Changing the Boston School Choice Mechanism*

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Abstract

In July 2005 the Boston School Committee voted to replace the existing Boston school choice mechanism with a deferred acceptance mechanism that simplifies the strategic choices facing parents. This paper presents the empirical case against the previous Boston mechanism, a priority matching mechanism, and the case in favor of the change to a strategy-proof mechanism. Using detailed records on student choices and assignments, we present evidence both of sophisticated strategic behavior among some parents, and of unsophisticated strategic behavior by others. We find evidence that some parents pay close attention to the capacity constraints of different schools, while others appear not to. In particular, we show that many unassigned students could have been assigned to one of their stated choices with a different strategy under the current mechanism. This interaction between sophisticated and unsophisticated players identifies a new rationale for strategy-proof mechanisms based on fairness, and was a critical argument in Boston's decision to change the mechanism. We then discuss the considerations that led to the adoption of a deferred acceptance mechanism as opposed to the (also strategy-proof) top trading cycles mechanism.

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1 Introduction

In July of 2005 the Boston School Committee, the governing body of the Boston Public Schools, voted to replace the existing school choice mechanism (henceforth the **Boston mechanism**) with an alternative mechanism that removes the incentives to "game the system" that handicapped the Boston mechanism. This followed two years of intensive discussion and analysis of the existing school choice system and the behavior it elicited, as well as a discussion of two different possible replacement school choice mechanisms.[1]

The authors of the present paper were first invited to meet with members of the Boston Public Schools (BPS) strategic planning team in October, 2003, following a Boston Globe story (Cook 2003) highlighting some of the vulnerabilities of the Boston mechanism as analyzed by Abdulkadiroğlu and Sönmez (2003) and Chen and Sönmez (2003), and comparing the Boston mechanism unfavorably with the clearinghouse used to match medical residents to hospitals (Roth 1984, Roth and Peranson 1998). In that meeting, we

1. presented theoretical, historical, and experimental evidence about the vulnerability of the Boston mechanism to preference misrepresentation (Abdulkadiroğlu and Sönmez 2003, Chen and Sönmez 2003, Roth 1991),
2. explained how such "gaming" may harm the system in many ways including through reduced efficiency, and
3. presented the outlines of two alternative mechanisms which are **strategy-proof** and hence immune to preference manipulation: a student-proposing deferred acceptance mechanism like that now used to match residents to hospitals, and a top trading cycles mechanism.

Loosely speaking, the Boston mechanism attempts to assign as many students as possible to their first choice school, and only after all such assignments have been made does it consider assignments of students to their second choices, etc. The problem with this is that if a student does not gain admission to his first choice school, it may be that his second choice is already filled to capacity with students who listed it as their first choice. That is, a student may fail to get a place in his second choice school that would have been available had he listed that school as his first choice. This has the potential both to change the preference rankings that some families submit, and to work to the disadvantage of families that fail to take into account such strategic considerations.

The Boston mechanism is one of a class of "priority mechanisms" that were tried to match medical graduates to "house officer" positions in various regions of the British National Health Service and eventually abandoned because that whole class of mechanisms suffers from the same kind of incentive problems as the Boston mechanism (Roth 1990, 1991).

[1] A school choice mechanism is a function that assigns students to schools for each school choice problem. Algorithms implement mechanisms. We will sometimes (ab)use the terms mechanism and algorithm as synonyms.

Based on the evidence, BPS staff were willing to entertain the possibility that Boston families might be engaged in strategic behavior through preference manipulation.[2] However, they wanted us to demonstrate this, and examine its consequences empirically, in the Boston data. To this end, they provided us with micro-level datasets on the choices, student characteristics, and school characteristics, and pressed us to make the empirical case for changing the Boston mechanism.

This paper presents the results of this exercise and describes the arguments for changing the Boston mechanism. We show that while the adoption of either strategy-proof mechanism would not have an adverse impact on student assignments if indeed there is little strategizing (or if the amount of strategizing only slowly declines after a new mechanism is introduced), there *is* evidence of strategic behavior among at least some of the families. While we cannot know if any particular preference list reflects a given family's true preferences, we will show that the pattern of submitted preferences, particularly in connection with the most desirable schools, reflects in broad outline the patterns that would be predicted if families are taking into account the strategic incentives that the Boston mechanism gives them.

However we also show that there are families that have been harmed under the Boston mechanism by reporting their preferences in a sincere way or by strategizing inadequately. For this purpose we concentrate particularly on students who did not receive any of their listed choices, and show that many of these students could have been assigned to one of their stated choices had they ranked the schools differently.

Since we do not know students' true preferences, we will not be able to assess directly any inefficiency in the current allocation. But to the extent that families are not reporting their true preferences, and only imperfectly strategizing, the indirect evidence will be strong that the outcome is inefficient.

Once presented with these various kinds of evidence, BPS staff was convinced that adoption of a strategy-proof mechanism is in the best interest of Boston Public Schools, and in May 2005 Superintendent Thomas Payzant recommended adoption of one of the strategy-proof mechanisms to the School Committee. On July 20, 2005, the School Committee unanimously voted to adopt the student-proposing deferred acceptance mechanism. Given the widespread use of the Boston mechanism and its variants in public school choice, the paper will also review aspects of the dialog that was ultimately successful in convincing policymakers to change the school choice mechanism, highlighting the role of strategy-proofness.

[2]Throughout the paper, we define strategic behavior or strategizing as submitting a preference list to the assignment mechanism that differs from the true preferences.

2 The Boston Mechanism

In Boston, students are assigned seats at public schools through a centralized student assignment mechanism.[3] In the spring of each school year, students who seek a spot in Kindergarten,[4] Grades 1, 6, and 9 are asked to submit a preference ranking of schools. (In Boston, students are allowed to rank no more than five schools.) Students in the remaining non-transition grades continue on in their current school unless they request and receive a transfer.

For most schools, for half of the seats at a given school the students are priority ordered as follows:

1. Students who are guaranteed a space at the school by virtue of already attending that school or a feeder school (**guaranteed priority**),
2. students who have a sibling at the school and live in the walk zone of the school[5] (**sibling-walk priority**),
3. students who have a sibling at the school (but who do not live in the walk zone of the school) (**sibling priority**),
4. students who live in the walk zone of the school (but who do not have a sibling at the school) (**walk zone priority**), and
5. other students in the zone.

A random lottery number is used to break the ties in each category (**random tie-breaker**).

For the other half of the seats, students are priority-ordered based on guaranteed and sibling priority and the random tie-breaker. (That is, students who live in the walk zone of a given school can have priority for half of the spaces, and the other half are allocated without any priority for living in the walk zone.) Students who are not in the main transition grades continue on in their current school through **guaranteed priority**. Guaranteed priority is also given to students in transition grades who attend combined elementary and middle schools (grades K2-8) or middle and high schools (grades 6-12). The district is divided into three zones, East zone, North zone and West zone. Most of the elementary and middle schools are registered as zone schools, which admit students only

[3]There are several special admission high schools that process applicants separately. These include schools that require an interview or presentation of a portfolio such as the Boston Arts Academy and three exam schools (Boston Latin School, Boston Latin Academy, and O'Bryant School of Mathematics and Science) which admit students based on grade point average or scores on an entrance examination.

[4]Boston public schools have three different entering levels for kindergarten. These grades, K0, K1, and K2, depend on the age of the student as of September 1st of the entering school year. K0 programs are for children who turn 3 years old by September 1st, K1 programs are for children who turn 4 years old, and K2 programs are for children who turn 5.

[5]Students who live within 1 mile from elementary school, within 1.5 miles from middle school, and within 2 miles from high school are considered to be in the walk zone of a school.

from their zone. A few elementary schools and middle schools are registered as **citywide schools**, to which students from all zones can apply. All high schools are citywide.[6]

Based on preferences, priorities and school capacities student assignments are determined with the following algorithm:

Step 1: In Step 1 only the first choices of the students are considered. For each school, consider the students who have listed it as their first choice and assign seats of the school to these students one at a time following their priority order until either there are no seats left or there is no student left who has listed it as his first choice.[7]

In general,

Step k: In Step k only the k^{th} choices of the students not previously assigned are considered. For each school with still available seats, assign the remaining seats to the students who have listed it as their k^{th} choice, one at a time following their priority order, until either there are no seats left or there is no student left who has listed it as her k^{th} choice.

The procedure terminates after any step k when every student is assigned a seat at a school, or if the only students who remain unassigned listed no more than k choices.

The following simple example illustrates how the Boston mechanism works.

Example 1: Let $I = \{i_1, i_2, i_3, i_4, i_5, i_6\}$ be the set of students, $S = \{a, b, c, d\}$ be the set of schools, and $q = (2, 2, 1, 1)$ be the school capacity vector. Student priorities at schools as well as their preferences are as follows:

$$\begin{array}{ll} & P_{i_1} : a \ldots \\ a : i_5 - i_1 - i_2 - i_3 \ldots & P_{i_2} : a \ldots \\ b : i_5 - i_6 - i_3 \ldots & P_{i_3} : a - b \ldots \\ c : i_4 - i_5 - i_6 \ldots & P_{i_4} : c \ldots \\ d : i_5 - i_6 \ldots & P_{i_5} : c - a - b - d \\ & P_{i_6} : c - a - b - d \end{array}$$

Step 1: Only the first choices of students are considered and those with higher priorities are accommodated. Each of students i_1 and i_2 is assigned a seat at school a; i_4 is assigned a seat at school c. At the end of Step 1, b has 2 and d has 1 seat available; students i_3, i_5, and i_6 are unassigned.

Step 2: Remaining students are considered for their second choices. There is no seat left at school a so students i_5, i_6 will not be accommodated in this round (too bad for student i_5 who lost the highest priority at school a) and student i_3 is assigned at seat at school b.

[6]The details of the entire priority structure are in Appendix 1.

[7]For schools with walk zone priority for half of the seats, the school is treated as two identical schools, each half the size of the original school, only one of which gives priority to students from the walk zone. Students are assumed to prefer the half that gives walk zone priority, and both halves of the school are adjacent in each student's preferences.

Therefore at the end of Step 2, each of schools b, d has 1 seat available and students i_5, i_6 are unassigned.

Step 3: Remaining students are considered for their third choices and student i_5 is assigned a seat at school b. At the end of Step 3, school d has 1 seat available and student i_6 is unassigned.

Step 4: The only remaining student i_6 is assigned a seat at his forth choice school d.

Therefore the outcome of the Boston mechanism is:

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 \\ a & a & b & c & b & d \end{pmatrix}.$$

Note that in the above example student i_5, who is assigned a seat at her third choice school b, can secure a seat at her second choice school a by ranking it as her top choice: The Boston mechanism is not **strategy-proof**; that is, students (or their parents) may improve their assignments by misrepresenting their preferences.

This itself would not be very disturbing had such manipulation been a remote possibility. For many mechanisms which are not strategy-proof, successfully manipulating the mechanism would require information that a student might not have. The problem is more severe for the Boston mechanism. Since a student who ranks a school as her second choice loses her priority to students who rank it as their first choices, it is very risky for the student to "waste" her first choice at a highly sought after school if she has relatively low priority. Hence the Boston mechanism gives students and their parents a strong incentive to misrepresent their preferences by improving the ranking of schools for which they have high priority.[8]

There are many signs that both the school district and families are aware that students may not always want to rank schools truthfully. The BPS school guide [2004, p3] explicitly advises parents to strategize when submitting their preferences (quotes in original):

> For a better chance of your "first choice" school . . . consider choosing less popular schools. Ask Family Resource Center staff for information on "underchosen" schools.

Moreover, other school districts that employ variants of the Boston mechanism make similar suggestions (for examples from other school districts, see Ergin and Sönmez (2003)).

Although it may be difficult to identify an optimal strategy, there are many strategies that might produce better results than truthful revelation of preferences for a family whose top choices are overdemanded schools. Ranking an underdemanded school as the

[8]While students are exogenously priority ordered at each school, the "effective priorities" are endogenous under the Boston mechanism in the sense that each student who ranks a school as her k^{th} choice is considered before each student who ranks it $(k+1)^{st}$. The exogenous priorities are only utilized to tie-break among students who have ranked a school at the same rank order. It is this ability of students to influence the effective priorities that makes the Boston mechanism vulnerable to preference manipulation.

first choice, as suggested by BPS, is one such possibility. Another possibility is ranking the first choice truthfully and choosing an underdemanded second choice (or, more generally, ranking as first choice a desirable overdemanded school for which the student has sufficient priority to have a chance of admission, and ranking a less preferred underdemanded school second).

In Boston, a parent group which meets to discuss student assignment, the West Zone Parent Group, recommends both kinds of strategies. Their introductory meeting minutes on 10/27/03 state:

> One school choice strategy is to find a school you like that is undersubscribed and put it as a top choice, OR, find a school that you like that is popular and put it as a first choice and find a school that is less popular for a "safe" second choice.

Whether either type of manipulation makes sense for a particular student depends on many factors including how popular her top choices are, her priorities at schools, and her attitude towards "risk." Among experimental subjects, both rules-of-thumb are fairly common (Chen and Sönmez 2003). The Appendix contains other illustrative quotes from this parent group about strategic decision-making in the submission of preferences.

Note that the restriction in Boston that students may list no more than five schools introduces another way in which students may not be able to state their true preferences, since a family that has visited more than five schools must immediately make a choice about which ones to list.

3 Two Strategy-Proof Mechanisms

Before looking at the Boston data, we first review the two alternative mechanisms we proposed, the deferred acceptance mechanism and the top trading cycles mechanism. In both mechanisms, students may submit preference lists containing as many schools as they wish.

The deferred acceptance algorithm was first studied by Gale and Shapley (1962) in the context of *two-sided matching markets*, i.e. markets in which there are two kinds of agents needing to be matched to one another. It produces *stable* matchings, i.e. matchings with the property that there do not exist two agents, not matched to one another, who would both prefer to be matched to one another. Its principal incentive properties were established for simple, one-to-one matching by Dubins and Freedman (1981) and Roth (1982), and for many to one matching problems of the kind we study here (in which each school admits many students, each of whom is admitted only to one school) by Roth (1985) and Roth and Sotomayor (1990).[9] For many-to-one matching problems, there does

[9]More recently, Abdulkadiroğlu (2005) extends the incentive results in a model with type-specific quotas, which also applies to the controlled school choice problem in which choice is restricted by racial quotas at

not exist any stable matching mechanisms that are strategy-proof for the schools, but the student-proposing deferred acceptance mechanism is strategy-proof for the students.

A decade before deferred acceptance algorithms were first formally studied, it turns out that the market for medical residents had developed a clearinghouse which uses an algorithm that is "outcome equivalent" to the hospital-proposing deferred acceptance algorithm (Roth 1984, 1995). (That is, they were using a different algorithm but the same mechanism.) More recently, versions of the deferred acceptance algorithm have become widely used in centralized clearinghouses that serve two-sided matching markets. Some examples include not only medical residents in the U.S., U.K., and Canada (Roth 1990, 1991), but a variety of more advanced medical positions (including the fellowship positions through which doctors become certified into different medical subspecialties; see e.g. Niederle and Roth 2003a,b, 2005), the markets for a number of other medical and health care professionals (see e.g. Table 1 in Roth and Rothblum, 1999), and even newly graduating Reform Rabbis (Bodin and Panken, 1999). And at the time we first spoke to BPS, New York City was in the process of replacing its decentralized system of high school admissions with a centralized clearinghouse based on the deferred acceptance algorithm (Abdulkadiroğlu, Pathak, and Roth 2005a,b). In this respect New York City schools resembled most of the other matching markets in which centralized clearinghouses have been introduced, in that a centralized clearinghouse was replacing a failing decentralized matching process, and the agents on both sides of the market, e.g. hospitals and medical graduates, or high schools and students, were both active players in the process.

The situation in Boston in 2003 was quite different. Boston already had a centralized school choice mechanism. And in Boston, only the students and their families were active players, the schools were passive, with priorities set by the central administration. That is, in Boston, school choice does not involve two kinds of agents who make choices: only the students make choices, by submitting their preference lists. The priorities that they have at each school are fixed in advance.

Abdulkadiroğlu and Sönmez (2003) observed that priorities students have at each school can be formally treated as school preferences, and hence two-sided matching mechanisms have their counterparts in the context of school choice problems as in Boston. They also observed that the stability axiom of two-sided matching markets is isomorphic in the context of school choice to what they referred as *elimination of justified envy*, i.e. there should not be a student who prefers to his assignment a school that either has a vacant seat or has admitted a student with lower priority. Since only students are strategic agents in Boston, the student-proposing deferred-acceptance mechanism is strategy-proof in this context, but its outcome may not be Pareto efficient (when one only considers the welfare of students), although it Pareto dominates any other matching that eliminates justified

schools. Hatfield and Milgrom (2005) obtains the incentive results in a model of matching with contracts which incorporates, as special cases, the college admissions problem, the Kelso-Crawford labor market matching model, and ascending package auctions.

envy. Abdulkadiroğlu and Sönmez (2003) also considered a Pareto efficient mechanism based on the method of top trading cycles, introduced in Shapley and Scarf (1974), and further studied by Roth and Postlewaite (1977). In the original context of "housing markets" Roth (1982b) showed that this method is strategy-proof and Abdulkadiroğlu and Sönmez (1999) showed how to extend it to more complex allocation problems. At the time we first met with BPS, discussions were underway to organize what eventually became the New England Program for Kidney Exchange, based on a proposal for a version of the top trading cycles mechanism outlined in Roth, Sönmez, and Ünver (2004).

So, at the time of our initial meeting, we were in a position to offer two different kinds of strategy-proof mechanisms that seemed suitable for consideration in Boston school choice. They work as follows.

1. Student-Proposing Deferred Acceptance mechanism

As we have already mentioned, it is costly under the Boston mechanism to list a first choice that you do not succeed in getting because, once other students are assigned their first-choice places, they cannot be displaced even by a student with higher priority. This is avoided under the **student-proposing deferred acceptance** mechanism. For a given list of priorities, student preferences and school capacities, this mechanism determines a student assignment with the following algorithm:

Step 1: Each student "proposes" to her first choice. Each school tentatively assigns its seats to its proposers one at a time in their priority order. Any remaining proposers are rejected.

In general, at

Step k: Each student who was rejected in the previous step proposes to her next choice if one remains. Each school considers the set consisting of the students it has been holding and its new proposers, and tentatively assigns its seats to these students one at a time in priority order. Any students in the set remaining after all the seats are filled are rejected.

The algorithm terminates when no student proposal is rejected, and each student is assigned her final tentative assignment.

In contrast with the Boston algorithm, the above deferred acceptance algorithm assigns seats only tentatively at each step, so students with higher priorities may be considered in subsequent steps. Consequently it is **stable** in the sense that there is no student who loses a seat to a lower priority student and receives a less-preferred assignment. Moreover all students prefer their outcome to any other stable matching (Gale and Shapley 1962) and the induced mechanism is strategy-proof (Roth 1985).

2. Top Trading Cycles mechanism.

If the intention of the school board is that priorities be "strictly enforced," the student proposing deferred acceptance mechanism is a leading candidate.[10] However, if welfare considerations apply only to students, there is tension between stability and Pareto optimality (Roth 1982, Balinski and Sönmez 1999, Abdulkadiroğlu and Sönmez 2003). If priorities are merely a device for allocating scarce spaces, it might be possible to assign students to schools they prefer by allowing them to trade their priority at one school with a student who has priority at a school they prefer. The **top trading cycles** mechanism (TTC) creates a virtual exchange for priorities. For a given list of priorities, student preferences and school capacities this mechanism determines a student assignment with the following algorithm:

Step 1: Assign counters for each school to track how many seats remain available. Each student points to her favorite school and each school points to the student with the highest priority. There must be at least one cycle. (A cycle is an ordered list of schools and students (student 1 - school 1 - student 2 - ... - student k - school k) with student 1 pointing to school 1, school 1 to student 2, ..., student k to school k, and school k pointing to student 1.) Each student is part of at most one cycle. Every student in a cycle is assigned a seat at the school she points to and is removed. The counter of each school is reduced by one and if it reaches zero, the school is removed.

In general, at

Step k: Each remaining student points to her favorite school among the remaining schools and each remaining school points to the student with highest priority among the remaining students. There is at least one cycle. Every student in a cycle is assigned a seat at the school she points to and is removed. The counter of each school in a cycle is reduced by one and if it reaches zero, the school is removed.

The procedure terminates when each student is assigned a seat or all submitted choices have been considered.

This version of the TTC mechanism was introduced by Abdulkadiroğlu and Sönmez (2003) and is an extension of **Gale's top trading cycles** mechanism described in Shapley and Scarf (1974). Many properties of TTC carry over to school choice including Pareto efficiency (Shapley and Scarf 1974) and strategy-proofness (Roth 1982b).

While the TTC is a Pareto efficient mechanism when only students are considered, and the student-proposing deferred acceptance mechanism is not, the former does not Pareto dominate the latter. One implication is, based on a stronger efficiency notion (such as a cardinal efficiency notion relying on the rank order of schools) the student-proposing deferred acceptance mechanism may perform better than the TTC for some problems. For

[10] In Turkey admissions to colleges (public or private) is through a similar centralized clearinghouse where priorities are "earned" based on some exams and hence should be strictly enforced. Hence student-proposing deferred acceptance mechanism is especially appealing in this context (Balinski and Sönmez 1999).

example, the student-proposing deferred acceptance mechanism may assign more students to their first choices than TTC. Moreover while each Nash equilibrium outcome of the complete information preference revelation game induced by the Boston mechanism is weakly Pareto dominated by each dominant-strategy equilibrium outcome of the student-proposing deferred acceptance mechanism (Ergin and Sönmez 2003), equilibrium outcomes induced by the Boston mechanism and TTC are not Pareto ranked.

4 Data

4.1 Data construction

All data for this paper were provided by Boston Public Schools from their assignment system or the school guide for the corresponding year. The data include student choices and assignments and school priorities and capacities.

The last major change to the student assignment algorithm in Boston occurred in 1999. Prior to that, the assignment mechanism was based on a system of quotas on race and other factors. The new assignment system adopted in 2000-01 and described in Section 2 has stayed mostly the same through the most recent assignment year 2004-05.[11] To avoid the complications with the transition from the old quota-based admissions plan, we focus our empirical analysis on the second year of the current system, school year 2001-02.[12]

In the 2001-2002 school year, students in transition grades submitted their application by February for a school spot beginning September 1st of that year. In this paper, we focus on students in the main transition grades: K2, 6, and 9.[13]

The actual assignment system has three rounds with the majority of students participating in the first round. Students who submit preferences in round 2 and 3 are mostly those who have missed the first assignment deadline. Although there might in principle be a strategic aspect to which round a student submits her application, students will be better off applying in the first round because this is when the largest number of school seats are available.[14] Indeed, Boston Public Schools strongly encourages students to apply

[11] The only changes have been minor modifications to a handful of walk-zone boundaries.

[12] Results for other school years remain largely the same.

[13] Although grades K0, K1, and 1 are also considered transition years by BPS, for simplicity we do not consider these students here. For K0 and K1, there are a very limited number of school spots and BPS does not guarantee students a spot. For instance, in 2001-02, there were 654 applicants for 141 K0 spots, and 78% of applicants were unassigned. There were 1,530 applicants for 546 K1 spots, and 64% were unassigned. We do not focus on grade 1 because the vast majority of students stay in the elementary school to which they were assigned a K2 place. In 2001-02, 83% of grade 1 spots were guaranteed to continuing students.

[14] If a school's capacity is filled after round 1, its seats are not available for new students in round 2. Moreover, if there is attrition due, say, to students leaving the system to enter private school or moving out of the school system, students in round 1 who were not admitted to that school have higher priority than students entering in later rounds. Therefore, if a student is able to obtain a spot by submitting preferences in the second round, she should also have been able to obtain a seat in the first round.

by the first deadline and informs families that their choices are more limited the longer they wait (see e.g. Boston School Guide, 2001, page 5). As a result, we focus on students submitting preferences in round 1. In 2001-02, 83% of assignments for grade K2, 94% of assignments for grade 6, and 89% of assignments for grade 9 took place in the first round.

The final population consists of students with a valid application form in the first round for 2001-02. More details on data construction are in the appendix.

4.2 Summary statistics

The city of Boston is divided into three zones: the East, North, and West. At the elementary and middle school level, students are only eligible to apply to schools in the zone where they live, and citywide schools. Figure 1 shows how these zones cover the geography of the city. The figure also shows the location of elementary schools for students applying for a school place in 2002. While the geography of the East and West zones are contiguous, the North zone is separated from them and divided into three parts by water.

Table 1 presents summary statistics on the number of students and schools across zones. There are almost three times as many elementary schools as middle schools, and only 12 high schools. Elementary schools average a little under 40 students per incoming class, while middle schools average almost 190 and high schools have more than 530 students per class.

Table 2 presents more information on student characteristics. At all entry points, the fraction of students receiving a free or reduced price lunch is between 60%-80%. Table 2 shows that the concentration of black students is highest in the East zone, and the overall school population is over 80% non-white. The North zone has the highest concentration of hispanic students and other students who are mainly Asian students living near Chinatown. The high percentage of these two groups also accounts for the relatively higher fraction of students who are bilingual in the North zone at elementary school.

The last column of Table 2 shows the percentage of students who submitted applications who withdrew from the public school system. The ratio is slightly higher in the West zone. Since the West zone tends to have higher overall income (i.e. fewer students receiving subsidized lunch), this may be due to a greater fraction of families who can afford to send their child to private or parochial school. However, there is no clear pattern at middle school where the overall percent who withdraw decreases to 11% and is comparable to the other zones.

Table 3 presents which stated choice students received from the Boston mechanism. In grades K2, 6, and 9, between 92%-95% of students were assigned to a school of their choice, with between 77% and 86% of students receiving their stated top choice. Commentators have have often used stated preferences to evaluate the performance of choice plans. For instance, in his description of public school choice in Massachusetts, Glenn (1991) writes:

> A majority of students are accepted into their first choice schools. For example,

> in 1991, 74 percent of sixth graders were assigned to their first choice school, 10 percent to their second choice, and 15 percent to schools they did not select.

Cookson (1994) discusses the school choice plan in Cambridge, which uses a version of the Boston mechanism, and states:

> 91% of all students entering the Cambridge public school system at the K-8 levels have gained admissions to schools of their choice, 75% to the school of their first choice, and 16% to either their second or third choice.

On the surface, the high fraction of students receiving their top choice might suggest that the mechanism is performing well, and/or that matching demand and supply is easy because preferences are dispersed. Indeed, the ability to tell the public that a high proportion of students receive their top choices may be a reason for the widespread popularity of the Boston mechanism. However, given the incentives of the Boston mechanism, treating stated choices as true choices does not give an accurate depiction of the performance of the mechanism. It would be a mistake, for instance, to conclude that 80% of students in Boston are satisfied with their assignment based on numbers that might not reflect the true preferences. Experimental evidence, suggests that a substantial fraction of participants might not reveal their choices truthfully under the Boston mechanism (Chen and Sönmez 2003).

The table also shows that the fraction of elementary school students receiving their first choice is about 10% lower in the West zone, and the fraction of students who are unassigned is higher. The greater competition in the West Zone may be due to parent groups such as the West Zone Parents Group advising families to choose their top choice as their true choice, and choosing a safe second choice. The higher fraction of unassigned is also consistent with parents in the West zone ranking hard to get schools and if they do not receive them, leaving the public school system.

Table 4 shows the priority through which students are assigned. At the elementary school level, about 16% of students are assigned to their guaranteed choice. This fraction increases to 29% and 52% at the middle and high school level. The reason for this is that at the K2 entry point, the students who are guaranteed their choice are mainly those who were fortunate enough to obtain a school placement for grade K1 and are continuing on in the same elementary school. For grade 6, there are a number of K2-8 schools and students in grade 5 at the school are guaranteed a spot for grade 6. For high school, the high fraction of guaranteed priority students is accounted for by both continuing students and because students who live in East Boston are guaranteed a spot at East Boston High School.

Among the students who do not use their guaranteed priority, the majority of students are assigned either through walk zone priority or without priority. Sibling-walk or sibling priority account for more assignments at elementary school than middle and high school, but for a smaller fraction of the total priority than walk or no priority students. At high school walk zone priority is lower because there are only 8 high schools, and applicants can apply from all over the city and are not restricted to their zone. Across grade levels, between 26% and 31% of students are assigned to a school without priority.

5 Strategic and Unsophisticated Behavior

We will present evidence that at least some families are responding to the incentives to manipulate their preferences. We will also consider how the different mechanisms would perform if in fact the preferences currently being submitted are in fact very close to the true preferences.

5.1 Applicant strategies

In Boston, students were allowed to rank up to five schools and the Boston School Guide recommends that parents "choose at least three schools" (2001, page 4). Table 5 presents the basic facts on student applicant forms. Inspection of the table shows that over 75% of students in grade K2 follow BPS's suggestion and rank at least three schools. This fraction drops to 61% and 46% for middle and high schools. This difference can be accounted for by the greater number of seats allocated via guaranteed priority for middle and high school. This fact also explains why there are a sizable number of students ranking only one school for middle and high school, since the majority of these students ranked only their guaranteed choice first. Many of these students are opting to stay in their grade K2-8 or 6-12 schools.

Between 7%-20% of students ranked the maximum allowed five schools on their choice form. Given that the Boston mechanism considers first choices before any second choices, the low number of students who ranked five schools is unsurprising. As Table 3 showed, less than 1% of students were assigned to their fifth choice.[15]

For a given year, define a school to be **overdemanded** if the number of students who rank that school as their first choice is greater than the number of seats at the school. These are schools whose seats are all assigned before the first round of the Boston algorithm is over (and hence no student who ranks these schools second or lower receives a seat at these schools).

The anecdotal evidence suggests that some parents at least partially understand this issue (recall the quotes in Section 2). We look for cross-sectional evidence in the population in the right hand side of Table 5 which shows what fraction of the first, second, and third choices of students are overdemanded that year. At elementary school, 53.4% of students rank an overdemanded school first, with competition fiercest in the West zone. Comparing this to fraction of second choices that are overdemanded, we see a significant

[15] This is one aspect of the data that we expect would change if a strategy-proof mechanism were introduced; it would then likely be worthwhile for families to investigate and list more schools.

drop. Consistent with the idea that ranking an overdemanded school second is a waste, only 35.7% of K2 applicants rank an overdemanded school as their second choice. Middle and high school display similar drops in the fraction ranking an overdemanded school first versus the fraction ranking an overdemanded school second. Of course, a drop in this percentage could be expected when schools have heterogenous quality and may be consistent with truthful preference revelation. For instance, students may believe that a certain school is good and be willing to forsake distance for quality and apply to the far-away overdemanded school, but for their second choice they will forsake quality for distance and apply to their underdemanded neighborhood school

More direct evidence of strategic behavior can be obtained by focusing on schools that are outliers. Recall that one bit of advice given to students by school districts and parent groups is to rank their true top choice as their first choice but then to rank a "safe" school as their second (and third etc.) choices.

Two striking examples in Panel *A* of Table 6 illustrate this phenomenon. At the Lyndon and Quincy schools, both widely recognized as good elementary schools in their zone, a very large number of students ranked the school first, but then there is a steep decline in the number who ranked the school second. At the Lyndon, 151 students ranked it first and only 45 ranked it second; at the Quincy 187 students ranked it first and only 35 ranked it second. In both cases, the 45 students ranking the Lyndon and the 35 students ranking the Quincy could not receive the school if they did not get their first choice because there were more applicants ranking the school first than each school's capacity. These two schools are outliers relative to the other schools in their respective zones where the distribution of students ranking schools across choices is much smoother.[16]

Panel B of Table 6 shows cross-sectional regressions between the difference in the number of students ranking a school first and the number ranking it second and various measures of whether a school is overdemanded. Specifications (1) and (2) proxy for overdemanded using the ratio of the number of applicants ranking the school first and the number of seats, while specifications (3) and (4) consider the difference between the number of students ranking the school first and its capacity. All four specifications corroborate the pattern suggested in Panel A: the extent of the preference discontinuity is directly related to whether or not a school is overdemanded. The magnitude of the effect, from specification (1), for instance, is if there are 2 applicants ranking the school as their first choice for each seat, then there will be 26 fewer applicants ranking that school second than a school where there is 1 applicant per seat.

Interestingly, the gap between the first and second choices of overdemanded zone schools is either not present or much smaller at overdemanded citywide schools. This is indicated by the significance of the citywide dummy in all specifications and is apparent

[16]At our first meeting with Boston Public Schools, we were met with some initial skepticism that parents would state their preferences strategically. We asked which was the most popular elementary school in the city, and were told it was the Lyndon. We asked if every nearby family ranked it first, and were told, of course not, you cannot get into the Lyndon without priority.

by looking at demand patterns at overdemanded citywide schools. For instance, at the Hernandez Elementary School which has 42 seats, there are 115 students who rank it first, 84 who rank it second, and 90 who rank it third. At the Young Achievers Elementary school which has 38 seats, there are 132 students who rank it first, 79 who rank it second, and 86 who rank it third. At Mission Hill, the other citywide elementary school, there are 19 seats, and 30 rank it first, 32 rank it second, and 40 rank it first. At the middle school level, the pattern is even more striking. The largest citywide middle school is the Timilty, which has 263 seats in 2002. At this school, there are 618 students who rank it first, 536 who rank it second, and 388 who rank it third. At each of these overdemanded citywide schools, the students who rank it second, third or lower have absolutely no chance of receiving an assignment there. This suggests that while most parents understand that ranking an overdemanded school second when they are at a priority disadvantage is a bad idea (i.e. when other students have higher priority at that school), fewer parents understand that ranking second an overdemanded citywide school at which they have no priority disadvantage is also inadvisable.[17]

The comparison of local and citywide schools can give us some further insight into the way parents take into account the Boston mechanism's incentives to misrepresent their preferences by improving the ranking of schools for which they have high priority.

5.2 Zone Schools vs. Citywide Schools

Consider Figure 2 which shows the location of each applicant for elementary school in 2001-02 with a dot on the map. Compare this figure to Figure 3, which only shows the students who apply to the six most overdemanded large elementary schools in the city. Comparing the two figures, we see a concentration of nearby students choosing overdemanded schools. Naturally, we should expect priorities to both influence and be correlated with preferences, since students value going to nearby schools or where their sibling attends. The maps only show that nearly all of the students who have priority for a nearby overdemanded school use that priority in their first choice.

A clearer way to try to identify the influence of priorities on preferences is to compare students who rank one of the zone schools as their top choices with students who rank one of the citywide schools as their top choices. Ranking an overdemanded zone school as the first choice is a gamble for students unless they have sibling or walk zone priority (and it may even be a gamble at some schools if the student only has walk zone priority). At these schools students who live within the walk zone of the school have an advantage compared to students who do not. Ranking an overdemanded citywide school as the first

[17] Because citywide schools draw from all over, one possibility is that parents seeking a citywide school are at an information disadvantage compared to parents seeking a local school. That is, parents considering a local school are likely to know parents of slightly older children who have navigated the process in previous years, but parents looking to send their children out of the local neighborhood may not get the same quality of culturally transmitted knowledge about the assignment process from their neighbors.

choice is also a gamble but unlike zone schools, only students with sibling priority have an advantage. So if parents have been manipulating their submitted preferences to take advantage of their walk zone priorities, one natural hypothesis is that students who rank overdemanded zone schools as their first choices tend to live much closer to their first choice school compared to students who rank an overdemanded city wide school as their first choices.

Fortunately, we can consider this thought experiment by comparing two schools of similar quality that are close to each other: the Lyndon school, which respects walk zone priority, and Young Achievers, which is a citywide school. Figure 4 shows the map of applicants who rank each school as their top choice. Given that the Young Achievers is a citywide school, we expect applicants to apply from all over the city. What is striking about this Figure is that very few of the students who live near the Lyndon rank the Young Achievers first instead.

Finally, we ask how many students apply to both of these schools as their first and second choice. Even though these schools are a six minute drive from each other,[18] less than 5% of the applicants who rank the Lyndon school first rank Young Achievers second, and none of the applicants who rank Young Achievers first rank the Lyndon second. Such ranking behavior is consistent with some parents realizing that ranking two overdemanded schools as their first and second choice is inadvisable.

5.3 Unsophisticated behavior

Without knowing the information available to students at the time they submit their rank order list and their true underlying preferences, it is difficult to identify students who are clearly making mistakes. However, as alluded to in the last section, ranking *two* overdemanded schools as a first and second choice is at least weakly a suboptimal response, since there is no chance of receiving a place at the second overdemanded school. This strategy is particularly inadvisable if the student only has random priority at her first choice so that their odds of receiving it are low.

In Table 7, we report the outcome of students who rank two overdemanded schools as their first and second choice and have random priority at their first choice. This is a risky strategy because when a student ranks an overdemanded school first with no priority, if their second choice was overdemanded, they will not receive it and can do no better than their third stated choice. The table shows that at elementary school, of the 391 students ranking two overdemanded schools, 38% receive their first choice while a third receive a lower choice, and 29% are unassigned. At middle school, about a third receive their top choice, a third receive a lower choice and a third are unassigned. In high school, slightly more students receive their first choice, but still a third are unassigned.

It is conceivable that a parent who lives in a neighborhood with only low-performing schools that are underdemanded may find it in their interest to rank schools that are better

[18]Travel time calculated from Yahoo!Maps, http://maps.yahoo.com

and further away. If they are fortunate with their random number, they may secure a spot and if they are not they can always hope to get their third, fourth or fifth choice. Another reason that someone may rank two overdemanded schools with random priority at the first is that they will take a shot for their first choice and if they do not receive it they will leave the public school system.

Since we cannot directly observe true preferences, we take a very conservative approach to identifying unsophisticated behavior, by concentrating on students who are unassigned by the mechanism, but who do not withdraw from the public school system. Because Massachusetts State law mandates that a child must attend school beginning in September of the calendar year in which they turn six years old, a student who does not receive one of her stated choices is assigned to the school closest to her home that has an available seat. Since the more desirable school places are usually assigned through Round 1 of the system, unassigned children are generally sent to schools that are worse than any of their stated choices.[19]

5.4 Unassigned students

Ranking two overdemanded schools with random priority at the first is pretty clearly a costly mistake when a student ends up unassigned and does not withdraw from Boston Public Schools [20]. As Table 7 shows, a significant fraction of those who select this action are unassigned. Indeed, a majority of all unassigned students submitted a rank order with this property.

In Table 8, we show the number and demographics of students who select this action, are unassigned and still continue to stay in Boston Public Schools. Notice first that 115 students were unassigned at elementary school, and 66 stayed in the system. At middle school, the fraction of unassigned who stayed in the system is even higher. These students are unlikely to have chosen two overdemanded schools with the intent of taking a gamble and getting a less desirable but available neighbhorhood school, because they are unassigned, and are unlikely to have gambled with the intent of leaving the system, because they remained in the system. In fact, it is likely these students are playing suboptimally.[21]

[19]We focus on unassigned students because the incentives for manipulation created by the Boston mechanism mean that we cannot simply assume that students would prefer to receive higher ranked choices. (If, say, the third stated choice gives the student higher utility than the second stated choice, but the perceived odds of receiving the third stated choice are much lower than the second stated choice, a student may find it optimal to flip the ordering of schools in her preference list.) However, as discussed below, the situation of unassigned students is much clearer, and we can safely assume that an unassigned student who remains in the system would have preferred to have received one of her stated choices.

[20]The nature of this mistake will become even clearer in the next section, when we show how it could be avoided by the majority of these students.

[21]Note that we are only considering a very narrow definition of suboptimal play. In principle, if we could confidently identify assigned students who had failed to play the Boston school choice game optimally, we could

The table shows the demographic patterns of these students. Comparing the distribution to the overall student population in Table 2, we see that a slightly higher fraction of students receive a subsidized lunch than the population, and there is a slightly higher fraction of black students.

Claiming that this action is suboptimal implies that there exists an alternate strategy that would have done better. One such strategy is ranking a school for which the student has walk zone priority as their top choice. The odds of receiving such a school will then be greater than ranking an overdemanded school with random priority. This alternative action would be undesirable, however, if all of the schools in the walk zone were subpar. Table 9 shows that this is not the case for these students. The table reports the difference in MCAS reading and math test passing rates at schools for which these students have walk zone priority, and the school that they were eventually assigned. For instance, of the 66 elementary school applicants, on average an applicant could have ranked a school in their walk zone with a 65% pass rate instead of the overdemanded first choice that they did not have priority for. This school has a 3% better reading score than the school that the student is eventually assigned. At high school, the difference is even more pronounced. A student could have ranked a school in the walk zone first with a reading pass rate of 47%, but by ranking two overdemanded schools as their top choice, they end up at a school with only 25% pass rate. The difference in reading and math test scores is statistically significant at middle and high school ($p = 0.01$), but not at elementary school.

The discussion of Table 9 hinted at alternative strategies that the unassigned students who ranked two overdemanded schools with random priority at the first could have played. We next consider all unassigned students, and ask if they could have been assigned to one of their ranked schools if they had reordered their preference list.

Table 10 presents summary statistics on the unassigned students. Comparing this table to Table 2, it appears that unassigned children may belong to any of the city's ethnic groups and zones, and that the main issue may be whether they ranked overdemanded schools at the top of their preference lists. The main difference however is with the number of students who withdraw. The fraction of the unassigned who withdraw is much higher than the withdrawing fraction in the overall student population. This may be due to at least two effects: a causal and a selection effect. Students who did not receive any choices may have left the public school system out of frustration. Or the students who were unassigned took a gamble and ranked overdemanded schools with the intention of leaving the system if they did not receive a desirable school. The fact that such a high fraction of these students ranked three or more choices may suggest that the selection effect is less important, though this is far from conclusive.

To demonstrate the undesirability of being unassigned, Table 11 compares the charac-

identify their demographic characteristics and application strategies. However, since the submitted preferences may differ from the true preferences, we choose to focus on a particular subset of students who are playing suboptimally because we can more confidently identify these students.

teristics of the school a child eventually attends when she is unassigned (Eventual School) to the average characteristics of that student's stated choices (Desired Schools). The two columns report the mean MCAS reading and math test scores at the eventual school the student attends and the their desired school. The difference between the eventual school's test scores and any of the desired school's test scores is statistically significant. In unreported calculations, we also calculated the distance and travel time difference. While the distances are on average larger for these unassigned kids, they are only marginally significantly larger. This is consistent with BPS trying to send unassigned children to the nearest school with unfilled places.

Both patterns indicate that the nearby school that still has capacity to which an unassigned student is placed is inferior than the applicant's desired choices. This table allows us to safely conclude that unassigned students are worse off than had they been assigned, and motivates our next exercise.

5.4.1 Individual Deviations (Missed Strategic Opportunities)

Table 13 asks how many of the unassigned students could have received one of their stated choices with the appropriate advice about how to edit their choice lists, by removing overdemanded schools from the top place(s) on their list.[22] (So, e.g. we do not consider whether students who listed only one school might have done better by listing more schools.) Given the evidence presented in Table 11 that the school unassigned students attend has significantly lower test scores than one of the stated choices, if an unassigned student could have been assigned she is no worse off and likely better off. Even for unassigned students who withdraw to private school, they likely prefer to have a school from the public school system to compare with their outside options rather than having no comparison school. It is important to note that this exercise is a non-equilibrium one because it considers individual deviations in isolation and if students can deviate they may cause other students to be worse off.[23] However, the exercise gives an indirect measure of

[22]It is also possible that a student who is assigned to her third stated choice or worse could have deviated in the way we consider here. We have considered such manipulations and we find 5% more elementary school students, 28% more middle school students and 35% more high school students could have successfully deviated. However, focusing on the unassigned is more conservative and only relies on the assumption that stated choices are better than being unassigned.

[23]Since students are not made aware of their random number at the time they submit their preferences, a possibly more appropriate measure may be the expected number of students who could deviate. This requires computing the outcome of the Boston mechanism for all possible deviations among students who could deviate over all possible random number combinations. Note as well as with a different draw of random numbers, there may be additional kids who are unassigned and we must also consider their potential deviations. With 3,326 students in Grade K2, for instance, this corresponds to 3326! possible permutations of students for a given student who could deviate and so direct computation is infeasible. We have no reason to believe that the number who could deviate based on the actual random number used by the Boston mechanism would differ significantly than the expected number.

the extent of pressure on the system and helps to understand the incentives to strategize among the unassigned.

To conduct this exercise, however, we must be able to reproduce the outcome of the Boston mechanism. Table 12 presents our replication of the Boston mechanism as a check on our understanding of the data. The table shows that while we are close to exactly replicating the outcome of Boston's mechanism, there are still some differences. For grade K2, we do not match 275 assignments (8.3%); for grade 6, we do not match 415 assignments (7.2%); and for grade 9, we do not match 376 assignments (5.9%). After extensive discussions with BPS staff about potential sources of discrepancies, we believe that these fractions are as close as we can get to reproducing their match.[24] This issue will only be relevant for Table 13 presenting an exercise involving only the unassigned students, and Table 14 which presents a comparison of the various choice mechanisms using the preference lists submitted under the Boston mechanism.

The first row of Table 13 shows that for the elementary school level, there are 202 student who are unassigned by BPS. Of these, 159 are candidates for the individual deviation exercise because they ranked more than 1 school and are also unassigned in our replication of BPS's allocation. If these students were to have edited the overdemanded schools from the top of their lists, 64% of them would have been assigned to one of their stated choices. Most of the unassigned either receive their second or third choice after the deviation. Returning to the population we have argued is playing suboptimally, we also consider what fraction could have individually deviated and been assigned. At elementary school, 61% could have been assigned, 52% at middle school and 67% at high school.

As a measure of the complexity of the advice that parents would need, the last four columns of Table 13 present the number of schools that must be removed from the top of the choice list. For the overwhelming majority, simply removing the school that they ranked first from their stated preference list would have let them be assigned. In middle school, where there are a greater number of unassigned students, 222 students could have received one of their stated choices, with over half getting their second stated choice. As in elementary school, the advice that these parents would have needed to receive is fairly simple: remove your top choice from your preference list. The last row of Table 13 confirms the same patterns hold in ninth grade.

In summary, Table 13 shows that even with a conservative approach, we can show that an unassigned student's incentives to deviate are strong. Simple rules of thumb such as not ranking an overdemanded school as your first choice would have led many unassigned

[24]Many of the decisions made in the dataset construction described in the Appendix such as how to treat empty choice forms were guided by an attempt to match the outcome produced by Boston Public Schools as closely as possible. Sources of discrepancy are related to the fact that some applicants are assigned administratively and by hand, a lack of a consistent sibling definition, priorities that are in place that are not reflected in Boston's application processing documents such as guaranteed school priority for students who transition to a different school that is in the same building, and unwritten policies related to the assignment of students to a program within a school.

students to receive one of their stated choices.

6 Naive Comparison of the Three Mechanisms: "First, Do No Harm..."

One of the primary concerns in the design and implementation of a practical matching mechanism that is likely to affect the welfare of thousands of participants is that, at the very least, it should not harm those it is intended to help. We had two related sources of potential concern about replacing the Boston mechanism with a strategy-proof mechanism intended to help families by allowing them to state their true preferences. The first concern is that, even if, as seems to be the case, many Boston families have been manipulating their preferences in response to the strategic incentives of the Boston mechanism, it might take several years after a strategy-proof mechanism is introduced before the word-of-mouth advice about how to behave in the old mechanism is replaced with confidence that it is safe to state true preferences. In the meantime, the strategy-proof mechanism might be allocating students in its first year or years of operation with preference lists very much like those currently submitted under the Boston mechanism. A second concern that would have the same outcome is that we might have misjudged the extent to which preferences are currently being manipulated. In both cases, we need to consider how the proposed new mechanisms would perform in case they are implemented and presented with preferences like those currently being submitted.[25]

In this section, we ignore the vulnerability of the Boston mechanism to preference manipulation and compare the outcomes of the three mechanisms using the stated preferences of Boston students. We will show that for each year and grade, the outcomes of all three mechanisms are very similar and therefore even if there is no strategic manipulation in Boston, or if it takes several years for families to learn that they no longer need to manipulate preferences, adoption of either alternative will not harm the performance of the assignment system.

Table 14 presents a comparison of the Boston mechanism, student-proposing deferred acceptance, and the top trading cycles mechanism, under the null hypothesis of no strategic behavior for stated preferences submitted during 2001-02. Since the Boston mechanism can be thought of as a version of the student-proposing deferred acceptance mechanism in which school priorities are adjusted so that after students are prioritized, those who rank a school first are elevated over those who rank it second, those who rank it second are elevated over those who rank it third, etc. it must be the case that the Boston mechanism is able to place more students in their top choice than the student-proposing deferred

[25] There is evidence that it takes time for participants to adjust to the use of a new matching mechanism. In the laboratory, a deferred acceptance mechanism increases the efficiency of allocation over time (Kagel and Roth, 2002), and this seems to have occured in the implementation of the New York City high school match, when we compare the second year of operation with the first (Abdulkadiroğlu, Pathak and Roth, 2005b).

acceptance mechanism. Moreover, since Table 3 showed that the Boston mechanism is able to assign a number of students to their top choice, stated preferences overall are such that there is not much concentration of demand for school spots. As a result, we anticipate no significant differences between the Boston mechanism and the two strategy-proof alternatives. The table confirms this expectation. For instance, Panel A shows the patterns at elementary school where 77.9% of students are assigned their top choice under our replication of the Boston mechanism, while 73.7% are under student-proposing deferred acceptance and 74.1% are under TTC. The Boston mechanism assigns slightly more students to their first choice, but the number of students one of their top three stated choices is roughly the same across mechanisms. Panels B and C show the pattern for middle and high school where the patterns are similar.

Comparing the student-proposing deferred acceptance mechanism to TTC, Table 14 shows that with stated preferences, the latter slightly outperforms the former based on comparing first choices. Note as well that fewer are assigned to their second choice under TTC. As mentioned before, there is no clear Pareto ranking between the student-proposing deferred acceptance mechanism and TTC. The intuition can be explained in an example with three students, where all students have walk-zone priority at a school A. Suppose the first two students have very good random numbers compared to the third student, and suppose that the first and third student both prefer A as their top choice. Under the student-proposing deferred acceptance mechanism, the first and third student are likely to receive school A, while under TTC the second student may trade her priority with another for whom school A is not her top choice. This leads the third student not to receive his top choice, and student-proposing deferred acceptance to assign more students to their top choice than TTC. Empirically, the slightly better performance of TTC over student-proposing deferred acceptance using stated choices is not robust. For alternate years and grades, DAA sometimes performs better.[26]

It is important not to read the comparison of the three mechanisms presented in Table 14 as a measure of what would happen under each scenario because the simulation uses stated choices. As we have argued above, there are good reasons to believe that people are responding to the incentives of the mechanism with manipulated preferences. Thus we anticipate that, once a strategy-proof mechanism is used, stated preferences will more closely reflect true preferences, which should avoid the inefficiencies that can result when families strategize with incomplete information. The table, however, shows that even if stated preferences change only slowly following the change to a strategy-proof mechanism, we should not anticipate adverse changes to the assignment.

[26]The comparison of DAA to TTC is also sensitive to exactly how the student population is defined.

7 Changing the mechanism

7.1 Policy discussion

The evidence summarized above convinced staff members at BPS and the Superintendent that the current student assignment algorithm should be changed. Central to their reform efforts and their arguments to the Boston School Committee, the ultimate decision making body for BPS, was the fact that there exists plausible alternate strategy-proof mechanisms.[27] In his memo to the School Committee on May 25, 2005, Superintendent Payzant wrote:[28]

> The most compelling argument for moving to a new algorithm is to enable families to list their true choices of schools without jeopardizing their chances of being assigned to any school by doing so.

Policymakers at Boston Public Schools recognized that the need to strategize in the current mechanism "provides an advantage to families who have the time, resources and knowledge to conduct the necessary research." This fairness argument turned out to be one of the most compelling arguments for policymakers. Superintendent Payzant writes:

> A strategy-proof algorithm *levels the playing field* by diminishing the harm done to parents who do not strategize or do not strategize well.

Furthermore, recommendations from BPS identified the following other benefits of a transition to a strategy-proof mechanism:

- A strategy-proof mechanism adds "transparency" and clarity to the assignment process, by allowing for clear and straightforward advice to parents regarding how to rank schools.[29]
- There will be potential efficiency gains (consistent with theory and experiments) when allocations are based on families' true preferences.
- School officials will be able to use the submitted preferences as indicators of family preferences, to determine which schools are in fact the most highly regarded, and to estimate the effect of policy changes. (e.g. what would happen if some walk zone boundaries were changed?)

[27] In many economic and abstract domains the only strategy-proof mechanisms are *dictatorial* (Gibbard 1973, Satterthwaite 1975). But see Abdulkadiroğlu and Sönmez (1999), Barbera and Jackson (1995), Barbera, Jackson and Neme (1997), Clarke (1971), Green and Laffont (1979), Groves (1973), Moulin (1994), Moulin and Shenker (1992), Papai (2000), Roth (1982), Roth, Sönmez and Ünver (2004, 2005a,b), Sprumont (1991), Vickrey (1961) for examples of non-dictatorial strategy-proof mechanisms in other resource allocation problems.

[28] See *Superintendent's Memorandum - May 25, 2005* at http://boston.k12.ma.us/assignment/.

[29] See *Recommendation to Implement a new BPS Assignment Algorithm - May 11, 2005* at http://boston.k12.ma.us/assignment/.

This last point was particularly important given the current parallel discussion in Boston about redefining the walk zone boundaries and school zones. Proposals have ranged from eliminating school choice altogether and having a system of neighborhood schools to allowing students to apply to any schools in the city.[30] One critical input to this debate is the anticipated transportation costs of busing students across Boston under each policy scenario. Because the current preference data BPS receives from the school assignment process has been manipulated due to the incentives the Boston mechanism gives participants, drawing credible inferences about these issues has proven difficult.

The following May 11, 2005-dated BPS Memorandum reflects this frustration:

> A resulting benefit for the system is that this alternative algorithm would provide the district with more credible data about school choices, or parent "demand" for particular schools. Using the current assignment algorithm, we cannot make assumptions about where families truly wish to enroll based on the choices they make, knowing many of those choices are strategic rather than reflective of actual preference.

Although we had no way of measuring, in the field data, the inefficiency of the Boston mechanism allocation, note that the fact that families appear not to be stating their true preferences, together with the fact that they are only imperfectly strategizing, strongly suggests that, as in the experimental laboratory (cf. Chen and Sonmez 2003), the resulting allocation is inefficient.

7.2 Choice of strategy-proof mechanism

While the Superintendent recommended the adoption of the student proposing deferred acceptance procedure to the School Committee, there was also discussion of the alternative strategy-proof mechanism based on TTC. Indeed, the TTC mechanism was initially recommended by a taskforce of community members focused on student assignment in 2003 after over a year of community meetings with parents and families.[31]

Regarding his recommendation to adopt the student-proposing deferred acceptance mechanism, the Superintendent states:

[30] See http://boston.k12.ma.us/assignment/ for some of the proposals. Last accessed September 11, 2005.

[31] At a public School Committee meeting on June 8th, 2005, when we were asked how the Committee and the public should think about the choice between TTC and deferred acceptance, we replied that a key question was "In case your child wants to go to the school for which my child has high priority, and my child wants to go to the school for which your child has a high priority, would anyone mind if they traded priorities?" We pointed out that this might result in a third family being excluded from a school even though a child with lower priority was admitted (but in this case the excluded child would have been unlikely to be admitted even in the absence of a trade). If this was not an objection that outweighed the benefits to the students who traded places, then we suggested TTC should be the choice, while if this were going to be a big problem, then DA might be preferred. Initially it appeared that the answer was going to be that no one minded, but as the discussion was broadened over the following weeks and months, the choice was made to go with the deferred acceptance algorithm.

> Another algorithm we have considered, Top Trading Cycles Mechanism, presents the opportunity for the priority for one student at a given school to be "traded" for the priority of a student at another school, assuming each student has listed the other's school as a higher choice than the one to which he/she would have been assigned. There may be advantages to this approach, particularly if two lesser choices can be "traded" for two higher choices. It may be argued, however, that certain priorities – e.g., sibling priority – apply only to students for particular schools and should not be traded away. Moreover, Top Trading Cycles is less transparent– and therefore more difficult to explain to parents – because of the trading feature executed by the algorithm, which may perpetuate the need or perceived need to "game the system."

Central to the theoretical discussion of TTC versus student proposing deferred acceptance was the tradeoff between justified envy and efficiency. The above quotation reflects an additional, related concern: the uneasiness of BPS to allow priorities to be traded, particularly where sibling priority is involved.[32] It also reflects concern that the mechanism that is adopted should be one that can be easily and clearly explained and defended.

Whether the TTC mechanism would indeed have been less transparent and more difficult to explain to parents would ultimately depend on how much effort the school district makes to communicate the relevant properties of the mechanism. With either strategy-proof mechanism, it should be easier to advise students to rank schools truthfully without requiring them to understand the technical details of the assignment algorithm.

Once the strategy-proof deferred acceptance mechanism has been in place for a few years, so that submitted preferences are more likely to correspond to true preferences, we hope to be able to more meaningfully compare TTC and deferred acceptance, and better understand how and how much the allocations that they produce may differ with respect to true preferences.

7.3 Implementing a new mechanism: Communication

Since the student-proposing deferred acceptance mechanism does not penalize students for ranking schools at which they do not have high priority, it is likely that students will rank more schools, including choices for which they have only a low probability of being admitted. As a result, BPS needs to prepare the public and itself for the political ramifications of fewer students receiving their top stated choice. And in general, communication

[32]There is sometimes a gap in intuitions between economists and non-economists about what kinds of things should and should not be traded. As with the reservations about the trading of sibling preferences, these perceptions can play a big role in what kinds of market designs are possible. For example, the need for kidney exchange arises in part from the fact that in most countries of the world it is not permitted to buy or sell organs (Roth et al. 2004,5). In a similar way, we do not expect to see school choice mechanisms in which families bid money for places in the most desirable schools in a direct way. Of course, economists have long recognized that there can be an implicit price to certain schools based on the costs of living in certain neighborhoods.

with the community is a big part of implementing a new mechanism. The advantages of strategy proofness would be lost if it is not effectively communicated to the community that under the new mechanism, in contrast to the old one, families will not suffer bad consequences from revealing their true preferences.

In his September 12, 2005 memo, Superintendent Payzant wrote the following list of items BPS needs to communicate to the community:

- The new school assignment formula enables families to *indicate their true choices of schools, in order of preferences, without regard to the size or popularity of the school.*
- Unlike in past years, families do not have to (in fact, should not) "strategize" about which schools to rank first, second or third based on the presumed likelihood of getting a seat.
- When ranking schools, families do *not* have to factor into their decision-making how many other families will choose a particular school, which priorities their student has to the school, how many siblings are likely to apply, or how high or low a random number their student may draw.
- Rather, families should indicate their true preferences, in order, based on their own research about the quality of the school and its appropriateness for the personal and educational needs of their child.
- The most effective means of learning about a school and determining whether or not it is a good fit is to visit the school and talk to the principal, teachers, parents and students.
- Families should list as many schools as possible (ideally, more than 6 schools) in order to increase the likelihood of receiving an assignment to one of their choices. Families should register as early as they are eligible to do so, for the best chance of getting schools of their choice.

8 Conclusion

One of the interesting challenges involved in moving from a school choice mechanism with bad incentives to one that makes it safe for families to list their true preferences is that the Boston school choice system was not broken in an obvious way. Rather, each year students were assigned to schools in an orderly manner, with a very high proportion getting their stated first choice. In this respect, Boston was like a patient with high blood pressure, a potentially deadly disease that has no easily visible symptoms.[33]

[33] This is in contrast to some other recent design projects in which there was an obvious market failure to be repaired, e.g. the lack of a thick market resulting in very few kidney exchanges (Roth et al. 2004, 2005), or the unraveling that reduced mobility in the market for gastroenterologists (Niederle and Roth, 2003, 2005), or the congestion in the old New York City high school admission process (Abdulkadiroğlu et al. 2005b). To carry

The challenge to investigating the nature of strategic play is that, since the Boston mechanism isn't strategy proof, our data didn't allow us to know the true preferences behind the submitted preferences. So, we can't hope to detect the full incidence either of strategic manipulation, or of the families who failed to manipulate as successfully as they could have.[34] Instead, we concentrated on showing that there are subsets of families who have manipulated their preferences, and others who could have profited by submitting different preferences. For the first conclusion, we showed that some families take account of their priorities when submitting their preferences by looking at the location of families listing as their first choice two nearby, overdemanded schools, one with and one without walk zone preference. For the second conclusion, we looked at the students who fared worst, those who received administrative assignments to schools with excess capacity, and showed that a large proportion of them could have been admitted to one of the schools they had listed, if they had not made the mistake of listing one or more overdemanded schools at the top of their lists.

While the available data do not permit us to assess the size of the potential welfare loss (as we can in laboratory data), what we could show was that many parents appeared to be responding to the strategic incentives to misrepresent their preferences, although many were without all the information that would be needed to do this well. This is precisely the behavior that, in the laboratory where we can measure efficiency losses, causes inefficiency in the resulting matchings. The quotes from the West Zone Parents Group in Appendix 2 further suggest that this misrepresentation of preferences was in many cases a costly process in itself (involving e.g. the gathering of intelligence on how many siblings would enter a given school that year). Moreover, remarks at the parent hearing on the allocation mechanism presented in Appendix 3 indicate that some parents resented having to strategize and that caused parents to lose trust in the public school system. In addition, the fact that preferences are manipulated, well or badly, means that the preference lists available to school officials are not reliable indicators of parents' preferences.

A strategy-proof mechanism has an advantage over the Boston mechanism in that, in contrast to the Boston mechanism, it would

- Allow families to list choices in order of true preferences.
- Make it easy to give parents correct advice about how to fill out their preference lists.
- Not penalize parents who are not sophisticated about the school choice process or well informed about which schools are overdemanded.

on the analogy, those problems were like a patient with a heart attack, where the best treatment might not be obvious, but there was little dispute that treatment was needed.

[34]The case of a family that receives its stated first choice makes both points clear. This could be their true first choice. But if it is not, then they could have profited from deviating from their true preferences, or been harmed, but this cannot be known from the data.

- Allow parents to spend their time visiting schools and assessing teachers, rather than researching the levels of competition for each school spot.
- Give school officials a more reliable indicator of parent preferences, which can be used not only to match students to schools, but to assess how policy changes such as a change in the borders of walk zones would influence which children would be likely to be assigned to which schools.

There are also a number of conclusions we can draw from the Boston experience that have implications for other school districts, and, more generally, for other market design problems.

First, there are quite a few other school districts with student assignment systems sharing the main features of the Boston mechanism, for example Cambridge, Charlotte-Mecklenberg, Denver, Miami-Dade, Rochester, Tampa-St. Petersburg, and White Plains.[35] Based on our analysis of the behavior of Boston families, it seems likely that in these other school districts parents are faced with solving a complex strategic problem, rather than just a problem of forming preferences over schools. A strategy-proof mechanism like top trading cycles or the deferred acceptance mechanism would lift this strategic burden from parents, and makes the school choice process more transparent. School choice is often a sensitive political issue, and transparency helps to remove some aspects of how to best assign children to schools from the political arena to the technical arena, and clarify which issues remain to be settled by the political process. There will always be such issues, since until there are enough top quality school places to satisfy all families, some aspects of school choice will be a distributive process, with only some students able to gain admission to the most desirable schools. But school assignment is far from zero sum, since different students (in different locations, and with different needs) will have different preferences.[36]

Our analysis of strategic behavior in Boston also serves to emphasize the fact (lost in some theoretical discussions of mechanism design) that, in complex environments, not all players may be responding optimally to the strategic incentives of the system, i.e. the system need not elicit equilibrium behavior. The policy discussion that developed in Boston showed that one advantage of strategy-proof mechanisms is that they level the playing field between the strategically sophisticated and well informed and those who may be unsophisticated or poorly informed. For policymakers in Boston, this proved to be a major point that led to the successful adoption of the new system.

In this connection, there are also good reasons to believe that, once the strategy-proof mechanism is put into operation in 2006, the adjustment to straightforward statement of preferences may not be immediate. So another desirable property of both the proposed

[35]See Hastings, Kane, and Steiger (2005) for a study of the effect of school choice in Charlotte-Mecklenberg on student achievement.

[36]Cullen, Jacob, and Levitt (2003), for instance, report that in an open enrollment plan in Chicago was successful at matching idiosyncratic tastes of parents and students and improving social circumstances.

mechanisms is that, if behavior is slow to adjust to the new dominant-strategy equilibrium of truthful revelation of preferences, no harm will be done during the transition period. That is, the success of the new mechanism does not depend on the complete and immediate success of the education and communication that must accompany the introduction of the new mechanism.

Overall, one lesson from the Boston experience is that mechanism design in a political environment requires that not only policy makers themselves be persuaded of the virtues of a new design, but that they be able to explain and defend the mechanism to the various constituencies they serve. Thus a desirable property of any proposed mechanism is that it should be simple, and easy to understand and communicate.

In summary, no mechanism can ensure that families will all receive their first choice school. However, there are better ways to allocate students to a fixed stock of school places than the current Boston mechanism. It proved possible to organize the game-theoretic arguments and empirical evidence in a way that could be effectively communicated to both policy makers and their public constituencies.[37] In the coming years, as the new Boston mechanism goes into operation, it should become possible to draw stronger conclusions about future modifications to the system, as it becomes safe for families to report their true preferences, so that more reliable information about family preferences and welfare will become available.

[37] This stands in contrast to the view espoused by Rubinstein (2005) in his presidential address to the Econometric Society, where he writes, "I believe that as an economic theorist I have very little to say which is of relevance in the real world and I do believe that there are very few models in economic theory that could be used to provide serious advice."

References

1. A. Abdulkadiroğlu (2005), "College Admissions with Affirmative Action," *International Journal of Game Theory*, 33: 535-549.

2. A. Abdulkadiroğlu, P. A. Pathak, and A. E. Roth (2005a), "The New York City High School Match," *American Economic Review*, Papers and Proceedings, 364-367.

3. A. Abdulkadiroğlu, P. A. Pathak, and A. E. Roth (2005b), "Redesigning the New York High School Match," in preparation, Harvard University.

4. A. Abdulkadiroğlu, P. A. Pathak, A. E. Roth, and T. Sönmez (2005), "The Boston Public School Match," *American Economic Review*, Papers and Proceedings, 368-371.

5. A. Abdulkadiroğlu and T. Sönmez (1999), "House Allocation with Existing Tenants," *Journal of Economic Theory*, 88: 233-260.

6. A. Abdulkadiroğlu and Tayfun Sönmez (2003), "School Choice: A Mechanism Design Approach," *American Economic Review*, 93: 729-747.

7. M. Alves and C. Willie (1990), "Choice, Decentralization and Desegregation: The Boston 'Controlled Choice' Plan" in William Clune and John White, eds., Choice and Control in American Education, volume 2: The Practice of Choice Decentralization and School Restructuring. New York: The Falmer Press, 17-75.

8. M. Balinski and T. Sönmez (1999), "A Tale of Two Mechanisms: Student Placement," *Journal of Economic Theory*, 84: 73-94.

9. S. Barbera and M. Jackson (1995), "Strategy-Proof Exchange," *Econometrica*, 63: 51-88.

10. S. Barbera, M. Jackson and A. Neme (1997), "Strategy-Proof Allotment Rules," *Games and Economic Behavior*, 18: 1-21.

11. L. Bodin and A. Paneken (2003), "High Tech for a Higher Authority: The Placement of Graduating Rabbis From Hebrew Union-Jewish Institute of Religion," *Interfaces*, 33: 1-11.

12. Boston Public Schools (2004), "Introducing the Boston Public Schools: A Guide for Parents and Students." Available at www.bostonpublicschools.org.

13. Y. Chen and T. Sönmez (2003), "School Choice: An Experimental Study," *Journal of Economic Theory*, forthcoming.

14. E. H. Clarke (1971), "Multipart Pricing of Public Goods," *Public Choice*, 2: 19-33.

15. G. Cook (2003), "School Assignment Flaws Detailed," *Boston Globe*, 9/12/2003.

16. P. W. Cookson (1994), *School choice: The struggle for the soul of American education*, New Haven: Yale University Press.

17. J. Cullen, B. Jacob, and S. Levitt (2003), "The Effect of School Choice on Student Outcomes: Evidence from Randomized Lotteries," NBER Working paper 10113.

18. L. E. Dubins and D. A. Freedman (1981), "Machiavelli and the Gale-Shapley Algorithm," *American Mathematical Monthly*, 88: 485-494.

19. H. Ergin and T. Sönmez (2003), "Games of School Choice Under the Boston Mechanism," *Journal of Public Economics*, forthcoming.

20. D. Gale and L. Shapley (1962), "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 69: 9-15.

21. A. Gibbard (1973), "Manipulation of voting schemes," *Econometrica*, 41: 587-601.

22. C. L. Glenn (1994), "Controlled Choice in Massachusetts Public Schools," *Public Interest*, 103: 88-105.

23. Green and Laffont (1977), "Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods," *Econometrica*, 427-38.

24. T. Groves (1973), "Incentives in Teams", *Econometrica*, 41: 617-31.

25. J. Hastings, T. Kane, and D. Staiger (2005), Evaluating a School Choice Lottery: The Importance of Heterogeneous Treatment Effects, working paper.

26. J. Hatfield and P. Milgrom (2005), "Matching with Contracts," *American Economic Review*, 95: 913-935.

27. J. Kagel and A. Roth (1999), "The Dynamics of Reorganization in Matching Markets: A Laboratory Experiment Motivated by a Natural Experiment," *Quarterly Journal of Economics*, 201-235.

28. A. Kelso and V. Crawford (1981), "Job Matching, Coalition Formation, and Gross Substitutes," *Econometrica*, 50: 1483-1504.

29. H. Moulin (1994), "Serial Cost Sharing of Excludable Public Goods," *Review of Economic Studies*, 61: 305-325.

30. H. Moulin and S. Shenker (1992), "Serial Cost Sharing," *Econometrica*, 50: 1009-1039.

31. M. Niederle and A. E. Roth (2003a), "Unraveling Reduces Mobility in a Labor Market: Gastroentrology with and without a Centralized Match," *Journal of Political Economy*, 111: 1342-1352.

32. Niederle and Roth (2003b), "Relationship between Wages and Presence of a Match in a Medical Fellowship," *Journal of the American Medical Association*, 290: 1153-1154.

33. M. Niederle and A. E. Roth (2005), "The Gastroentrology Fellowship: Should there be a Match?" *American Economic Review*, Papers and Proceedings, 372-375.

34. S. Pápai (2000), "Strategyproof Assignment by Hierarchical Exchange," *Econometrica*, 68: 1403-1433.

35. T. W. Payzant (2005a), "Student Assignment Mechanics - Algorithm Options," School Committee Memorandum, May 11, 2005. Last accessed November 1, 2005. http://boston.k12.ma.us/assignment/

36. T. W. Payzant (2005b), "Student Assignment Mechanics: Algorithm Update and Discussion," School Committee Memorandum, May 25, 2005. Last accessed November 1, 2005. http://boston.k12.ma.us/assignment/

37. T. W. Payzant (2005c), "Communications Strategy for the School Choice Process" School Committee Memorandum, September 12, 2005. Last accessed November 1, 2005.

38. A. E. Roth (1982a), "The Economics of Matching: Stability and Incentives," *Mathematics of Operations Research*, 7: 617-628.

39. A. E. Roth (1982b), "Incentive Compatibility in a Market with Indivisible Goods," *Economics Letters*, 9: 127-132.

40. A. E. Roth (1985), "The College Admissions Problem is Not Equivalent to the Marriage Problem," *Journal of Economic Theory*, 36: 277-288.

41. A. E. Roth (1984), "The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory." *Journal of Political Economy*, 92: 991-1016.

42. A. E. Roth (1990), "New Physicians: A Natural Experiment in Market Organization." *Science*, 250: 1524-1528.

43. A. E. Roth (1991), "A Natural Experiment in the Organization of Entry Level Labor Markets: Regional Markets for New Physicians and Surgeons in the U.K." *American Economic Review*, 81: 415-440.

44. A. E. Roth (2002), "The Economist as Engineer: Game Theory, Experimental Economics, and Computation as Tools of Design economics." *Econometrica*, 70: 1341-1378.

45. A. E. Roth and E. Peranson (1999), "The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design." *American Economic Review*, 89: 748-780.

46. A. E. Roth and A. Postlewaite (1977), "Weak Versus Strong Domination in a Market with Indivisible goods." *Journal of Mathematical Economics*, 4: 131-137.

47. A. E. Roth and U. G. Rothblum (1999), "Truncation Strategies in Matching Markets: In Search of Advice for Participants," *Econometrica*, 67: 21-43.

48. A. E. Roth, T. Sönmez, and U. Unver (2004), "Kidney Exchange," *Quarterly Journal of Economics*, 119: 457-488.

49. A. E. Roth, T. Sönmez, and U. Unver (2005a), "A Kidney Exchange Clearinghouse in New England," American Economic Review, Papers and Proceedings, 95,2, May, 376-380.

50. A. E. Roth, T. Sönmez, and U. Unver (2005b), "Pairwise Kidney Exchange," Journal of Economic Theory, 125, 2, December, 151-188.

51. A. E. Roth and M. Sotomayor (1990), *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*, Cambridge: Cambridge University Press, 1990.

52. A. Rubinstein (2005), "Dilemmas of an Economic Theorist," Econometric Society Presidential Address, available at: http://arielrubinstein.tau.ac.il/papers/Pa.pdf.

53. M. Satterthwaite (1975), "Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions," *Journal of Economic Theory*, 10: 187-217.

54. L. Shapley and H. Scarf (1974), "On Cores and Indivisibility," *Journal of Mathematical Economics*, 1: 23-28.

55. Y. Sprumont (1991), "The Division Problem with Single-Peaked Preferences: a Characterization of the Uniform Allocation Rule," *Econometrica*, 59: 509-519.

56. W. Vickrey (1961), "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance*, 16: 8-37.

9 Appendices

Appendix 1: Data definitions and priorities

We utilize a proprietary database of student choices and geographic characteristics for the school years 2000-2004 provided by Boston Public Schools for this study.

Student population: The population consists of students who submit preferences in Round 1 with a complete form, and students who receive a guaranteed assignment (to their present school) in round R in June of the school year. We assume that these latter students have elected to stay in their present school. To keep only students with complete application forms, if the assignment method is I for incomplete application or R for returned application, the record is deleted. Each year there are also student records with empty choice forms. The majority of these students continue to stay in their current school, which is either a combined elementary and middle school (grades K2-8) or middle and high school (grades 6-12). For students with an empty first choice, we have the following rules: If the method is A for guarantee assignment (to present school), the first choice is set to their guaranteed school. If the method is D for administratively assigned, the record is deleted. If the method is E for East Boston guarantee student, we append the guaranteed school at the end of the choice list, if it is not already there. If the method is G, the blank first choice is set to the assigned school. If the method is H and the first choice is blank, then the record is deleted. Students with methods D or H and blank first choices are mainly those with special needs. If the method is N and first choice is blank, the record is deleted. If the method is U for unassigned and the first choice is blank, the record is deleted.

For purposes of simulating the algorithms, student preference lists are modified for each choice to actually rank two programs, one corresponding to the half of the school where walk zone priority applies and the other to where it does not. The walk zone half is always placed ahead of the other half for students who have walk zone priority at the school, and vice versa for students who do not.

Unassigned students are those who are labeled unassigned, administratively assigned, or hand assigned (methods U,D, and H, respectively), have been assigned in the final pass (run=90) or to their sixth choice, which is not in the dataset (run=95), or are assigned to a September school that is not one of their choice schools.

Capacities: School capacities are calculated as the number of assigned students to a school from the student population defined above. At the elementary and middle school level, each school is split into a walk zone priority half and the half where walk zone priority does not apply. If there is an additional seat, it is given to the walk zone half.

Priorities: The actual priority ordering of the Boston mechanism was constructed from the *Controlled Choice Application Rules of the Records Management Unit* of the Boston Public Schools and extensive discussions with the staff of Boston Public schools.

For the half of the school seats where walk zone priority applies, students are ordered into indifference classes where random numbers are used to break ties. Among students who rank a school first, students applying from the walk zone are ordered by priority according to guaranteed, sibling-present school, sibling, present school, students who live in a geocode without a walk zone school (non-walk priority) and no priority. Then students without walk zone priority ranking the school first are ordered according to guaranteed, sibling-present school, sibling, present school, non-walk priority, and random. The next set of indifference classes are students who rank the school second. Among the walk zone students, first are those with siblings followed by those without. The next set of students rank the school second without walk zone priority. These students are ordered according to sibling, non-walk priority, and no priority. The next class are those who rank the school as their third choice. These students are ordered as walk-sibling, walk, sibling, no priority. The same ordering applies for the remaining choices ordered by choice.

At the half of the school where walk zone priority is ignored, the students are ordered according to to guarantee, sibling-present school, sibling, present school, and no priority. Then, students who rank the school second according to sibling, non-walk priority, and no priority. Then, students who rank the school third according to sibling and no priority. The same ordering applies for the remaining choices.

The dataset provided by Boston Public Schools includes a sibling indicator if a student was assigned to their school via sibling priority. This is how we identify which school students are eligible for sibling priority. Note that it is possible we miss situations where a student applies to a school with sibling priority and does not receive a spot. This only happens, however, if there are more siblings applying to the school as their first choice than priority seats available and thus only applies to less than one percent of seats. After substantial experimentation constructing our own sibling definitions, we believe that the BPS sibling indicator is the most accurate measure of sibling priority. The dataset also includes a student's geocode and a mapping of geocodes to walk zone priority from which we determined whether a student was eligible for walk zone priority at a given school. Finally, the random number we use to break indifferences in priority is the same number provided by BPS.

Appendix 2: Quotes from West Zone Parents Group

The West Zone Parents Group (WZPG) is a volunteer group of parents who either have children in Boston Public Schools or are considering enrolling their children. The group is open to anyone who wants to participate. Members meet to share information and help one another understand the process of choosing and registering for a school in Boston.

The group hosts an email group, WESTZONEPARENTS, on the internet site Yahoo!Groups that serves as a supplement to meetings and is a way for people to communicate and share information between meetings. The archived history of the group is available, and was last accesssed on June 17, 2005.

The following excerpts from posted messages are regarding strategic behavior related to the Boston assignment mechanism:

- WZPG Parent correspondence, 1/22/2004, Subject: your input

  For those of you considering putting the Haley as a first choice, ... you may want to put a safer school second than you had been planning to, in case the momentum builds even more than you had expected. You'll probably be fine and automatically get it as a first cohice, but you may want to still play it safe.

- WZPG Parent correspondence, 1/28/2005, Subject: Re: Philbrick School

  Have you gotten any sense if a lot of people are choosing the Philbrick as a 1st choice? We really like Philbrick ... but are not in the walk zone. We are putting Manning 1st since we're in the walk zone and Philbrick 2nd but I'm getting very nervous that Philbrick has gotten so popular that it might only be a good #1 selection. We're also looking for a good safety for 4th place, perhaps Hale or Mendell.

- WZPG Parent correspondence, 1/28/2005, Subject: Re: Philbrick School

  I think there are probably 2-3 siblings entering K2 [at the Philbrick]. I know of 2 people who are putting it as a first choice... I don't know what to say—according to last year's numbers, putting it second would be safe, but the year we applied, only first choice people got in. I think it would be okay if your third choice were a VERY safe bet.

- WZPG Parent correspondence, 1/30/2005, Subject: Re: Philbrick School

  We're also having trouble with deciding on a secure 2nd choice. We are struggling with deciding between the Manning and the Haley for our 1st choice. They are both in our walk zone but for logistics and time ... the Manning works best for our family. We really loved the Haley but I'm afraid that it is not a safe 2nd choice anymore.

- WZPG Parent correspondence, 4/5/2005, Subject: Sumner K1- poll

  Our son is in a great Montessori program and I am reluctant to pull him out even though I was very impressed with the Sumner... our "true" first choice would be a K2 that is closer to our home...

Appendix 3: Comments from Public Hearing

On June 8th, 2005 the Boston School Committee held a public hearing on the assignment algorithm. There were six comments, two of which argued for increased transparency in the system, one which argued against the change, one who was upset with their assignment and the remaining two were from community members who had been involved in the controlled choice a decade earlier but were no longer affiliated with Boston Public Schools. These last two statements were as follows (Source: Audio Recording of Public Hearing on Assignment Algorithm, June 8, 2005, obtain from School Committee Executive Secretary Laurie Ciardi):

Comment 1:

It [changing assignment algorithms] is a long time coming. I'm extremely pleased to see the adopting of an assignment method that does not penalize students for ranking one school over another. I was very involved with the student assignment process: I was on a taskforce back in 1985, was co-chair of a taskforce on K2s, and have been pushing for a method like for 20 years. I totally agree with a method where you don't penalize students for ranking a particular school.

Comment 2:

[..] I have participated in the selection process 12 times with 4 different children. I was one of the gang of 6 who drafted the assignment process with Chuck Willie and Michael Alves. I have been waiting 16 years for this vote. This [The assignment algorithm] is one issue that I had lost on.

I find the current system of maximizing first choice to be insidious and destructive. I urge each school committee member to vote enthusiastically for this new algorithm proposal. [...] My wife and I take dozens of phone calls around choice time in Dorchester. We have to tell people that it doesn't make sense to choose our children's elementary school. And that is absurd. And the people who get that advice get very angry. [...] Because to get into the O'Hearn you need to be luckier than megabucks. So I have to say [to these parents], don't make your first choice your first choice. That's enraging. It is at the bottom of the anger that you [the School Committee] get from West Roxbury.

But it is even more cruel and unusual for 'non-savvy' parents. And I've never met anyone who was savvy after ten minutes who wasn't just angry. It's the Timilty problem. I've never chosen the Timilty middle school for my four children. I'm very happy with the McCormick and Rogers, but I knew it would be a bad mistake because if you choose the Timilty your risk of getting an administrative assignment goes up astronomically. That's wrong because if you don't know this, your chances of choosing your first choice and getting an administrative assignment skyrocket.

It angers the parents who figure it out because they are told not to make their first choice the first one. And it hurts those who don't figure it out because they choose a popular school and end up in the administrative assignment bin.

This new system [...] will heal the problem and quelch the anger. We will be able to give parents good advice. They will make more choices because they will not be confused. More importantly, they will not be harmed.

The maximizing first choice system is harmful to parents. I urge you to change it by supporting the Superintendent's recommendation.

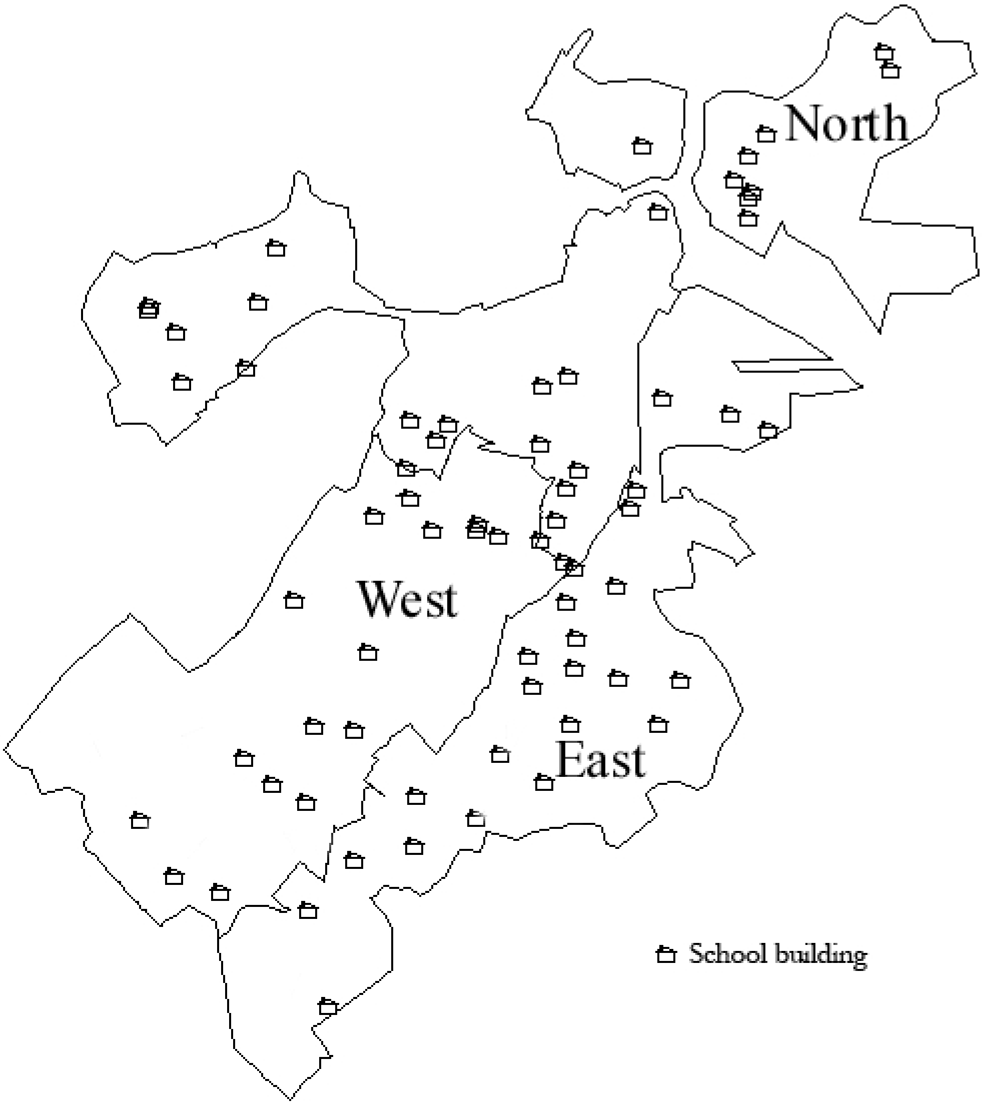

Figure 1: Boston Public School Assignment Zones and Elementary Schools in 2001-02

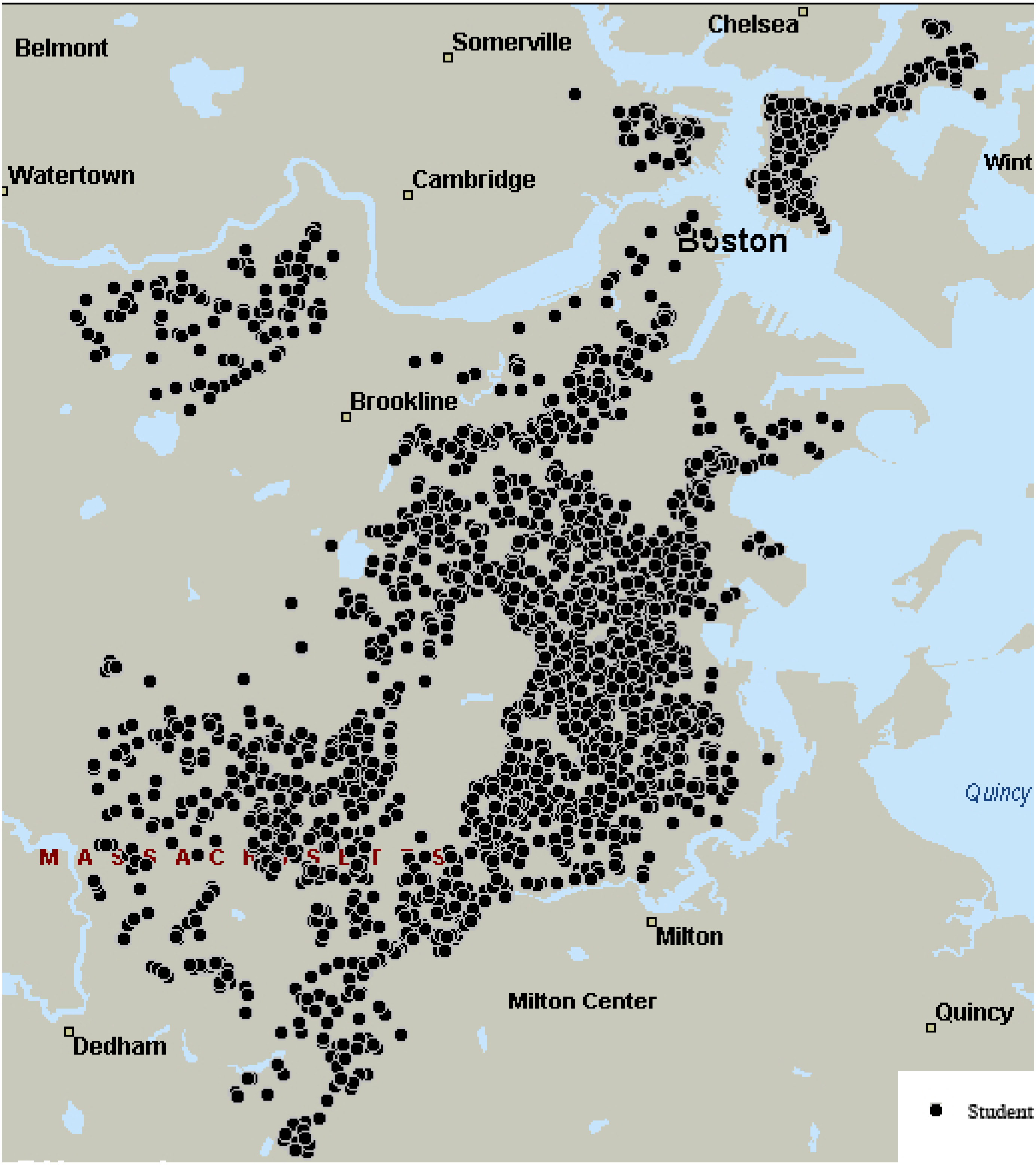

Figure 2: All Applicants for Elementary School in 2001-02

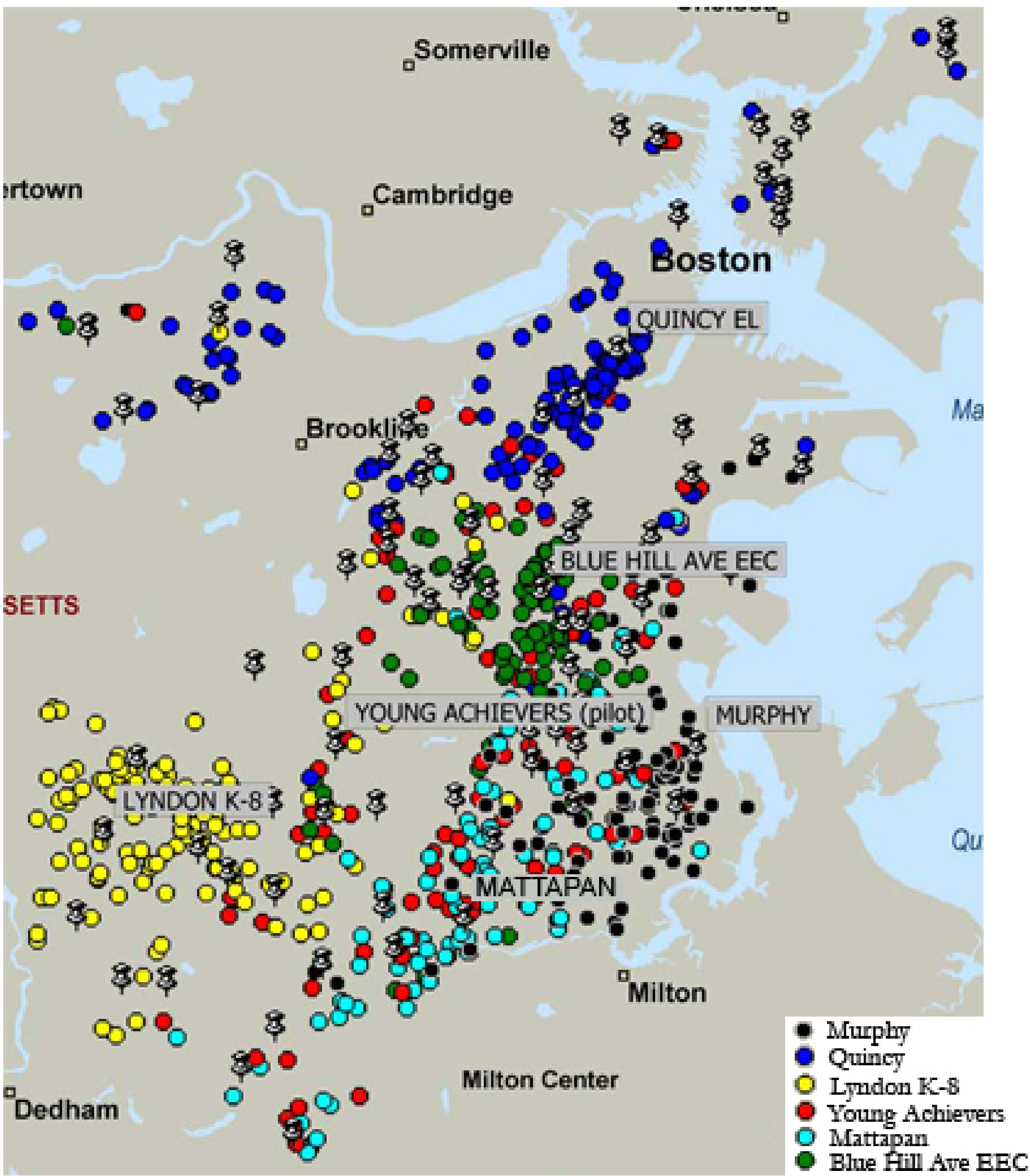

Figure 3: Overdemanded Elementary Schools and Applicants in 2001-02

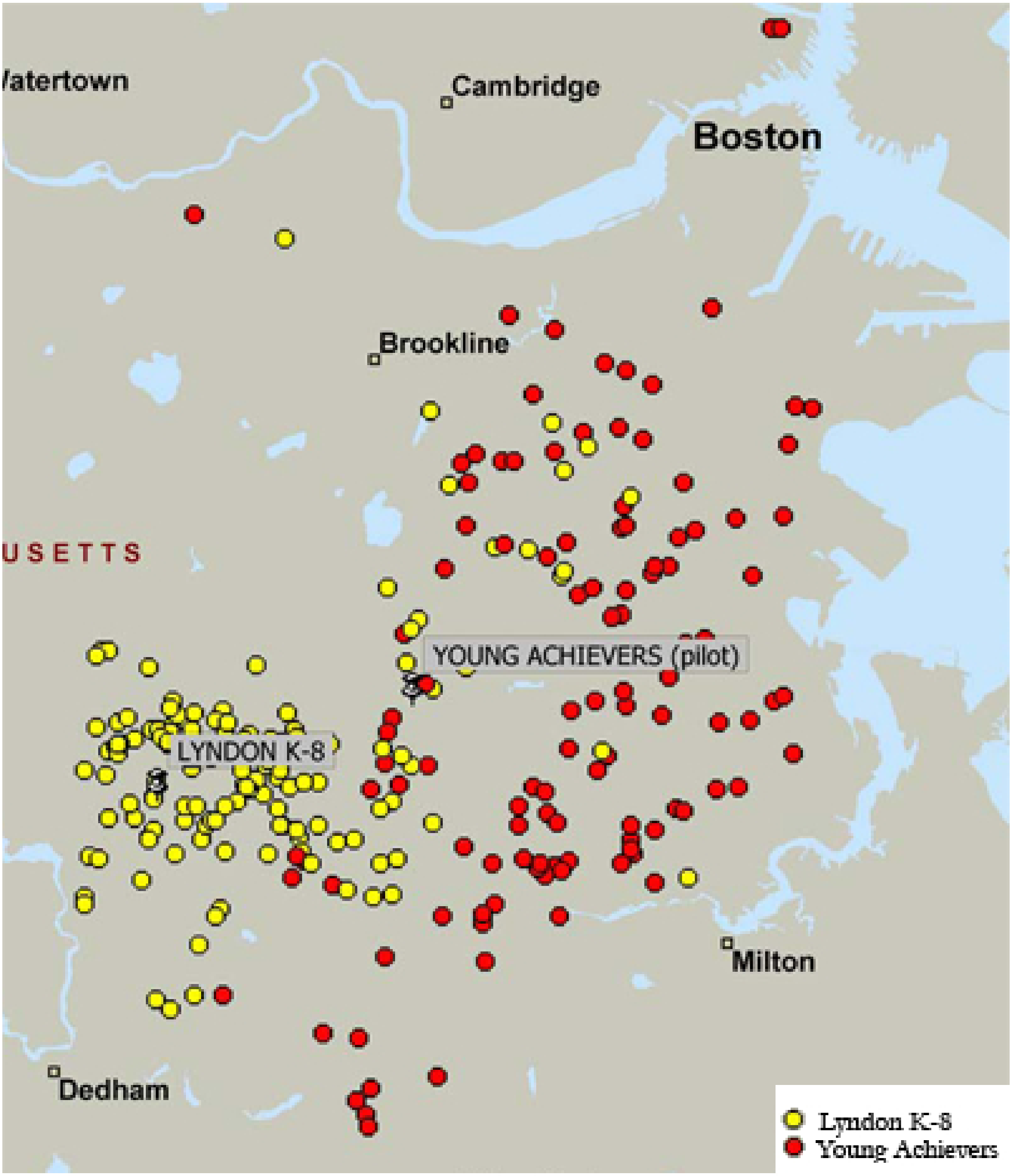

Figure 4: All Applicants in 2001-02 Ranking Lyndon or Young Achievers as First Choice

Table 1— 2001-2002 District Characteristics[a]

	Number of Schools	Number of Applicants
Elementary School[b]		
East	30	1,264
North	28	1,072
West	24	990
Citywide	3	–
Total	85	3,326
Middle School		
East	10	2,383
North	8	1,568
West	6	1,478
Citywide	5	–
Total	29	5,429
High School		
Total	12	6,380

[a]Notes: Statistics tabulated using data obtained from Boston Public Schools. School population defined as all schools open to students through assignment process. Student population defined as students with valid application form applying for the 2001-2002 school year, who have submitted their preferences in round 1 by February for a school spot in fall 2002. Additional details on definition of student population in appendix.

[b]Elementary school defined as schools offering grade K2 (must be 6 years old on or before September 1, 2002) and includes Early Learning Centers (ELC) and K-8 programs. Middle schools defined as grade 6 and high defined as grade 9. Continuing schools such as those which cover grades K-8 or 6-12 are treated as two separate schools.

Table 2— 2001-2002 Student Characteristics[a]

	Number	Subsidized Lunch	Female	Race: Black	Race: White	Race: Hisp.	Race: Other	Bilingual	Withdrew[b]
Elementary school applicants									
East	1,264	71%	50%	60%	12%	18%	9%	12%	14%
North	1,072	75%	51%	22%	17%	45%	15%	25%	12%
West	990	60%	51%	33%	26%	37%	4%	15%	16%
Total	3,326	69%	50%	40%	18%	32%	10%	17%	14%
Middle school applicants									
East	2,383	77%	48%	64%	10%	18%	8%	8%	10%
North	1,568	80%	50%	27%	16%	43%	14%	11%	10%
West	1,478	77%	52%	48%	13%	37%	3%	12%	11%
Total	5,429	78%	49%	48%	13%	30%	9%	9%	10%
High school applicants									
Total	6,380	65%	49%	48%	16%	27%	8%	11%	14%

[a]Notes: Statistics tabulated using data obtained from Boston Public Schools. Gender, race, bilingual and withdrawal status calculated from student file in the following year. Details on definition of student population in appendix.

[b]Students who withdraw may either attend parochial, private or charter schools or have moved outside BPS, withdrew with unknown status or left the system for other reasons (expulsion, illness, pregnancy, etc.)

Table 3— 2001-2002 Stated Choice Received[a]

	Top Choice		2nd Choice		3rd Choice		4th Choice		5th Choice		Unassigned[b]	
	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent
Elementary school applicants												
East	1,010	80%	101	8%	47	4%	27	2%	16	1%	63	5%
North	883	82%	100	9%	25	2%	13	1%	3	0%	48	4%
West	705	71%	100	10%	59	6%	21	2%	14	1%	91	9%
Total	2,598	78%	301	9%	131	4%	61	2%	33	1%	202	6%
Middle school applicants												
East	1,894	79%	128	5%	125	5%	29	1%	14	1%	193	8%
North	1,153	74%	157	10%	65	4%	22	1%	3	0%	168	11%
West	1,110	75%	130	9%	104	7%	10	1%	9	1%	115	8%
Total	4,157	77%	415	8%	294	5%	61	1%	26	0%	476	9%
High school applicants												
Total	5,497	86%	428	7%	100	2%	42	1%	11	0%	302	5%

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on definition of student population in appendix.
[b]Unassigned students are those who are either hand assigned or receive a school that is not on their preference list.

Table 4— 2001-2002 Priority Eligibility at Assigned School[a]

	Guaranteed		Sibling-Walk		Sibling		Walk[b]		No Priority	
	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent
Elementary school applicants										
East	218	17%	147	12%	126	10%	317	25%	393	31%
North	159	15%	156	15%	71	7%	305	29%	333	31%
West	137	14%	84	8%	113	11%	266	27%	299	30%
Total	514	16%	387	12%	310	9%	888	27%	1,025	31%
Middle school applicants										
East	669	28%	56	2%	97	4%	585	25%	783	33%
North	495	32%	49	3%	66	4%	390	25%	400	26%
West	418	28%	53	4%	62	4%	358	24%	472	32%
Total	1,582	29%	158	3%	225	4%	1,333	24%	1,655	30%
High school applicants										
Total	3,297	52%	105	2%	150	2%	844	13%	1,682	26%

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on definition of student population in appendix.
[b]Students who live in geocodes with no walk zone schools, but are assigned via non-walk priority are counted as receiving walk priority.

Table 5— 2001-2002 Applicant Strategies[a]

	# schools ranked 1	2	3	4	5	Over-demanded[b] 1st	Over-demanded 2nd	Over-demanded 3rd
Elementary school applicants								
East	19%	3%	40%	14%	23%	49%	35%	29%
North	18%	9%	51%	10%	11%	53%	30%	22%
West	14%	3%	48%	13%	22%	60%	43%	35%
Total	17%	5%	46%	13%	19%	53%	36%	28%
Middle school applicants								
East	31%	4%	48%	7%	10%	68%	44%	37%
North	38%	9%	41%	6%	5%	45%	33%	25%
West	31%	5%	47%	9%	8%	62%	42%	32%
Total	33%	6%	46%	7%	8%	60%	40%	32%
High school applicants								
Total	52%	2%	32%	6%	8%	38%	21%	16%

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on definition of student population in appendix. There are 2 students in grade 6 who rank five schools and are assigned to their unranked guaranteed North zone middle school. We treat these students as ranking five choices despite appending their guaranteed choice school to their preference list for the mechanism simulations.

[b]A choice is overdemanded if a student ranked a program as their nth choice for $n = 1, 2, 3$ when the number of students who ranked that school first that year was greater than the total capacity. If a choice is empty, it is by definition not overdemanded.

Table 6— 2001-2002 Grade K2 Preference Discontinuities[a]

Panel A: Discontinuity at Specific Schools

School	Seats	Number ranking 1st	Number ranking 2nd	Number ranking 3rd
Lyndon	50	151	45	30
Other West Zone	940	839	655	604
Quincy	112	187	35	28
Other North Zone	960	885	679	574

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on definition of student population in appendix.

Table 6— **2001-2002 Grade K2 Preference Discontinuities (cont.)**

Panel B: Discontinuity in the Cross-section[a]

Dependent Variable:
Δ_i = Number Ranking i First - Number Ranking i Second

	(1)	(2)	(3)	(4)
$Overdemand_1$[b]	26.43	26.61	-	-
	(4.92)	(5.00)		
$Overdemand_2$[c]	-	-	0.941	0.944
			(0.093)	(0.094)
Citywide dummy	-19.60	-19.09	-33.40	-32.57
	(15.88)	(16.20)	(12.07)	(12.30)
Zone dummies	N	Y	N	Y
N	85	85	85	85
R^2-adj	0.260	0.279	0.554	0.566

[a]Note: Table presents regressions of the difference in the number of students ranking a school first and the number of students ranking a school second on proxies for whether that school was overdemanded, a citywide dummy variable, and zone dummies. Specifications (1) and (3) include an intercept. Standard errors in parenthesis.

[b]$Overdemand_1$ is the ratio of the number of students ranking school first to school's total capacity.

[c]$Overdemand_2$ is the difference between the number of students ranking school first and the school's total capacity.

Table 7— 2001-2002 Ranking Two Overdemanded Schools with Random Priority at the First[a]

		Received 1st Choice		Received Other Choice		Unassigned	
	Total	Number	Percent	Number	Percent	Number	Percent
Elementary school applicants	391	147	38%	129	33%	115	29%
Middle school applicants	1,035	351	34%	327	32%	357	34%
High school applicants	555	253	46%	114	21%	188	34%

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on definition of student population in appendix. Boston Public schools reports that in this population there are 3 students who receive their second choice in elementary school, 24 who receive their second choice in middle school, and 9 in high school. It is impossible for these students to receive their second choice under the Boston mechanism as described; this discrepancy is the result either of a miscoded assignment or an error in the priority structure for these students. In our simulated Boston mechanism presented in Table 12, none of these students receive their second choice.

Table 8— 2001-2002 Unassigned Students with Top Two Choices Overdemanded with Random Priority at First in System[a]

		Zone			Subsidized		Race				
	Total	East	North	West	Lunch	Female	Black	White	Hisp.	Other	Bilingual
Elementary school applicants											
Unassigned	66	32	8	26	71%	47%	60%	20%	20%	0%	6%
Middle school applicants											
Unassigned	249	127	61	61	81%	51%	62%	7%	26%	6%	2%
High school applicants											
Unassigned	143	82	21	40	71%	66%	59%	5%	28 %	7%	1%

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on definition of student population in appendix. When demographic information is missing, it is unreported.

Table 9— 2001-2002 Differences in MCAS Passing Rates at Schools for Unassigned Students with Top Two Choices Overdemanded with Random Priority at First in System[a]

	MCAS Reading Passing Rate[b]		MCAS Math Passing Rate	
	Average for Walk Priority School	Average for Eventually Assigned School	Average for Walk Priority School	Average for Eventually Assigned School
Elementary school applicants	65%	62%	51%	50%
Middle school applicants	68%	63%	20%	16%
High school applicants	47%	25%	20%	14%

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on definition of student population in appendix.

[b]Massachusetts Comprehensive Assessment System (MCAS) reading and math scores correspond to school-wide averages in 2000. For grade K2, the score refers to the fourth grade test, for grade 6, the score refers to the eighth grade test, and for grade 9, the score refers to the tenth grade test. This is the information available in the 2001 School Guide used when applicants make their choices.

Table 10— 2001-2002 Unassigned Student Characteristics[a]

	Number	Fraction Ranking ≥ 3	Subsidized Lunch	Female	Race: Black	Race: White	Race: Hisp.	Race: Other	Bilingual	Withdrew[b]
Elementary school applicants										
East	63	86%	48%	45%	62%	21%	14%	3%	5%	37%
North	48	73%	50%	54%	19%	31%	40%	10%	21%	46%
West	91	91%	73%	51%	24%	56%	14%	7%	2%	61%
Total	202	85%	61%	50%	34%	39%	20%	7%	8%	50%
Middle school applicants										
East	193	91%	69%	52%	73%	6%	16%	5%	2%	26%
North	168	81%	74%	58%	43%	12%	35%	9%	2%	26%
West	115	85%	51%	49%	51%	15%	32%	3%	3%	35%
Total	476	86%	67%	53%	57%	10%	27%	6%	2%	28%
High school applicants										
Total	302	93%	64%	61%	60 %	8%	25%	6%	1%	23%

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on definition of student population in appendix. When demographic information is missing, it is unreported.

[b]Students who withdraw may either attend parochial, private or charter schools or have moved outside BPS, withdrew with unknown status or left the system for other reasons (expulsion, illness, pregnancy, etc.)

Table 11— 2001-2002 Differences in Schools' MCAS[a] Passing Rates Schools for Unassigned

	MCAS[b] Reading Pass Rate	MCAS Math Pass Rate
Elementary school applicants		
Eventual school[c]	63%	50%
Desired schools[d]	73%	67%
Middle school applicants		
Eventual school	63%	16%
Desired schools	82%	36%
High school applicants[e]		
Eventual school	23%	14%
Desired schools	32%	16%

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on definition of student population in appendix. Passing rate at the ranked schools is larger than eventual school with p-value of 0.01.

[b]Massachusetts Comprehensive Assessment System (MCAS) reading and math scores correspond to school-wide averages in 2000. For grade K2, the score refers to the fourth grade test, for grade 6, the score refers to the eighth grade test, and for grade 9, the score refers to the tenth grade test. This is the information available in the 2001 School Guide used when applicants make their choices.

[c]Eventual school is the school the student is in in June of the following school year, if the student is still in the system.

[d]Desired school is any school on an unassigned student's rank order list. The pass rates for desired schools are averaged across schools ranked.

[e]Specialized high schools such as Boston Latin are not included in this sample because they are not assigned through the centralized admissions process.

Table 12 — 2001-2002 Replication of Boston Mechanism[a]

Stated Choice Received	1st	2nd	3rd	4th	5th	Unassigned	Fraction Different
Elementary school applicants							
Boston	2,599	301	131	61	33	202	
Our Replication	2,590	309	103	23	12	289	8.3%
Middle school applicants							
Boston	4,170	411	298	61	26	473	
Our Replication	4,197	417	269	44	17	485	7.6%
High school applicants							
Boston	5,497	428	100	42	11	301	
Our Replication	5,483	407	158	38	6	285	5.9%

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on construction of student population in the appendix.

Table 13— Individual Deviations for Unassigned Students[a]

	Number of applicants	Number unassigned by BPS	Number who could deviate[b]	Percent who benefit	Stated choice received after deviation				# of choices removed			
					2nd	3rd	4th	5th	1	2	3	4
Elementary school applicants	3,326	202	159	64%	49	33	12	7	78	16	6	1
Middle school applicants	5,429	473	362	61%	115	72	23	12	168	40	11	3
High school applicants	6,380	301	198	84%	89	69	7	2	115	45	6	1

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on implementation of algorithms and the definition of student population in the appendix. An individual deviation for this table is defined as submitted a rank order list that differs from the actual list by removing the first n schools from the top of the list, where $n = 1, 2, 3, 4$. Students who benefit are those who receive one of their stated choices after an individual deviation.

[b]Students who could deviate are those who are both unassigned by BPS and by our replication of BPS's allocation who rank at least two schools and are unassigned.

Table 14— 2001-2002 Naive Comparison of Mechanisms[a]

Stated Choice	Boston Mechanism		Student-proposing Deferred Acceptance		Top Trading Cycles Mechanism	
	number	percent	number	percent	number	percent
Panel A: Elementary school applicants						
1st	2,590	77.9	2,451	73.7	2,464	74.1
2nd	309	9.3	419	12.6	410	12.3
3rd	103	3.1	173	5.2	171	5.1
4th	23	0.7	55	1.7	55	1.7
5th	12	0.4	23	0.7	23	0.7
Unassigned	289	8.7	205	6.2	203	6.1
Panel B: Middle school applicants						
1st	4,197	77.3	3,922	72.2	3,938	72.5
2nd	417	7.7	701	12.9	689	12.7
3rd	269	5.0	328	6.0	317	5.8
4th	44	0.8	75	1.4	68	1.3
5th	17	0.3	23	0.4	23	0.4
Unassigned	485	8.9	380	7.0	394	7.3
Panel C: High school applicants						
1st	5,486	86.0	5,261	82.5	5,328	83.5
2nd	407	6.4	624	9.8	587	9.2
3rd	158	2.5	236	3.7	202	3.2
4th	38	0.6	36	0.6	31	0.5
5th	6	0.1	7	0.1	6	0.1
Unassigned	285	4.5	216	3.4	226	3.5

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on construction of student population in the appendix. All three mechanisms defined in main text and priority structure defined in the appendix. The numbers for the Boston mechanism refer to our replication of the mechanism, not the outcome from the actual mechanism.