 0$ makes self-fulfilling and/or solvency crises more likely. This is because the government needs to borrow more than under the case where the productivity shocks are i.i.d; however, the government restraints itself in part because it is aware that a rising debt makes solvency and liquidity crises more likely. The opposite intuition holds for a positive productivity shock.

The debt has been assumed to have maturity of one period. This assumption has been made for the only purpose of making the analysis simpler. Lengthening the maturity of debt can reduce the size of the self-fulfilling region but it is not going to eliminate it. Hence, the qualitative results of the paper should hold true.

Since the model is one of perfect information, foreign lenders know exactly the fundamentals and therefore know in which debt region they place the borrowing country. Along the same lines, an international lender of last resort can distinguish between a liquidity and a solvency crisis. In reality, such distinction may be hard to know. It would be interesting to extend the model to a setting with incomplete information and moral hazard.

The welfare analysis of bailouts has been carried out under the assumption that there are no costs involved with IMF intervention. The existence of costs, certainly a more realistic assumption, will not change qualitatively the results.

At last, the model easily allows the study another proposal that has been suggested within the reform of the international financial architecture: "bailing in", namely forcing the debt-holders that caused a liquidity crisis to assume some of the losses (if any) generated by the crisis.

Appendix

A Borrowing with commitment

Assume the government can commit to repay what it has borrowed and study the equilibrium when the government can borrow and lend from the international lenders. Since productivity is stochastic (and if $\rho = 0$ in (7) the shocks are i.i.d.), we must account for the possibility that, due to a sequence of bad shocks, the government issues such a large stock of debt that its price is zero (even if such event may have a very, very small probability). The counterpart to this is having the transversality condition satisfied in expected terms. Let's consider first the foreign lenders. When commitment is feasible, the lender anticipates full repayment by the borrower. The optimal lending decision solves the following problem

$$J_t^c(b_t) = \max_{b_{t+1}} x_t + \beta E_t J_{t+1}^c(b_{t+1}) \tag{A.1}$$

subject to (7) and

$$q_t b_{t+1} \leq \overline{x}. \tag{A.2}$$

Condition (A.2) arises because the new debt must be purchased before the old debt is repaid by the government. The first-order condition for the problem is

$$q_t = \beta E_t[z_{t+1}]. \tag{A.3}$$

The foreign lenders purchase the bonds offered by the government as long as their expected gross rate of return is at least $1/\beta$. Notice that $E_t z_{t+1}$ captures the probability that the debt will be repaid; under commitment, this probability is below 1 only to account for the likelihood that the transversality condition is violated.

The government chooses its borrowing, and thereby its taxes, so as to maximize the utility of consumers. More precisely, the government solves the following dynamic programming problem:

$$V^c(B_t) = \max_{\tau_t} u(c_t) + \beta E_t V^c(B_{t+1}), \tag{A.4}$$

where a superscript c stands for "commitment", subject to (5). Notice that we do not need to impose a no-Ponzi-scheme-condition on the government because, if the government issues too much debt, its price simply goes to zero. The first-order condition for this problem is

$$u'(c_t) = E_t u'(c_{t+1}) + \frac{Cov(u'(c_{t+1}, z_{t+1})}{E_t[z_{t+1}]} \tag{A.5}$$

where I have used condition (A.3). The new amount debt and therefore the optimal tax at t satisfy the intertemporal Euler equation amended to take into account the probability that default from bad luck occurs; the covariance term is positive and gives the intuitive result that the optimal tax at t is higher than it would be if default had probability truly equal to zero: the government borrows a little less to avoid default.

To find an analytical solution, I assume that the probability of a sequence of bad shocks is negligible, hence $E_t z_{t+1} = 1$; it should be kept in mind, however, that this is *not* the exact solution if the productivity shocks are i.i.d. In this case, (A.5) simplifies as the covariance term disappears, so that the consumer's marginal rate of substitution of present for future consumption is equal to the price of future consumption in terms of present consumption.

Consumers choose how much to consume and to invest knowing what the lump-sum tax (or transfer) is; by consolidating the representative consumer and the government, we see that investment in k_{t+1} satisfies

$$\frac{1}{\beta} = E_t[A_{t+1} \alpha k_{t+1}^{\alpha-1}]. \tag{A.6}$$

Consumers invest up to where the expected marginal returns from capital are equal to $1/\beta$. Under the stochastic process for productivity (4), the following constant level of capital (and therefore investment) satisfies (A.6)

$$k_{t+1} = k^c = [A\alpha\beta]^{\frac{1}{1-\alpha}}. \tag{A.7}$$

The actual return from capital and production fluctuate with the productivity realizations, i.e. production is procyclical and production at t is

$$y_t^c = (A + \epsilon_t)\,[A\alpha\beta]^{\frac{\alpha}{1-\alpha}}. \tag{A.8}$$

Unlike the autarkic economy, there is only one source of variability in production - the productivity shock - because capital is constant here. Given the productivity realization ϵ_t, the government levies a lump-sum tax or transfers resources to the consumers so as to keep their consumption and their investment in capital constant. More precisely, suppose the initial stock of outstanding debt is b_t; current and expected future taxes depend on the outstanding stock of debt via the constraint

$$E_t \sum_{s=t}^{\infty} \beta^{(s-t)} \tau_s = b_t, \tag{A.9}$$

The higher the stock of outstanding debt, the higher current and expected future taxes. In period t, the optimal tax is

$$\tau_t^c = (1-\beta)\, b_t + \beta\{\epsilon_t k_t^\alpha + A[k_t^\alpha - (k^c)^\alpha]\}. \tag{A.10}$$

The optimal tax at t consists of the interests on the initial stock of debt (the first term on the right-hand side), the stochastic part of output plus output in excess of the average optimal level (the second term on the right-hand side); for $s > t$, the second term simplifies to $\beta \epsilon_s k^c$, as stock of capital is then equal to its optimal level. Notice that taxes are procyclical. The current account:

$$CA_t = -b_{t+1} + b_t = -\frac{1-\beta}{\beta}\, b_t + \frac{\tau_t^c}{\beta}$$

is also procyclical and depends on past productivity shocks via b_t. Consumption is constant; for initial conditions b_t and k_t, the constant level of consumption is given by

$$c^c = A[(1-\beta)k_t^\alpha + \beta(k^c)^\alpha] - (1-\beta)b_t + \beta\epsilon_t k_t^\alpha \tag{A.11}$$

and its expected value as of time $t-1$ is

$$E_{t-1}c^c = A[(1-\beta)k_t^\alpha + \beta(k^c)^\alpha] - (1-\beta)b_t.$$

The lifetime utility of the representative agent in the open economy with commitment is

$$U_t^c = \frac{1}{1-\beta} u(c^c). \tag{A.12}$$

Given b_t, ϵ_t and k_t, consumers' welfare is higher in the open economy with commitment than in the autarkic economy if $U_t^c \geq U^a(k_t, \epsilon_t)$. Several factors affect the welfare comparison of the two equilibria. First, the outstanding stock of debt raises current and future taxes in the open economy. Second, consumption variability lowers welfare in the autarkic economy; on the other hand, there is no uncertainty in consumption in the open economy with commitment. Third, if the initial level of capital k_t is low in the open economy with commitment, the government borrows to bring capital and expected production to its optimal level; however, future taxes will be correspondingly higher. In the autarkic economy, a low initial capital stock implies low investment and therefore a low expected output tomorrow. Forth, investment is lower on average in autarky because of (10).

B Equilibrium

In period t, the aggregate state of the economy is $s_t = (B_t, K_t, z_{t-1}, \epsilon_t)$, with $b_t = B_t$ (market clearing last period) and $k_t = K_t$. The state of each agent consists of the aggregate state, any individual state variable and any variable that has already been chosen. Let $\tau(s_t, B_{t+1}, q_t, \phi_t), z(s_t, B_{t+1}, q_t, \phi_t), b(s_t)$ be the government policy functions, $q(s_t, b_{t+1})$ be the price function and $K(s_t, B_{t+1}, \tau_t, z_t)$ be function that describes aggregate capital. Consider the consumers, who are the last to act and know everything that has happened in the period. Their state is defined by $k_t, s_t, B_{t+1}, \tau_t, z_t$; if the government defaulted, the economy is in autarky and the consumers solve the problem described in section 3; otherwise, their value function is defined by

$$U(k_t, s_t, B_{t+1}, \tau_t, z_t) = \max_{k_{t+1}, c_t} u(c_t) + \beta E_t U(k_{t+1}, s_{t+1}, B_{t+2}, \tau_{t+1}, z_{t+1})$$

subject to

$$c_t \leq (A + \epsilon_t) k_t^\alpha - \tau_t - k_{t+1}$$
$$c_t, k_{t+1} \geq 0.$$

The consumer policy function are $c(k_t, s_t, B_{t+1}, \tau_t, z_t)$ and $k(k_t, s_t, B_{t+1}, \tau_t, z_t)$.

Consider next the foreign lenders; their state is defined by s_t, B_t, b_{t+1} and the sunspot variable ϕ_t; the value function for the foreign lenders is given by

$$J(s_t, B_t, b_{t+1}, \phi_t) = \max_{B_{t+1}} x_t + \beta E_t J(s_{t+1}, B_{t+1}, b_{t+2}, \phi_{t+1}) \tag{B.13}$$

subject to (7), (8) and

$$B_{t+1} \geq -\Delta.$$

The foreign lenders' policy function is denoted $B(s_t, B_t, b_{t+1}, \phi_t)$.

Consider now the government decisions. First, the government chooses the new debt offering when its state is simply s_t and subject to the lenders' supply schedule $q(s_t, b_{t+1})$. The government is strategic and takes into account the effect of its decision at this stage on its own tax-default decisions later in the period, and on the consumption-investment decision by the consumers. The value function for the government is defined by

$$V(s_t) = \max_{b_{t+1}} u(c_t) + \beta E_t V(s_{t+1})$$

The policy function for the government at this stage is denoted $b(s_t)$.

Later on in the period, the government decides whether to default on the outstanding stock of debt and, residually via the budget constraint, the tax on consumers. The policy functions $\tau(s_t, b_{t+1}, q_t, \phi_t), z(s_t, b_{t+1}, q_t, \phi_t)$ are the solution to

$$V(s_t, B_{t+1}, q_t) = \max_{\tau_t, z_t} u(c_t) + \beta E_t V(s_{t+1}, B_{t+2}, q_{t+1})$$

subject to (5), with $z_t = 0$ or 1.

The equilibrium is a list of value functions U, J, V for consumers, foreign lenders and the government; policy functions c, k for consumers, B for foreign lenders, b, τ, z for the government; a price function q and aggregate capital K such that policy functions are the solution to the value functions of each respective agent, with $K(s_t, B_{t+1}, \tau_t, z_t) = k(s_t, B_{t+1}, \tau_t, z_t)$ and $b \in B(s_t, B_t, b_{t+1})$ for all b_{t+1} such that $-\Delta \leq b_{t+1} \leq \overline{x}/q(s_t, b_{t+1})$.

Notice that I have restricted my attention to Markov-equilibrium, so that agents' future actions can be derived completely by their policy functions.

C Proof

The proof is as follows. Let

$$V_\epsilon^n \equiv \frac{\partial V^n(s_t,0,q_t)}{\partial \epsilon} \quad V_\epsilon^d \equiv \frac{\partial V^d(s_t,0,q_t)}{\partial \epsilon} \quad V_b^n \equiv \frac{\partial V^n(s_t,0,q_t)}{\partial b}$$

$$V_b^d \equiv \frac{\partial V^d(s_t,0,q_t)}{\partial b} \quad V_{K_t}^n \equiv \frac{\partial V^n(s_t,0,q_t)}{\partial K_t} \quad V_{K_t}^d \equiv \frac{\partial V^d(s_t,0,q_t)}{\partial K_t},$$

where s is the aggregate state and it includes the current productivity shock. Notice that

$$\frac{d\epsilon'}{db_t} = \frac{V_b^d - V_b^n}{V_\epsilon^n - V_\epsilon^d} > 0 \quad \frac{d\epsilon'}{dK_t} = \frac{V_{K_t}^d - V_{K_t}^n}{V_\epsilon^n - V_\epsilon^d} < 0.$$

To see that $V_b^d - V_b^n > 0$, just notice that $\tau_t = b_t$ if the government repays the debt (hence consumption is low) whereas $\tau_t = 0$ if the government defaults. As for the denominator, $V_\epsilon^n = u'(c_t^n)K_t^\alpha$ and $V_\epsilon^d = u'(c_t^d)K_t^\alpha$; given K_t, if $B_{t+1} = 0$, taxes must necessarily be weakly positive so that $c_t^n \le c_t^d$ which implies that $V_\epsilon^n \ge V_\epsilon^d$. This also implies that

$$V_{K_t}^d - V_{K_t}^n = [u'(c_t^d) - u'(c_t^n)]\alpha K_t^{\alpha-1} < 0.$$

D Proof

I am going to show that $\epsilon'(b_t,k_t) > \epsilon''(b_t,k_t)$. Suppose not; then

$$V^n(s_t', B_{t+1}, q_t) < V^d(s_t', B_{t+1}, q_t) \quad \text{and} \quad V^n(s_t', 0, q_t) > V^d(s_t', 0, q_t). \tag{D.14}$$

In equilibrium, $q_t = 0$ when $b_{t+1} > \underline{b}(K_{t+1})$, which implies that $V^d(s_t', 0, q_t) = V^d(s_t', B_{t+1}, q_t)$ and $V^n(s_t', 0, q_t) = V^n(s_t', B_{t+1}, q_t)$.

References

[1] Atkeson, Andrew: "International Lending with Moral Hazard and Risk of Repudiation," *Econometrica,* Vol. 59, 1991.

[2] Buiter, Willem and Sibert, Anne: "UDROP: A Small Contribution to the New International Financial Architecture," mimeo 1999.

[3] Bulow, Jeremy and Kenneth Rogoff: "Cleaning up Thirld World Debt Without Getting Taken to the Cleaners" *Journal of Economic Perspectives,* Vol. 4, No. 1, 1990.

[4] Bulow, Jeremy and Kenneth Rogoff: "Sovereign Debt: Is to Forgive to Forget?" *American Economic Review,* Vol. 79, 1989.

[5] Chari, V.V. and Kehoe, Patrick: "Sustainable Plans," *Journal of Political Economy,* Vol. 98, no. 4, 1990.

[6] Chari, V.V. and Kehoe, Patrick: "Sustainable Plans and Mutual Default," *Review of Economic Studies,* Vol. 60, no. 1, 1993.

[7] Cline, William: *International Debt Reexamined,* Institute for International Economics, 1995.

[8] Cole, Harold and Timothy Kehoe: "Self-Fulfilling Debt Crises," forthcoming *Review of Economic Studies.*

[9] Eaton, Jonathan: "Debt Relief and the International Enforcement of Loan Contracts," *Journal of Economic Perspectives,* Vol. 4, No. 1, 1990.

[10] Eaton, Jonathan and Raquel Fernandez: "Sovereign Debt" in Gene Grossman and Kenneth Rogoff, eds. *Handbook of International Economics,* Vol. 3, 1995.

[11] Eaton, Jonathan and M. Gersovitz: "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies,* Vol. 48, 1981.

[12] Eichengreen, Barry: *Toward a New Financial Architecture,* Institute for International Economics, Washington DC, 1999.

[13] Fernandez, Raquel and Robert Rosenthal: "Strategic Models of Sovereign-Debt Renegotiations," *Review of Economic Studies,* Vol. 57, 1990.

[14] Fischer, Stanley: "On the Need for an International Lender of Last Resort," IMF mimeo, 1999.

[15] Kenen, Peter: "Organizing Debt Relief: The Need for a New Institution," *Journal of Economic Perspectives,* Vol. 4, No. 1, 1990.

[16] Kletzer,Kenneth: "Sovereign Debt Renegotiation under Asymmetric Information," in Jacob Frenkel, Michael Dooley and Peter Wickham, eds. *Analytical Issues in Debt*, 1989.

[17] Krugman, Paul: "Market-Based Debt-Reduction Schemes," in Jacob Frenkel, Michael Dooley and Peter Wickham, eds. *Analytical Issues in Debt*, 1989.

[18] Jeanne, Olivier: "Sovereign Debt Crises and the Global Financial Architecture," mimeo 1999.

[19] Lindert, Peter and Peter Morton: "How Sovereign Debt Has Worked," in Sachs,Jeffrey D. ed. *Developing country debt and economic performance*, Volume 1, The international financial system. National Bureau of Economic Research Project Report series Chicago and London: University of Chicago Press 1989; 39-106.

[20] Miller, Marcus and Lei Zhang: "Sovereign Liquidity Crises: the Strategic Case for a Payments Standstill," mimeo, 1998.

[21] Obstfeld, Maurice: "Rational and Self-Fulfilling Balance of Payments Crises," *American Economic Review,* Vol. 76, 1986.

[22] Sachs, Jeffrey: "The debt overhang of developing countries," in Guillermo Calvo, Roland Findlay, Pentti Kouri, and Jorge Braga de Macedo, eds. *Debt, stabilization and development: Essays in memory of Carlos Diaz-Alejandro,* 1989.

[23] Sachs, Jeffrey: "A Strategy for Efficient Debt Reduction," *Journal of Economic Perspectives,* Vol. 4, No. 1, 1990.

[24] Stokey, Nancy: "Credible Public Policy," *Journal of Economic Dynamics and Control,* Vol. 15, no. 4, 1991.