Sequential Auctions with Endogenously Determined Reserve Prices

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Abstract

This paper models an auction game in which two identical licenses for participating in an oligopolistic market are sold in a sequential auction. There is no incumbent. The auction for the first license is a standard first-price, sealed-bid type with an exogenously set reserve price, while the second uses the price of the first unit as the reserve price. This auction rule mimics the license auction for the Turkish Global Mobile Telecommunications in 2000. For some parameter values of the model, this auction setup generates less or equal revenue as selling the monopoly right with the second-price, sealed-bid auction. However, for other parameter values, the seller may get higher revenues.

Key Words: Auctions, Sequential First-Price Auctions, Endogenous Reserve Prices.

JEL Classification Codes: D44, D82, L10

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1 Introduction

Auctions may be designed in many different ways, such as with or without a reserve price, with or without an entry fee etc. If an individual is auctioning an item, her objective is maximizing her profit. However, if the auctioneer is a government or a regulatory agency and the auctioned item is a license that may affect the market structure, the auctioneer might have other objectives as well. Some governments are worried about the competitiveness of the market and/or price of the service or product after the auction. They pay attention to the effects on the consumer and aim to enhance the total welfare of the public. Others, however, may simply focus on extracting maximum revenue. If there is more than one license to be auctioned, the seller has to make an additional decision about which auction method to use. The seller has to choose, for example, between selling them independently, simultaneously, sequentially, as a bundle, or any variant of these.

The Global System for Mobile Communications (GSM) – cell phone – license auctions throughout the world have been an important part of governments' agendas for the last few years. This paper focuses on, among others, the Turkish GSM auction in 2000, which has unique features like making reserve prices and values of licenses endogenous. The Turkish government tried to sell two licenses in a sequential auction. The first license is sold in the usual first price sealed-bid auction. Then, in the second auction, bids are required to be at least the winning price of the first license. In these auctions, the Turkish government sold only the first license, but for a very high price. The second license has never been sold. Firms that have a license compete in the cell phone industry. Value of a license for a firm depends on the number and the types of firms in the industry, which makes the value of a license endogenous.

Klemperer (2001) considers Turkish GSM auction in 2000 as a failure due to not thinking through the rules carefully. He claims that this auction setup is biased towards creating monopoly. Since auction results determine the seller's revenue and the structure of the Turkish GSM market, and because of the criticisms like Klemperer's (2001), it is important to analyze this auction setup.

This paper models the Turkish GSM license auction done in 2000 for two licenses as follows: There are two stages of the procedure, auction stage and competition stage. In the auction stage, two licenses are auctioned; only the firms that obtain a license can participate in the competition stage. There is a regulatory agency, the seller, who sells two licenses in a sequential auction format. The initial auction is for the first license and uses the first-price, sealed-bid rule, i.e. the firm that bids the maximum amount receives the first license by paying its bid. The seller then sets this price as a reserve price in the second auction for the second license. The winner of the first license cannot participate in the second auction. Any firm satisfying predetermined criteria can bid. To simplify, the paper first focuses on the two-bidder case. Then the model is extended to the n-firm case. The competition among firms that have a license is modeled by a reduced-form industry profit function, i.e. the profit of a firm depends on the number of firms in the industry, which makes the values of licenses endogenous. After solving the model, the auction revenue for the seller is compared with that from other types of auctions.[1]

The results show that this auction setup is not a superior design for a large set of values of the modeling parameters. For some parameter values, it produces the same revenue as the second-price, sealed-bid auction for a monopoly right, and for other parameter values it might produce less revenue. However, for some modeling parameter values[2], this auction design may generate more revenue than that from selling one license.[3]

In auctioning identical objects sequentially economists pay lots of attention to the phenomenon of declining prices, the so called "declining price anomaly." Ashenfelter (1989) analyzes a data set for U.K. and U.S. wine auctions. These auctions are held in a sequential form for lots of identical bottles of wine, where, contrary to the setup used in this paper, the sequences of the auction are totally independent. He finds that "prices are at least twice as likely to decline as to increase," naming this as an anomaly since according to the law of one price an identical unit should be sold for the identical price. Likewise von der Fehr (1994) analyzes the declining price anomaly by adding participation costs to an independent private value setting in sequential auctions[4]. Implications of his model are parallel to those of other declining price anomaly studies. Bernhart

[1] Auction revenue is compared because the seller's objective is to maximize revenue due to the fact that everybody including government members talked about the price and its ratio to budget deficit after the auction results are revealed.

[2] These parameter values may be interpreted as the case of strong network externalities.

[3] I compare the revenue from the auction design in this paper to that with selling one license because there is a general tendency to think that the design discussed here creates monopoly. For example Klemperer (2001) says about the Turkish GSM auction that "*... One firm then bid far more for the first license than it could possibly be worth if the firm had to compete with a rival holding the second license. But ... no rival would be willing to bid that high for the second license, which therefore remained unsold, leaving the firm without a rival ...*" Hence it is natural to compare this design that of with selling one license, the monopoly right, to see which setup is more beneficial for the seller. Naturally one can ask why this setup is not compared to selling two licenses simultaneously or sequentially without any reserve price and/or without any link between the sequence of the auctions. Since one participant's valuation depends on others' types, the players' strategies and therefore the equilibrium are for now not obvious and are left for future research.

[4] English Clock auctions in which bidders increase their bids openly and the auction ends when only one bidder remains, and the bidder who accepts the highest bid gets the object.

and Scoones (1994) also find that the mean sales price falls, even with risk-neutral bidders. A recent paper by Van Den Berg, Van Ours, and Pradhan (2001) also finds declining prices in analyzing the data for the Dutch Dutch-rose auctions.

Under the assumption of the fixed reserve price or entry fee Jehiel and Moldovanu (2000) examine the positive and negative externalities created by the auctioned object – a patent. The authors study an auction whose outcome influences the future interactions among agents, where the type of the agent, which is private information at the time of the auction, determines the impact of future interactions. Similar to the findings of the current study, in their model one's willingness to pay depends on his or her belief about the potential auction outcome. They derive equilibrium bidding strategies for second-price, sealed-bid auctions for one indivisible object, focusing on two buyers. McAfee and Vincent (1997) solve for the equilibria in sequential auctions with posted reserve prices, whereas in this paper the reserve price is endogenous.

The Spanish Treasury Bill auctions use a rule in the auction setup similar to that presented in this paper. Mazon and Nunez [5](1999) state that during the second round of the Spanish Treasury Bill auctions, if it takes place, each market maker can participate in the auction and has to submit her bids at prices higher or equal to the price prevailing in the first-round auction.[6]

This paper is organized as follows: Section 2 describes the model. Section 3 solves the model assuming asymmetric information at the firm level and describes the results first for the two-firm auction as a benchmark and then for the n-firm case, deriving the bidding behaviors of participants. Section 3 also shows that altering some values of the model's parameters changes the solution to the model dramatically. Section 4 solves for the revenue function of the seller and also presents a comparison with other types of auctions. Section 5 concludes the paper. Proofs of the propositions are given in Appendix A, and Appendix B describes the Turkish 2000 GSM license auction.

2 The Model

Consider a market in which only firms with licenses can participate. The government has two identical licenses to sell. The market is inactive prior to the

[5]The nature of the Treasury Bill auctions treated as divisible good auctions are different than license auctions.

[6]This price is WAP, the weighted average price of accepted bids. Since the Treasury Bill auctions have totally different structure than the license auctions, the results of Mazon and Nunez (1999) are not discussed here. For more information about the Spanish Treasury Bill auctions see Mazon and Nunez (1999) and Alvarez (2001).

sale of the licenses. There are two risk-neutral potential entrants (firms) -later extended to n firms- that bid for these two licenses. This paper analyzes an auction game in which the licenses are sold in two consecutive auctions. Each buyer can purchase no more than one of the auctioned licenses. The risk-neutral seller uses a first-price, sealed-bid auction design with an additional rule that "bids in the second auction, for the second license, shall begin from the winning price of the first auction": in technical terms, the winning price of the first auction is taken as the reserve price in the second auction. If nobody beats the reserve price in the second auction, the second license remains unsold. If both firms bid the same amount, the usual tie rule applies, i.e. each will receive the license with a probability of 1/2. After the auctions, the licensed firm(s) are allowed to produce services.

Each firm may be either high cost or low cost, denoted by c_H and c_L respectively, where $c_H > c_L$. The probability of a firm's being high-cost type, p, and the complementary probability of its being low-cost type, $(1-p)$, are common knowledge; but firms' types are private information.

After the auction stage is over, licensed firms compete in selling services. We assume that in this stage, firms' types are revealed. Thus, in competition stage, licensed firms obtain their operational profits dependent on licensed firms' type. The operational profits of firms are described as reduced form payoffs. Reduced form payoffs allows variety of market structure including markets with network externalities. In the auction stage, if only one license is sold, the winning firm will be a monopoly and its profit is the monopoly profit of its type, H or L. If both licenses are sold, there will be a duopoly, and the profit for the I type firm is IJ, where J is the type of the other firm in the duopoly.[7] A firm that does not receive a license, the loser, receives zero profits.

3 Solving for Equilibrium

A firm's cost is a private information. Neither the seller nor the firm's rivals know other firms' cost type in the auction stage, the other firms and the seller know only that a particular firm is a high-cost type with probability p and a low-cost type with probability $(1-p)$. Although there may be asymmetric equilibria, this paper examines only symmetric equilibria. There are no Nash equilibria in symmetric pure strategies; therefore, symmetric equilibrium in mixed strategies is studied.

The equilibrium strategies depend on the gross profit orders of the various

[7] Throughout the paper, I stands for the monopoly profit of type-I, and IJ stands for the duopoly profit of type-I when the other firm is type-J.

outcomes. There are three cases to consider. In the first, LL and LH are less than H, which is always less than L, i.e. $LL < LH < H < L$, the second $H < LL < LH < L$, and the third $LL < H < LH < L$.[8]

In the first case, a type-L firm can deter the entry of a type-H firm by bidding just a bit more than H. This bid also deters the entry of a type-L firm at the second auction, because at the second auction it cannot beat the reserve price, which in this scenario is at least H, which in turn is greater than LL. In the second case, a type-H firm will bid H. A type-L firm, knowing this, may bid just above H. But, if the other firm is of type L, it can come in and get the second license. Type-L may eliminate this possibility for some parameter values by increasing its bids above LL. However, it may produce the same expected profit for type-L if it lowers its bid below LL (keeping it above H). Though it lowers the possibility of being a monopoly, in this strategy, the payment is also low. For some other modeling parameter values, it may not be possible to deter the entry of a second firm to the market. All these results will be explained in sections 3.1.1, 3.1.2, and 3.1.3. In the third case, as in the first case, type-H bids H and type-L bids something above H. Briefly, there are three cases: The first one is $LL < LH < H < L$, the second is $H < LL < LH < L$, and the third is $LL < H < LH < L$. The strategies of players and the revenue for the seller all depend on which case is realized, as stated earlier. Before proceeding with the solutions, the following Lemma is established to show that a type-H firm always bids H with probability one.

Lemma 1 *Type-H plays H with probability 1 and gets zero expected net profit.*

Proof. *See Appendix A.1.* ■

The following section gives the solution for each case.

3.1 Solution for the Two-Firm Auction

3.1.1 The first case, $LL < LH < H < L$

Since the monopoly profit of a type-L firm (shortly L-firm) is greater than the monopoly profit of an H-firm, an L-firm can deter the entry of the H-firm at the second auction simply by bidding slightly higher than H in the first auction.[9]

[8] As HL is less than other profits and does not change the orderings, there is no need to include it in the orderings.

[9] Indeed L-firm bids more than H. If not, its rival gets the first license. This leaves no options for the L-firm but losing the second license as well. Therefore, if the L-firm bids anything less than H or bids H, it makes zero profit. However, a bid higher than H gives positive expected profit to the L-firm.

Since an H-firm knows this and since its rival may be higher cost too, an H-firm bids its monopoly profit, H, to make its expected profit maximum, which is zero. It makes zero expected net profit if it receives the first license – it pays H, and receives H. Therefore, for an H-firm, bidding its monopoly profit is a weakly dominating strategy.

Now, let us find the strategy for the L-firm. Since only symmetric equilibria are considered, L-firm's bid, B, should satisfy

$$B \geq pH + (1 - p)\, B, \tag{1}$$

which implies

$$B \geq H. \tag{2}$$

The maximum profit of an L-firm is L. Therefore the bid of an L-firm cannot be outside the interval determined by the monopoly profit of the high-cost type and its own monopoly profit, i.e.,

$$B \in [H, L]\,. \tag{3}$$

The expected profit of a type-L firm, when its bid is x, is

$$[p + (1 - p)\, F(x)]\, (L - x)\,, \tag{4}$$

where $F(x)$ is the cumulative distribution function for bidding x to be determined by the mixed strategy of the L-firm. Since there are no point masses in the equilibrium density as shown in Appendix A.2, the cumulative distribution function is continuous on the interval specified below. If $f(x)$ is the density corresponding to $F(x)$, then $f(x) = F^{'}(x)$ almost everywhere.

Since bidding L gives zero profit for sure, the type-L firm wants to bid less than L in order to make positive expected profits, implying that there is a point in the specified interval above which type-L does not want to bid. Denote this point as $\overline{b}$ therefore $B \in \left[H, \overline{b}\right]$.

Since bidding H and $\overline{b}$ should result in the same profit for L-firm as they are two points of the interval on which L-firm mixes its strategies, we have

$$p\, (L - H) = \left(L - \overline{b}\right), \tag{5}$$

which implies

$$\overline{b} = (1 - p)\, L + pH. \tag{6}$$

Bidding $x \in \left(H, \overline{b}\right)$ and bidding $\overline{b}$ should give the same expected profit for some cumulative distribution function $F(x)$ and probability density function $f(x)$, i.e.,

$$[p + (1-p)\, F(x)]\,(L-x) = L - \overline{b}. \tag{7}$$

Inserting $\overline{b}$ from (6) into (7) gives

$$[p + (1-p)\, F(x)]\,(L-x) = p\,(L-H)\,. \tag{8}$$

Solving (8) for $F(x)$ yields

$$F\,(x) = \left(\frac{(L-H)}{(L-x)} - 1\right)\frac{p}{1-p} \quad , \tag{9}$$

and the density of $F(x)$ is

$$f(x) = \begin{cases} \frac{(L-H)p}{(L-x)^2(1-p)} & x \in (H, \overline{b}] \\ 0 & otherwise. \end{cases} \tag{10}$$

Thus,

Proposition 2 *With $LL < LH < H < L$, in the first auction, the L-firm randomizes its bid x in the interval $(H, \overline{b}]$ according to the probability density function $f(x)$ given by (10) and the H-firm bids H. Only the first license is sold.*

Proof. *See Appendix A.3.* ■

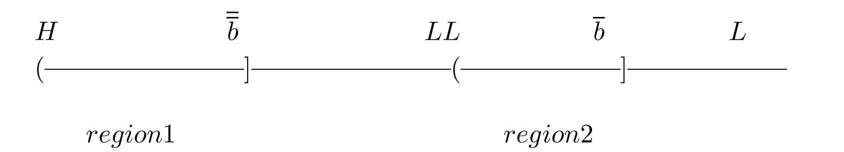
By playing such a strategy, each player maximizes expected profit. The L-firm bids above the monopoly profit of the H-firm to deter its entry into the market and to eliminate the possibility of its receiving a license either in the first or second auction. Also, since $LL < H$, the L-firm's bid x precludes rival type-L firm from bidding at the second auction, because to receive the license at the second auction, a type-L firm would have to pay at least x, which exceeds LL, the post-auction profit.

3.1.2 The second case, $H < LL < LH < L$

In the previous case, in which $LL < LH < H < L$, the type-H firm bids H, which is its equilibrium strategy. Again, in this case, type-H bids H. What about the type-L firm? Bidding more than H eliminates the entry of a type-H firm in the second auction. However, if the price of the first auction is lower than the duopoly profit LL, there is a threat of entry by a type-L firm at the second auction.

Now, consider raising the bid above the duopoly profit, LL. This eliminates the threat of entry by a type-L firm at the second auction for a cost of paying more. However, letting the second license be sold and lowering its own bid below LL, a type-L firm can make the same expected profit as bidding above LL. Here, there are two forces working in opposite directions: bidding high increases the probability of receiving the first license, and, if receiving the first license precludes a second license being sold, gives profit of $L - (high\ bid)$; whereas bidding low decreases the probability of selling only one license, and if two licenses are sold, profits fall to $LL - (low\ bid)$. Note that profits in both cases can be the same; multiplication of the difference between a high payment and a high return, L, with a higher probability of receiving the first license can be equal to the product of the difference between a low payment and a low return, LL, with a lower probability of receiving the first license. So a natural guess is that the symmetric equilibrium strategy has two separate supports, $(H, \bar{\bar{b}}]$ and $(LL, \bar{b}]$, where $\bar{\bar{b}}$ and $\bar{b}$ are some upper bounds of these supports. Indeed, it can be shown that any symmetric Bayesian Nash Equilibrium takes this form and is unique.

*Figure*1: Bidding regions on the profit line.



Now there are two subcases: In the first, the strategies give some weight to both supports, $(H, \bar{\bar{b}}]$ and $(LL, \bar{b}]$; in the second, playing on $(LL, \bar{b}]$ is not feasible owing to the model's parameter values, i.e. players bid only on $(H - \bar{\bar{b}}]$, possibly for a different $\bar{\bar{b}}$. We can calculate the symmetric Bayesian Nash Equilibrium.

Subcase i: strategies on two separate regions $\left(p \leq \frac{L-LL}{L-H} \right)$

Let there be a probability distribution function $F(x)$ on the profit line such that

$$F(x)=\begin{cases} 0 & x\leq H \\ F_1(x) & x\in(H,\overline{\overline{b}}] \\ F_1\left(\overline{\overline{b}}\right)+F_2(x) & x\in(LL,\overline{b}] \\ 1 & x>\overline{b} \end{cases}$$

where $F_1(x)$ and $F_2(x)$ are parts of the distribution function $F(x)$ in *regions* 1 and 2 respectively. Appendix A.2 shows that these probability-like distribution functions are continuous. Also, let $F_1\left(\overline{\overline{b}}\right)=q$. As discussed earlier, a type-H firm bids H.

Since the expected profits of bidding $\overline{\overline{b}}$ and LL are the same,

$$p\left(L-\overline{\overline{b}}\right)+(1-p)F_1\left(\overline{\overline{b}}\right)\left(LL-\overline{\overline{b}}\right)=p(L-LL)+(1-p)F_2(LL)(L-LL),$$

which gives

$$\overline{\overline{b}}=LL-\frac{(1-p)q}{(1-p)q+p}(L-LL). \tag{11}$$

Similarly, equating $p(L-LL)+(1-p)F_2(LL)(L-LL)$ to $L-\overline{b}$ gives

$$\overline{b}=(1-t)L+tLL, \tag{12}$$

where $t=p+(1-p)q$.

The condition $\overline{\overline{b}}\geq H$ is always satisfied.[10]

To find $F_1(x)$, we equate the expected profit of bidding any x in $(H,\overline{\overline{b}}]$ to the expected profit of bidding $\overline{\overline{b}}$. This yields

$$F_1(x)=\frac{t(L-LL)-p(L-x)}{(1-p)(LL-x)} \qquad \mathit{for}\ x\ \epsilon\ (H,\overline{\overline{b}}]. \tag{13}$$

In order to find $F_2(x)$, equate the expected profit of bidding x in $(LL,\overline{b}]$ to the expected profit of bidding $\overline{b}$, and then solve it for $F_2(x)$. This gives

$$F_2(x)=\left[\frac{t(L-LL)}{(L-x)}-p\right]\frac{1}{1-p}-q \qquad \mathit{for}\ x\ \epsilon\ (LL,\overline{b}]. \tag{14}$$

Since $F_1(H)=0$,

$$q=\frac{p}{1-p}\frac{(LL-H)}{(L-LL)}. \tag{15}$$

[10] $\overline{\overline{b}}\geq H$, since $\left(1-\frac{L-LL}{L-H}\right)(LL-H)\geq 0$, which is always true since $H<LL$ and $H<L$.

However, for some values of p, q can be greater than 1, which is not possible since q is a portion of a cumulative distribution function whose value can be at most one. The values of p that make $q \leq 1$ are

$$p \leq \frac{L - LL}{L - H}. \tag{16}$$

Therefore, when $p \leq \frac{L-LL}{L-H}$, the distribution function is of the following form

$$F(x) = \begin{cases} 0 & for\ x \leq H \\ \frac{p}{1-p}\frac{x-H}{LL-x} & for\ \ x \in (H, \overline{\overline{b}}] \\ \frac{p}{1-p}\left[\frac{x-H}{L-x}\right] & for\ \ x \in (LL, \overline{b}] \\ 1 & for\ \ \ x > \overline{b} \end{cases}$$

whose density is

$$f(x) = \begin{cases} \frac{p}{1-p}\frac{LL-H}{(LL-x)^2} & for\ \ x \in (HL, \overline{\overline{b}}] \\ \frac{p}{1-p}\frac{L-H}{(L-x)^2} & for\ \ x \in (LL, \overline{b}] \\ 0 & otherwise, \end{cases} \tag{17}$$

where

$$\overline{\overline{b}} = \frac{LL^2 + H(L - 2LL)}{L - H},$$
$$\overline{b} = (1 - p)L + pH.$$

The results are summarized in the following proposition:

Proposition 3 *When $H < LL < LH < L$ and $p \leq \frac{L-LL}{L-H}$, the L-firm randomizes its bid x in the intervals $\left(H, \overline{\overline{b}}\right]$ and $\left(LL, \overline{b}\right]$ according to the probability density function $f(x)$ given by (17) and the H-firm bids H. Under these bidding strategies, one or two licenses can be sold depending on the realization of bids at the first auction. If, in the first auction, at least one bid is in $\left(LL, \overline{b}\right]$ or both firms are type-H, then only one license is sold, whereas if both bids are in $\left(H, \overline{\overline{b}}\right]$, then two licenses are sold at a price equal to the maximum bid of the first auction.*

Proof. *See Appendix A.4.* ■

Subcase ii: the upper region vanishes $\left(p > \frac{L-LL}{L-H}\right)$

For values of p greater than the right-hand side of (16), q, given by (15), is greater than one. But this is not possible, because q is just a portion of the cumulative distribution function $F(x)$. Hence if p is greater than the critical value, q should be taken as 1. Therefore,

$$q = \begin{cases} \frac{p}{1-p}\frac{(LL-H)}{(L-LL)} & if \;\; p \leq \frac{L-LL}{L-H} \\ 1 & otherwise \end{cases} . \quad (18)$$

Here the story changes dramatically. Now there is no *region* 2. Players play only in *region* 1 with a new definition of upper bound of *region* 1, $\bar{\bar{b}}_1$. This indicates that type-L gives all the weight to *region* 1. Since p is high, L-firm is unlikely to have a type-L rival in the auction. Trying to deter the entry of type-L rival no longer has priority, since trying to do so cannot give as much expected profit as squeezing the bids to *region* 1. Again by a method similar to that used to find $\bar{\bar{b}}$ previously, the upper bound of the support for the case with $q = 1$, $\bar{\bar{b}}_1$, becomes

$$\bar{\bar{b}}_1 = (1-p)LL + pH, \quad (19)$$

so that the distribution function becomes

$$G(x) = \frac{p}{1-p}\frac{(x-H)}{(LL-x)} \qquad for \; x \in (H, \bar{\bar{b}}_1]. \quad (20)$$

As a result, the distribution function when $p > \frac{L-LL}{L-H}$ is

$$G(x) = \begin{cases} 0 & for \;\; x \leq H \\ \frac{p}{1-p}\frac{x-H}{LL-x} & for \;\; x \in (H, \bar{\bar{b}}_1] \\ 1 & for \;\; x > \bar{\bar{b}}_1, \end{cases}$$

whose density function is

$$g(x) = \begin{cases} \frac{p}{1-p}\frac{LL-H}{(LL-x)^2} & for \; x \in (H, \bar{\bar{b}}_1] \\ 0 & otherwise. \end{cases} \quad (21)$$

Therefore

Proposition 4 *When $H < LL < LH < L$ and $p > \frac{L-LL}{L-H}$, L-firm randomizes its bid x in the interval $\left(H, \bar{\bar{b}}_1\right]$ according to the probability density function $g(x)$ given by (21), and the H-firm bids H. Under this bidding strategy, one or two licenses may be sold depending on the types of the firms. If both firms are type-H or only one is type-L, then only one license is sold. However, if both firms are type-L, then two licenses are sold at a price equal to the maximum bid of the first auction.*

Proof. *Similar to the proof of Proposition 2. See Appendix A.3.* ■

3.1.3 The third case, $LL < H < LH < L$

As in the previous cases, type-H bids H. Unlike the second case, type-L bids $x \in (H, \overline{b}]$ for some $\overline{b}$. To see this, let type-L bid $x \in (LL, H)$. Then he loses the first auction and the winner may be type-H or type-L. If the winner of the first auction is also type-L, then he cannot get the second license, because if he bids b above the price of the first license which is at least H, his net profit is $(LL - b) < 0$.[11] If the winner is type-H, then he may get the second license by bidding H, but the net expected profit is $p(LH - H)$, which is less than $(p + (1-p)F(y))(L - y)$[12] if he bids y, something greater than H. Therefore he doesn't want to bid $x \in (LL, H)$. By a similar argument in the previous cases, type-L mixes his bid on the interval $(H, \overline{b}]$ for some $\overline{b}$. As a result, the equilibrium strategy, in this case, is: type-H bids H and type-L bids $x \in (H, \overline{b}]$, with some probability distribution function $F(x)$ given by

$$F(x) = \left(\frac{(x-H)}{(L-x)}\right)\frac{p}{1-p}, \tag{22}$$

whose density function is

$$f(x) = \begin{cases} \frac{(L-H)p}{(L-x)^2(1-p)} & x \in (H, \overline{b}] \\ 0 & otherwise. \end{cases}, \tag{23}$$

where

$$\overline{b} = (1-p)L + pH. \tag{24}$$

As seen from equations (22), (23), and (24), the functional forms of the bid functions are exactly the same with the bid functions derived in the first case.[13]

Proposition 5 *When $LL < H < LH < L$, the L-firm randomizes its bid x in the interval $\left(H, \overline{b}\right]$, according to the probability density function $f(x)$ given by (23), and the H-firm bids H. Under these bidding strategies, only one license is sold.*

Proof. *Similar to the proof of Proposition 2. See Appendix A.3.* ■

The following theorem summarizes the results from these propositions:

[11] Since $LL < H$ and b is a number greater than H.

[12] This is equal to $p(L - H)$.

[13] Recall that only the functional forms are the same. Qualitatively, the solution is different from the solution of the first case due to the change in the ordering of profits.

Theorem 6 ***(2-firm Equilibrium Strategies)*** *In the auction game described above, there exists a unique symmetric equilibrium strategy of the following form:*[14]

If **LL** $<$ **LH** $<$ **H**, *or* **LL** $<$ **H** $<$ **LH**, *the L-firm randomizes its bid x in the interval $(H, \overline{b}]$ according to the probability density function*

$$f(x) = \begin{cases} \frac{(L-H)p}{(L-x)(1-p)} & for \ \ x \in \left(H, \overline{b}\right] \\ 0 & otherwise, \end{cases}$$

where

$$\overline{b} = (1-p)\,L + pH.$$

If **H** $<$ **LL** $<$ **LH**, *we have two subcases:*

Subcase i $\left(\textit{when } p \leq \frac{L-LL}{L-H}\right)$*: Type-H bids H and type-L randomizes its bid x in the intervals $\left(H, \overline{\overline{b}}\right]$ and $(LL, \overline{b}]$ according to the probability density function*

$$f(x) = \begin{cases} \frac{p}{1-p}\frac{LL-H}{(LL-x)^2} & for \ x \in (H, \overline{\overline{b}}] \\ \frac{p}{1-p}\frac{L-H}{(L-x)^2} & for \ x \in (LL, \overline{b}] \\ 0 & otherwise, \end{cases}$$

where

$$\overline{\overline{b}} = \frac{LL^2 + H(L-2LL)}{L-H},$$

and

$$\overline{b} = (1-p)L + pH.$$

Subcase ii $\left(p > \frac{L-LL}{L-H}\right)$*: type-H bids H and type-L randomizes its bid x in the interval $\left(H, \overline{\overline{b}}_1\right]$ according to the probability density function*

$$g(x) = \begin{cases} \frac{p}{1-p}\frac{LL-H}{(LL-x)^2} & for \ x \in (H, \overline{\overline{b}}_1] \\ 0 & otherwise, \end{cases}$$

where

$$\overline{\overline{b}}_1 = (1-p)LL + pH.$$

[14] i.e. This pair of strategies is unique among the symmetric strategies. There may be equilibria with asymmetric strategies.

3.2 Generalization to n-firm case

This section generalizes the solution of the two-firm auction to n firms. In the n-firm case, the strategy in the second auction changes as there are more than one potential rivals in the auction. We can show that in the second auction, if any firm bids it must be type-L. To see this, assume that if there is an entrant in the second auction, then it is type-L. Under this assumption, in the first auction, as argued for the two-firm case, type-H plays H, and type-L plays a mixed strategy. There might be two possibilities for the winning price in the first auction: it can be either H, or something bigger than H.[15] If it is H, this implies all the firms happen to be type-H, therefore, as one has to beat the price of the first one in the second auction, nobody can get the second license. If it is greater than H, then there might be type-L bidders, and only they are able to make a bid in the second auction depending on the profit ordering we discussed before. This shows that the assumption we made is indeed a valid assumption. In the second auction, if there are bidders, they bid their duopoly profits, LL. By backward induction, we get the following solution for the model in n-firm:

3.2.1 Solution when $LL < LH < H < L$ or $LL < H < LH < L$

If there are n firms participating in the auction, then using the same logic as in the sections 3.1.1 and 3.1.3, the respective equations become

$$\overline{b}_n = \left(1 - p^{n-1}\right) L + p^{n-1} H, \tag{25}$$

where $\overline{b}_n$ is the upper limit of bidding for firm L with n participants in the first auction;

$$F_n\left(x\right) = \left(\left(\frac{L-H}{L-x}\right)^{\frac{1}{n-1}} - 1\right) \frac{p}{1-p} \tag{26}$$

is the cumulative distribution function of bidding; and

$$f_n(x) = \begin{cases} \frac{p}{(n-1)(1-p)} \left(\frac{L-H}{(L-x)^n}\right)^{\frac{1}{n-1}} & x \in \left(H, \overline{b}_n\right] \\ 0 & otherwise \end{cases} \tag{27}$$

is the density function. Type-H bids its monopoly profit and type-L randomizes its bid x over the interval $\left(H, \overline{b}_n\right]$ according to the probability density function $f_n(x)$ given by (27).

The corresponding proposition for the n-firm auction becomes

[15] Type-H bids H, type-L bids more than H.

Proposition 7 *When $LL < LH < H < L$ or $LL < H < LH < L$ with n firms, type-L randomizes its bid x in the interval $\left(H, \overline{b}_n\right]$, according to the probability density function $f_n(x)$ given by (27), and type-H bids H. Only the first license is sold.*

Proof. *See Appendix A.5.* ■

3.2.2 Solution when $H < LL < LH < L$

A straightforward extension of the two-firm results gives the following propositions for the n-firm auctions for this case:

Proposition 8 *When $H < LL < LH < L$ and $p \leq \left(\frac{L-LL}{L-H}\right)^{\frac{1}{n-1}}$ with n firms, type-L randomizes its bid x in the intervals $\left(H, \overline{\overline{b}}_n\right]$ and $\left(LL, \overline{b}_n\right]$ according to the probability density function*

$$f_n(x) = \begin{cases} \frac{p}{1-p}\frac{1}{n-1}\left(\frac{LL-H}{(LL-x)^n}\right)^{\frac{1}{n-1}} & for\ x \in (H, \overline{\overline{b}}_n] \\ \frac{p}{1-p}\frac{1}{n-1}\left(\frac{L-H}{(L-x)^n}\right)^{\frac{1}{n-1}} & for\ x \in \left(LL, \overline{b}_n\right] \\ 0 & otherwise, \end{cases}$$

where

$$\overline{\overline{b}}_n = \frac{LL^2 + H(L - 2LL)}{L - H},$$

and

$$\overline{b}_n = (1 - p^{n-1})L + p^{n-1}H,$$

and type-H bids H. Under these bidding strategies, one or two licenses can be sold depending on the realization of bids at the first auction. If in the first auction any one of bids is in $\left(LL, \overline{b}_n\right]$, if all the firms are type-H, or if there is only one type-L, then only one license can be sold, whereas if there is more than one type-L and all the bids are in $\left(H, \overline{\overline{b}}_n\right]$, then two licenses are sold at a price equal to the maximum bid of the first auction and LL. So there is a possibility of selling both licenses.

Proof: A straightforward extension of the proof of *Proposition 3* given in Appendix A.4.

Proposition 9 *When $H < LL < LH < L$ and $p > \left(\frac{L-LL}{L-H}\right)^{\frac{1}{n-1}}$ with n firms, firm L randomizes its bid x in the interval $\left(H, \overline{\overline{b}}_{n,1}\right]$, according to the probability density function*

$$g_n(x) = \begin{cases} \frac{p}{1-p}\frac{1}{n-1}\left(\frac{LL-H}{(LL-x)^n}\right)^{\frac{1}{n-1}} & for\ x \in (H, \overline{\overline{b}}_{n,1}] \\ 0 & otherwise, \end{cases}$$

where

$$\overline{\overline{b}}_{n,1} = LL - p^{n-1}(LL - H),$$

and type-H bids H, where $\overline{\overline{b}}_{n,1}$ denotes the upper bound of the bidding interval. Under this bidding strategy, one or two licenses can be sold depending on the types of the firms. If all the firms are type-H or only one is type-L, then only one license is sold. Otherwise two licenses are sold at a price equal to the maximum bid of the first auction and LL.

Proof: A straightforward extension of the proof of *Proposition 2,* given in Appendix A.3.

4 The Seller's Revenue

4.1 The 2-Firm Case:

4.1.1 When $LL < LH < H < L$ or $LL < H < LH < L$

Expected revenue of the seller for the two-firm auction is given by

$$R_2 = p^2 H + p^2 \left[\left(\frac{L-H}{L-\overline{b}}\right)^2 (2\overline{b} - L) - 2H + L\right]. \tag{28}$$

4.1.2 When $H < LL < LH < L$

With the symmetric strategies as specified in Theorem 6 when there are two firms, the revenue of the seller for $p \leq \frac{L-LL}{L-H}$ is given by

$$E(Revenue) = p^2 H + 2p(1-p)\left[\int_H^{\overline{\overline{b}}} xf(x)dx + \int_{LL}^{\overline{b}} xf(x)dx\right] \tag{29}$$

$$+ (1-p)^2 \begin{bmatrix} 4q^2 \int_H^{\bar{\bar{b}}} xF(x)f(x)dx + 2q(1-q)\int_{LL}^{\bar{b}} xf(x)dx \\ +2(1-q)^2 \int_{LL}^{\bar{b}} xF(x)f(x)dx \end{bmatrix}.$$

Also, for the values of $p > \frac{L-LL}{L-H}$, the expected revenue is given by

$$\begin{aligned} E(Revenue) &= p^2 H + 2p(1-p)\left[\int_H^{\bar{\bar{b}}_1} xg(x)dx\right] \\ &+4(1-p)^2 \int_H^{\bar{\bar{b}}_1} xG(x)g(x)dx. \end{aligned} \tag{30}$$

The expected revenue functions can be calculated analytically, but they are very long and not easy to interpret. Therefore, the expected revenue functions are calculated for all possible values of p and various values of LL, H, and L . The curve named as 2-licenses in Figure 1 gives the expected revenue functions for four different value combinations of those parameters. As seen from the graphs, for some values of parameters, as the probability of type-H increases, the revenue first decreases and then increases to some extent, reaches a local maximum, and then decreases. At first glance, it seems that revenue should decrease as p increases. However, because of the rule about the reserve price of the second auction, there is another effect pushing up the revenue for some p values. Although the bids are becoming smaller as p increases, the probability of selling two licenses and hence the probability of receiving twice the maximum bid of the first auction increase. The result is that the revenue increases for these values of p. The amount of increase at the revenue depends on the expected number of licenses sold; for a larger expected number of licenses sold, there is a larger increase in revenue. In light of these facts, we have the following theorem for the revenue of the seller.

Theorem 10 ***(Revenue)*** *In the auction game described above, under the unique symmetric equilibrium strategies given in Theorem 6, the seller has the following expected revenue functions:*

If $LL < LH < H < L$ or $LL < H < LH < L$, only the first license is sold. The seller's expected revenue is

$$R = p^2 H + p^2 \left[\left(\frac{L-H}{L-\overline{b_2}} \right)^2 (2\bar{b}_2 - L) - 2H + L \right].$$

If $H < LL < LH < L$, one or two licenses can be sold depending on the realization of bids at the first auction. If, in the first auction, any one of the bids is in $(LL, \bar{b}]$, then only one license is sold, whereas if both bids are in $\left(H, \bar{\bar{b}}\right]$, then *two licenses are sold at a price equal to the maximum bid of the first auction. Under these conditions and with* $p \leq \frac{L-LL}{L-H}$*, the seller's expected revenue is*

$$E(Revenue) = p^2 H + 2p(1-p)\left[\int_H^{\overline{\overline{b}}} x f_1(x)dx + \int_{LL}^{\overline{b}} x f_2(x)dx\right]$$

$$+(1-p)^2\left[\begin{array}{c} 4q^2 \int_H^{\overline{\overline{b}}} xF_1(x)f_1(x)dx + 2p(1-q)\int_{LL}^{\overline{b}} xf_2(x)dx \\ +2(1-q)^2 \int_{LL}^{\overline{b}} xF_2(x)f_2(x)dx \end{array}\right],$$

and with $p > \frac{L-LL}{L-H}$*, the seller's expected revenue is*

$$E(Revenue) = p^2 H + 2p(1-p)\left[\int_H^{\overline{\overline{b}}_1} xg(x)dx\right] + 4(1-p)^2 \int_H^{\overline{\overline{b}}_1} xG(x)g(x)dx,$$

where

$$\overline{\overline{b}} = \frac{LL^2 + H(L-2LL)}{L-H},$$

$$\overline{\overline{b}}_1 = (1-p)LL + pH,$$

and

$$\overline{b} = (1-p)L + pH.$$

4.2 The n-Firm Case

Extension of the 2-firm case expected revenue function for the seller yields the following theorem for the n-firm expected revenue functions:

Theorem 11 ***(n-firm Revenue)*** *In the auction game described above, under the unique symmetric equilibrium strategies given in Propositions 8 and 9, the seller has the following expected revenue functions:*

If $LL < LH < H$ *or* $LL < H < LH$*, only the first license is sold. The seller's expected revenue is*

$$R_n = p^n H + p^n\left[\left(\frac{L-H}{L-\overline{b}_n}\right)^{\frac{n}{n-1}}\left(n\overline{b}_n - (n-1)L\right) - nH + (n-1)L\right]. \quad (31)$$

If $H < LL < LH$, one or two licenses can be sold depending on the realization of bids at the first auction. If, in the first auction, any one of the bids is in $(LL, \overline{b}]$, then only one license is sold, whereas if all the bids are in $\left(H, \overline{\overline{b}}\right]$, then two licenses are sold at a price equal to the maximum bid of the first auction and LL. Under these conditions and with $p \leq \left(\frac{L-LL}{L-H}\right)^{\frac{1}{n-1}}$, the seller's expected revenue is

$$R_n = \begin{cases} p^n H + \binom{n}{1} p^{n-1}(1-p)\left[\int_H^{\overline{\overline{b}}_n} x f_1(x)dx + \int_{LL}^{\overline{b}_n} x f_2(x)dx\right] \\ \quad + \sum_{i=2}^{n} \binom{n}{i} p^{n-i}(1-p)^i \left[2q^i \int_H^{\overline{\overline{b}}_n} ixF_2^{i-1}(x) f_2(x)dx \right. \\ \quad \left. + \sum_{j=1}^{i} \binom{i}{j} q^{i-j}(1-q)^j \int_{LL}^{\overline{b}_n} jxF_2^{j-1}(x) f_2(x)dx \right] \end{cases} \tag{32}$$

and with $p > \left(\frac{L-LL}{L-H}\right)^{\frac{1}{n-1}}$, the seller's expected revenue is

$$R_n = \begin{cases} p^n H + \binom{n}{1} p^{n-1}(1-p) \int_H^{\overline{\overline{b}}_{n,1}} x g(x)dx \\ \quad + 2\sum_{i=2}^{n} \binom{n}{i} p^{n-i}(1-p)^i \int_H^{\overline{\overline{b}}_{n,1}} ixG^{i-1}(x) g(x)dx \end{cases} \tag{33}$$

where F_1, F_2, f_1, f_2, G, g, $\overline{\overline{b}}_n$, $\overline{b}_n$, and $\overline{\overline{b}}_{n,1}$ are defined as in Propositions 8 and 9.

4.3 Comparison with other auctions

As seen from the results, the seller can sell only one license by using this auction format if H, the monopoly profit of type-H, is greater than LL, the duopoly profit of a type-L firm when the other firm is type-L. This result occurs because there is no threat to the winner of the first auction. Once a firm has won the first license, its bid deters entry to the second auction. The winner of the first auction may be type-H or type-L. If it is type-H, then all the other firms are also type-H, and since the winner pays its monopoly profit, no firm can pay more at the second auction. If the winner is type-L, then the remaining firms for the second auction can be either type-L or type-H. However, the types of the firms do not matter anymore, since a type-H cannot bid more than its monopoly profit, H, which is less than the payment for the first one, and a type-L cannot beat the price of the first license in the second auction. If it wins, it would get only duopoly profit but would pay more.

Now, let us compare the results with a second-price, sealed-bid auction setup to sell only one license, the monopoly right. In that type of auction, the revenue for the seller is

$$R_n = H(p^n + np^{n-1}(1-p)) + L(1 - p^n - np^{n-1}(1-p)). \quad (34)$$

If we put the definition for $\overline{b}_n$, (25), into the revenue function, (31), we get exactly the same function as in (34). This tells us that if $LL < H$ holds, then there is no difference between using this Turkish style auction and the second-price, sealed-bid auction to sell the monopoly right.

If $H < LL < LH$ holds, then the story changes dramatically. For n-firm auction it is very difficult to compare the revenues for the seller. Hence, comparison is carried out for 2-firm auction for $H < LL < LH$ case.

If $H < LL < LH$ holds, there is a symmetric mixed strategy equilibrium with two separate supports. The expected profits are equal throughout these supports. In these cases, the seller's revenue is given in (29) and (30) under the parametric restrictions specified previously. Since analytically it is very difficult to compare (29) and (30) with (34) for n=2, the same values of H, LL, and L are used to calculate the revenue of the seller both if the seller sells only one license with the second-price, sealed-bid auction and this paper's setup for comparison. The revenue functions are the solid curves in Figure 1.

As seen in Figure 1, selling the monopoly right under the second-price, sealed-bid auction format generates more revenue than the Turkish style auction setup. However, if there is strong network externalities such that the industry profit of a duopoly is greater than the unregulated monopoly, then, as seen in Figure 2, the model is able to give more revenue for the seller than selling only one license for some parameter values. This happens, as I stated earlier at the model's setup, owing to the probability of selling both licenses. The following theorem summarizes the results about revenues.

Theorem 12 *If $LL < H < L$, the 2-firm Turkish auction produces exactly the same revenue for the seller as selling the monopoly right, whereas if $H < LL < L$ and $L \geq 2LL$, it produces less revenue for the seller. If $L \leq 2LL$ holds, then the auction is able to produce higher revenue for some parameter values.*

5 Conclusion:

This paper analyzed the use of endogenous reserve pricing in sequential auctions. The reserve price is made endogenous by relating the consecutive auctions in a way that the winning price of the first auction is the reserve price in the second auction. Since the result of the first auction is not known a priori, this makes the reserve price of the license in the second auction endogenously determined. This setup is used in GSM licenses in Turkey. In the Turkish case, the seller, the government, argued that this setup creates more revenue for the government.

This study, first, sets up a model with two cost types – high and low – of firms, whose type is not known by the rivals, and solves it for symmetric n-firm case. According to the solution of the model, one firm is attempting to become a monopoly in the industry. The low-cost type firm has the resources to reach that aim for some values of the parameters. If the firms have the same cost, one of them still becomes a monopoly in the industry. At the equilibrium, the high-cost firm bids its monopoly profit always, and for some values of the parameters, the low-cost firm chooses a mixed strategy on a region above the high-cost firm's bid. As one expects, increasing the number of firms in the race forces the firms to bid more aggressively. Not only does the upper boundary of the bidding interval $\overline{b}$ increase, but also the participants give more weight to the upper sections of the bidding interval. In any case, the efficient firm receives the license, and the government expectedly receives more revenue as the number of firms in the auction increases.

For some other parametric values, although the high-cost firm still bids its monopoly profit, the low-cost firm changes its strategy. Since the threat of resulting in a duopoly is credible, the low-cost firms arrange their bidding behaviors accordingly, taking this threat into account, and bid from two separate regions. In addition, both licenses can be sold if the participants play the symmetric strategies specified in this paper.

This auction setup creates the same revenue as selling the monopoly right with the second-price, sealed-bid auction for some profit orders, whereas, for others, it may create higher revenue for the seller. As a result, it may be a good idea, with some regulation on the duopoly market outcome, to design such an auction instead of auctioning only one license with a second-price sealed-bid auction.

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A Appendix

A.1 Proof of Lemma 1

Proof. There are two possibilities: The first is that type-H gives a positive weight to z, the infimum of the support. Then, a type-H firm can increase the expected net profit by bidding $z + \epsilon$; the cost of doing this is just ϵ, whereas the benefit is getting rid of the type-H rival with certainty, which creates much more profit than ϵ. Since every type-H knows this, each one keeps increasing its bid as much as possible, which is its willingness to pay T.[16] But at bid level H, the expected net profit is zero since the profit minus the payment is zero. Hence, if a type-H bids some level z with positive probability, then this level is T[17] and the expected net profit of type-H is zero. The second possibility is that a type-H may bid in an interval, $[y, z]$. Now suppose that profit over this interval is greater than zero. But at bid y, the winning probability is zero, which means the expected net profit is zero at level y. So the profit cannot be positive, and therefore y cannot be part of the support. But y is arbitrarily chosen; hence the interval collapses to point z, which means, that type-H gives a positive probability to this point. By the first case, the expected net profit is zero.

A.2 The distribution function F(x) is continuous

Proof. There is no point mass in the density function. If the amount x were bid with positive probability mass, i.e. if there is a jump in the graph of F(x), there would be a positive probability of a tie at x. If a deviant bids slightly higher, i.e. x+ε , with the same probability with which the other bids x, it loses an amount ε however, it increases the probability of receiving the license with the amount of the jump. Thus, giving a positive probability to point x cannot be part of a symmetric equilibrium. Therefore, the distribution function of bidding has no jumps from zero to the potential maximum bid, which implies that it is a continuous function in this interval. Can there be a jump at the potential maximum bid? No, because both bidding the potential maximum and bidding a bit less than it give the same payoff. Hence, there is no significance in increasing the weight of the potential maximum bid. ■

[16] The profit, if a type-H firm gets the license, would be T, and the payment is T, so the difference is zero. Since every firm may be type-H, we may end up with this result with some positive probability. Therefore $expectednetprofit = (somepositiveprobability)(T - T) = 0$.

[17] The monopoly profit level of that type.

A.3 Proposition 2:

Proof: *If a firm of type-H bids less than H, given the other players' bids as specified in the proposition, then the H-firm is going to lose the auction and receive zero. If a type-L firm bids less than H, then it is going to lose the first auction, given that the other players play the proposed strategy. Therefore, L-firm does not want to lower its bid below H. What about bidding more than $\overline{b}$? Is it better than bidding any x in $\left(H, \overline{b}\right]$? If the L-firm bids $\overline{b} + \epsilon$, then it is going to receive*

$$L - \overline{b} - \epsilon. \tag{35}$$

However, if it bids any x from the specified interval, it is going to receive

$$p(L - H). \tag{36}$$

Placing $\overline{b}$ from (6) into (35) gives $p(L-H)-\epsilon$, which is obviously less than the value in (36). Bidding x in the specified interval is better for the L-firm than bidding outside this interval.

Last, let us see that bidding any x in $\left(H, \overline{b}\right]$ gives the same expected profit.

$$E\ (profit) = (L - x)\left[p + (1 - p)F(x)\right]. \tag{37}$$

Placing $F(x)$ from (9) into (37) gives

$$E\ (profit) = p(L - H), \tag{38}$$

which is constant irrespective of the choice of x. Therefore, any x in $\left(H, \overline{b}\right]$ gives the same expected profit. Since no firm wants to deviate, the specified bidding strategy is an equilibrium.

A.4 Proposition 3:

Proof. *If an H-firm bids less than H, given that the other player bids the specified amount in the proposition, then the H-firm is going to lose the auction and receive zero. If the L-firm bids less than H, then it is going to lose the first auction, given that the other player plays the proposed strategy. Therefore, the L-firm does not want to lower its bid below H. What about bidding $x \in (\overline{\overline{b}}, L]$? Is this a good idea for the L-firm? Now, bidding $x \in (\overline{\overline{b}}, L]$ cannot increase $F_1(x)$ and does not change $F_2(x)$, i.e. there is no change in the winning probability. However, bidding $x \in (\overline{\overline{b}}, L]$ decreases the expected profit because the bidder is now going to pay more if it wins, although the expected revenue stays the same* *implying less expected net profit. To see this, let the L-firm bid* $\overline{\overline{b}} + \epsilon$. *Then it receives*

$$p(L - \overline{\overline{b}} - \epsilon) + (1 - p)q(LL - \overline{\overline{b}} - \epsilon).$$

However, bidding $\overline{\overline{b}}$ *gives*

$$p(L - \overline{\overline{b}}) + (1 - p)q(LL - \overline{\overline{b}}),$$

which is obviously greater. So there cannot be any bid in $(\overline{\overline{b}}, L]$.

Can there be any bid x *in* $(\overline{b}, L]$ *? Again by bidding* $\overline{b}$*, the bidder is definitely going to receive the license, and therefore there is no reason to increase the payment, since this will only decrease the expected net profit of the bidder. To see this, let the L-firm bid* $\overline{b} + \epsilon$. *Then it will receive*

$$L - \overline{b} - \epsilon = (p + (1 - p)q)(L - LL). \tag{39}$$

However, if it bids any x *from the specified interval, it will receive*

$$\begin{aligned} &p(L - x) + (1 - p)(F_2(x) + q)(L - x) \\ &\quad = (p + (1 - p)q)(L - x) + (1 - p)F_2(x)(L - x), \end{aligned} \tag{40}$$

which is obviously greater than (39). Therefore, bidding in the specified interval is better than bidding outside this interval.

Last, let us see that bidding any x *in* $(\overline{\overline{b}}, L]$ *and* z *in* $(LL, \overline{b}]$ *give the same expected profit. Any* x *in* $(\overline{\overline{b}}, L]$ *gives*

$$p(L - x) + (1 - p)F_1(x)(LL - x) = (p + (1 - p)q)(L - LL). \tag{41}$$

Also, any z *in* $(LL, \overline{b}]$ *gives*

$$p(L - x) + (1 - p)(F_2(x) + q)(L - x) = (p + (1 - p)q)(L - LL). \tag{42}$$

Since the terms in equations (41) and (42) are the same and independent from the choice of x *or* z*, any bid in either of these intervals produces the same expected profit.*

A.5 Proposition 7

Proof. *The logical flow is the same as in the previous proof. The H-firm bids* H*, because any other bid gives negative net profit. If the L-firm bids less than* H*, then it is going to lose the first auction, given that the other players play the proposed strategy. Therefore, the L-firm does not want to lower its bid below* H.

What about bidding more than $\overline{b}_n$? Is it better than bidding any x in $\left(H, \overline{b}_n\right]$? If the L-firm bids $\overline{b}_n + \epsilon$, then it will receive

$$L - \overline{b}_n - \epsilon. \tag{43}$$

However, if it bids any x from the specified interval, it will receive

$$p^{n-1}(L - H). \tag{44}$$

Placing $\overline{b}_n$ from (25) into (43) gives $p^{n-1}(L - H) - \epsilon$, which is obviously less than the value in (44). Bidding x in the specified interval is better for L-firm than bidding outside this interval.

Does any x in $\left(H, \overline{b}\right]$ give the same expected profit?

$$\begin{aligned} E(profit) = (L - x) \Big[& p^{n-1} + \tbinom{n-1}{1} p^{n-2}(1-p)F_n(x) \\ & + \tbinom{n-1}{2} p^{n-3}(1-p)^2 F_n^2(x) \\ & + ... \\ & + \tbinom{n-1}{n-2} p(1-p)^{n-2} F_n^{n-2}(x) + (1-p)^{n-1} F_n^{n-1}(x) \Big], \end{aligned}$$

which can be written as

$$= (L - x) \left[p + (1-p)F_n(x) \right]^{n-1}. \tag{45}$$

Placing $F_n(x)$ from (26) into (45) gives

$$= p^{n-1}(L - H), \tag{46}$$

which is constant irrespective of the choice of x. Therefore, any x in $\left(H, \overline{b}_n\right]$ gives the same expected profit. As a result, no firm wants to deviate, and thus the specified bidding strategy is an equilibrium.

Group	Bid
1	2,525
2	1,350
3	1,224
4	1,207
5	1,017

Table 1: Bids in the second generation GSM license auction in Turkey in 2000, in million $US.

B Appendix

B.1 2000 Turkish Mobile Phone License Auction

In Turkish GSM licenses on April 2000, the government offered two licenses to the market. The auction setup was the following: first, firms bid for the first license, for which there is no binding reserve price. The maximum bid will get the first license paying the bid amount. In the auction for the second license, bids have to start from the price of the first license, i.e. the winning price of the first license is the reserve price of the second license. Again, the maximum bid, if it is above the reserve price, wins the second license.

Five groups participated in the auction. Each group was composed of a group of domestic firms and a foreign partner. These groups were: (1) Isbankasi-Telecom Italia; (2) Dogan-Dogus-Sabanci Holding Companies and Telefonica Spain; (3) Genpa-Atlas Construction, Atlas Finance, Demirbank and Telenor Mobile Communications, Norway; (4) Fiba-Suzer -Nurol Holding Companies, Finansbank, Kentbank and Telecom France; and (5) Koctel Telecommunication Services and SBC Communications Inc., US. Their bids are given in Table 1.

According to the bids given in the Table 1, group 1 received the first license. Other groups were invited to the second auction but did not participate, since the reserve price for the second auction was set at 2.525 billion US$, the price of the first unit. As a result, only one of the two licenses was sold. So far, there has not been any attempt to auction the second license.

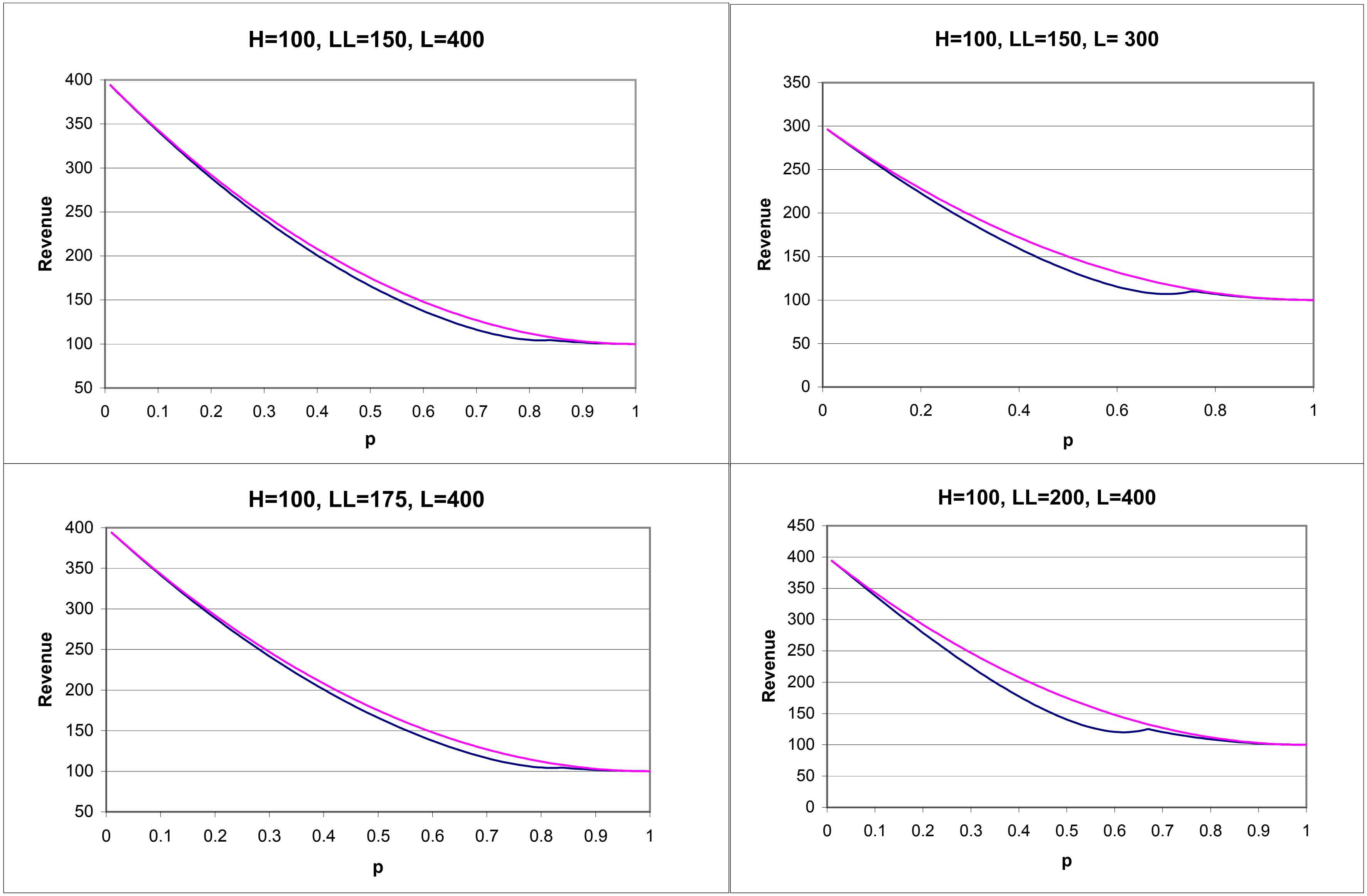

Figure 1: Comparison of the revenue generated by the model and selling the monopoly right when L is greater or equal to LL. From these figures we can conclude that the model cannot create more revenue for the seller than selling only one license. The bold line is the revenue for the auction described in this paper. The other line is the revenue of selling the monopoly right.

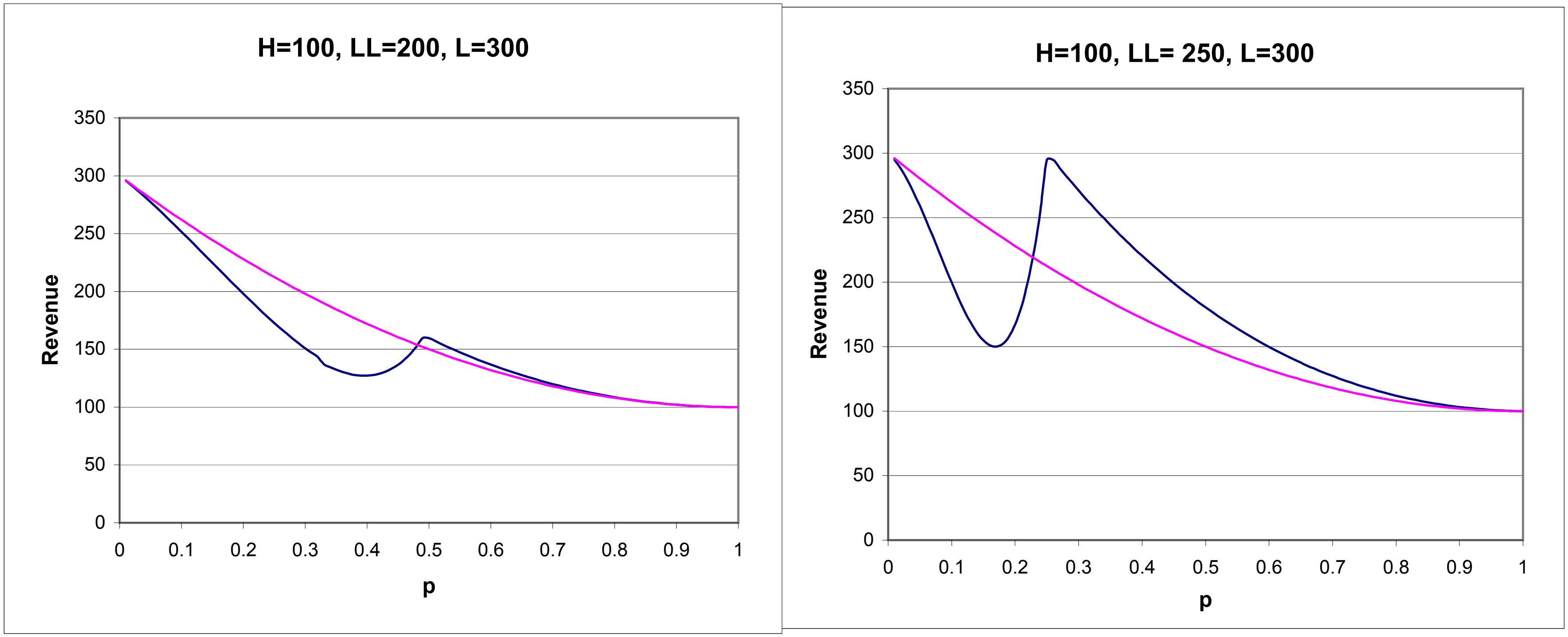

Figure 2: But if we compare these two figures, it seems that the model is able to generate more revenue for the seller. However, to get this result, industry profit has to be greater than the monoply profit, which may be possible under some regulatory conditions. The bold line is for the revenue of the auction described in this paper. The other line is for the revenue of selling monopoly right.