

# Technological Change and Public Financing of Education

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## Abstract

We study investment in education in an overlapping generation model with altruism where credit market imperfections ration borrowing and cause persistent underinvestment in human capital. We characterize the optimal government policy and the policy that would emerge under majority voting in response to a technological change that raises the returns to education. The optimal government policy consists in a transfer of resources from future to current generations to finance investment in education and an increase in consumption for the current old generation. The policy chosen under majority voting accomplishes a generational transfer only if a majority of individuals are credit constrained. We consider two policy instruments: a labor income tax and an education subsidy. Current voters prefer a reduction in the current income tax rate to an education subsidy, as the former can finance an increase in their consumption.

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# 1 Introduction

We study investment in education in an environment where individuals cannot commit to repay their loans, so that uncollateralized borrowing is limited and investment in human capital is suboptimally low. We consider some alternative government policies, study their welfare impact and determine which of them would emerge under majority voting. We find that, in response to a technological change that raises the returns to education, agents vote for government policies that transfer resources from future to current generations.

Investment in human capital requires, like other forms of investment, up-front disbursements. This is true for compulsory education, where disbursements take the form of tuition fees if children attend private school, and for college and graduate education, where disbursements take the form of tuition fees, room and board, and forgone income while in school. As for other forms of investment, education often depends on the availability of credit for these up-front disbursements.

Unlike investment in physical capital, however, education is hard to collateralize as the human capital it generates cannot be repossessed by the creditor in the event of default. In environments characterized by limited contract enforceability or, alternatively, by uncertainty and asymmetric information, private credit markets fail to provide adequate financing of education and cause underinvestment in human capital.

Education is an important factor contributing to economic growth. Cross-country evidence shows that the growth rate of real per capita GDP depends positively on the initial stock of human capital, as measured by educational attainment in secondary and higher schooling.<sup>1</sup> Hence, credit market imperfections that limit private financing of education and inhibit human capital accumulation have negative implications for economic growth and welfare.

Failures of private markets create the need for well-designed government policies. Investment in education is one of such cases, as government policies that either make financing of education available or provide public education improve welfare. However, well-designed government policies can be hard to implement. These policies require a lot of information or simply create bad incentives that undo their positive effects. Moreover, heterogeneous individuals prefer different policies and voting will result in choosing policies that are optimal for some but fail to be so for others.

This paper develops a two-period overlapping generations model with altruism where individuals acquire education when young and work when old. We call the union of a parent and her offspring a family. Parents decide how much education and/or financial assets to bequeath to their offsprings. Individuals cannot commit to repay their loans and the rule of law prohibits carrying liabilities from parent to child;

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<sup>1</sup>See Barro [1].

default is punished by seizure of the defaulting family's financial assets and exclusion from the credit market for the remainder of life. As a result, in this economy old individuals cannot borrow in equilibrium; young individuals can only borrow up to the financial assets owned by the family – hence, net borrowing is zero.

In response to a skill-biased technological change that raises the productivity of education, parents would like to invest more in their offsprings' education. First, we consider a benchmark economy where individuals can commit to repay their loans and liabilities can be transmitted from one generation to the next. In this economy, individuals borrow to raise investment in education *and* to raise current consumption; the transition to the new steady state is instantaneous.

Then, we consider our economy without commitment. Families with large initial holdings of financial assets instantaneously transition to their new long-run equilibrium allocation. Families with low holdings of financial assets would like to borrow to invest in education and raise current consumption, but cannot. Compared with the benchmark economy, these families underinvest in education and converge slowly to their new long-run equilibrium allocation.

In the presence of credit market imperfections that inhibit investment in education, government policies can be welfare improving. Theoretically, the government could replicate the welfare-maximizing allocation that emerges in the benchmark economy by lending to the families that are borrowing constrained and subsequently levying lump-sum taxes for the repayment. Section 3 studies this policy.

If individuals are heterogeneous in terms of ability and wealth, the welfare-maximizing allocation requires ability- and wealth-specific policies. Such policies are hard to implement in reality. We consider two alternative policy instruments. The first is a labor income tax decided by majority voting. If the median voter is credit constrained following an increase to the returns to education, the current labor income tax falls and then gradually increases. Lower income taxes in the short run free family resources that can be allocated to raise education and consumption, thereby speeding up the convergence to the new steady state. Even though an income tax decided by majority voting is a rudimentary tool to achieve public financing of human capital, it brings large welfare improvements over the allocation with no government policy at all.

The second policy consists in an education subsidy financed by constant labor income taxes. Like the previous policy, it brings a welfare improvement over the allocation with no government policy. However, this policy is dominated by the labor income tax/subsidy policy and the intuition is simple. An education subsidy makes resources available for the only purpose of investing in education while a variable labor income tax/subsidy makes resources available without constraining their allocation between education and consumption. In fact, current old individuals want to increase their own consumption in response to an increase in their offsprings' consumption, and the latter policy allows them to do so.

A testable implication of our model is that higher investment in education, such as longer schooling time and higher enrollment rates in college education, has a negative effect on the government budget. This is true for emerging economies that are building up their stock of human capital, and for more mature economies when they experience an increase in the returns to education. Moreover, this implication applies to both public and private education systems. Continental Europe has largely adopted a public education system; in response to a skill-biased technological change, it should experience higher government spending in education and therefore smaller government surpluses. The United States have opted for a mixed system of private and public education; it should experience lower income taxes and higher public subsidies to education. Section 2 tests this hypothesis using data from 138 countries over the period 1960 to 1990 and finds that an increase in per capita education is indeed accompanied by lower government surpluses.

A related literature studies the implications of private and public investment on education on growth and the evolution of income inequality. Glomm and Ravikumar [12] compare economies with public and private education systems and find that, if median income is below the mean, majority voting results in public education, which redistributes income from more to less affluent individuals. We also study and compare public and private financing of education under majority voting but, unlike Glomm and Ravikumar, we do so in a setting characterized by credit market imperfections that prevent borrowing. Our focus is on the role of government policies in the face of market failures.

Drazen [10] was the first to notice that government bonds may improve welfare in a model with human capital bequests; his result, however, arises in a partial equilibrium framework. Cukierman and Meltzer [8] analyze the conditions conducive to larger debt accumulation within a general equilibrium model. Public debt crowds out capital and therefore affects the returns to the factors of production. Under certain conditions, a change in the interest and wage rates generates a majority of individuals favoring a larger debt. Cukierman and Meltzer, however, do not model human capital or credit constraints. Galor and Zeira [11] show that, in the presence of imperfect credit markets, the initial distribution of wealth affects output in the short- and in the long-run through investment in human capital. Credit markets' imperfections are the central feature of our model too; unlike Galor and Zeira, we concentrate on how government policies can raise investment in human capital.

This paper is organized as follows. Section 2 presents evidence on the recent skill-biased technological change, on the returns to college education and on college enrollment. Section 3 presents the model; the effects of a skill-biased technological change with nondistortionary taxes are studied in Section 4 whereas Section 5 considers the case of distortionary taxation. Section 6 presents some numerical examples and Section 7 concludes. The mathematical proofs are presented in Appendix A.

## 2 Some evidence

A technological change has changed the nature of work in the last twenty years by raising the importance of education.<sup>2</sup> Employers have increased their demand of specific skills, such as computer literacy, interpersonal and teamwork skills, and problem-solving skills. Even though the supply of well-educated workers was growing rapidly than before the 1970s, demand simply outpaced supply in the 80s and 90s. As a result, a substantial gap has emerged between the earnings of individuals with a college education and those without it. Between 1979 and 1999, the median real weekly earnings of U.S. male college graduates aged 25 and over working full-time rose by 15%, while the earnings of full-time working males with only a high school diploma fell by 12%. As shown in figure 1, the ratio of real earnings for college and high-school U.S. male graduates has raised by more than 70% from 1979 to 1998.

This increase in the earnings gap for different educational levels is leading to a dramatic increase in the overall educational attainment of the population. The median number of years that an adult American has spent in school rose from 8.6 in 1940 to nearly 13 in the 1990s; the college enrollment ratio for individuals between 18 and 24 years old, shown in figure 1, has tracked the earnings differential quite closely, going from 25% to 37% over the same time period.

Higher returns to education are a phenomenon common to all industrialized economies. The occupational structure of the work force has changed dramatically in the developed world in the last thirty years: in 1970, blue-collar workers accounted for a greater proportion of the labor force than white-collar workers in fifteen out of twenty two OECD countries; within ten years the situation was reversed, and by 1990 white-collar occupations largely outnumbered blue-collar ones in all countries except Spain.<sup>3</sup>

Higher returns to human capital have materialized as skill-biased unemployment in Europe, where real wages are more rigid.<sup>4</sup> Unemployment in Europe rose from 2% to around 12% between 1974 and today, and this increase is concentrated in the less educated fraction of the labor force. Figure 2 shows unemployment by educational attainment in 1988, which was a peak of the economic cycle. This educationally-driven pattern of unemployment becomes even more pronounced during recessions, as reported in OECD [23].

According to many, such as Krugman and Lawrence [16], Lawrence and Slaughter [18], and Bhagwati [4], the rise in the education premium is due to a skill-biased

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<sup>2</sup>See Katz and Murphy [15] for a detailed analysis.

<sup>3</sup>See OECD [23].

<sup>4</sup>In 1985, Coe [7] estimated that short-run real wage rigidity - measured as the percentage increase in the unemployment rate necessary to offset a real shock that would otherwise increase the price level by 1% - after 1973 was 1.4 in Europe and 0.67 in the United States.

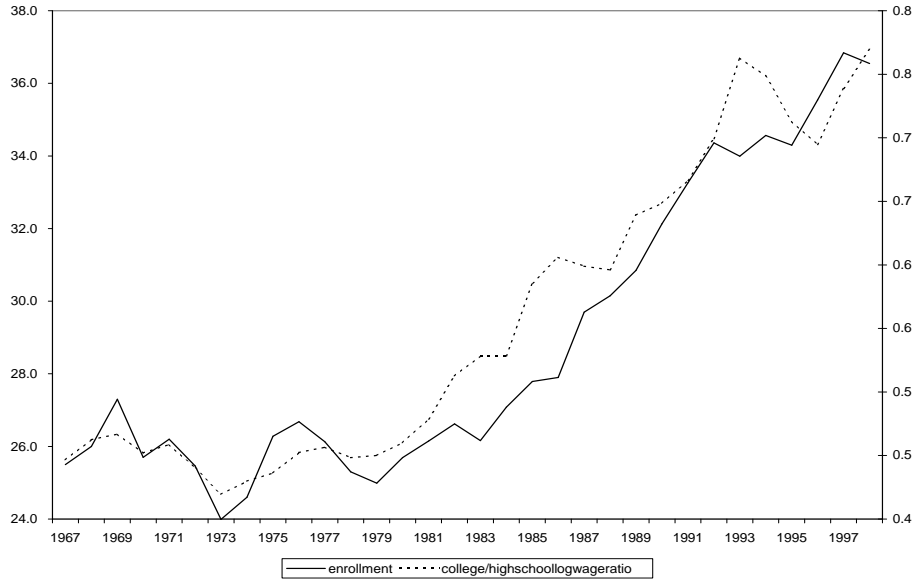


Figure 1: College enrollment rates and college/high school log-earnings ratio

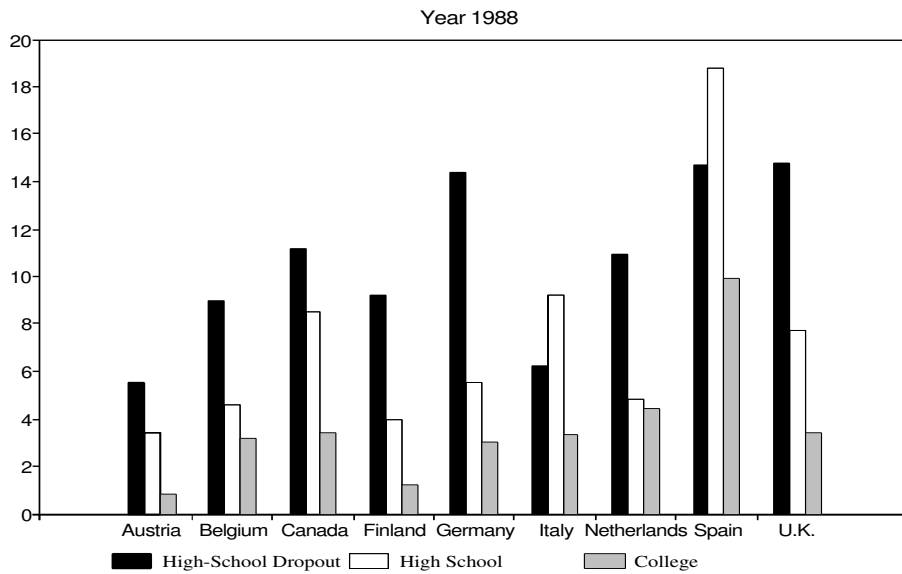


Figure 2: Male unemployment rate by level of attainment

technological change. Greenwood and Yorukoglu [13] relate this technological change to the introduction of new information technologies. Empirical support in favor of this hypothesis is found by Bound and Johnson [6] and Katz and Murphy [15]. Wage inequality between college-educated and high school-educated workers has risen in other industrial countries characterized by low wage rigidity, such as the United Kingdom

and Japan.<sup>5</sup> Krusell et al. [17] attribute the increase of the skill premium to capital-skill complementarities and suggest that increased wage inequality results from growth driven by efficient technologies embodied in capital equipment.

Other works consider trade with emerging economies as the cause of skill-driven earnings inequality. Some works look at quantities; Borjas, Freeman and Katz [5] estimated the quantities of educated and uneducated labor embodied in U.S. exports and imports of manufactured goods and conclude that trade flows explain at most 15% of the increased wage inequality between 1980 and 1988. Sachs and Shatz [24] consider the hypothetical scenario in which net trade balance remains a constant share of domestic spending between 1978 and 1990 and estimate that trade in manufacturing goods with low-wage countries reduced U.S. manufacturing employment by 4.8% over the same period. Baghwati [4] and Leamer [19] argue that prices rather than volumes are the correct indicators of trade pressure. Leamer [19] finds a rapid decline in the relative price of unskilled labor intensive goods in the 50s and 70s, but not in the 80s.

No clear consensus has been reached yet as to the role of technological change and globalization in explaining growing wage inequality in the United States. This paper considers an increase in the skill premium that originates from a technological change; the result would remain unchanged if trade rather than technology were at the origin of the premium.

The link between the skill premium and public financing of education is inadequate credit markets for human capital: it is extremely difficult to collateralize debt incurred to invest in education because human capital is embodied in a person and cannot be repossessed by the creditor in the event of default. If credit market imperfections limit borrowing to invest in education, individuals with insufficient resources will benefit from a policy that raises their available resources via transfers or tax cuts or education subsidies.

There is much evidence that credit constraints to finance education are important in the United States. College enrollment rates for lower and middle income families students have followed closely the dynamics of federal and state financial aid programs in the last twenty years. Dick, Edlin and Topper [9] find that a one dollar fall in the “Expected Family Contribution” (EFC), a figure which measures what parents can reasonably be expected to pay for their children’s education, does not generally lead to an extra dollar in financial aid. They also find that the financial aid system for college education is overall regressive, as a dollar fall in EFC lowers the value of aid awards by more for families with low means than for those with high means. Behrman, Pollak and Taubman [2] find an inverse relationship between sibship size and sib schooling. They also discover earnings similarities for the children of veterans but not for veterans themselves, for whom the GI Bill assured equal access to financing for education.

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<sup>5</sup>See Machin [21] and Katz, Loveman, and Blanchflower [14].

In the United States, financial aid programs reach only a fraction of the student population and provide limited support.<sup>6</sup> The average undergraduate cost of tuition, room, and board was \$7,448 for 4-year public universities, and \$22,502 for 4-year private universities in the academic year 1995-96 in the United States.<sup>7</sup> These figures do not include the opportunity cost of forgone earnings while in college. In the same academic year, 49% of the student population received some form of financial aid including work study; 39% received grants and 25% received loans. The average financial aid was \$6,832, the average grant \$3,864 and the average loan \$4,345.<sup>8</sup> More importantly, financial aid does very little to eliminate differences in income. In the academic year 1995-96, the average financial aid to students from families with income less than \$20,000 was \$7,198 while financial aid to students from families with income above \$100,000 was \$6,051, a mere \$1,147 difference. Loans to students from the former income group averaged \$3,778 while loans to students from the latter group averaged \$4,082, a negative \$-304 difference that indicates that loans to finance education reflect students' parental income more than actual need.

Tuition fees are lower in Europe than in the United States because college education is highly subsidized by governments; this results, however, in higher average tax rates and lower disposable income. Student loans hardly exist in continental Europe and, in general, credit markets are less developed than in the United States; room, board, and foregone wages while at school, which are also components of the cost of education, are higher than in the United States because the cost of living and low-skill wages are also higher.

The cost of borrowing also plays an important role in determining if and to what extent the government should intervene in financing education. Even if capital markets provide financing for education, individuals may still prefer public to private financing of education if the interest rate on the former is lower.

We investigate the relationship between government budget balances and investment in education using a panel dataset from 138 countries over the period 1960 to 1996. We use three different measures of investment in education: ER measures enrollment rates in school, all levels (source: UNESCO); AS measures average years of high school (source: Barro-Lee dataset); AV measures average years of college education (source: Nehru and Dhareshwar). ER is a broader measure of investment of education, as it measures enrollment in primary, secondary and tertiary education; AV is a narrow measure that only looks at college education. The dependent variable in the regressions is SUR, which measures government overall surplus as a % of GDP; the other regressors are: Y, the growth of real GDP; DEM, an indicator of democracy ranging

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<sup>6</sup>See U.S. Department of Education [25] and McPherson and Schapiro [22].

<sup>7</sup>These figures are from the U.S. Department of Education [26].

<sup>8</sup>Separate data on average financial aid for public and private institutions is not available.



Dependent variable: SUR						
<i>Variables</i>						
ER	-1.70 (-3.92)					
AS		-1.74 (-3.48)				
AV			-1.77 (-3.79)			
ER*DEM				-0.18 (-4.18)		
AS*DEM					-0.21 (-4.29)	
AV*DEM						-0.18 (-4.04)
Y	0.16 (11.39)	0.17 (10.61)	0.17 (11.43)	0.17 (10.69)	0.17 (5.37)	0.18 (5.72)
SUR(-1)	0.68 (39.72)	0.67 (34.4)	0.67 (39.42)	0.66 (33.93)	0.64 (27.73)	0.63 (27.27)
period:	1960-96	1960-90	1960-87	1960-96	1960-90	1960-87
# obs.	1625	1237	923	1625	1237	923

Table 1: All countries

from 0 to 10, with 0 low and 10 high (source: Polity98, Regime Characteristics); fixed effects, which are not reported here. Real GDP growth is included in the regression because the government surplus is affected by the business cycle. Since current GDP growth is an endogenous variable determined, among other things, by current fiscal policy, we instrument Y with its lagged value, Y(-1). We use iterative generalized least squares to estimate our system of regressions.

We estimate two specifications. The first uses the investment-in-education variable by itself as regressor, while the second interacts it with the democracy variable. The first three columns of Table 1 show the results of the first specification; the last three show the results for the second specification. As expected, the coefficient of Y is positive and significant in all regressions; the lagged surplus is also significant and positive with a coefficient around 0.6, as government surpluses are persistent. Investment in education is negative and significant, both by itself and when interacted with an index for democracy. The results are very similar in both specifications; the  $R^2$ s of the equations in the system are slightly higher in the specification where the variables are interacted. We interpret this evidence as suggesting that governments, and especially

Dependent variable: PSUR			
<i>Variables</i>	<i>Coefficients</i>	<i>Coefficients</i>	<i>Coefficients</i>
ER	-2.01 (-3.93)		
AS		-1.77 (-3.24)	
AV			-0.9 (-1.73)
Y	0.22 (5.09)	0.26 (5.47)	0.19 (4.13)
PSUR(-1)	0.79 (33.4)	0.72 (25.22)	0.69 (21.29)
period:	1960-96	1960-90	1960-87
# obs.	666	563	506

Table 2: OECD countries

democratically elected ones, engage in public financing of education by running smaller government surpluses.

Next, we study the relationship between investment in education and government surplus among industrialized countries only. This is a smaller but more homogeneous set of countries in terms of institutions, income per capita and average level of education in the population; moreover, we can use primary surplus data for these countries. The dependent variable in the regression is PSUR, general government primary surplus as % of GDP; the regressors are as in table 1, except for the democracy dummy, which has been dropped. As before, the variable Y is instrumented by its lagged value Y(-1); we use AV, AS and ER as dependent variables and estimate the regressions using iterative generalized least squares. The results, presented in table 2, indicate that investment in education has a strong negative effect on primary surpluses: a one percent increase in enrollment rates in all school levels reduces the primary surplus to GDP ratio by two percent.

### 3 The Model

Consider an economy with overlapping generations, each composed of a continuum of individuals. Time is discrete and individuals live for two periods; population is stationary because each person gets one offspring at the beginning of the second period of life and such union is called a family. A person in the first period of her life is referred to as the offspring or young individual whereas a person in the second period of her

life is called the parent or old individual. An individual may go to college when young and works when old. For simplicity, consumption only takes place in the second period of life.

Each old individual supplies one unit of labor inelastically for the production of a perishable good and produces a quantity that depends on her productivity. It is assumed here that: a) productivity depends on innate economic ability that differs among individuals; b) investment in education increases individual productivity. For simplicity, innate economic ability is assumed to be an hereditary characteristic and every offspring has exactly her parent's innate aptitudes.<sup>9</sup>

Old individuals' resources are the outcome of their production activity. Denoting respectively by  $y$ ,  $\alpha$ , and  $e$  an old individual's production, innate ability and education, production possibilities can be written as follows:<sup>10</sup>

$$y = h(\alpha, e) \tag{1}$$

The production technology  $h(\cdot, \cdot)$  is common to all workers in the economy and strictly increasing in both arguments. The variable  $e$  represents education beyond compulsory schooling and can be thought of as college education; hence,  $e \geq 0$  and higher levels of  $e$  identify better college education.

Education enhances productivity in this economy. However, education is costly: sending an offspring to college entails a fixed cost  $F$  and a variable cost equal to the level of education  $e$  in terms of forgone current consumption. The fixed cost reflects those components of the cost of education that are largely independent of its quality, such as foregone consumption while at school, room and board, and books. The existence of fixed costs is an important assumption - this nonconvexity is the source of multiple equilibria - but not a necessary one for the results of this paper. The variable cost of education represents tuition fees and it is assumed there exists a continuum of colleges whose tuition fees are perfectly correlated with the quality of education they provide. If credit markets were complete, young individuals would take a loan to pay for their education and repay the debt the following period. However, the assumption of complete credit markets for education is unrealistic, as argued in the previous section. To keep the analysis simple, here I assume that credit markets for education do not exist because future labor earnings cannot serve as collateral on an educational loan; parents therefore trade off the benefits of sending their offspring to college with the costs stemming from current forgone consumption.

Within a family all economic decisions are taken by the parent. Family's labor income depends on the productivity of the old individual, which in turn depends on

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<sup>9</sup>A positive correlation with the parent's innate ability would be enough to guarantee the results to carry through.

<sup>10</sup>This formalization is similar to Loury [20].

her education and innate economic ability. Each period total family income is divided among consumption, investment in the offspring's education, and investment in non-human capital, namely a financial bequest to the offspring. These investments are the only two possible means to transfer resources through time. It is also assumed that there is no legal mechanism to leave negative bequests. Each parent's utility depends on the family's well-being, which consists of the parent's current consumption and her offspring's well-being. Let  $V_t$  be the utility of the old individual at time  $t$ . Then

$$V_t = u(c_t) + \beta V_{t+1} \quad (2)$$

where  $c_t$  is consumption at time  $t$ ,  $\beta$  is the rate at which each generation cares about the utility of the next generation, with  $0 < \beta < 1$ .  $u(c)$  is the utility stemming from consumption and its properties are described later. (2) can be solved forward and, provided  $\lim_{s \rightarrow \infty} \beta^s V_{t+s} = 0$ ,

$$V_t = \sum_{i=0}^{\infty} \beta^i u(c_{t+i}) \quad (3)$$

To sum up, four elements characterize this economy: parental altruism, productivity-enhancing education, heterogeneity in innate ability among the population and the fact that one cannot borrow against future income to finance education. Some technical assumptions are now imposed on these elements. Innate ability is distributed on a bounded support and it is perfectly autocorrelated within each family. Consumption is always desirable, but especially so when consumption levels are low. Concerning production, it is assumed there are diminishing returns to education and a strictly positive marginal product of ability. More formally:

**Assumption 1** *Innate ability is distributed on the unit interval and perfectly autocorrelated within a family. The distribution has a continuous, strictly positive density function,  $f : [0, 1] \rightarrow \mathbf{R}_+$ . Families can be ranked according to their ability. A family with innate ability  $\alpha$  is referred to as the  $\alpha$ -family.*

**Assumption 2**  *$u : \mathbf{R}_+ \rightarrow \mathbf{R}$  is strictly increasing, strictly concave, twice continuously differentiable satisfying  $u_c(0) = +\infty$ .*

**Assumption 3**  *$h : [0, 1] \times \mathbf{R}_+ \rightarrow \mathbf{R}_+$  is twice continuously differentiable, strictly increasing in both arguments, strictly concave in  $e$ , satisfying*

1.  $\forall \alpha, \exists \gamma(\alpha) > 0, \gamma_\alpha(\alpha) > 0$  such that  $h_e(\alpha, e) < \gamma(\alpha)$ ;
2.  $h(\alpha, 0) = l + \epsilon(\alpha)$ , where  $l > 0, \epsilon(\alpha) > 0$  and continuous, with  $\epsilon_\alpha(\alpha) > 0, \forall \alpha \in [0, 1]$ ;
3.  $h_e(\alpha, 0) = M(\alpha) > 0, M_\alpha(\alpha) \geq 0$ ;

4.  $h_{e\alpha}(\alpha, e) > 0$ .

Throughout the paper a subscript on a function denotes the partial derivative with respect to the subscript. The first condition in Assumption 3 puts an upper bound on the returns from education. Intuitively, unbounded returns from education may generate forever increasing levels of education and the economy may fail to have a steady state. The second condition puts a lower bound on production that increases with innate ability. The third condition states that the marginal returns from education at zero are a non-decreasing function of innate ability. The fourth condition states that education and innate ability are complementary in production.

It is now possible to characterize the family's optimal allocation of income. The decision to allocate income among consumption, education, and financial bequest is taken by parents to maximize their utility  $V_t$ . The old individual with innate ability  $\alpha$  solves the following maximization problem:

$$\max_{e_t, s_t} V_t = \sum_{i=0}^{\infty} \beta^i u(c_{t+i}) \quad (4)$$

$$\text{s.t.} \quad c_t + e_t + F(e_t) + s_t = y_t + s_{t-1}(1+r) \quad (5)$$

$$y_t(\alpha) = h(\alpha, e_{t-1}) \quad (6)$$

$$c_t > 0, \quad e_t \geq 0, \quad s_t \geq 0 \quad (7)$$

$$F(0) = 0; \quad F(e) = F > 0, \quad \forall e > 0 \quad (8)$$

where  $s_t$  is the financial bequest left by the parent to her offspring at time  $t$  and  $y_t$  is earnings at time  $t$ .  $F(e)$  is the fixed cost of education and it is described in (8). Resources at  $t$  are defined as  $w_t = y_t + s_{t-1}(1+r)$ , which are the sum of earnings and bequests (including interests) received from parents. (5) is the budget constraint faced by the old individual and (6) describes the relationship between earnings and productivity. I consider a small open economy that takes the world interest rate as given; bequests earn the world rate of interest  $r$ . The first-order conditions from the maximization problem are:

$$FOC_{e_t} : \begin{cases} -u_c(c_t) + \beta u_c(c_{t+1}) h_e(\alpha, e_t) \leq 0 \\ e_t \frac{\partial V_t}{\partial e_t} = 0 \end{cases} \quad (9)$$

$$FOC_{s_t} : \begin{cases} -u_c(c_t) + \beta (1+r) u_c(c_{t+1}) \leq 0 \\ s_t \frac{\partial V_t}{\partial s_t} = 0. \end{cases} \quad (10)$$

(9) and (10) imply that the optimal allocation of resources among consumption, education, and bequests is such that their marginal rate of returns are equalized unless

the nonnegativity constraints are binding. If resources and ability are large enough (in a sense to be defined later), the returns from the three activities are equal.

Empirical evidence suggests that, at least up to a certain amount, investment in human capital, yields higher returns than investment in nonhuman capital. Therefore, it is assumed that the rate of return to education is higher than the interest rate up to a certain amount of education:

**Assumption 4**  $M(\alpha) > (1 + r)$ .

Assumption 3 and 4 state that marginal returns from education are higher than  $(1 + r)$  when education is equal to zero and diminish as education increases. Therefore, there exists an amount of education  $\bar{e}(\alpha)$  at which the marginal return from education is equal to  $(1 + r)$ , as shown in Figure 3. Investment in education for an  $\alpha$ -individual never exceeds  $\bar{e}(\alpha)$ . Because of the complementarity of education and innate ability in production,  $\bar{e}(\alpha)$  is an increasing function of  $\alpha$ .

**Definition 1** *An individual is financially constrained if she would choose to leave a negative bequest.*

An individual is financially constrained when the upper part of (10) is negative. In words, she would altruistically leave a negative financial bequest to her offspring if the legal system allowed her to.

**Definition 2** *Let  $\bar{w}_t(\alpha, F) \equiv \inf \{w_t : -u_c(c_t) + \beta(1 + r) u_c(c_{t+1}) = 0\}$ .*

In words,  $\bar{w}_t(\alpha, F)$  is the lowest level of resources for the  $\alpha$ -individual at which the upper part of (10) is equal to zero and the individual is *not* financially constrained.

**Definition 3** *Let  $\underline{w}_t(\alpha, F) \equiv \inf \{w_t : -u_c(c_t) + \beta u_c(c_{t+1}) h_e(\alpha, e_t) = 0 | e_t > 0\}$ .*

In words,  $\underline{w}_t(\alpha, F)$  is the lowest level of resources at which the upper part of (9) is equal to zero and the  $\alpha$ -individual invests in education.

The properties of investment in education, bequests, and consumption that maximize (4) are summarized in the following proposition:

**Proposition 1** *Under Assumptions 1-4 there exist unique  $e_t^*$ ,  $s_t^*$ , and  $c_t^*$  s.t.  $V_t$  is maximized, which have the following properties:*

1.  $e_t^* = e(\alpha, w_t, F)$ ,  $e_\alpha \geq 0$  if  $h_{e\alpha} > J(\alpha)$ ,  $0 \leq e_w < 1$ ,  
 $e_t^* = 0 \forall w \in [0, \underline{w}(\alpha, F))$ , and  $e_t^* = \bar{e}(\alpha) \forall w \in [\bar{w}(\alpha, F), \infty)$ ;
2.  $s_t^* = s(\alpha, w_t, F)$ ,  $s_\alpha \geq 0$  if  $\beta(1+r) \geq 1$ ,  $0 \leq s_w < 1$ ,  $s_t^* = 0 \forall w \in (\underline{w}_t(\alpha, F), \bar{w}(\alpha, F))$ ;
3.  $c_t^* = c(\alpha, w_t, F)$ ,  $c_\alpha > 0$ ,  $0 \leq c_w < 1$ .

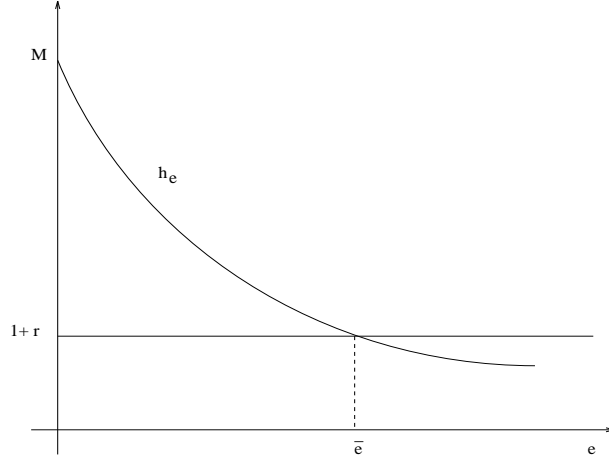


Figure 3: Marginal returns

where  $J(\alpha) = h_\alpha [u_{cc}(c_t) - \beta u_{cc}(c_{t+1})h_e] / [\beta u_c(c_{t+1})]$ . Moreover

1.  $\frac{\partial \underline{w}_t(\alpha, F)}{\partial \alpha} < 0$ ,  $\frac{\partial \underline{w}_t(\alpha, F)}{\partial F} > 0$ ;
2.  $\frac{\partial \bar{w}_t(\alpha, F)}{\partial \alpha} < 0$ ,  $\frac{\partial \bar{w}_t(\alpha, F)}{\partial F} > 0$ ,

Proof: See Appendix A.

In the absence of fixed costs of education ( $F = 0$ ), Assumption 3 and 4 guarantee that  $\underline{w}_t = 0$  and all individuals invest in education, even if the financial bequests received from their parents are zero. If at  $t$  the  $\alpha$ -individual has enough resources, namely if  $w_t \geq \bar{w}_t(\alpha, F)$ , then investment is at its steady state level and financial bequests may be positive, namely  $e_t = \bar{e}(\alpha)$  and  $s_t \geq 0$ ; on the other hand, if resources are low so that  $w_t < \bar{w}_t(\alpha, F)$ , then the individual is financially constrained and  $0 < e_t < \bar{e}(\alpha)$ .

With fixed costs of education ( $F > 0$ ), individuals with low abilities or low resources do not invest in education at all. More precisely, if  $w_t < \underline{w}_t(\alpha, F)$ , then the  $\alpha$ -individual does not invest in education. With low abilities, income may not be enough to cover the fixed cost of education; a sufficient condition for not investing in education is simply  $h(\alpha, \bar{e}) - l - \epsilon(\alpha) < F$ . Notice that an individual is not financially constrained in the sense of definition 1 if  $\beta(1+r) \geq 1$ : investment in education is effectively not an option. If  $w_t \in [\underline{w}_t(\alpha, F), \bar{w}_t(\alpha, F))$ , then the  $\alpha$ -individual invests in education and is financially constrained:  $0 < e_t < \bar{e}(\alpha)$  and  $s_t = 0$ ; if resources are high and  $w_t \geq \bar{w}_t(\alpha, F)$ , investment in education is at its steady state level and financial bequests may be positive:  $e_t = \bar{e}(\alpha)$  and  $s_t \geq 0$ . Section 6 presents some numerical simulations.

Optimal investment in education varies among families with different innate abilities in a way that depends on the properties of  $h(\cdot, \cdot)$ . Notice that the earnings of an old individual depend on her innate ability and her parent's resources in the following manner:

$$y_t = h(\alpha, e^*(\alpha, w_{t-1}, F)). \quad (11)$$

Proposition 1 states that optimal investment in education depends positively on innate ability if  $h_{e\alpha} > J(\alpha)$ . A higher innate ability affects investment in education by raising both parent's and offspring's earnings. If  $\beta(1+r) = 1$ , these two effects cancel each other out in the steady state, as parent's and offspring's future consumption are equal. During the transition to the steady state, the balance of the two effects depends on the third derivative of the utility function. With isoelastic utility functions, the condition  $h_{e\alpha} > J(\alpha)$  is certainly satisfied even along the transition path.<sup>11</sup>

**Definition 4** Let  $\underline{\alpha}(F) \equiv \sup \{\alpha : e^*(\alpha, w, F) = 0, \forall w\}$ .

Given the fixed cost of education  $F$ , individuals with ability levels below  $\underline{\alpha}(F)$  do not invest in education even if their financial bequests are high. For these individuals, the returns to education are not high enough to pay  $F$ .

For a given initial distribution of resources, this model has a unique steady state where all family variables, i.e. consumption, education and resources, remain constant over time. This result can be summarized in the following proposition:

**Proposition 2** Let  $\beta(1+r) = 1$  and let  $\gamma(\alpha) = 1/e_w(\alpha, w, F)$ ,  $\forall \alpha \in [0, 1]$ . Under Assumptions 1-4, for a given initial distribution of financial bequests, there exists a unique stable long-run equilibrium. The  $\alpha$ -individual has in the steady state

1. education  $\bar{e}(\alpha, w, F) = \bar{e}(\alpha)$  if  $w_t \geq \underline{w}_t(\alpha, F)$ ,  $\bar{e}(\alpha, w, F) = 0$  otherwise;
2. financial bequest  $\bar{s}(\alpha, w, F) = s_{t-1} - \frac{h(\alpha, \bar{e}(\alpha, w, F)) - y_t(\alpha)}{1+r}$  if  $w_t < \underline{w}_t(\alpha, F)$  and  $w_t \geq \bar{w}_t(\alpha, F)$ ,  $\bar{s}(\alpha, w, F) = 0$  otherwise;
3. earnings  $\bar{y}(\alpha, w, F) = h(\alpha, \bar{e}(\alpha, w, F))$ ;
4. consumption  $\bar{c}(\alpha, w, F) = h(\alpha, \bar{e}(\alpha)) - \bar{e}(\alpha) - F + r\bar{s}(\alpha, w, F)$  if  $w_t \geq \underline{w}_t(\alpha, F)$ ,  $\bar{c}(\alpha, w, F) = h(\alpha, 0) + r\bar{s}(\alpha, w, F)$  otherwise

where  $\bar{c}_\alpha > 0$  and  $\bar{y}_\alpha > 0$ .

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<sup>11</sup>Earnings are an increasing function of innate ability if  $h_\alpha + h_e e_\alpha > 0$ . Therefore  $e_\alpha > 0$  is sufficient but not necessary for the condition to be satisfied.



Proof: see Appendix A.

Attention is restricted to the case where  $\beta(1+r) = 1$  to rule out equilibrium paths characterized by ever growing or decreasing consumption levels. Then a unique stable equilibrium exists if there are diminishing returns to education, i.e. if  $h_e e_w < 1$ . Technically, this restriction makes earnings at time  $(t+1)$  a contraction mapping with respect to earnings at time  $t$  and guarantees the existence of a unique fixed-point.

Now we characterize the equilibrium that would arise if individuals could borrow to invest in education. While this is not a realistic equilibrium, it is a useful benchmark. More importantly, optimal government policy replicates this allocation. Suppose that individuals can borrow from abroad to finance their investment in education and can leave negative financial bequests to their offsprings; hence,  $s_t(\alpha, w_t, F)$  can be negative but still satisfy the No-Ponzi game condition. The following proposition characterizes this steady state.

**Proposition 3** *If individuals can borrow and leave negative financial bequests, the steady state is reached instantaneously; all families with  $\alpha > \underline{\alpha}(F)$  invest in education. Given the initial distribution of resources and education at  $t$ , at the steady state*

1. *education is  $\bar{e}(\alpha)$  if  $\alpha > \underline{\alpha}(F)$  and 0 otherwise;*
2. *the net financial position is  $\bar{s}(\alpha) = s_{t-1} - \frac{h(\alpha, \bar{e}(\alpha)) - y_t(\alpha)}{1+r}$  if  $\alpha > \underline{\alpha}(F)$ ,  
 $\bar{s}(\alpha) = s_{t-1} - \frac{h(\alpha, 0) - y_t(\alpha)}{1+r}$  otherwise;*
3. *consumption is  $\bar{c}(\alpha) = h(\alpha, \bar{e}(\alpha)) - \bar{e}(\alpha) - F + r\bar{s}(\alpha)$  if  $\alpha > \underline{\alpha}(F)$ ,  
 $\bar{c}(\alpha) = h(\alpha, 0) + r\bar{s}(\alpha)$  otherwise*

*for the  $\alpha$ -family.*

When borrowing and leaving negative financial bequests is feasible, the steady state is reached instantaneously. No matter the current level of education and financial bequest received from parents, all individuals with innate abilities above  $\underline{\alpha}(F)$  invest optimally in education. In fact, no individual is financially constrained in this setting and those who do not invest in education do so because the returns to education are simply not high enough for them. Those individuals who were financially constrained in the previous setting can now borrow to both raise their current consumption to the steady state level and to invest in education.

## 4 Technological change with nondistortionary taxes

Consider now a skill-biased technological change that raises the marginal returns to human capital. The new technology, embodied in the production function  $\hat{h}(\alpha, e)$ , is

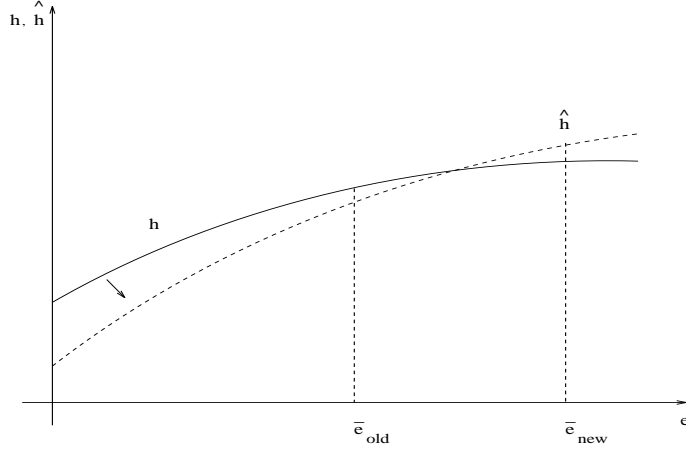


Figure 4: Technological change

characterized by higher marginal productivity of education for any  $\alpha$ ,  $\hat{h}_e > h_e$  for all  $e$ , as shown in Figure 4. The change in technology enhances earnings for individuals with education strictly above  $\hat{e}$ , but lowers them for everyone else. This characterization of the technological change is consistent with the decline in real wage for low-skill workers since the late 1970s but is not fundamental for the results of this model, which hold true as long as the marginal returns to education increase. In terms of figure 4, all we need is that the new production function is steeper than the old one and the two curves need not intersect; in fact, the new production function may lie completely above the old one.<sup>12</sup> The function  $\hat{h}(\cdot, \cdot)$  has the same properties of  $h(\cdot, \cdot)$ .

Consider the standard setting where borrowing and leaving negative financial bequests is not permitted. Optimal investment in education is now described by

$$FOC_{e_t} : \begin{cases} -u_c(c_t) + \beta u_c(c_{t+1}) \hat{h}_e(\alpha, e_t) \leq 0, \\ e_t \frac{\partial V_t}{\partial e_t} = 0 \end{cases} \quad (12)$$

The optimal financial bequest is still described by (10).

The skill-biased technological change affects the aggregate level of education in three ways. First, it raises investment in education for those who can afford it:  $\hat{h}_e$  is higher and makes individuals invest more. Second, it changes the fraction of families that can afford to invest in education. Higher returns to education reduce  $\underline{w}_t(\alpha, F)$  but lower productivity for individuals with innate abilities below  $\hat{\alpha}$  raises  $\underline{w}_t(\alpha, F)$ ; the overall effect on the fraction of individuals that can afford to invest in education depends on

<sup>12</sup>I am assuming that the new technology is efficient in the sense that total production is higher than under the old technology under the existing distribution of innate ability and education.

which of these two effects dominates. Third, it changes the fraction of families for which education is not an option: this is a change in  $\underline{\alpha}(F)$ , which raises due to higher returns to education but may fall for individuals with innate ability below  $\hat{\alpha}$  due to lower productivity.

The skill-biased technological change affects  $\underline{\alpha}(F)$ ,  $\underline{w}_t(\alpha, F)$ ,  $\hat{w}_t(\alpha, F)$  that, in turn, affect the new steady state values for education, bequest, production and consumption, respectively  $\bar{e}_n(\alpha, w, F)$ ,  $\bar{s}_n(\alpha, w, F)$ ,  $\bar{y}_n(\alpha, w, F)$ ,  $\bar{c}_n(\alpha, w, F)$ . For a given initial distribution of resources, the economy has a unique steady state that is as in proposition 2 with  $\hat{h}(\alpha, e)$  in place of  $h(\alpha, e)$ .

The following proposition compares the steady state before and after the technological change; to simplify the analysis, I assume that  $\hat{e} = 0$ : the skill-biased technological change makes all individuals with a positive amount of education more productive, but it leaves those with no education as productive as before.<sup>13</sup> As a result, more people invest in education than before ( $\underline{w}_t(\alpha, F)$  falls).

**Proposition 4** *In the steady state that follows the skill-biased technological change,  $\underline{w}_t(\alpha, F)$  falls and more individuals invest in education; for all  $w_t \geq \underline{w}_t(\alpha, F)$ , we have that*

1.  $\bar{e}_n(\alpha) > \bar{e}(\alpha)$ ;
2.  $\bar{s}_n(\alpha, w, F) - \bar{s}(\alpha, w, F) = \frac{h(\alpha, \bar{e}(\alpha)) - \hat{h}(\alpha, \bar{e}_n(\alpha))}{1+r} < 0$ ;
3.  $\bar{y}_n(\alpha, w, F) > \bar{y}(\alpha, w, F)$ ;
4.  $\bar{c}_n(\alpha, w, F) > \bar{c}(\alpha, w, F)$ .

Proof: see Appendix A. The skill-biased technological change raises education, output and consumption in the long run for all the individuals who can afford to invest in education.

**Fact 1** *The long-run earnings distribution becomes more polarized after the technological change.*

The long-run effects of skill-biased technological change are unambiguous: individuals who invest in education are better off in the long run. As for those who do not invest in education, they are as well off as before the technological change if  $\hat{e} = 0$  but worse off if  $\hat{e} > 0$ . The long-run distribution of earnings becomes more dispersed as a consequence of heterogeneity in innate ability and higher returns to education. In fact, more polarization in earnings occurs independently of fixed costs of education and

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<sup>13</sup>This is because the function  $h(\alpha, e)$  is continuous – see Assumption 3.

even if the new production function lies completely above the old one (and everyone's productivity improves): it is the consequence of a steeper productivity profile.

The transition to the new steady state depends on the completeness of capital markets, the legal system and initial resources. If individuals can borrow to invest in human capital or if parents can leave negative financial bequests to their offsprings, the transition to the new steady state is instantaneous. For example, a parent could borrow enough to increase her offspring education to its new long-run equilibrium level and equalize the level of current and future family consumption; the loan repayment could be equally shared by all future family members. If families have large initial financial resources, the transition to the steady state is also instantaneous. Here we consider the case where families have zero initial financial resources. In this case, educated individuals may be financially constrained following the technological change and the transition to the new steady state may be gradual and characterized by increasing levels of consumption and education.

Government debt may be welfare improving in this *non-Ricardian* setting: current lifetime utility of a financially constrained family is raised by any policy that reallocates resources from the future to the present. The remainder of this section studies the optimal government policy and the policy that is chosen under majority voting.

Suppose that bequests cannot be negative and borrowing to finance education is not available. For simplicity, there is no public consumption, the government borrows at the constant world interest rate  $r$ . The government's budget constraint in period  $t$  is given by

$$b_{t+1} = (1 + r) b_t + \int_{\alpha} \tau_t(\alpha) df(\alpha) \quad (13)$$

where  $b_t$  is the stock of public debt at time  $t$  and  $\tau_t(\alpha)$  is the transfer to the  $\alpha$ -family (if  $> 0$ ) or tax (if  $< 0$ ) at time  $t$ . Public debt cannot be repudiated at any time and the government meets the No Ponzi Game condition.

Consider first the case where government transfers are nondistortionary and let  $G \in [0, 1]$  be the set of financially constrained individuals. There are many transfer policies that improve the welfare of at least half the voting population and would therefore be chosen in a pairwise election against the status quo, which I assume to be zero transfers. Here we consider specifically the policy that mimics the decentralized allocation described in proposition 3. The budget constraint for  $\alpha$ - individual at time  $t + i$  is

$$y_{t+i} + \tau_{t+i} + (1 + r)s_{t+i-1} = c_{t+i} + e_{t+i} + F(e_{t+i}) + s_{t+i} \quad (14)$$

and the transfer policy adopted by the government is

$$\tau_{t+i}(\alpha) = \begin{cases} \max \left\{ \frac{\hat{h}(\alpha, \bar{e}_n(\alpha)) - \hat{h}(\alpha, \bar{e}(\alpha))}{1+r} - s_{t-1}, 0 \right\}, & i = 0 \\ \min \left\{ -r \left( \frac{\hat{h}(\alpha, \bar{e}_n(\alpha)) - \hat{h}(\alpha, \bar{e}(\alpha))}{1+r} - s_{t-1} \right), 0 \right\} & i > 0 \end{cases} \quad (15)$$

Given the transfer policy, the financial bequest at  $t$  is

$$s_t(\alpha) = \max \left\{ 0, s_{t-1}(\alpha) - \frac{\bar{y}_n(\alpha) - \hat{y}_t(\alpha)}{1+r} \right\}. \quad (16)$$

Following the technological change, the transfer policy in (15) sets the economy at a new steady state where

1. education is  $\bar{e}_n(\alpha)$ ;
2. consumption is  $\bar{c}'_n(\alpha) = [\bar{y}_n(\alpha) + r\hat{y}_t(\alpha)]/(1+r) - [\bar{e}_n(\alpha) + F] + r\bar{s}(\alpha)$ ;
3. the financial bequest is  $\bar{s}(\alpha) = s_{t-1}(\alpha) - \frac{\bar{y}(\alpha) - y_t(\alpha)}{1+r}$

for the  $\alpha$ -family. In words, following the technological change, the  $\alpha$ -individual invests at  $\bar{e}_n(\alpha)$  and consumes at  $\bar{c}'_n(\alpha)$  by either running down her financial assets or, if those are not sufficient, by receiving a bond-financed transfer from the government; all  $\alpha$ -generations share the interest burden on the public debt issued to finance their initial transfer.

The transfer policy in (15) does not redistribute across families, it allows an instantaneous transition to the new steady state and it achieves the first best of proposition 3;<sup>14</sup> as long as  $G$  is of positive measure, the government runs a budget deficit at  $t$ ; public debt increases at  $t$  and remains constant after that.

Let the status quo be no transfer to any family. All families who invest in education - i.e. whose wealth is above  $\underline{w}$  - strictly prefer this transfer policy to the status quo; all families whose wealth is below  $\underline{w}$  are unaffected by it and therefore weakly prefer it to the status quo. In a pairwise comparison with the status quo, the policy in (15) is adopted if  $G$  is of positive measure and indifferent votes randomize their vote or abstain or vote in favor of it. The following proposition summarizes this result.

**Proposition 5** *Let transfers be nondistortionary and let  $G$  be of positive measure. Following the technological change, the deficit-financed transfer policy in (15) achieves the first best of proposition 3; this policy is adopted in a majority vote against the status quo.*

Once again, there are many other transfer policies that redistribute resources across families, would beat (15) in a pairwise comparison and would therefore be adopted in a majority vote.

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<sup>14</sup>The families who cannot afford to invest in education at  $t$  but could afford it at the new equilibrium, i.e. such that  $\hat{h}(\alpha, \bar{e}_n(\alpha)) \geq \underline{w}(\alpha, F) > \hat{h}(\alpha, \bar{e}(\alpha))$ , also invest in education with this transfer policy.

It is important to notice that the proceeds from issuing bonds are allocated not only to raise investment in education, but also the consumption of currently old individuals; alternative transfer policies, such as education subsidies or non-contingent subsidies of the type  $\bar{e}_n - \bar{e}$  may fail to achieve the first best of proposition 3. Consider the following education subsidy policy

$$\tau_{t+i}(\alpha) = \begin{cases} \max \{ \bar{e}_n(\alpha) - \bar{e}(\alpha) - s_{t-1}(\alpha), 0 \}, & i = 0 \\ \min \{ -r[\bar{e}_n(\alpha) - \bar{e}(\alpha) - s_{t-1}], 0 \} & i > 0 \end{cases} \quad (17)$$

where the subsidy  $\tau_t(\alpha)$  can only be spent in education. This policy fails to achieve the first best because current consumption at  $t$  is lower than optimal. Alternatively, suppose the policy in (17) is a non-contingent transfer; since the transfer is allocated in part on education and in part on current consumption, education is below  $\bar{e}_n(\alpha)$  and the steady state is not reached.

## 5 Technological change with distortionary taxes

A large body of empirical evidence suggests that the size and timing of taxation/transfers has real effects on the economy. There are several reasons for such real effects: taxes distort economic behavior; marginal future taxes depend only in part from current taxation; deferred taxation changes the tax burden levied on an individual.<sup>15</sup> This section considers the case of income taxation. The budget constraint for the  $\alpha$ -individual is now given by

$$y_t(1 - T_t) + (1 + r)s_{t-1} = c_t + e_t + F(e_t) + s_t, \quad (18)$$

and the government budget constraint at  $t$  is

$$b_{t+1} = (1 + r)b_t - T_t \int_{\alpha} y_t(\alpha) df(\alpha). \quad (19)$$

Income taxes are distortionary in this setting as they affect the incentive to invest in education. High future income taxes reduce the returns to education and therefore reduce investment in education; more precisely, in response to an increase in next period tax rate  $T_{t+1}$ , the  $\alpha$ -individual raises current consumption and, if she is not financially constrained, her financial bequest.

The first-order condition with respect to the financial bequest,  $s_t$ , is unchanged; the first-order condition with respect to the investment in education is

$$FOC \ e_t : -u_c(c_t) + \beta u_c(c_{t+1}) h_e(\alpha, e_t)(1 - T_{t+1}) \leq 0. \quad (20)$$

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<sup>15</sup>For a detailed discussion, see Bernheim [3].

**Proposition 6** *Following the technological change, the economy converges to its new steady state where*

1. education is  $\bar{e}_n(\alpha)$  s.t.  $\hat{h}_e(\alpha, \bar{e}_n)(1 - \bar{T}_n) = 1 + r$ ;
2. production is  $\bar{y}_n(\alpha) = \hat{h}(\alpha, \bar{e}_n)$ ;
3. consumption is  $\bar{c}_n(\alpha) = \bar{y}_n(\alpha)(1 - \bar{T}_n) - [\bar{e}_n(\alpha) + F] + r\bar{s}_n(\alpha)$ ;
4. the financial bequest is

$$\bar{s}_n(\alpha) = \max \left\{ 0, \bar{s}(\alpha) - \sum_{n=0}^{\infty} (1+r)^{-n} [\bar{y}_n(\alpha)(1 - \bar{T}_n) - y_{t+s}(\alpha)(1 - T_{t+s})] \right\};$$

5. the tax rate is  $\bar{T}_n = \frac{r\bar{b}_n}{\bar{Y}_n}$ , where  $\bar{Y}_n = \int_{\alpha} \bar{y}_n(\alpha) df(\alpha)$  and  $\bar{b}_n$  is public debt.

Proof: see appendix A.

The economy converges to a steady state where education is higher than before the technological change provided the income tax is not too high:  $\bar{e}_n > \bar{e}$  if and only if  $(1 - \bar{T})/(1 - \bar{T}_n) < \hat{h}_e(\bar{e})/h_e(\bar{e})$ . The intuition is simple: higher returns to education encourage investment in education while a higher income tax discourages it. In the steady state, public debt is constant and the tax revenues finance the interest payment on the outstanding stock of debt.

Now we characterize the optimal path of the income tax for the  $\alpha$ -individual and the transition to the steady state. We assume that the individual can commit to a sequence of tax rates  $\{T_{t+j}\}_{j=0}^{\infty}$ . The  $\alpha$ -individual's optimal tax at time  $t$  solves

$$V(b_t, \alpha) = \max_{T_t} u(c_t(\alpha)) + \beta V(b_{t+1}, \alpha) \quad (21)$$

subject to (18) and the government budget constraint (19).

**Proposition 7** *Individual preferences are singled peaked with respect to the income tax rate. The solution to (21) exists, is unique and it satisfies*

$$u_c(c_s(\alpha)) \frac{y_s(\alpha)}{Y_s} = u_c(c_{s+1}(\alpha)) \frac{y_{s+1}(\alpha)}{Y_{s+1}}, \quad \forall s \geq t \quad (22)$$

Proof: see Appendix A.

Equation (22) simply says that the  $\alpha$ -individual equalizes over time the distortion created by income taxes, namely  $u_c(c)y/Y$ . A marginal increase in taxes at  $t$  reduces consumption and raises its marginal utility by  $u_c(c_t(\alpha))y_t(\alpha)$ ; it also raises tax revenues by  $Y_t$  but reduces current investment in education, thereby reducing future income and raising the future marginal utility from consumption.

Let's consider now the case where income tax rates are chosen by majority voting among the old individuals. In this simple setting without uncertainty and where innate ability is a family characteristic, voting at time  $t$  on a sequence of taxes  $\{T_{t+j}\}_{j=0}^{\infty}$  or voting each period  $t+j$  on the current tax  $T_{t+j}$  is equivalent. Let  $\{T_{t+j}(\alpha^m)\}_{j=0}^{\infty}$  be the optimal tax sequence for the median individual, i.e. the individual with the median innate ability  $\alpha^m$ , and let the status quo consist of the sequence of tax rate prevailing before the technological change,  $\{\bar{T}\}$ . We also assume the following.

**Assumption 5** *Ranking families according to their innate abilities,  $\alpha$ , or to their financial bequests before the technological change,  $\bar{s}(\alpha)$ , gives the same ordering.*

Individuals with higher innate abilities have (weakly) higher financial bequests, i.e. they are richer. This assumption is sufficient to establish a monotone relationship between innate ability  $\alpha$  and the optimal tax rate  $T_{t+j}(\alpha)$ .

**Proposition 8** *If a majority of individuals is financially constrained following the technological change, the optimal tax sequence for the median individual is adopted by a majority vote in a pairwise election against the status quo.*

Proof: Proposition 8 follows directly from assumption 5 and the fact that individual preferences are single peaked with respect to the proportional income tax rate and the median voter ideal policy is adopted in a majority voting.

If the median voter is financially constrained following the technological change, then a majority of individuals is also financially constrained; the median voter ideal tax rate is adopted in a majority voting and the economy gradually converges to the steady state.

## 6 Simulations

This section presents a simple numerical simulation of the model. The production function is given by

$$y = h(\alpha, e) = y_0 + \alpha \left( 1 + A \frac{e^\theta}{\theta} \right), \quad \theta \in (0, 1),$$

where  $y_0 + \alpha$  is output produced by the type- $\alpha$  individual with no education and  $A$  is the education-augmenting technological progress. We consider three individuals with different innate abilities: low ability, with  $\alpha = 0.6$ , medium ability, with  $\alpha = 1$ , and high ability with  $\alpha = 1.5$ . At the initial steady state, there are no taxes, all individuals invest in education and leave zero bequests to their offsprings. We study the effect of a 40% increase in education productivity, consisting in an increase of  $A$  from 0.6 to 1. Table 3 reports the parameter and initial values.



<i>Parameter</i>	$\alpha = 0.6$	$\alpha = 1$	$\alpha = 1.5$
$\theta$	0.5	0.5	0.5
$F$	0.1	0.1	0.1
$y_0$	0.1	0.1	0.1
$r$	0.05	0.05	0.05
$A$	0.6	0.6	0.6
$A'$	1	1	1
$e_0$	0.118	0.327	0.735
$s_0$	0	0	0
$T_0$	0	0	0
$V$	-3.137	12.533	25.305

Table 3: Parameter and initial values

<i>Parameter</i>	$\alpha = 0.6$	$\alpha = 1$	$\alpha = 1.5$
Without taxes/subsidies			
$\bar{e}$	0.327	0.907	2.041
$\bar{s}$	0	0	0
$V$	-1.258	13.624	26.293
With taxes/subsidies			
$\bar{T}$	0.12	0.12	0.12
$\bar{e}$	0.318	0.884	1.989
$\bar{s}$	0.047	0.084	0
$V'$	-1.224	19.872	33.072
$V' - V$	3%	41%	26%

Table 4: Steady state with and without taxes

Figure 5 shows education and consumption during the transition to the new steady state for the medium-ability individual (left column), low-ability individual (center column) and high-ability individual (right column). All three individuals are financially constrained following the technological change, so that the transition is gradual; notice that 90% of the transition is accomplished in four to five periods. Welfare, measured as the family's utility  $V_t$ , improves for all individuals in this economy; however, the largest welfare gains accrue to the low-ability individual. Table 4 reports the values for education, consumption and saving at the new steady.

Next, we consider the same economy with the distortionary tax of section 5. More precisely, the government levies a labor income tax that is decided every period by majority voting among the old individuals in the population. In this economy, the medium-ability individual is the median voter in every period and her optimal tax/subsidy is

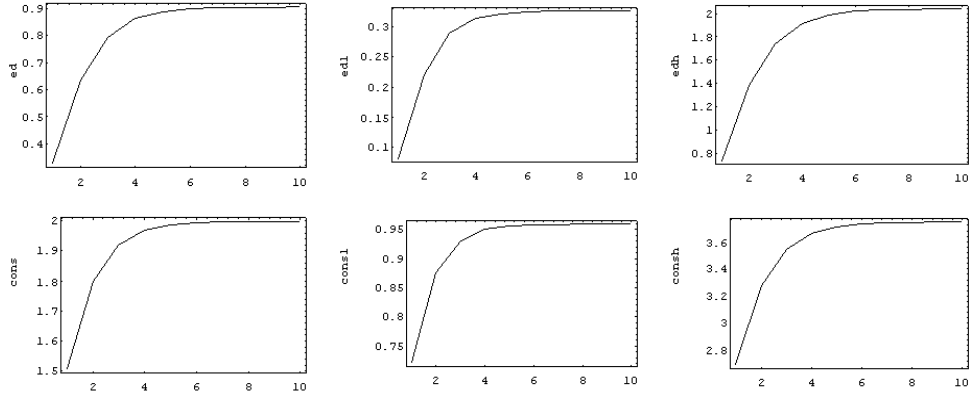


Figure 5: Skill-biased technological change with no taxes

the one adopted by the government.

Table 4 summarizes the new steady state values and Figure 6 shows the transition to the new steady state for this economy. The first row is the labor income tax. The immediate response of the government is a 35% income subsidy followed by income taxes that increase and stabilize at about 13% in the new steady state. The income subsidy allows families to invest in education and raise current consumption; as a result, medium- and low-ability individuals are financially unconstrained following the technological change; in fact, their consumption profile is flat and they leave positive financial bequests to their offsprings. High-ability individuals need to invest more in education than the others and the subsidy is not enough to make them financially unconstrained following the technological change. Hence, the consumption profile is increasing for high-ability individuals and their financial bequests are zero throughout the transition and in the new steady state.

In the short run, education for medium- and low-ability individuals overshoots its long-run equilibrium level. This is in response to raising income taxes, which lower the optimal investment in education. Notice that long-run investment in education is lower with government intervention than without it. Even though the fiscal tool considered here is quite basic and it is decided by majority voting, its introduction is welfare improving for all individuals, as shown in the last row of Table 4. The largest welfare gain accrue to the medium-ability individuals; this is not surprising, as they get to choose their optimal labor income tax.

## 7 Conclusions

This paper emphasizes the importance of public funding of education when credit markets are incomplete and there is limited access to borrowing for investment in

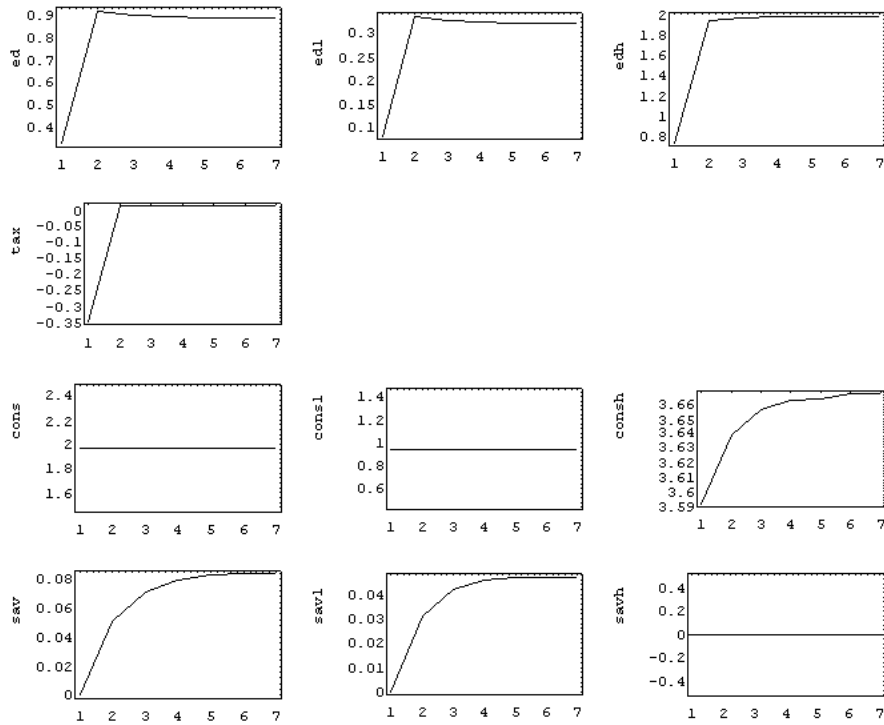


Figure 6: Skill-biased technological change with taxes

education. In response to a technological change that makes human capital more productive, households want to raise their investment in education, but may be unable to do so for that purpose. Hence, investment in education is suboptimally low and there is a lengthy convergence to the long-run equilibrium.

In this setting, the optimal policy for the government is to transfer resources from future to current generations and allow an investment in education that would not otherwise take place. If taxes are distortionary, this can be achieved by a sequence of decreasing tax cuts that eventually turn into tax increases. Somewhat surprising, we find that an education subsidy is suboptimal. The current working generation wants to raise her consumption in response to their offspring's increase in future consumption. By restricting the subsidy to education, the current old generation may be unable to raise her own consumption.

## A Mathematical Proofs

**Proof of Proposition 1:** By applying the *implicit function rule* to (9) I obtain

$$e_\alpha = \frac{u_{cc}(c_t)h_\alpha - \beta u_{cc}(c_{t+1})h_\alpha h_e - \beta u_c(c_{t+1})h_{e\alpha}}{u_{cc}(c_t) + \beta u_{cc}(c_{t+1})h_e^2 + \beta u_c(c_{t+1})h_{ee}}$$

and  $e_\alpha > 0$  if  $h_{e\alpha} > J(\alpha)$ , where

$$J(\alpha) = \frac{h_\alpha u_{cc}(c_t) - \beta h_\alpha h_e u_{cc}(c_{t+1})}{\beta u_c(c_{t+1})}.$$

With isoelastic utility function, the condition  $h_{e\alpha} > J(\alpha)$  is satisfied because  $J(\alpha) < 0$  and  $h_{e\alpha} > 0$  (see assumption 3.4). Similarly

$$e_w = \frac{u_{cc}(c_t)}{u_{cc}(c_t) + \beta u_{cc}(c_{t+1})h_e^2 + \beta u_c(c_{t+1})h_{ee}}$$

and  $0 < e_w < 1$ . Now applying the *implicit function rule* to (10) gives

$$s_\alpha = \frac{h_\alpha [u_{cc}(c_t) - \beta(1+r)u_{cc}(c_{t+1})]}{u_{cc}(c_t) + \beta(1+r)^2 u_{cc}(c_{t+1})}.$$

Therefore  $s_\alpha \gtrless 0$  depending on whether  $\beta(1+r) \gtrless 1$ . This also gives

$$s_w = \frac{u_{cc}(c_t)}{u_{cc}(c_t) + \beta(1+r)^2 u_{cc}(c_{t+1})}$$

and  $0 < s_w < 1$ . Concerning consumption, by differentiating (5) it results

$$c_\alpha = h_\alpha + e_\alpha(h_e - 1) + s_\alpha r.$$

To eliminate paths characterized by ever increasing or decreasing consumption, restrict attention to the case  $\beta(1+r) = 1$ . Hence  $s_\alpha = 0$  and  $c_\alpha > 0$ . Consumption can be expressed in terms of resources as  $c_t = w_t - e(\alpha, w_t) - s(\alpha, w_t) - F(e_t)$ . Hence

$$c_w = 1 - e_w - s_w = \frac{\beta^2(1+r)^2 u_{cc}(c_{t+1}) [u_{cc}(c_{t+1})h_e^2 + u_c(c_{t+1})h_{ee}] - u_{cc}(c_t)^2}{[u_{cc}(c_t) + \beta u_{cc}(c_{t+1})h_e^2 + \beta u_c(c_{t+1})h_{ee}] [u_{cc}(c_t) + \beta(1+r)^2 u_{cc}(c_{t+1})]}$$

and  $0 < c_w < 1$ .

Now I prove that  $\underline{w}(\alpha, F)$  is decreasing in  $\alpha$ , whereas  $\bar{w}(\alpha, r)$  is increasing in  $\alpha$ . The precise definition of the former is

$$\underline{w} \equiv \{\inf w_t : -u_c(c_t) + \beta u_c(c_{t+1})h_e = 0 \mid e_t > 0\}.$$

Differentiating the above expression with respect to  $\alpha$  yields

$$-u_{cc}(c_t)h_\alpha + \beta u_{cc}(c_{t+1})h_\alpha h_e + \beta u_c(c_{t+1})h_{e\alpha} > 0.$$

Therefore  $\underline{w}(\alpha, F)$  is a decreasing function of  $\alpha$ . In a similar manner,

$$\bar{w}(\alpha, r) \equiv \{w_t : h_e(\alpha, e) = (1+r)\},$$

and differentiating this expression with respect to  $\alpha$  gives

$$h_{e\alpha} > 0.$$

Hence  $\bar{w}(\alpha, r)$  is an increasing function of  $\alpha$ .

Under Assumptions 2-3, both  $e_\alpha(\alpha, w)$  and  $e_w(\alpha, w)$  are continuous on the range  $(\underline{w}(\alpha, F), \bar{w}(\alpha, r)]$ ,  $\forall \alpha \in [0, 1]$ . Hence  $e(\alpha, w)$  is continuous on  $(\underline{w}(\alpha, F), \bar{w}(\alpha, r)]$ . Moreover  $e(\alpha, w) = \bar{e}(\alpha)$  on  $(\bar{w}(\alpha, r), \infty)$ . Therefore  $e(\alpha, w)$  is continuous on  $(\underline{w}(\alpha, F), \infty)$ . The presence of a fixed cost makes education discontinuous at  $\underline{w}(\alpha, F)$ .

A similar argument holds for  $s(\alpha, w)$ . Under Assumption 2 both  $s_\alpha(\alpha, w)$  and  $s_w(\alpha, w)$  exist and are continuous on  $(\bar{w}(\alpha, r), \infty)$ . Hence  $s(\alpha, w)$  is continuous on this range. Since  $s(\alpha, w) = 0$  on  $[l + \epsilon(\alpha), \bar{w}(\alpha, r)]$ ,  $s(\alpha, w)$  is continuous.

Consumption is discontinuous at  $\underline{w}(\alpha, F)$ . Investment in education is inhibited by the presence of fixed costs at low level of resources. When resources surpass  $\underline{w}(\alpha, F)$ , investment in education begins and consumption falls discretely by the amount of the fixed costs of education. The intuition is that the reduction in current consumption is compensated by future increasing consumption levels.

**Proof of Proposition 2:** For a given  $\alpha$ , if  $w_t < \underline{w}_t(\alpha, F)$ , then  $e_t = \bar{e}(\alpha, F) = 0$ . Hence,  $\bar{y}(\alpha, w, F) = h(\alpha, 0)$ ,  $\bar{s}(\alpha, w, F) = s_{t-1} + (y_t - h(\alpha, 0))/(1+r)$ ,  $\bar{c}(\alpha, w, F) = h(\alpha, 0) + r\bar{s}(\alpha, w, F)$ .

If  $w_t > \bar{w}_t(\alpha, F)$ , then  $e_t = \bar{e}(\alpha, F) = \bar{e}(\alpha)$ . Hence,  $\bar{y}(\alpha, w, F) = h(\alpha, \bar{e}(\alpha))$ ,  $\bar{s}(\alpha, w, F) = s_{t-1} + (y_t - h(\alpha, \bar{e}(\alpha)))/(1+r)$ ,  $\bar{c}(\alpha, w, F) = h(\alpha, \bar{e}(\alpha)) + r\bar{s}(\alpha, w, F)$ .

If  $w_t \in [\underline{w}_t(\alpha, F), \bar{w}_t(\alpha, F)]$ , define  $f(w|\alpha) \equiv h(\alpha, e(\alpha, w, F))$ . Hence  $f(w|\alpha) : \mathbf{R}_+ \rightarrow \mathbf{R}_+$ . Let  $\rho(x, z) = |x - z|$ , for all  $x, z \in \mathbf{R}_+$ . The set of all real positive numbers with distance  $\rho(x, z)$  is a metric space. It follows from the condition  $\gamma(\alpha) = 1/e_w(\alpha, w)$  and from Assumption 3, part 1, that  $f_w(w|\alpha) < 1$ , for all  $w \in \mathbf{R}_+$ . Therefore  $f(w|\alpha)$  is a contraction mapping of  $\mathbf{R}_+$  into  $\mathbf{R}_+$ .

Let  $g : \mathbf{R}_+ \rightarrow \mathbf{R}_+$  such that  $g(y) = y + k$ , where  $k \geq 0$ .  $k$  represents the non-earnings share of resources, if any. Then  $f \circ g : \mathbf{R}_+ \rightarrow \mathbf{R}_+$ , and  $f \circ g$  is a contraction mapping of  $\mathbf{R}_+$  into  $\mathbf{R}_+$ . By the Fixed-Point Theorem, there exists a unique  $\tilde{y} \in \mathbf{R}_+$  such that  $\tilde{y} = (f \circ g)(\tilde{y})$ .

The argument above holds for all  $\alpha \in [0, 1]$ . Hence the sequence  $\{\tilde{y}(\alpha)\}$  is increasing because  $\alpha_0 < \alpha_1 \Rightarrow f(w|\alpha_0) < f(w|\alpha_1)$ , for all  $w \in \mathbf{R}_+$  and for all  $\alpha_0, \alpha_1 \in [0, 1]$ .

**Proof of Proposition 4:** steady state education is  $\bar{e}_\alpha \equiv \{e : h_e = 1 + r\}$ . Since  $\hat{h}(\alpha, e)$  is strictly concave in  $e$ , and  $\hat{h}_e > h_e, \forall e$ , it follows that  $\bar{e}_\alpha$  increases for all  $\alpha$ .

**Proof of Proposition 6:** At the new steady state

1.

$$u_c(\bar{c}_n, \bar{l}_n) = \beta u_c(\bar{c}_n, \bar{l}_n) \hat{h}_e(\alpha, \bar{e}_n, \bar{L}_n)(1 - \bar{T}_n)$$

and

$$u_c(\bar{c}_n, \bar{l}_n) = \beta(1 + r)u_c(\bar{c}_n, \bar{l}_n).$$

Hence, at the steady state

$$1 + r = \hat{h}_e(1 - \bar{T}_n).$$

$$\hat{h}_e(1 - \bar{T}_n) = h_e(1 - \bar{T})$$

2. This follows from the individual budget constraint (18)
3. Public debt must be constant in the steady state for the transversality condition and the No Ponzi Game condition to be satisfied. This implies that tax revenues must exactly cover the interest payments on the outstanding stock of debt.

**Proof of Proposition 7:** The utility function  $V_t$  is concave in  $T_t$  (this can be easily proved by taking the second-order derivative with respect to  $T_t$ , but the proof is available upon request from the author); thus, individual preferences are single peaked in  $T_t$ .

## References

- [1] Barro, Robert J. and Xavier Sala-i-Martin: *Economic Growth*, McGraw-Hill, 1995.
- [2] Behrman, Jere R., Robert A. Pollak and Paul Taubman: "Family Resources, Family Size, and Access to Financing for College Education," *Journal of Political Economy*, Vol. 97, no. 2, 1989.
- [3] Bernheim, B. Douglas: "Ricardian Equivalence: An Evaluation of Theory and Evidence," *NBER Macroeconomics Annual*, Vol. 2, 1987.
- [4] Bhagwati, Jagdish and Dehejia, Vivek: "Trade and the Wages of the Unskilled: Is Marx Striking Again?" in *Trade and Wages*, J. Bhagwati and M. Koster ed. American Enterprise Institute, 1994.

- [5] Borjas, George J. Richard B. Freeman and Lawrence F. Katz: "On the Labor Market Effects of Immigration and Trade," in *Immigration and the work force: Economic consequences for the United States and source areas*. Borjas G. and R. Freeman eds.; University of Chicago Press, 1992.
- [6] Bound, John and George Johnson: "Changes in the Structure of Wages in the 1980's: An Evaluation of Alternative Explanations," *American Economic Review*, Vol. 82, no. 3, 1992.
- [7] Coe, David T.: "Nominal Wages, the NAIRU and Wage Flexibility," *OECD Economic Studies* 5, Paris, 1985.
- [8] Cukierman, Alex and Allan H. Meltzer: "A Political Theory of Government Debt and Deficits in a Neo-Ricardian Framework," *American Economic Review*, Vol. 79, no. 4, 1989.
- [9] Dick, Andrew W., Aaron S. Edlin and Michael Topper: "The Implicit Taxes in College Financial Aid," Conference paper presented at the winter meetings of the American Economic Association, January 1995.
- [10] Drazen, Allan: "Government Debt, Human Capital, and Bequests in a Life-Cycle Model," *Journal of Political Economy*, Vol. 86, no. 3, 1978.
- [11] Galor, Oded and Joseph Zeira: "Income Distribution and Macroeconomics," *Review of Economic Studies*, Col. 60, 1993.
- [12] Glomm, Gerhard and B. Ravikumar: "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," *Journal of Political Economy*, Vol. 100, 1992.
- [13] Greenwood, Jeremy and Mehmet Yorukoglu: "1974," mimeo, 1996.
- [14] Katz, Lawrence F., Gary W. Loveman, and D.G. Blanchflower: "A Comparison of Changes in the Structure of Wages in Four OECD Countries," NBER Working Paper no. 4297, Cambridge, Mass., 1993.
- [15] Katz, Lawrence F. and Kevin M. Murphy: "Changes in Relative Wages, 1963-1987: Supply and Demand Factors," *Quarterly Journal of Economics*, Vol. 107, 1992.
- [16] Krugman, Paul R. and Robert Z. Lawrence: "Trade, Jobs, and Wages," *Scientific American*, April 1994.

- [17] Krusell, Per, Lee Ohanian, Jose' V. Rios-Rull and Giovanni Violante: "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis," *Econometrica*, Vol. 68, No. 5, September 2000.
- [18] Lawrence, Robert Z. and Matthew J. Slaughter: "Trade and U.S. Wages: Giant Sucking Sound or Small Hiccup?" *Brookings Papers on Economic Activity, Microeconomics*, Vol. 2, 1994.
- [19] Leamer, Edward E.: "Trade, Wages and Revolving Door Ideas," NBER Working Paper no. 4716, Cambridge, Mass., 1994.
- [20] Loury, Glenn C.: "Intergenerational Transfers and the Distribution of Earnings," *Econometrica*, Vol. 49, no. 4, 1981.
- [21] Machin, Stephen: "Changes in the Relative Demand for Skills in the U.K. Labour Market," in *The Skills Gap and Economic Activity*, edited by Alison Booth and Dennis Snower, Cambridge, CUP, 1994.
- [22] McPherson, Michael, S. and Morton O. Schapiro: *Keeping College Affordable*, The Brookings Institution, Washington, D.C, 1991.
- [23] OECD: *Employment Outlook*, Paris, 1992.
- [24] Sachs, Jeffrey D. and Howard J. Shatz: "Trade and Jobs in U.S. Manufacturing," *Brookings Papers on Economic Activity*, Vol. 1, 1994.
- [25] U.S. Department of Education: *Digest of Education Statistics*,, 1990.
- [26] U.S. Department of Education: *Financing Undergraduate Education: 1990*, National Center for Education Statistics, 1993.