A The Steady State

We denote constant, steady-state levels of variables by dropping the time subscript and assume $f_E = f_E^*$, $f_X = f_X^*$, $\tau = \tau^*$, $L = L^*$, and $Z = Z^* = 1$. Under these assumption, the steady state of the model is symmetric: $\tilde{Q} = Q = TOL = 1$ and the levels of all other endogenous variables are equal across countries.

Solving for $\tilde{z}_X$

Given the solution for the average export productivity $\tilde{z}_X$, we can obtain the cutoff level $z_X$ from $\tilde{z}_X = \nu z_X$, where $\nu \equiv \{k / [k - (\theta - 1)]\}^{1/(\theta - 1)}$. We can solve for $\tilde{z}_X$ as follows. The Euler equation for share holdings yields:

$$\tilde{v} = \frac{\beta (1 - \delta)}{1 - \beta (1 - \delta)} \left( \tilde{d}_D + \frac{N_X}{N_D} \tilde{d}_X \right).$$

Combining this equation with the free entry condition $\tilde{v} = f_E w$ implies:

$$\tilde{d}_D + \frac{N_X}{N_D} \tilde{d}_X = \frac{[1 - (1 - \delta) \beta]}{(1 - \delta) \beta} f_E w.$$  \hfill (A.1)
The steady-state zero profit export cutoff equation is:

$$\ddot{d}_X = \frac{\theta - 1}{k - (\theta - 1)}.$$  \hfill (A.2)

Also, steady-state profits from selling at home and abroad are $\ddot{d}_D = (\tilde{\rho}_D)^{1-\theta} C/\theta$ and $\ddot{d}_X = (\tilde{\rho}_X)^{1-\theta} C/\theta - \dot{w} f_X$, respectively. These two equations imply:

$$\ddot{d}_D = \left( \frac{\tilde{\rho}_X}{\tilde{\rho}_D} \right)^{\theta-1} \left( \ddot{d}_X + \dot{w} f_X \right).$$  \hfill (A.3)

Optimal pricing yields $\tilde{\rho}_D = [\theta/ (\theta - 1)] \tilde{z}_D^{-1} w$ and $\tilde{\rho}_X = [\theta/ (\theta - 1)] \tau \tilde{z}_X^{-1} w$. Hence, $\tilde{\rho}_X/\tilde{\rho}_D = \tau \tilde{z}_D/\tilde{z}_X$, and substituting this into (A.3), we have:

$$\ddot{d}_D = \left( \frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta-1} \left( \ddot{d}_X + \dot{w} f_X \right),$$

or, taking (A.2) into account,

$$\ddot{d}_D = \left( \frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta-1} \left( \dot{w} f_X \frac{\theta - 1}{k - (\theta - 1)} + \dot{w} f_X \right).$$  \hfill (A.4)

The steady-state share of exporting firms in the total number of domestic firms is:

$$\frac{N_X}{N_D} = (z_{\min})^k (\tilde{z}_X)^{-k} \left[ \frac{k}{k - (\theta - 1)} \right]^\frac{k}{\theta - 1}.$$  \hfill (A.5)

Substituting equations (A.2), (A.4), and (A.5) into (A.1), using $\tilde{z}_D = \{k/ [k - (\theta - 1)]\}^{\frac{1}{\theta-1}} z_{\min}$, and rearranging yields:

$$(\tilde{z}_X)^{1-\theta} (\tau z_{\min})^{\theta-1} \left[ \frac{k}{k - (\theta - 1)} \right]^2 + (\tilde{z}_X)^{-k} (z_{\min})^k \left[ \frac{k}{k - (\theta - 1)} \right]^\frac{k}{\theta - 1} \frac{\theta - 1}{k - (\theta - 1)} = \left[ 1 - (1 - \delta) \beta \right] f_E \frac{f_X}{(1 - \delta) \beta}.$$  

This equation can be rewritten as:

$$\xi_1 (\tilde{z}_X)^{1-\theta} + \xi_2 (\tilde{z}_X)^{-k} = \xi_3,$$  \hfill (A.6)
where

\[ \xi_1 \equiv (\tau z_{\text{min}})^{\theta-1} \left[ \frac{k}{k-(\theta-1)} \right]^2 > 0, \]
\[ \xi_2 \equiv (z_{\text{min}})^k \left[ \frac{k}{k-(\theta-1)} \right]^{\frac{k}{\theta-1}} \theta - \frac{1}{k-(\theta-1)} > 0, \]
\[ \xi_3 \equiv \frac{[1-(1-\delta)\beta] f_E}{(1-\delta)\beta f_X} > 0. \]

The left-hand side of equation (A.6) is a hyperbola. This guarantees existence and uniqueness of \( \tilde{z}_X > 0 \), the exact value of which we obtain numerically.

**Solving for \( \tilde{\rho}_X \)**

The law of motion for the total number of domestic firms implies:

\[ N_E = \frac{\delta}{1-\delta} N_D. \quad \text{(A.7)} \]

Steady-state aggregate accounting yields \( C = wL + N_D \tilde{d}_D + N_X \tilde{d}_X - N_E w f_E \). Using (A.1) and (A.7), this can be rewritten as:

\[ \frac{C}{w} = L + N_D f_E \frac{1-\beta}{(1-\delta)\beta}. \quad \text{(A.8)} \]

Equation (A.2) and the expression for average export profits, \( \tilde{d}_X = (\tilde{\rho}_X)^{1-\theta} C/\theta - w f_X \), imply:

\[ \frac{C}{w} = \tilde{\rho}_X^{\theta-1} \frac{\theta k}{k-(\theta-1)} f_X. \quad \text{(A.9)} \]

The price index equation \( N_D \tilde{\rho}_D^{1-\theta} + N_X \tilde{\rho}_X^{1-\theta} = 1 \) yields:

\[ \frac{\tilde{\rho}_X^{\theta-1}}{N_D} = \left( \frac{\tilde{\rho}_X}{\tilde{\rho}_D} \right)^{\theta-1} + \frac{N_X}{N_D}, \]

or, using \( \tilde{\rho}_X/\tilde{\rho}_D = \tau \tilde{z}_D/\tilde{z}_X \) and equation (A.5),

\[ \frac{\tilde{\rho}_X^{\theta-1}}{N_D} = \left( \frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta-1} + \left( \frac{z_{\text{min}}}{\tilde{z}_X} \right)^k \left[ \frac{k}{k-(\theta-1)} \right]^{\frac{k}{\theta-1}}. \quad \text{(A.10)} \]

Together, equations (A.8), (A.9), and (A.10) yield the following equation for \( \tilde{\rho}_X \):

\[ \tilde{\rho}_X^{1-\theta} = \left[ \frac{\theta k}{k-(\theta-1)} f_X - K^{-1} f_E \frac{1-\beta}{(1-\delta)\beta} \right] L^{-1}. \]
where $K$ is the right-hand side of equation (A.10).

**Special Case: All Firms Export**

In this case, equation (A.9) no longer holds since the zero cutoff profit condition (A.2) no longer applies. Using $\tilde{d}_D = (\tilde{\rho}_D)^{1-\theta} C/\theta$ and $\tilde{d}_X = (\tilde{\rho}_X)^{1-\theta} C/\theta - w f_X$, equation (A.1) can be written as:

$$\tilde{\rho}_X^{1-\theta} \frac{C}{\theta} \left( \frac{\theta}{\theta - 1} \right) - w f_X = \left[ \frac{1 - \delta}{1 - \beta} \right] f_E w,$$

which implies:

$$C/w = \tilde{\rho}_X^{\theta - 1} \frac{\theta}{\theta - 1} \left( f_X + \frac{1 - \delta}{1 - \beta} f_E \right).$$

Equation (A.11) now replaces equation (A.9) when solving for $\tilde{\rho}_X$. This yields the following expression for $\tilde{\rho}_X$:

$$\tilde{\rho}_X^{1-\theta} = \left[ \frac{\theta}{\theta - 1} \right] \left( f_X + \frac{1 - \delta}{1 - \beta} f_E \right) - K^{-1} f_E \frac{1 - \beta}{(1 - \delta) \beta} L^{-1}.$$

**Solving for the Remaining Variables**

The solutions for other endogenous variables are straightforward

- $N_D = K^{-1} \tilde{\rho}_X^{-1}$;
- $\tilde{\rho}_D = \tilde{\rho}_X \frac{\theta}{\theta - 1} \tilde{z}_D$ using $\tilde{\rho}_X/\tilde{\rho}_D = \tau \tilde{z}_D/\tilde{z}_X$;
- $w = \tilde{\rho}_X \frac{\theta - 1}{\theta - 1} \tilde{z}_X$ using $\tilde{\rho}_X = [\theta/(\theta - 1)] \tau \tilde{z}_X^{-1} w$;
- $C = w \left[ L + N_D f_E \frac{1 - \beta}{(1 - \delta) \beta} \right]$ using (A.8);
- $N_E = \frac{\delta}{\delta - \gamma} N_D$;
- $N_X = N_D (z_{\min})^k (\tilde{z}_X)^{-k} \left[ \frac{\theta}{k - (\theta - 1)} \right]^k \right] \tilde{z}_X^{\kappa}$ using (A.5);
- $\tilde{d}_D = \frac{1}{\beta} (\tilde{\rho}_D)^{1-\theta} C$;
- $\tilde{d}_X = \frac{1}{\beta} (\tilde{\rho}_X)^{1-\theta} C - w f_X$;
- $\tilde{v} = w f_E$ (using the free entry condition);
- $1 + r = 1/\beta$ (using the Euler equation for bond holdings).
Symmetry of the steady state ensures 
\[ \tilde{z}_X = \tilde{z}_D, \tilde{\rho}_X = \tilde{\rho}_D, N_D^* = N_D, N_E^* = N_E, N_X^* = N_X, \]
\[ \tilde{d}_D^* = \tilde{d}_D, \tilde{d}_X^* = \tilde{d}_X, \tilde{v}^* = \tilde{v}, \]
in addition to \( C^* = C, w^* = w, \) and \( r^* = r. \)

B Labor Market Clearing

Recall that a firm with productivity \( z \) produces \( Z_t z \) units of output per unit of labor employed. Consider separately the labor used to produce goods for the domestic and export markets: let \( l_{D,t}(z) \) and \( l_{X,t}(z) \) represent the amount of labor hired to produce goods for each market. These only represent labor used in production; in addition, each new entrant hires \( f_{E,t}/Z_t \) units of labor to cover the entry cost, and each exporter hires \( f_{X,t}/Z_t \) units of labor to cover the fixed export cost in every period. The profits earned from domestic sales for a firm with productivity \( z \) are then given by:

\[
d_{D,t}(z) = \rho_{D,t}(z) Z_t z l_{D,t}(z) - w_t l_{D,t}(z) = \frac{1}{\theta - 1} w_t l_{D,t}(z),
\]

using \( \rho_{D,t}(z) = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z} \) from optimal pricing. This relationship holds for a firm with average productivity \( \tilde{z}_D, \) and also for averages across all domestic firms. This implies that the average amount of production labor hired to cover domestic sales is \( (\theta - 1) \tilde{d}_{D,t}/w_t. \) The total amount of such labor hired at home is thus \( N_{D,t} (\theta - 1) \tilde{d}_{D,t}/w_t. \)

The profits earned from export sales for an exporting firm with productivity \( z \) are given by:

\[
d_{X,t}(z) = Q_t \rho_{X,t}(z) \frac{Z_t z l_{X,t}(z)}{\tau_t} - w_t \left[ l_{X,t}(z) + \frac{f_{X,t}}{Z_t} \right] = \frac{1}{\theta - 1} w_t l_{X,t}(z) - w_t f_{X,t},
\]

using \( \rho_{X,t}(z) = Q_t^{-1} \tau_t \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z} \) from optimal pricing. Note that only \( Z_t z l_{X,t}(z)/\tau_t \) export units are sold, although \( Z_t z l_{X,t}(z) \) are produced (the remaining fraction having “melted” away in an iceberg fashion while crossing the border). Again, this relationship holds for a firm with average export productivity \( \tilde{z}_{X,t}, \) and also for averages across all exporters. The average amount of production labor hired to cover export sales is thus \( (\theta - 1) \tilde{d}_{X,t}/w_t + (\theta - 1) f_{X,t}/Z_t. \) Multiplying by \( N_{X,t} \) yields the total amount of such labor for the home economy.

The total amount of production labor hired in the home economy is then

\[
\frac{\theta - 1}{w_t} N_{D,t} d_{D,t}(z) + \frac{\theta - 1}{w_t} N_{X,t} \tilde{d}_{X,t} + \frac{\theta - 1}{Z_t} N_{X,t} f_{X,t}.
\]

Adding the total amount of labor hired by new entrants, \( N_{E,t} f_{E,t}/Z_t, \) and that hired by exporters
to cover the fixed costs, $N_{X,t}f_{X,t}/Z_t$, yields the aggregate labor demand for the home economy:

$$L_t = \frac{\theta - 1}{w_t} N_{D,t} \tilde{d}_{D,t} + \frac{\theta - 1}{w_t} N_{X,t} \tilde{d}_{X,t} + \frac{\theta}{Z_t} N_{X,t} f_{X,t} + \frac{1}{Z_t} N_{E,t} f_{E,t}.$$  

Equating $L_t$ to labor supply ($L$) yields the equilibrium condition for home’s labor market. The derivation for foreign is analogous.

**Balanced Trade Implies Labor Market Clearing**

We now demonstrate that balanced trade under financial autarky implies labor market clearing.

Using the home price index equation $1 = N_{D,t} (\tilde{\rho}_{D,t})^{1-\theta} + N^*_{X,t} (\tilde{\rho}^*_{X,t})^{1-\theta}$, the balanced trade condition $Q_t N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} C_t^* = N^*_{X,t} (\tilde{\rho}^*_{X,t})^{1-\theta} C_t$ can be written:

$$Q_t N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} C_t^* = \left[ 1 - N_{D,t} (\tilde{\rho}_{D,t})^{1-\theta} \right] C_t.$$

This condition can be re-written as

$$C_t = \theta N_{X,t} \left( \tilde{d}_{X,t} + \frac{w_t f_{X,t}}{Z_t} \right) + \theta N_{D,t} \tilde{d}_{D,t},$$

since $\tilde{d}_{D,t} = (\tilde{\rho}_{D,t})^{1-\theta} C_t/\theta$ and $\tilde{d}_{X,t} = Q_t (\tilde{\rho}_{X,t})^{1-\theta} C_t^*/\theta - w_t f_{X,t}/Z_t$. Combining this with aggregate accounting ($C_t = w_t L + N_{D,t} \tilde{d}_{D,t} + N_{X,t} \tilde{d}_{X,t} - N_{E,t} w_t f_{E,t}/Z_t$) yields the labor market clearing condition for the home economy:

$$L = \frac{\theta - 1}{w_t} N_{D,t} \tilde{d}_{D,t} + \frac{\theta - 1}{w_t} N_{X,t} \tilde{d}_{X,t} + \frac{\theta}{Z_t} N_{X,t} f_{X,t} + \frac{1}{Z_t} N_{E,t} f_{E,t}.$$  

The proof for the foreign economy follows the same steps.