

# TENANCY RENT CONTROL AND CREDIBLE COMMITMENT IN MAINTENANCE\*

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## Abstract

Under tenancy rent control, rents are regulated within a tenancy but not between tenancies. This paper investigates the effects of tenancy rent control on housing quality, maintenance, and rehabilitation. Since the discounted revenue received over a fixed-duration tenancy depends only on the starting rent, intuitively the landlord has an incentive to spruce up the unit between tenancies in order to “show” it well, but little incentive to maintain the unit well during the tenancy. The paper formalizes this intuition, and presents numerical examples illustrating the efficiency loss from this effect.

**Keywords:** tenancy rent control, rent control, maintenance, housing quality, rehabilitation, credible commitment

**JEL Classification:** R21, R38

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# Tenancy Rent Control and Credible Commitment in Maintenance

## 1 Introduction

Tenancy rent control is a form of rent control in which rents are regulated within a tenancy but may be raised without restriction between tenancies; more specifically, the starting rent for a tenancy is unregulated but the path of nominal rents within a tenancy, conditional on the starting rent, is regulated, typically causing rents to rise less rapidly over the tenancy than they would in the absence of controls<sup>1</sup>. Many, perhaps most, jurisdictions around the world that previously had traditional first- and second-generation rent control programs (Arnott (1995)) have moved in the direction of tenancy rent control as a method of partial decontrol<sup>2</sup>.

In jurisdictions that have stricter forms of rent control, tenancy rent control may be an attractive method of partial decontrol. Because the starting rent adjusts to clear the market, tenancy rent control does not generate the excess demand phenomena (such as key money, waiting lists, and discrimination) of stricter rent control programs, and should have less adverse effects on tenant mobility and the matching of households to housing units<sup>3</sup>. Tenancy rent control continues to provide sitting tenants with improved security of tenure; for one thing, rent regulation within tenancies precludes economic eviction; for another, because tenancy rent control, like other forms of rent control, provides landlords with an incentive to evict tenants, it is invariably accompanied by conversion (rehabilitation, demolition and reconstruction, and conversion to condominium) restrictions<sup>4</sup>. As well, tenancy rent control may be a politically attractive method of partial decontrol since it continues to provide rent protection to sitting tenants, who are typically the strongest opponents of decontrol. These benefits must be weighed against the costs. The most obvious costs are the tenant lock-in created by tenancy rent control and the unfairness of the preferential treatment of sitting tenants. There are also less obvious costs. The workability of tenancy rent control makes it more difficult to move to complete decontrol, should this be deemed desirable. Also, because a rent control administration is kept in place, it is relatively easy to return to harder controls should the political winds change. Landlords, fearing this, may curtail investment<sup>5</sup>.

This paper focuses on another less obvious cost of tenancy rent control – its adverse effect on maintenance, construction, demolition and reconstruction, and rehabilitation. Pollakowski (1999) provides an empirical analysis of the effects of New York City’s rent control system on housing

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<sup>1</sup>This defines the “ideal type”, which is what will be modeled in this paper. Many jurisdictions have forms of rent control that are intermediate between tenancy rent control, according to the above definition, and more traditional forms of rent control. In some, rent increases are regulated both within and between tenancies, but less severely between tenancies than within tenancies. In others, rent increases are unregulated between tenancies but are subject to a variety of regulatory provisions within a tenancy, such as a guideline rent increase (which allows rents to rise by a certain percentage per year) with a cost-pass through provision (which allows the landlord to apply for a rent increase above the guideline rent increase if justified by cost increases).

<sup>2</sup>Basu and Emerson (2000, 2003) and Arnott (2003) list some of these jurisdictions. Borsch-Supan (1996) models the current German system and Iwata (2002) the current Japanese system, both of which are termed “tenant protection” systems.

<sup>3</sup>There is a large literature on the adverse effects of rent control. Three particularly good papers that avoid polemical rent-control bashing are Frankena (1975), Glaeser and Luttmer (2003), and Olsen (1988).

<sup>4</sup>Miron and Cullingworth (1983) and Hubert (1991) examine the effects of rent control on security of tenure.

<sup>5</sup>These less obvious costs are evident in the Ontario experience with rent control (e.g., Smith, 2003).

maintenance there. Arnott and Johnston (1981) provides an informal, diagrammatic discussion of the effects of several rent control programs (though not tenancy rent control) on housing quality and maintenance. This paper will adapt the model of Arnott, Davidson, and Pines (1983) to examine how the application of tenancy rent control to a single atomistic landlord-builder affects his profit-maximizing behavior<sup>6</sup>.

Assume, as we will throughout the paper in order to abstract from the tenant lock-in effect, that tenancy duration is exogenous. There are two conflicting intuitions concerning the effects of tenancy rent control on the atomistic landlord's behavior. A lay person with good economic intuition would probably argue that tenancy rent control gives the landlord an incentive to spruce up his units between tenancies so that they "show" well and hence can be let at a higher starting rent, but little incentive to maintain the units well during tenancies since, after the starting rent has been agreed upon, maintaining well has no effect on the rent stream during the tenancy. An economist might however reasonably object that, with tenancy duration exogenous, there is nothing to prevent the landlord from following the program that is profit maximizing in the absence of tenancy rent control – which we shall term the *efficient program*. If the landlord follows this program, the tenant should be willing to pay as much over her tenancy as she would have for an uncontrolled unit. This line of reasoning suggests that, were it not for the tenancy lock-in, the landlord's profit-maximizing program would be unaffected by the application of tenancy rent control.

The resolution of the two conflicting intuitions lies in the ability of the landlord to credibly commit to the efficient program. If he is able to credibly commit to a maintenance program, he will credibly commit to the efficient program and the tenant will agree to pay the same in rent in discounted terms over the duration of the tenancy as in the absence of rent control. The landlord will therefore be making the same revenue and incurring the same costs as in the absence of rent control, and can surely do not better than this. If, however, the landlord is unable to credibly commit to pursuing the efficient program, once the lease is signed he has an incentive to pursue a different maintenance program, which we term the *opportunistic program*. Since the signing of the lease fixes the discounted rent the landlord will receive over the current tenancy, the only incentive he has to maintain is to improve the quality of the unit at the end of the lease, as this will increase the discounted rent he receives on subsequent tenancies. Compared to the efficient program, the opportunistic program entails both a reduction in average maintenance and a postponement of maintenance within a tenancy. Before the lease is signed, a prospective tenant should in this situation realize that under tenancy rent control the landlord will pursue the opportunistic rather than the efficient maintenance program and hence not be willing to pay as high a starting rent as she would if he were to pursue the efficient program.

The crux of the matter is therefore the landlord's ability, under tenancy rent control, to commit to a particular maintenance program. Three commitment mechanisms might be partially effective. The first is contracting on maintenance. One problem with this commitment mechanism is that, since maintenance is such an amorphous concept, maintenance clauses in the lease would be highly incomplete; for example, if the contract were to require the landlord to replace appliances every ten years, he might replace with appliances that are used and reconditioned or of minimal quality. Another problem is that it would be costly for a tenant to document that her landlord had not met

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<sup>6</sup>Since the analysis is "very" partial equilibrium, it will ignore the effects of tenancy rent control on the level of rents and on other markets such as the labor market.

While the paper focuses on tenancy rent control, the techniques employed can be applied to examine the effects of other forms of rent control on the landlord's optimal program (indeed, Arnott and Johnston (1981) does so, albeit informally).

the maintenance terms of the contract. The second commitment mechanism, reputation, is likely to be ineffective since the typical prospective tenant knows little or nothing about different landlords' maintenance performance when she is searching for a unit. The third mechanism, maintenance regulation, suffers from problems similar to those for contracting on maintenance. In our judgment, such commitment devices are generally ineffective, and in our analysis we shall assume them to be completely ineffective. The efficiency costs that we identify are reduced to the extent that these commitment mechanisms are indeed effective.

Section 2 analyzes the landlord's profit-maximizing program in the absence of rent control. Section 3 examines how tenancy rent control in the absence of credible commitment in maintenance distorts the profit-maximizing program. Section 4 provides some calibrated examples focusing on the magnitude of the efficiency loss caused by tenancy rent control. And section 5 concludes.

## 2 The Profit-Maximizing Program without Rent Control

A competitive landlord owns a vacant lot of fixed area on which only a single unit of housing can be constructed<sup>7</sup>. Housing is durable and its quality is endogenous. Four quality-changing technologies are available: construction, maintenance, rehabilitation, and demolition. The economic environment is stationary over time and described by the quality-changing technologies, the rent function relating market rent to quality, and the interest rate. The maintenance technology is autonomous – the unit's rate of quality change depends on its current quality and the current level of maintenance expenditure but not on the unit's age *per se*. The landlord chooses the profit-maximizing program. Under these assumptions, phase plane analysis may be employed.

A rather thorough analysis of this problem is presented in Arnott, Davidson, and Pines (1983). Here we focus on a special – but also probably the most realistic – case, in which, in the absence of controls, at the beginning of the program it is profit maximizing to construct and downgrade. Three qualitatively different active programs may be profit maximizing:

1. Initial construction, followed by downgrading to saddlepoint quality (program  $\mathcal{S}$ ).
2. A construction-downgrading-demolition cycle (program  $\mathcal{D}$ ).
3. Initial construction followed by a downgrading-rehabilitation cycle (program  $\mathcal{R}$ ).

### 2.1 Program $\mathcal{S}$

Under program  $\mathcal{S}$ , at time 0 the landlord constructs a single housing unit of quality  $q_c$  on his lot and then downgrades the unit asymptotically to saddlepoint quality  $q^S$ . Where  $q(t)$  is quality at time  $t$ ,  $P(q)$  the exogenous rent function,  $m(t)$  maintenance expenditure at time  $t$ ,  $r$  the interest rate,  $\alpha$  construction cost per unit of quality,  $g(q, m)$  the depreciation function, and  $T$  the terminal

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<sup>7</sup>The analysis can be extended to endogenize structural density (Arnott, Davidson, and Pines (1986)).

time, the profit-maximizing program is the solution to

$$\begin{aligned} & \max_{q_c, m(t)} \int_0^{\infty} (P(q(t)) - m(t)) e^{-rt} dt - \alpha q_c \\ & \text{s.t.} \quad \begin{aligned} & i) \quad \dot{q} = g(q, m) \\ & ii) \quad q_c \equiv q(0) \quad \text{free} \\ & iii) \quad \lim_{T \uparrow \infty} q(T) \quad \text{free} \end{aligned} \end{aligned} \tag{1}$$

Note that quality is measured as some fraction of construction costs, and that tenant maintenance is not considered. We impose non-negativity conditions on  $q$  and  $m$ . Where  $'$ s denote derivatives and subscripts partial derivatives, we also impose reasonable restrictions on the functions  $P$  and  $g$ : i)  $P(0) = 0$ ,  $P'(q) > 0$  and  $P''(q) < 0$ ; and ii)  $g_q < 0$ ,  $g_m(q, 0) = \infty$ ,  $g(q, 0) < 0$ ,  $g_m(q, \infty) = 0$ ,  $g_m > 0$ ,  $g_{mm} < 0$ . Thus, rent increases with quality but at a diminishing rate; there are positive but diminishing returns to maintenance; holding fixed the rate of quality deterioration, more has to be spent on maintenance as quality increases; and with zero maintenance, the unit deteriorates. In our numerical examples, the first-order conditions of the  $\mathcal{S}$  program will define a unique interior maximum.

We solve the problem using optimal control theory (Kamien and Schwartz (1991)). The current-value Hamiltonian corresponding to (1) is

$$\mathcal{H}^o = P(q(t)) - m(t) + \phi(t) g(q(t), m(t)), \tag{2}$$

where  $\phi(t)$  is current-value co-state variable on  $\dot{q} = g(q, m)$ . The first-order condition<sup>8</sup> for maintenance is

$$-1 + \phi(t) g_m(q(t), m(t)) = 0. \tag{3}$$

Since  $\phi(t)$  is the marginal value of quality at time  $t$ , and  $g_m(q(t), m(t))$  the amount by which quality is increased by an extra dollar's expenditure on maintenance,  $\phi g_m$  is the marginal benefit from maintenance. Thus, at each point in time, maintenance should be such that marginal benefit equals marginal cost. The conditions imposed on  $g_m$  guarantee that there is a unique, interior optimal level of maintenance expenditure for all non-negative values of  $q$  and  $\phi$ ; thus, we may write  $m = m(q, \phi)$  with  $m_\phi > 0$ . Inserting this function into (2) yields the maximized current-value Hamiltonian:

$$\mathcal{H}(q, \phi) = P(q) - m(q, \phi) + \phi g(q, m(q, \phi)). \tag{4}$$

The equation of motion of the co-state variable is

$$\dot{\phi} = r\phi - \mathcal{H}_q = r\phi - P' - \phi g_q. \tag{5}$$

The assumptions thus far have not ruled out the possibility that the optimal saddlepoint program entails upgrading to saddlepoint quality via maintenance alone. We assume that the maintenance and construction technologies are such that the optimal saddlepoint program entails construction at the start of the program. The transversality condition with respect to  $q_c$  is then

$$\phi(0) = \alpha; \tag{6}$$

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<sup>8</sup>Throughout the analysis we shall omit second-order conditions as we compare the profit-maximizing programs with and without rent controls, for which the second-order conditions will hold. We shall also omit non-negativity conditions. In the numerical examples of section 4, we explicitly verify that non-negativity conditions hold.

construction quality should be increased up to the point where the marginal value of quality equals its marginal cost.

We are now in a position to construct the phase plane corresponding to this program. We assume that: i) the  $\dot{q} = 0$  locus is positively sloped; ii) the  $\dot{\phi} = 0$  locus is negatively sloped; and iii) the  $\dot{q} = 0$  locus and  $\dot{\phi} = 0$  locus intersect in the positive orthant. Thus, there is a unique saddlepoint,  $S = (q^S, \phi^S)$ . We assume furthermore that  $\phi^S > \alpha$ , unless otherwise noted. Figure 1 displays a phase plane consistent with these assumptions. As is the case for all the figures, Figure 1 is drawn for the functional forms and parameters used in the series of numerical examples presented in Section 4.

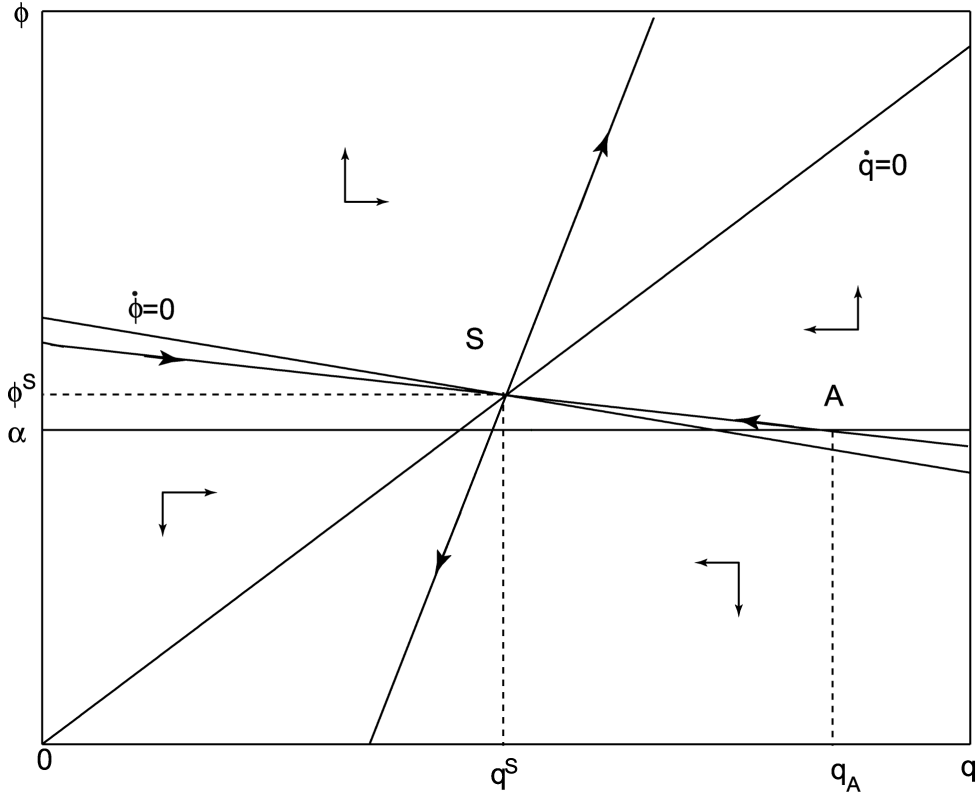


Figure 1: Phase plane for construction with downgrading to the steady state. Construction cost ( $\alpha \cdot 10^5$ ) is \$60,000.

We also have the infinite horizon transversality conditions associated with terminal quality and terminal time. Arnott, Davidson, and Pines (1983) proves that, under the assumptions made, these conditions imply that the optimal trajectory must terminate at the saddlepoint. Putting together the necessary conditions for optimality, we obtain that the  $\mathcal{S}$  program entails construction at that quality at which the right stable arm intersects the  $\phi = \alpha$  line, followed by downgrading along the stable arm to the saddlepoint.

For an autonomous optimal control problem with discounting, the value of the program at any time along an optimal trajectory equals the value of the Hamiltonian at that time divided by the interest rate:

$$V(t) = \frac{\mathcal{H}(q(t), \phi(t))}{r}.$$

The economic interpretation is that the value of the Hamiltonian gives the economic return per unit time from owning the program, which includes the net (of expenses and depreciation) earnings stream it generates plus capital gains, and competitive asset pricing requires that the net return per unit time from owning an asset equal the asset price times the discount rate.

With some abuse of notation, we denote the value of the maximized Hamiltonian at a point labeled  $X$  in the phase plane by  $\mathcal{H}(X)$ . The value of the program immediately after initial construction is then  $\frac{\mathcal{H}(A)}{r}$ , so that the value of the program immediately before initial construction, which is the value of the  $\mathcal{S}$  program, is  $V^{\mathcal{S}} = \frac{\mathcal{H}(A)}{r} - \alpha q_A$ .

## 2.2 Program $\mathcal{D}$

Consider next program  $\mathcal{D}$ , which entails a construction-demolition cycle, where  $q_s$  is the starting quality for each cycle. The landlord's profit-maximizing program is the solution to<sup>9</sup>

$$\begin{aligned} \max_{q_s, q_T, T, m(t)} & \frac{1}{1 - e^{-rT}} \left\{ \int_0^T (P(q(t)) - m(t)) e^{-rt} dt - \alpha q_s \right\} \\ \text{s.t.} \quad & i) \quad \dot{q} = g(q, m) \\ & ii) \quad q_s \equiv q(0) \text{ free} \\ & iii) \quad q_T \equiv q(T) \text{ free} \\ & iv) \quad T \text{ free} \end{aligned} \tag{7}$$

Let  $J(q_s, q_T, T)$  denote the maximized value of the expression in curly brackets, which is the present value of net revenue from a single cycle as a function of  $q_s$ ,  $q_T$ , and  $T$ . Then (7) can be rewritten as

$$\max_{q_s, q_T, T} \frac{1}{1 - e^{-rT}} J(q_s, q_T, T).$$

We assume that the  $\mathcal{D}$  program entails construction at the beginning of each cycle. Eqs. (2) through (6) continue to apply. The transversality condition for  $q_T$  is

$$\phi(T)q(T) = 0, \tag{8}$$

which indicates that the building's quality should be run down until the optimal trajectory intersects one of the axes in the phase plane. If the optimal trajectory intersects the  $q$ -axis, as will be the case in all our numerical examples, the condition is that  $\phi(T) = 0$ ; the building's quality should be run down until, at the end of the cycle, the marginal value of quality is zero. The transversality condition for  $T$  is

$$\mathcal{H}(q(T), \phi(T)) + r\alpha q_s = \mathcal{H}(q(0), \phi(0)); \tag{9}$$

the left-hand side is the marginal benefit from postponing demolition and reconstruction, the right-hand side the marginal cost. We can provide a useful geometric depiction of this transversality condition. Now,

$$\mathcal{H}(q(0), \phi(0)) - \mathcal{H}(q(T), \phi(T)) = \int_{q_T}^{q_s} \left( \mathcal{H}_q + \mathcal{H}_\phi \left( \frac{d\phi}{dq} \right) \right)^* dq,$$

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<sup>9</sup>The analysis can be straightforwardly extended to treat demolition costs.





below which it is. Thus, a  $\mathcal{D}$  program exists for construction costs below a critical level, but not otherwise.

If a  $\mathcal{D}$  program exists, which is more profitable, the  $\mathcal{D}$  program or the  $\mathcal{S}$  program? We have already demonstrated that the value of the  $\mathcal{S}$  program immediately prior to construction is  $V^{\mathcal{S}} = \frac{\mathcal{H}(A)}{r} - \alpha q_A$ . An analogous line of reasoning establishes that the value of the  $\mathcal{D}$  program is  $V^{\mathcal{D}} = \frac{\mathcal{H}(C)}{r} - \alpha q_C$ . Now,  $\mathcal{H}_A - \mathcal{H}_C = \int_{q_C}^{q_A} \mathcal{H}_q(q, \alpha) dq = \int_{q_C}^{q_A} (r\alpha - \phi) dq$  (from (5)). Thus,  $\left(\frac{\mathcal{H}_A}{r} - \alpha q_A\right) - \left(\frac{\mathcal{H}_C}{r} - \alpha q_C\right) = \int_{q_C}^{q_A} -\phi dq$  along  $\phi = \alpha$ , which can be seen to be negative. Thus, if a  $\mathcal{D}$  program exists, it is more profitable than the  $\mathcal{S}$  program. It can also be shown that if a  $\mathcal{D}$  program does not exist, the optimal  $\mathcal{S}$  program is more profitable than any construction-demolition cycle program. Thus, the construction-demolition cycle program is more profitable than the saddlepoint program when construction costs are below the critical value, and the saddlepoint program is more profitable than any construction-demolition cycle program when construction costs are above the critical value, which accords with intuition.

### 2.3 Program $\mathcal{R}$

The final option is a rehabilitation cycle, which entails constructing at quality  $q_c$ , downgrading to quality  $q_T$ , rehabbing up to quality  $q_s$ , and then repeating the downgrading-rehabilitation cycle. Discounted net rents

$$\begin{aligned} & \max_{q_c, q_s, q_T, T_c, T, m(t)} \int_0^{T_c} (P(q(t)) - m(t)) e^{-rt} dt - \alpha q_c \\ & + \frac{e^{-rT_c}}{1 - e^{-rT}} \left[ \int_0^T (P(q(t)) - m(t)) e^{-rt} dt - R(q_s, q_T) \right], \end{aligned}$$

are maximized with respect to  $q_c$ ,  $q_s$ ,  $q_T$ ,  $T_c$ ,  $T$ , and  $m(t)$  where  $T_c$  is the length of time from construction to the first rehab,  $T$  the length of the rehabilitation cycle, and  $R(q_s, q_T)$  the cost of rehabbing a unit of quality  $q_T$  to quality  $q_s$ . It is assumed that it remains profitable to construct initially, so that (3) through (6) continue to apply. The transversality conditions are

$$\begin{aligned} q_c & : \phi_c = \alpha \\ T & : \mathcal{H}(q_T, \phi_T) + rR(q_s, q_T) = \mathcal{H}(q_s, \phi_s) \\ q_s & : \phi(0) = \phi_s = \frac{\partial R}{\partial q_s} \\ q_T, q_T & : \phi(T_c) = \phi(T) = \phi_T = \frac{\partial R}{\partial q_T} \\ T_c & : \mathcal{H}(q_{T_c}, \phi_{T_c}) = \mathcal{H}(q_T, \phi_T) \end{aligned}$$

In our numerical examples, we shall assume that the function  $R(q_s, q_T)$  is strongly separable in  $q_s$  and  $q_T$ , i.e.  $R(q_s, q_T) = R_1(q_s) - R_2(q_T)$ . Figure 3 plots a configuration of the phase plane for which the rehabilitation cost function is linear in the two quality levels. Here too the timing transversality condition can be displayed as an equal areas condition, that Area NQR equals Area RTUV. Adapting the argument used in the previous two subsections, it can be shown that the

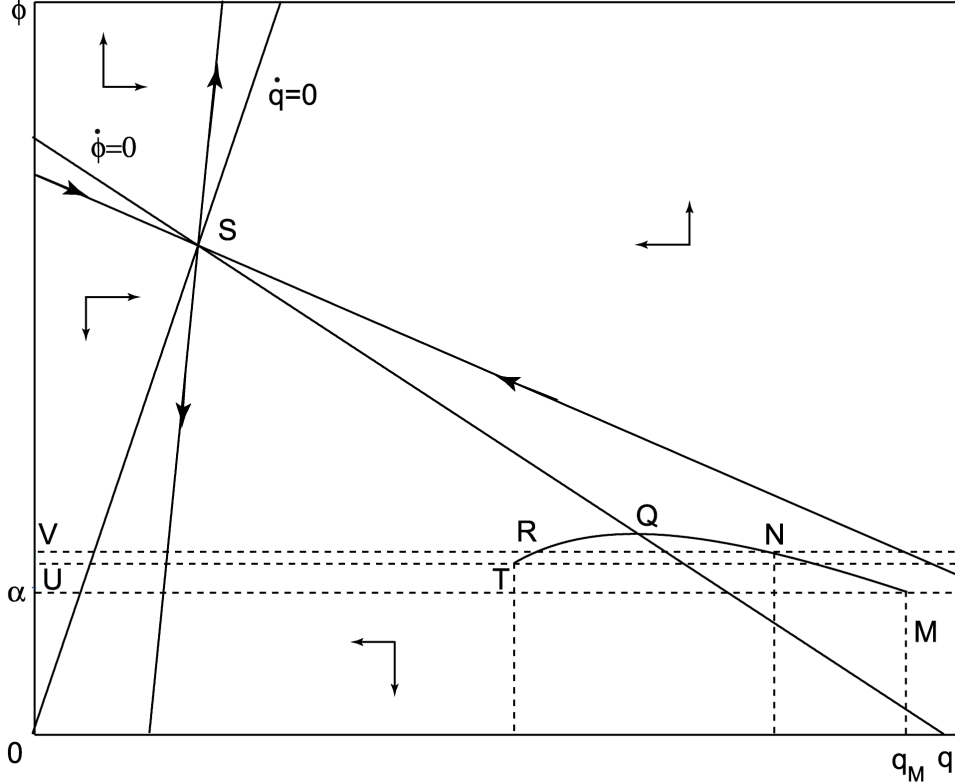


Figure 3: Phase plane for a rehabilitation cycle. Construction cost ( $\alpha \cdot 10^5$ ) is \$20,000.

value of the  $\mathcal{R}$  program is  $\mathcal{H}_M/r - \alpha q_M$ .

Applying the same line of reasoning as in the previous subsection, it can be shown that if the  $\mathcal{R}$  program exists, it is more profitable than the  $\mathcal{S}$  program, and that if the  $\mathcal{R}$  program does not exist, the  $\mathcal{S}$  program is more profitable than any program entailing rehabilitation. It remains to compare the profitabilities of the  $\mathcal{R}$  program and the  $\mathcal{D}$  program, if both exist. Both start on the  $\phi = \alpha$  line between where it intersects the right stable arm and the  $\dot{\phi} = 0$  line. The argument employed in the previous subsection to prove that, if the  $\mathcal{D}$  program exists, it is more profitable than the  $\mathcal{S}$  program, can be adapted to prove that if both the  $\mathcal{D}$  and the  $\mathcal{R}$  program exist, the one which starts further to the left on the  $\phi = \alpha$  line is the more profitable. An upward shift of the  $R_1(q_s)$  function or a downward shift of the  $R_2(q_T)$  function reduces the profitability of the  $\mathcal{R}$  program relative to the  $\mathcal{S}$  program and the  $\mathcal{D}$  program.

In section 4 we shall present a series of related numerical examples, indicating different sets of parameter values for which each of programs  $\mathcal{S}$ ,  $\mathcal{D}$ , and  $\mathcal{R}$ , are profit maximizing.

### 3 The Profit-maximizing Program with Tenancy Rent Control

We model tenancy rent control as a ceiling on the time path of rents over the duration of a tenancy, conditional on the starting rent<sup>10</sup>. Letting  $p_s$  denote the starting rent,  $u$  the length of time into the tenancy, and  $F(p_s, u)$  (with  $\partial F/\partial p_s > 0$ ) the rent control function – the maximum allowable rent  $u$  years into a tenancy, conditional on  $p_s$  – a tenancy rent control program imposes the constraint that  $\hat{P}(u) \leq F(p_s, u)$ , where  $\hat{P}(u)$  is the rent charged by the landlord  $u$  years into the tenancy.

We shall examine the effects of tenancy rent control applied to a single housing unit when all other units are uncontrolled; the analysis is therefore partial equilibrium. We make a number of simplifying assumptions:

**Assumption 1** *The length of a tenancy is exogenous at  $L$ .*

This assumption is made for two reasons. First, we wish to abstract from the effect of tenancy rent control on tenancy duration, in order to focus on its effects on landlord maintenance and conversion. Second, the assumption takes into account that tenancy rent control is invariably accompanied by restrictions on eviction<sup>11</sup>. Since tenancy rent control front-end loads rent over a tenancy, shorter tenancies are more profitable for landlords. In the absence of restrictions on eviction, tenancy rent control would therefore provide landlords with an incentive to evict tenants<sup>12</sup>. Under the assumption, the landlord is able to rehabilitate or to demolish-and-reconstruct only between tenancies.

**Assumption 2** *The rent control function is such that the landlord finds it profit maximizing to charge the maximum controlled rent over the duration of a tenancy, i.e.  $\hat{P}(u) = F(p_s, u)$ .*

This assumption states that, under the opportunistic program, the time path of controlled rents over a tenancy are sufficiently “front-end loaded” relative to the time path of market rents that the tenancy rent control constraint binds strictly throughout the tenancy. While not primitive, this assumption greatly simplifies the analysis since otherwise the possibility would have to be considered that the rent control constraint binds over some quality intervals of a tenancy but not over others.

**Assumption 3** *Tenants are identical.*

**Assumption 4** *Tenants face perfect capital markets and discount financial flows at the same rate as the landlord.*

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<sup>10</sup>There are tenancy rent control programs that restrict the percentage increase in rent from one year to the next. Under such a program, a landlord might find it profit maximizing to charge less than the maximum allowable rent increase for some time interval during a tenancy, in which case the ceiling on the time path of rents would thereafter be determined by the rent level at the time the percentage increase regulation again becomes binding. Thus, our modeling of tenancy rent control entails a simplification.

<sup>11</sup>We use the term eviction to mean that the tenant is required to leave her unit even though she would prefer not to, rather than in the legal sense.

<sup>12</sup>Tenancy rent control rules out economic eviction (raising rents to force a tenant out) but at least in North America, where annual tenancies are the norm, a landlord can evict a tenant in some jurisdictions simply by choosing not to renew the lease, and in others by citing as just causes minor lease violations or his intention to lease the unit to a family member, convert it to owner occupancy, or rehabilitate it.

With identical tenants, the market rent as a function of quality adjusts so that a renter receives the same utility at all quality levels. Thus, under tenancy rent control, a tenant is indifferent between living in a controlled and uncontrolled unit if and only if the discounted value of controlled rents over the tenancy equals the discounted value of market rents for the same quality path, discounted at her discount rate. The assumption that the tenant’s discount rate is the same as the landlord’s is made to simplify the analysis.

Under the above assumptions, the opportunistic program is independent of the form of the rent control function. A proof runs as follows. Suppose that the profit-maximizing program with a particular rent control function has been solved for. Now modify the rent control function, holding constant the program but allowing the starting rents for each tenancy to adjust so that tenants remain indifferent between controlled and uncontrolled housing. The profitability of the program remains unchanged and the landlord cannot improve profitability by altering the program. Without ambiguity, we may then let  $\hat{q}(u; q_s)$  denote the time path of quality over a tenancy under the opportunistic program, conditional on starting quality  $q_s$ . And the condition that, with the opportunistic program, over each tenancy the discounted value of controlled rents equals the discounted value of market rents may be written as

$$\int_0^L F(p_s, u) e^{-ru} du = \int_0^L P(\hat{q}(u; q_s)) e^{-ru} du.$$

Thus, under the above assumptions, it is the imposition of tenancy rent control rather than its severity<sup>13</sup> that matters since it is the imposition of tenancy rent control that undermines the credibility of the efficient program.

In the analysis of the previous section, without rent control, there were three qualitatively different optimal programs for the landlord, the  $\mathcal{S}$  program, the  $\mathcal{D}$  program, and the  $\mathcal{R}$  program. The same three qualitatively different optimal programs are present under tenancy rent control.

### 3.1 Program $\hat{\mathcal{S}}$

Program  $\hat{\mathcal{S}}$  under tenancy rent control is the analog of program  $\mathcal{S}$  in the absence of rent control. Under our assumptions concerning the characteristics of the maintenance and construction technologies, program  $\mathcal{S}$  entails construction followed by downgrading to steady-state quality. Program  $\hat{\mathcal{S}}$ , too, entails construction followed by downgrading from one tenancy to the next, but maintenance follows a sawtooth pattern, increasing within each tenancy and then falling discontinuously from the end of one tenancy to the start of the next. The program converges to a steady-state tenancy maintenance cycle in which quality is highest at the beginning and end of each tenancy, rather than to a steady-state quality.

We decompose solution of the opportunistic program under tenancy rent control during a single tenancy into two stages. In the first stage, we solve the program taking as given not only the initial quality of the unit and the duration of the tenancy but also the terminal quality. In the

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<sup>13</sup>A tenancy rent control program is more severe than another if it permits a lower nominal percentage increase in rent every year during a tenancy.

Assumption A.2 is that the tenancy rent control program is sufficiently severe that the landlord finds it profit maximizing to charge the maximum controlled rent over the duration of the tenancy. If the tenancy rent control program is sufficiently “lax” that the landlord finds it profit maximizing to charge the maximum controlled rent over no portion of the tenancy, the program has no effect. Intermediate situations are analytically messy.

second stage, we solve for the profit-maximizing terminal quality. The landlord decides on this program after the lease has been signed, and therefore after his discounted rent over the tenancy has been determined. The first-stage problem entails the minimization of discounted maintenance expenditures needed to achieve terminal quality,  $q_L$ , taking as given the starting quality,  $q_s$ , and the tenancy duration,  $L$ . This is an elementary optimal control program with a well-known solution. Define  $J(q_s, q_L, L)$  to be the value of this program. We shall use three properties of the solution:

$$\partial J / \partial q_s = \phi(0) \quad \partial J / \partial q_L = -\phi(L)e^{-rL} \quad \dot{\phi} = r\phi - \phi g_q \quad (12)$$

where  $\phi(t)$  is the current value of the co-state variable on  $\dot{q} = g(q, m)$ . The first solution property indicates that  $\phi(0)$  is the marginal value of quality at the start of the tenancy, after the tenancy contract has been signed. The second indicates that  $\phi(L)$  is the marginal value of terminal quality at terminal time, so that  $\phi(L)e^{-rL}$  is the marginal value of terminal quality discounted to the beginning of the tenancy. Since the first stage of the problem entails deciding on the maintenance path over the tenancy, after the contract has been signed, we refer to  $\phi$  as the marginal value of quality via maintenance or the *ex post* (viz., after the tenancy contract has been signed) marginal value of quality. The last solution property is that over a tenancy the marginal value of quality via maintenance grows<sup>14</sup> at the rate  $r - g_q$  through the tenancy.

The second stage of the solution of the opportunistic program entails the choice of  $q_L$ . To derive this, we work with a value function. Under tenancy rent control, the value of a housing unit is a function not only of quality but also of how much time remains in the current tenancy contract<sup>15</sup>. Let  $\widehat{V}(q)$  denote the value of a housing unit of quality  $q$  between tenancies, and  $Z(q_s)$  the revenue received over a tenancy contract, discounted to the beginning of the tenancy contract. The landlord decides on the maintenance program, and hence  $q_L$ , after signing the tenancy contract, and therefore after the revenue received over the tenancy has been determined. Then the value function for  $\widehat{V}(q)$  may be written as

$$\widehat{V}(q_s) = Z(q_s) + \max_{q_L} [J(q_s, q_L, L) + \widehat{V}(q_L)e^{-rL}]. \quad (13)$$

Terminal quality is chosen to maximize the expression in square brackets. The corresponding first-order condition is

$$\partial J / \partial q_L + \widehat{V}'(q_L)e^{-rL} = 0. \quad (14)$$

Comparing the second equation in (12) and (14) yields

$$\phi(L) = \widehat{V}'(q_L). \quad (15)$$

Differentiating (13) with respect to  $q_s$  yields

$$\begin{aligned} \widehat{V}'(q_s) &= Z'(q_s) + \partial J / \partial q_s \quad (\text{using the envelope theorem}) \\ &= Z'(q_s) + \phi(0) \quad (\text{using (12)}). \end{aligned} \quad (16)$$

Eq. (16) requires some care in interpretation.  $\widehat{V}'(q_s)$  is the *ex ante* (before the tenancy contract

<sup>14</sup>Suppose the landlord buys an extra unit of quality today at a price of  $\phi$ . Instantaneously, he must make the competitive return on that unit,  $r\phi$ , and the return comprises two components, the capital gain,  $\dot{\phi}$ , minus the depreciation,  $-\phi g_q$ .

<sup>15</sup>Since the housing market remains competitive under rent control, it must still be the case that owning the program for an increment of time between  $u$  and  $u + du$  within a tenancy provides income of  $rV(q(u), u)$ , where  $V(q(u), u)$  is the market value of a controlled housing unit of quality  $q$   $u$  units of time into a tenancy. From this relationship, the rent control function, and the boundary condition that  $\widehat{V}(q_s) = V(q_s, 0)$ ,  $V(q(u), u)$  may be calculated.

has been signed) marginal value of quality at the start of a tenancy, while  $\phi(0)$  is the *ex post* (after the tenancy contract has been signed) marginal value of quality at the start of a tenancy. Eq. (16) indicates that, at starting quality, the *ex ante* marginal value of quality exceeds the *ex post* marginal value of quality by  $Z'(q_s)$ , marginal discounted revenue. Thus, there is a downward jump discontinuity in the marginal value of quality at the time the lease is signed. Now return to (15). It states that, in contrast, the marginal value of quality immediately before the termination of the tenancy equals the marginal value of quality immediately afterwards, in both cases equaling the increase in the property price from a unit increase in terminal quality.

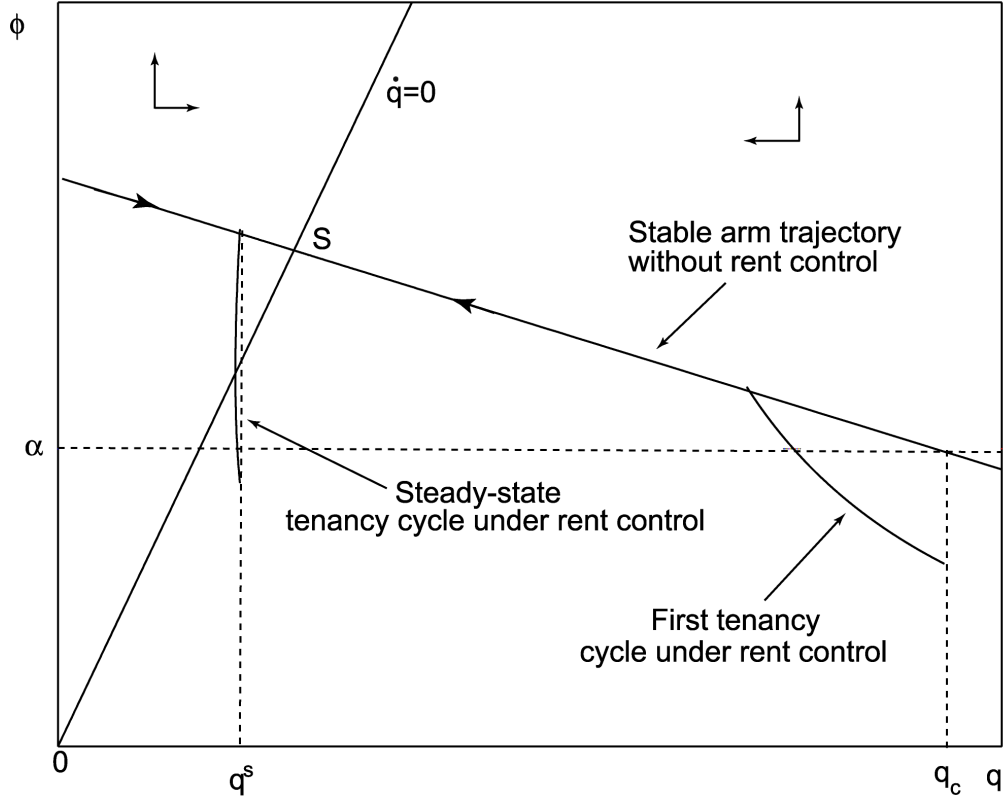


Figure 4: Phase plane for construction-downgrading to the steady-state cycle under rent control. Construction cost ( $\alpha \cdot 10^5$ ) is \$40,000.

The value of the  $\widehat{S}$  program immediately prior to construction is

$$\widehat{V}^{\widehat{S}} = \max_{q_c} [\widehat{V}(q_c) - \alpha q_c]. \quad (17)$$

Assuming an interior solution, the corresponding first-order condition for profit-maximizing construction quality is

$$\widehat{V}'(q_c) - \alpha = 0 \quad (18)$$

Comparing (16) and (18), for the first tenancy, since  $q_c = q_s$ ,

$$\phi(0) = \alpha - Z'(q_c). \quad (19)$$

Construction occurs at that quality level, for which the *ex ante* marginal value of quality via construction equals the marginal cost, while the *ex post* marginal value of construction quality falls

short of marginal construction cost by  $Z'(q_c)$ .

In the steady state, quality varies within a tenancy, but the starting and terminal qualities remain constant from one tenancy to the next. Let  $q_\sigma$  denote the optimal starting and terminal quality of a steady state cycle. Since in a steady-state tenancy  $q_s = q_L = q_\sigma$ ,

$$\widehat{V}(q_\sigma) = \frac{1}{1 - e^{-rL}} \{Z(q_\sigma) + J(q_\sigma, q_\sigma, L)\}.$$

Figure 4 displays the phase diagram of the  $\widehat{\mathcal{S}}$  program for the numerical example, and plots the optimal trajectory for two tenancies, the first tenancy that occurs immediately after construction and the steady-state tenancy. For comparison it also plots the optimal (stable arm) trajectory without rent control. With the depreciation function we employ, maintenance expenditures are positively related to  $\phi$  and independent of  $q$ . The diminished incentive to maintain under tenancy rent control is reflected in the lower position, on average, of the optimal trajectory under tenancy rent control. The incentive under tenancy rent control to postpone maintenance expenditures towards the end of the tenancy is also evident.

### 3.2 Program $\widehat{\mathcal{D}}$

Program  $\widehat{\mathcal{D}}$  under tenancy rent control is the analog of program  $\mathcal{D}$  in the absence of rent control. Recall that, under our assumptions concerning the construction and maintenance technologies, program  $\mathcal{D}$  entails constructing at a quality above saddlepoint quality, downgrading smoothly to demolition, and then repeating the cycle, which has an endogenous length of  $T$ . Recall, too, that if an optimal demolition program exists, it is more profitable than the optimal saddlepoint program. The program  $\widehat{\mathcal{D}}$  differs from program  $\mathcal{D}$  in two important respects. First, because of the assumed fixed duration of a tenancy under tenancy rent control, demolition can occur only between tenancies, so that the length of the demolition cycle must be some integer multiple of  $L$ . Thus, there are two types of cycles, the construction-demolition cycle and the maintenance cycle within each tenancy. Since terminal time is not, therefore, a continuous variable, there will not be a timing transversality condition. Instead, optimal cycle length can be computed by comparing the profit obtained when demolition occurs after every tenancy, after every second tenancy, and so on. Second, under tenancy rent control the commitment problem arises.

Our solution of the  $\widehat{\mathcal{D}}$  program proceeds in two stages<sup>16</sup>. In the first stage, the profit-maximizing program is calculated conditional on the number of tenancies in a construction-demolition cycle. Let  $\widehat{V}^n(q_c)$  denote the value of a housing unit that has just been constructed at quality  $q_c$ , conditional on  $n$  tenancies in the cycle, and  $\mathcal{V}^n$  the value of the optimal program conditional on  $n$  tenancies within a demolition cycle. In the second stage, the corresponding profit levels are compared for different numbers of tenancies within the cycle. In this subsection, we ignore the complications that would arise if the non-negativity constraint on  $q$  would bind.

We start by solving for the optimal program, conditional on the unit being demolished after each tenancy. Once the tenancy contract has been signed, the landlord has no incentive to maintain. Spending on maintenance does not increase the revenue received over the tenancy and the value of the structure is zero at the end of the cycle since it is about to be demolished. The value of the

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<sup>16</sup>Eqs. (13) - (19) apply to the demolition case as well. We proceed as we do in order to provide more insight into the economics, and to motivate the numerical solution algorithm we employ.

program is

$$\mathcal{V}^1 = \max_{q_c} \left\{ \widehat{V}^1(q_c) - \alpha q_c \right\} = \max_{q_c} \frac{1}{1 - e^{-rL}} \{Z(q_c) - \alpha q_c\},$$

from which the first-order condition for profit-maximizing construction quality is straightforward to obtain.

We now solve for the optimal program, conditional on the structure being demolished after two tenancies. It is profit maximizing for the landlord to spend nothing on maintenance during the second tenancy. Let superscript  $i$  on  $q$  denote the order of tenancy within a demolition cycle, so that  $q_L^1$  is terminal quality for the first tenancy, for example. Then

$$\widehat{V}^2(q_c) = Z(q_c) + \max_{q_L^1} [J(q_c, q_L^1, L) + Z(q_L^1)e^{-rL}] + (\widehat{V}^2(q_c) - \alpha q_c)e^{-2rL}.$$

Thus,

$$\widehat{V}^2(q_c) = \frac{1}{1 - e^{-2rL}} \{Z(q_c) + \max_{q_L^1} [J(q_c, q_L^1, L) + Z(q_L^1)e^{-rL}] - \alpha q_c e^{-2rL}\}.$$

Calculate first  $Z(q_L^1)$ . Then solve the maximization problem in square brackets, which yields  $q_L^1$  as a function of  $q_c$ , from which an expression for  $\widehat{V}^2(q_c)$  is obtained. Since the value of the program prior to construction, conditional on construction at quality  $q_c$ , is  $\widehat{V}^2(q_c) - \alpha q_c$ , the final step is to choose  $q_c$  to maximize  $\widehat{V}^2(q_c) - \alpha q_c$ .

This line of reasoning suggests an algorithm for solving for the profit-maximizing program with  $n$  tenancies during a construction-demolition cycle. Let  $v^i(q_s^i, n)$  be the value of revenue net of maintenance expenditures received from the beginning of tenancy  $i$  until the structure is demolished, discounted to the beginning of tenancy  $i$ , conditional on  $q_s^i$  and the number of tenancies within a demolition cycle. Proceed by backward recursion<sup>17</sup>. First, calculate  $v^n(q_s^n, n)$  ( $= Z(q_s^n)$ ). Second, solve

$$\max_{q_s^n} J(q_s^{n-1}, q_s^n, L) + v^n(q_s^n, n)e^{-rL}.$$

Denote by  $q_s^n(q_s^{n-1})$  the value of  $q_s^n$  that solves this maximization problem, as a function of  $q_s^{n-1}$ . Then

$$v^{n-1}(q_s^{n-1}, n) = Z(q_s^{n-1}) + J(q_s^{n-1}, q_s^n(q_s^{n-1}), L) + v^n(q_s^n(q_s^{n-1}), n)e^{-rL}.$$

Return to step 2, but replacing  $n$  by  $n - 1$ , and  $n - 1$  by  $n - 2$ . Proceed recursively backwards until  $v^1(q_s^1, n)$  – the value discounted to construction time of the net revenue received over the life of the building as a function of  $q_s^1 = q_c$ , conditional on  $n$  tenancies – is obtained. Then<sup>18</sup>

$$\mathcal{V}^n = \max_{q_c} \frac{1}{1 - e^{-rnL}} \{v^1(q_c, n) - \alpha q_c\}.$$

If the optimal number of tenancies is finite, then  $n^* = \arg \max_n \{\mathcal{V}^n\}$ , and  $\mathcal{V}^{n^*}$  is the value of the  $\widehat{D}$  program. If the optimal number of tenancies is infinite, we say that an optimal demolition program does not exist.

Figure 5 plots one cycle of the  $\widehat{D}$  program for the numerical example for which the profit-maximizing

<sup>17</sup>This algorithm is inapplicable to the optimal saddlepoint program, since the optimal saddlepoint program contains an infinite number of tenancies.

<sup>18</sup>Alternatively, we may write  $\widehat{V}^n(q_c) = v^1(q_c, n) + (-\alpha q_c + \widehat{V}^n(q_c))e^{-rnL}$ , and obtain  $\mathcal{V}^n$  as the value of  $\widehat{V}^n(q_c) - \alpha q_c$  maximized with respect to  $q_c$ .



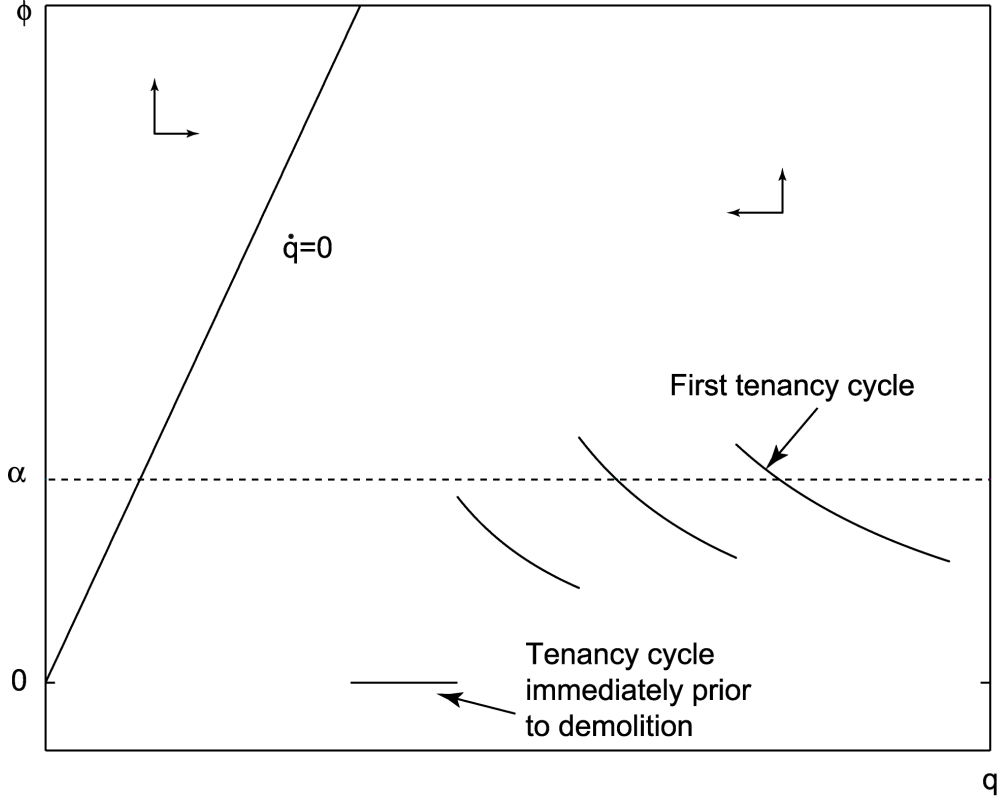


Figure 5: Phase plane for a demolition cycle under tenancy rent control. Construction cost ( $\alpha \cdot 10^5$ ) is \$30,000.

number of tenancies within a demolition cycle is four. Note that  $\phi = 0$  throughout the last tenancy.

### 3.3 Program $\widehat{\mathcal{R}}$

Program  $\widehat{\mathcal{R}}$  under rent control is the analog of program  $\mathcal{R}$  in the absence of rent control. Recall that, under our assumptions concerning the construction and maintenance technologies, program  $\mathcal{R}$  entails constructing at quality  $q_c$  above saddlepoint quality, downgrading the unit to quality  $q_T$ , upgrading it via rehabilitation to quality  $q_s$ , downgrading it along the original trajectory from  $q_s$  to  $q_T$ , and then repeating the rehabilitation cycle *ad infinitum*. We also showed that if program  $\mathcal{R}$  exists, it is more profitable than program  $\mathcal{S}$ , and that, if both program  $\mathcal{R}$  and program  $\mathcal{D}$  exist, the one with the lower construction quality is the more profitable. Program  $\widehat{\mathcal{R}}$  differs from program  $\mathcal{R}$  in two respects. First, because under tenancy rent control rehabilitation is permitted only between tenancies and because tenancy duration is  $L$ , the period from initial construction to the first rehabilitation must be some integer multiple of  $L$ , as must the period between subsequent rehabilitations. Because of this, the starting and terminal quality of a rehabilitation cycle will in general vary from one rehabilitation to the next. Second, as with the other two rent control programs, downgrading does not occur smoothly because of the commitment problem.

In the optimal demolition program with rent control, all the cycles are the same. This is not in general true of the optimal rehabilitation program; the number of tenancies may be different for different rehabilitation cycles. In our numerical examples, however, since we assume that the mar-

ginal benefit of increasing quality via rehabilitation is independent of the quality level from which rehabilitation is undertaken, the first rehabilitation is followed by the stationary rehabilitation cycle. In this case, the construction of a solution algorithm is relatively straightforward. First, one solve for the opportunistic stationary rehabilitation cycle, conditional on one, two, etc. tenancies between rehabilitations, and then for the unconditional opportunistic stationary cycle. And second, solve for the optimal program up to the first rehabilitation, conditional on one, two, etc. tenancies to that point, and then for the unconditional optimal program.

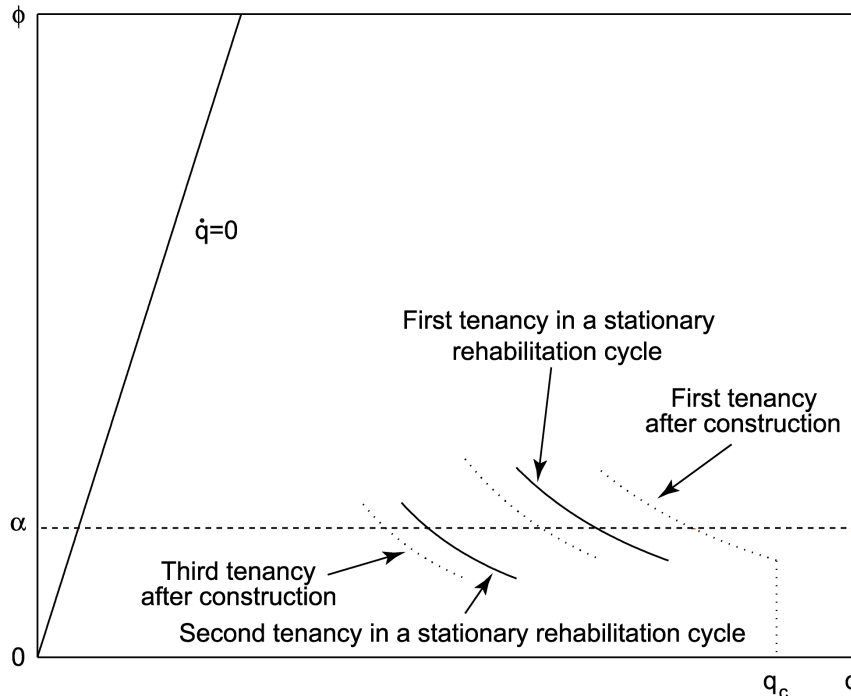


Figure 6: Phase plane for a rehabilitation cycle under tenancy rent control. Construction cost ( $\alpha \cdot 10^5$ ) is \$20,000.

Among the  $\hat{S}$ ,  $\hat{D}$ , and  $\hat{R}$  programs, the overall optimal program is the one with the highest value. The deadweight loss due to rent control is simply the difference between the value of the optimal program without rent control minus the value of the optimal program with rent control.

## 4 Numerical Examples

This section presents a series of related numerical examples with the aim of quantifying the effects of tenancy rent control. The efficiency cost caused by the commitment problem is of special interest.

### 4.1 Choice of functional forms and parameters

We had hoped to draw on the empirical literature in our choice of functional forms and parameters. Unfortunately, there seem to be no empirical studies that have employed the Arnott, Davidson, and Pines (1983) conceptual framework as the basis for empirical analysis. As a result, we adopt

the more modest goal of developing numerical examples whose parameters and functional forms are “reasonable”. We choose the functional forms so as to obtain equations of motion that are the solutions to linear differential equations, as well as (for the case of rent control) closed-form value functions. And we choose the parameters to generate plausible results for the steady-state, demolition, and rehabilitation programs.

As in the theoretical analysis, we measure quality as proportional to construction costs. We assume the following functional forms for the rent function, the construction cost function, and the maintenance/depreciation function:

$$P(q) = eq - \frac{fq^2}{2} \quad C(q) = \alpha \quad \dot{q} = -\delta q + 2am^{1/2}$$

The rent equation generates a linear, downward-sloping marginal-willingness-to-pay-for-quality function. The maintenance/depreciation function is about the simplest possible. In the absence of maintenance, quality depreciates exponentially at the rate  $\delta$ . A given level of maintenance expenditure slows down the rate of quality depreciation by an amount that is independent of quality, and there are diminishing returns to maintenance. The optimal expenditure on maintenance is given by  $a^2\phi^2$ ; maintenance expenditure is therefore increasing in  $\phi$  and independent of  $q$ . Substituting the expression for optimal maintenance into the depreciation function gives the maximized depreciation function,

$$\dot{q} = -\delta q + 2a^2\phi. \quad (20)$$

In the absence of rent control, these equations imply a co-state equation of the form

$$\dot{\phi} = (r + \delta)\phi - e - fq, \quad (21)$$

and with tenancy rent control<sup>19</sup>,

$$\dot{\phi} = (r + \delta)\phi. \quad (22)$$

In the absence of rent control, these equations of motion correspond to a phase plane with a linear, upward-sloping  $\dot{q} = 0$  line and a linear, downward-sloping  $\dot{\phi} = 0$  line, whose intersection point, the saddlepoint is at

$$q^S = \frac{2a^2e}{\delta(r + \delta) + 2a^2f} \quad \phi = \frac{e\delta}{\delta(r + \delta) + 2a^2f}.$$

With rent control, the  $\dot{\phi} = 0$  line coincides with the  $q$ -axis, so that the  $\dot{q} = 0$  and  $\dot{\phi} = 0$  lines do not intersect in the interior of the phase plane.

We take as our units of measurement years and hundreds of thousands of dollars. We start by setting the following parameters:

$$\delta = 0.03, \quad r = 0.0375, \quad a = 0.2121, \quad e = 0.055, \quad f = 0.005, \quad \text{and } L = 10.$$

These parameters imply a saddlepoint quality of 2.0, saddlepoint maintenance of 0.02 (\$2000 per year), saddlepoint rent of 0.10 (\$10000 per year), and a value of the co-state variable (the marginal

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<sup>19</sup>Thus, both with and without rent control, the state and co-state equations are together a pair of linear first-order differential equations in  $q$  and  $\phi$ . In the absence of rent control, substituting one into the other generates linear, second-order differential equations for  $q$  alone and  $\phi$  alone. And with rent control, (22) is a linear, first-order differential equation in  $\phi$  alone, and substituting the solution to (22) into (20) results in a linear, first-order differential equation in  $q$  alone.

value of quality) at the saddlepoint of 0.667.  $\alpha$  is varied across examples.

Our rehabilitation function has a very simple form:  $R(q_s, q_T) = \beta_1 q_s - \beta_2 q_T = \beta_2(q_s - q_T) - (\beta_1 - \beta_2)q_s$ , where  $\beta_1 = 0.25$ ,  $\beta_2 = 0.24$ . Thus, besides a linear cost of quality upgrade, the landlord has to pay a fee proportional to the ‘target’ quality  $q_s$ .

## 4.2 Numerical solution procedures

The details of the numerical solution procedures employed are presented in the Appendix of the version of the paper available at <http://fmwww.bc.edu/ec/arnott.php>. Here we just describe in broad terms the general approaches. In the absence of rent control, the solution procedure centers on solving for the solution parameters of the second-order linear differential equation for  $\phi$ , since everything else may be solved for once these parameters are obtained. One parameter is obtained from the initial condition that  $\phi(0) = \alpha$ . How the other parameter is determined depends on the type of program. In the case of the saddlepoint program, the second parameter is obtained from the  $\phi$ -coordinate of the saddlepoint; in the case of the demolition program, the second parameter and the period of the demolition cycle are solved simultaneously from  $\phi(T) = 0$  and the terminal time (or equal-areas) transversality condition; in the case of the rehabilitation program, the second parameter, as well as  $\phi_s$  and  $\phi_T$ , are solved simultaneously from the transversality conditions for  $\phi_s$ ,  $\phi_T$ , and the terminal time transversality condition.

The approaches taken to solve the optimal programs with tenancy rent control are more complex. It is convenient to express the unknown parameters in the functions  $\phi(t)$  and  $q(t)$  in terms of  $q(0)$  and  $q(L)$ . This allows us to obtain the discounted revenue received over a tenancy,  $Z(q_0)$ , and the net value of a tenancy cycle,  $J(q_0, q_L, L)$ . For program  $\widehat{\mathcal{S}}$ , we make a conjecture about the form of  $\widehat{V}(q)$ . Then, using (14) to find  $q_L(q_s)$  and plugging it into (13), we apply the method of undetermined coefficients to solve for  $\widehat{V}(q)$ . The final step is to find the construction quality  $q_c$  using (18). The solution algorithm for the demolition program with rent control was described in Section 3.2 and that for the rehabilitation program sketched in Section 3.3.

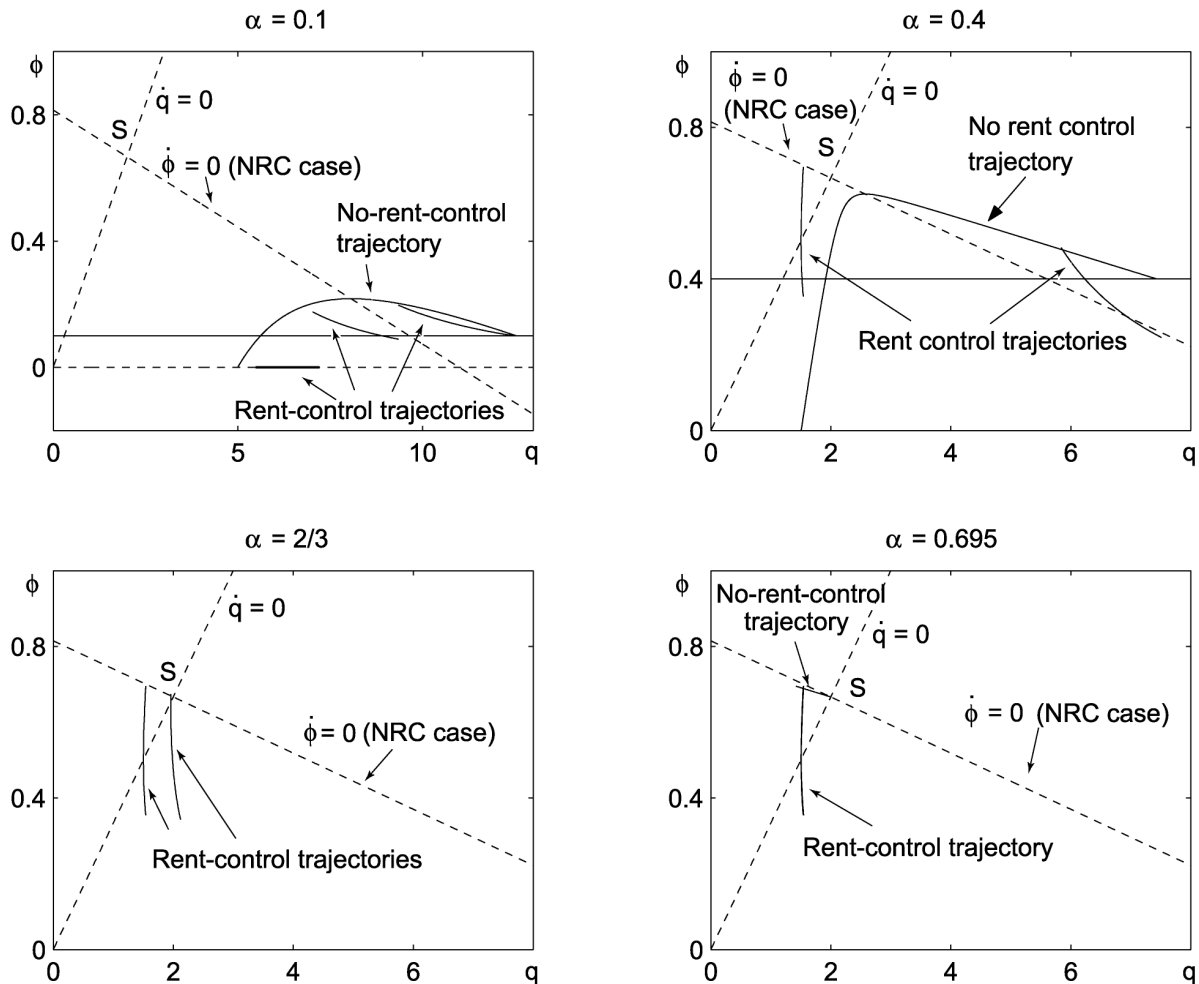
## 4.3 Examples without rehabilitation

In this subsection, we assume that rehabilitation is unprofitable and that  $\alpha$  is not so high as to make initial construction unprofitable. In the absence of rent control, the optimal program is therefore either the optimal saddlepoint program or the optimal demolition program, with the saddlepoint program being optimal for  $\alpha$  above 0.4166 and the demolition program for  $\alpha$  below that level. With rent control, the optimal program entails either convergence to a steady-state cycle or a demolition program, with the former occurring when construction costs are high relative to maintenance. We proceed by lowering  $\alpha$  from one example to the next.

- $\alpha = 0.695$

The fourth panel of Figure 7 displays the phase diagram for this example, both with and without rent control. The  $\dot{\phi} = 0$  locus in the absence of rent control is shown as the dotted line; with rent control, it coincides with the  $q$ -axis. Recall that the level of maintenance is proportional to  $\phi$ . In the absence of rent control, the optimal program entails construction at  $q = 1.416$ , followed by upgrading to steady-state quality,  $q^S = 2.0$ . Construction occurs at that quality at which the marginal value of quality,  $\alpha$ , equals the marginal cost of construction. The value of the

program is 0.751. With rent control, the optimal program entails a steady-state tenancy cycle, with construction at  $q = 1.542$ . As explained earlier,  $\phi$  jumps downwards discontinuously immediately after a tenancy contract is signed, reflecting the commitment problem, and then rises continuously within the tenancy. With increasing maintenance over the tenancy, quality initially falls and then rises until it reaches construction quality by the end of the tenancy. The value of the program is 0.694. Thus, the efficiency loss due to tenancy rent control is 7.6% of the value of the uncontrolled program. Observe that the average quality of housing is lower under rent control, consistent with intuition.



Note: NRC is an abbreviation for no rent control.

Figure 7: Phase planes with and without rent control. No rehabilitation.

- $\alpha = 0.667$

The third panel of Figure 7 shows the optimal trajectories for this example without and with rent control. The optimal program in the absence of rent control entails constructing at saddlepoint quality and holding quality constant at that level. The value of the Hamiltonian at the saddlepoint is 0.08 (rent of 0.10 minus maintenance costs of 0.02 and of course no depreciation). Housing value is 2.133 and construction costs are 1.333, so that the value of the program prior to construction is 0.800 and the land to housing value ratio 0.375. Are these numbers reasonable? The “cap rate”

(the percentage of net rent to value) is low, under the model's assumptions simply equaling the interest rate; if uncertainty and property taxes were considered, the cap rate would be reasonable. Maintenance expenditures are 0.94% of housing value, which accords broadly with the 1-percent rule that maintenance expenditures are typically about 1% of property value. The Figure shows two rent-control trajectories. The path on the right is for the first tenancy, that on the left for the steady-state tenancy. Construction occurs just above saddlepoint quality. Maintenance increases within each tenancy, but starting quality falls from one tenancy to the next, converging to steady-state starting quality below saddlepoint quality. The value of the program is 0.746, implying a deadweight loss due to tenancy rent control of 6.8% of value.

- $\alpha = 0.4$

It was noted earlier that, with the assumed functional forms and parameter values, in the absence of rent control the optimal demolition program is more profitable than the optimal saddlepoint program when  $\alpha$  is below 0.4166. Thus, in this example, displayed in the second panel of Figure 7, the optimal program without rent control is a demolition cycle. Construction occurs at a quality considerably above saddlepoint quality. This is followed by downgrading to demolition quality, at which point the structure is demolished and the cycle exactly repeated. The value of the program is 2.058. In contrast, with tenancy rent control, convergence to a steady-state cycle remains optimal<sup>20</sup>. Construction occurs at high initial quality, followed by downgrading from one tenancy to the next (but with rising maintenance within each tenancy) converging to a steady-state cycle. The value of the program is 2.030, so that in this case the deadweight loss due to rent control is only 1.4% of the uncontrolled program value.

- $\alpha = 0.1$

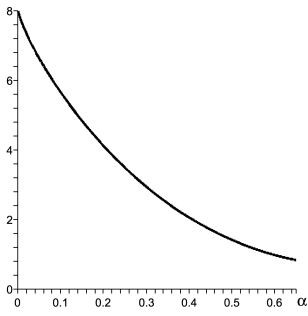
In this example, shown in the first panel of Figure 7, construction is sufficiently cheap relative to maintenance that a demolition cycle is profit maximizing both with and without rent control. The range of qualities over a demolition cycle is similar for the two programs. The level of maintenance is lower under rent control at every quality level; as a result, depreciation is more rapid and the demolition cycle shorter. The values of the program without and with rent control are 5.663 and 5.549, respectively, implying a deadweight loss due to rent control of 2.0% of the value of the uncontrolled program.

Figure 8 focuses on the deadweight loss resulting from the application of rent control. Panel A shows the value of the optimal program without rent control as a function of  $\alpha$ . There is a slope discontinuity in the value of this optimal program at  $\alpha = 0.4166$ , where the switch occurs between the range of qualities where the saddlepoint program is optimal and where the demolition cycle is optimal. There are several slope discontinuities in the value of the optimal program with rent control.<sup>21</sup> The one corresponding to the highest value of  $\alpha$  corresponds to the switch point between the range of qualities for which the steady-state cycle is optimal and for which the demolition cycle is optimal. The ones at lower values of  $\alpha$  correspond to switch points for which different numbers of tenancies within a demolition cycle are optimal. Panel B shows the absolute loss in program value from the application of tenancy rent control, and Panel C the corresponding proportional loss.

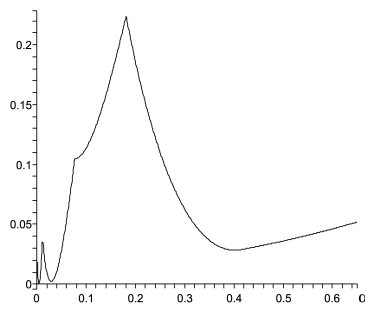
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<sup>20</sup>The critical construction cost level below which the optimal program entails demolition is therefore lower with rent control than without. With  $\alpha = 0.4$ , the deadweight loss due to the commitment problem is therefore higher with the optimal demolition program than with the optimal steady-state program.

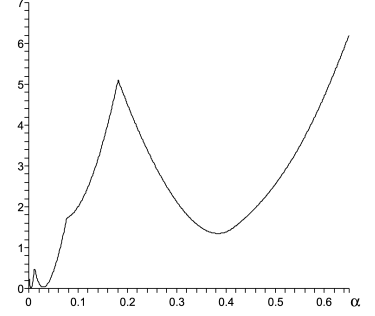
<sup>21</sup>This function is not drawn since to the naked eye, it is hard to distinguish for that drawn in Panel A.



Panel A: Value of the optimal programs without rent control as a function of construction cost,  $\$10^5$



Panel B: Absolute deadweight loss due to rent control,  $\$10^5$

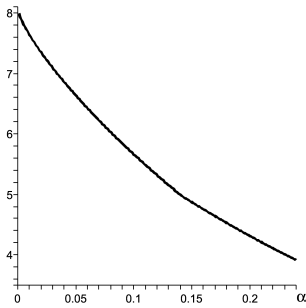


Panel C: Relative deadweight loss due to rent control, %

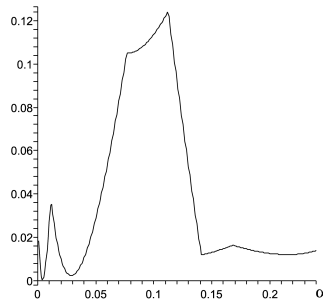
Figure 8: Values of optimal programs without rent control, and deadweight loss due to rent control - Case without rehabilitation

#### 4.4 Examples with rehabilitation

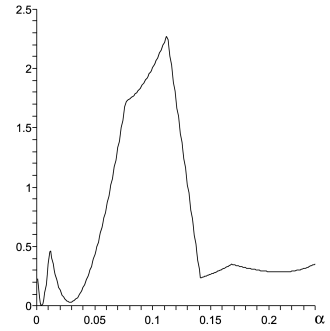
In examples with rehabilitation, we consider a limited range of  $\alpha$  ( $0 < \alpha < 0.24$ ). Due to our choice of the functional form, for higher values of  $\alpha$  construction becomes unreasonably expensive compared to rehabilitation. Figure 9 presents the value of the optimal program without rent control as a function of  $\alpha$ , and the absolute and relative deadweight loss due to rent control.



Panel A: Values of the optimal programs without rent control as a function of construction cost,  $\$10^5$



Panel B: Absolute deadweight loss due to rent control,  $\$10^5$



Panel C: Relative deadweight loss due to rent control, %

Note: Figure 8 has a different  $\alpha$ -axis scale than Figure 9.

Figure 9: Values of optimal programs without rent control, and deadweight loss due to rent control - Case with rehabilitation

For  $\alpha \leq 0.112$  the optimal program with or without rent control is demolition (programs  $\mathcal{D}$  and  $\widehat{\mathcal{D}}$ ). Under rent control: for  $\alpha \leq 0.011$ , the  $\widehat{\mathcal{D}}$  program has only one tenancy cycle between demolitions; for  $0.011 < \alpha \leq 0.077$ , two tenancy cycles; and for  $0.077 < \alpha \leq 0.112$ , three tenancy cycles. This explains the non-smoothness of deadweight loss when demolition is optimal. For  $0.112 < \alpha \leq 0.141$ ,  $\mathcal{D}$  is still the optimal program without rent control but under rent control rehabilitation is more

profitable. For  $0.141 < \alpha \leq 0.24$ , the optimal program is rehabilitation with or without rent control. For  $0.141 < \alpha \leq 0.168$ , the  $\widehat{\mathcal{R}}$  program entails three tenancies before the first rehab, while for  $0.168 < \alpha \leq 0.24$  only two tenancies precede the first rehab. As a result, there is a ‘kink’ in panels B and C at  $\alpha = 0.168$ . At these relatively low values of  $\alpha$  steady-state programs are never optimal. The relative loss is limited and does not exceed 2.5% of the value of an optimal program. Absolute loss may reach \$12,000 per unit per year.

## 5 Conclusion

In recent years an increasing number of jurisdictions around the world have adopted what has come to be known as tenancy rent control, typically as a method of partial decontrol of a previously stricter form of rent control. Under tenancy rent control, rents are controlled within a tenancy but are free to vary between tenancies. Tenancy rent control appears attractive, as a way of providing security of tenure to sitting tenants without the excess demand distortions created by stricter control programs. How attractive tenancy rent control in fact is depends on the magnitude of the distortions it creates. Since tenancy rent control typically results in the contract rent exceeding the market rent in the early years of the tenancy and falling short of it in later years, it provides an incentive for tenants to stay in their apartments longer than they otherwise would. In this paper we examined the effects of tenancy rent control on a landlord’s choice of the quality path of his housing units, which includes his decisions on construction quality, maintenance, rehabilitation, and demolition and reconstruction, under the assumptions that tenancy duration is exogenous and that the controls are applied to only a single housing unit. We showed that the application of tenancy rent control gives rise to a potential commitment (or time inconsistency) failure. We contrasted two programs, the efficient program and the opportunistic program. The efficient program is the profit-maximizing program in the absence of rent control. The opportunistic program is the profit-maximizing program over a tenancy *once the tenancy contract has been signed*. The signing of the contract results in the present value of revenue from the tenancy being independent of the landlord’s maintenance expenditure, and hence reduces his incentives to maintain. Before the tenancy contract is signed, the landlord would like to commit to following the efficient program, but none of the commitment mechanisms available – contract, reputation, and regulation – is likely to be very effective. In our analysis, we assumed that these mechanisms are completely ineffective, so that the landlord follows the opportunistic program. Building on the Arnott-Davidson-Pines model of housing quality and maintenance, we compared the properties of the efficient and opportunistic programs. Section 4 presented a series of related numerical examples, with the aim of quantifying the deadweight loss due to the commitment failure. For reasonable parameter values, we found that the deadweight loss is modest but not insignificant, ranging from zero to eight percent of the pre-control value of the program.

There are several open questions left for future research.

1. The paper considered the application of tenancy rent control to a single housing unit when the rest of the market is uncontrolled. How do the results change when the entire market is controlled?
2. The paper built on the Arnott-Davidson-Pines filtering model. Since there is no empirical work based on this model, the numerical examples used simple functional forms and “reasonable” parameter values. How would the results change if estimated functional forms were used instead?
3. The paper assumed, under tenancy rent control, that tenancy duration is exogenous. But, by front-end loading rents, tenancy rent control should increase tenancy duration. How important is



this distortion compared to the commitment-in-maintenance distortion considered here, and how do the two distortions interact?

4. The paper noted that tenancy rent control improves security of tenure for tenants. What is the social value of doing so?

5. The paper compared the unrestricted market equilibrium to the market equilibrium under tenancy rent control. But since tenancy rent control has typically been employed as a method of partial decontrol, it is perhaps more relevant to ask: What is the magnitude of the efficiency gain when a stricter form of rent control is replaced by tenancy rent control?

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## A Technical Appendix

In this appendix we find optimal programs without and with rent control for the functional forms used in our numerical examples. As was stated in Section 4, in the numerical examples we employ the following rent function, construction cost function, and maintenance/depreciation function:

$$\begin{aligned} P(q) &= eq - \frac{fq^2}{2} = 0.055q - \frac{0.005q^2}{2} \\ C(q) &= \alpha q \\ \dot{q} &= -\delta q + 2am^{1/2} = -0.03q + 2(0.045\phi^2)^{1/2}. \end{aligned}$$

### A.1 No-rent-control programs

We start by solving the system of differential equations (20) and (21):

$$\begin{aligned} \dot{q} &= -\delta q + 2a^2\phi, \\ \dot{\phi} &= (r + \delta)\phi - e + fq. \end{aligned}$$

This system can be reformulated as follows:

$$\begin{aligned} \ddot{\phi} - r\dot{\phi} - (2a^2f + \delta(r + \delta))\phi + \delta e &= 0, \\ q &= \frac{\dot{\phi} + e - \phi(r + \delta)}{f}. \end{aligned} \tag{A-1}$$

The solution to the second order differential equation for  $\phi$  has the following form:

$$\phi(t) = C_1e^{\gamma_1 t} + C_2e^{\gamma_2 t} + B \tag{A-2}$$

where

$$\begin{aligned} B &= \frac{\delta e}{2a^2f + \delta(r + \delta)} = \phi^S, \\ \gamma_1 &= \frac{r + \sqrt{r^2 + 4(2a^2f + \delta(r + \delta))}}{2}, \\ \gamma_2 &= \frac{r - \sqrt{r^2 + 4(2a^2f + \delta(r + \delta))}}{2}. \end{aligned}$$

With  $\phi(t)$ , we can find  $q(t)$  using (A-1):

$$q(t) = \frac{1}{f} [C_1e^{\gamma_1 t}(\gamma_1 - r - \delta) + C_2e^{\gamma_2 t}(\gamma_2 - r - \delta) + e - B(r + \delta)].$$

Recalling that

$$q^S = \frac{2a^2e}{\delta(r + \delta) + 2a^2f}$$

and rearranging  $\frac{e - B(r + \delta)}{f}$ , we obtain that

$$q(t) = \frac{1}{f} [C_1e^{\gamma_1 t}(\gamma_1 - r - \delta) + C_2e^{\gamma_2 t}(\gamma_2 - r - \delta)] + q^S. \tag{A-3}$$

Whether the  $\mathcal{S}$  or  $\mathcal{D}$  program is optimal, the transversality condition (6) holds:

$$\phi(0) = \alpha.$$

Using this condition, we solve for  $C_2$ :

$$\begin{aligned} C_1 + C_2 + \phi^S &= \alpha, \\ C_2 &= \alpha - C_1 - \phi^S. \end{aligned}$$

The other transversality condition that allows us to solve for  $C_1$  is different for the  $\mathcal{S}$  and  $\mathcal{D}$  programs, which we consider in turn.

### A.1.1 Program $\mathcal{S}$

The steady-state program implies that

$$\begin{aligned} \lim_{t \rightarrow \infty} q(t) &= q^S \\ \lim_{t \rightarrow \infty} \phi(t) &= \phi^S. \end{aligned} \tag{A-4}$$

Notice that  $\gamma_1 > 0$  while  $\gamma_2 < 0$ . Therefore, (A-4) can hold only if  $C_1 = 0$ . This condition completely defines  $q(t)$  and  $\phi(t)$ :

$$\begin{aligned} \phi(t) &= C_2 e^{\gamma_2 t} + \phi^S, \\ q(t) &= \frac{1}{f} C_2 e^{\gamma_2 t} (\gamma_2 - r - \delta) + q^S, \\ C_2 &= \alpha - \phi^S. \end{aligned}$$

### A.1.2 Program $\mathcal{D}$

To find  $C_1$ , we use the transversality condition  $\phi(T) = 0$ :

$$\begin{aligned} C_1 e^{\gamma_1 T} + C_2 e^{\gamma_2 T} + B &= 0, \\ C_1 &= -\frac{B + C_2 e^{\gamma_2 T}}{e^{\gamma_1 T}}. \end{aligned}$$

The last unknown is  $T$ . It is determined by the equal-areas condition:

$$\mathcal{H}(T) = \mathcal{H}(0) - r\alpha q(0) \tag{A-5}$$

where

$$\mathcal{H}(t) = eq(t) - \frac{fq(t)^2}{2} + a^2 \phi(t)^2 - \delta q(t)\phi(t). \tag{A-6}$$

Equation (A-5) involves sums of exponents of  $T$ , so it cannot be solved analytically. We find its solution numerically for a given value of  $\alpha$ . It appears that this equation has two solutions in the region where  $T$  is positive. We choose the one that results in the higher value of the program.

### A.1.3 Program $\mathcal{R}$

Our rehabilitation technology is  $R(q_s, q_T) = 0.25q_s - 0.24q_T$ . In this problem, there are two different pairs of laws of motion for  $q$  and  $\phi$ ,  $\{q_c(t), \phi_c(t)\}$  for a tenancy immediately after construction, which we call a construction cycle, and the other,  $\{q(t), \phi(t)\}$ , for all subsequent tenancies, which we call rehabilitation

cycles. Both pairs are described by (A-3) and (A-2), respectively, but with different unknown constants, which we will denote as  $\{C_{c1}, C_{c2}\}$  for a construction cycle and  $\{C_1, C_2\}$  for rehabilitation cycles. We start by finding the laws of motion for rehabilitation cycles. The transversality conditions

$$\begin{aligned}\phi(0) &= \frac{\partial R(q_s, q_T)}{\partial q_s} = 0.25, \\ \phi(T) &= -\frac{\partial R(q_s, q_T)}{\partial q_T} = 0.24\end{aligned}$$

allow us to solve for the unknown constants on which  $q(t)$  and  $\phi(t)$  depend. Then we find the optimal duration of the rehabilitation cycle  $T$ , using the equal-area condition:

$$(\mathcal{H}(0) - \mathcal{H}(T))/r = R(q(0), q(T)). \quad (\text{A-7})$$

Here  $\mathcal{H}(\cdot)$  is defined in (A-6) and depends on the laws of motion for the rehabilitation cycle. We solve this equation numerically using Maple 9.5 and obtain that the optimal duration of the rehabilitation cycle is (approx.) 16.61 years. We verify that there are no other solutions for positive  $T$  by examining behavior of the left-hand side and the right-hand side of (A-7). Notice that  $T$  does not depend on the cost of construction. Then we find the laws of motion  $q_c(T)$  and  $\phi_c(T)$  for the construction cycle using the following transversality conditions:

$$\begin{aligned}\phi_c(0) &= \alpha, \\ \phi_c(T_c) &= \phi(T).\end{aligned}$$

Finally, we numerically solve for the length of the construction cycle  $T_c$  for each specific  $\alpha$  using the following equation:

$$q_c(T_c) = q(T).$$

## A.2 Programs with rent control

Under programs with rent control, the differential equation for  $\phi$  is different from that without rent control. Solving the system (20) and (22)

$$\begin{aligned}\dot{q} &= -\delta q + 2a^2 \phi, \\ \dot{\phi} &= (r + \delta)\phi,\end{aligned}$$

we obtain the following solutions:

$$\begin{aligned}\phi(t) &= c_1 e^{(r+\delta)t}, \\ q(t) &= \frac{2a^2 c_1}{r + 2\delta} e^{(r+\delta)t} + c_2 e^{-\delta t}.\end{aligned}$$

We solve for  $c_1$  and  $c_2$  in terms of initial and terminal quality of a tenancy cycle,  $q_s$  and  $q_L$ :

$$\begin{aligned}q(0) &= \frac{2a^2 c_1}{r + 2\delta} + c_2 = q_s, \\ c_2 &= q_0 - \frac{2a^2 c_1}{r + 2\delta}.\end{aligned}$$

Since the analytical solutions to programs with rent control contain quite messy expressions, we give only solutions for the values of parameters used in our numerical examples and round all values to the third digit.

$$\begin{aligned}
q(t) &= \frac{q_L - q_s e^{-\delta L}}{e^{(r+\delta)L} - e^{-\delta L}} \left( e^{(r+\delta)t} - e^{-\delta t} \right) + q_0 e^{-\delta t} \\
&= 0.817(e^{0.0675t} - e^{0.03t})q_L + (-0.606e^{0.0675t} + 1.606e^{0.03t})q_s, \\
\phi(t) &= c_1 e^{(r+\delta)t} = \frac{(q_L - q_s e^{-\delta L})(r + 2\delta)}{(e^{(r+\delta)L} - e^{-\delta L})2a^2} e^{(r+\delta)t} \\
&= (-0.656q_s + 0.886q_L)e^{0.0675t}.
\end{aligned}$$

Recall that optimal maintenance is  $m(t) = a^2 \phi^2$ . Thus, the value of a tenancy cycle is

$$J(q_s, q_L, L) = - \int_0^L m(t) e^{-rt} dt = - \int_0^L a^2 \phi(t)^2 e^{-rt} dt.$$

It is straightforward to calculate this function but the expression is cumbersome; examining the  $\phi(t)$  function, one can see that

$$J(q_s, q_L, L) = G_1 q_s^2 + G_2 q_L^2 + G_3 q_s q_L.$$

where  $G_1$ ,  $G_2$ , and  $G_3$  are some known functions of parameters.

Using the definition of the rent function  $P(\cdot)$ , we also calculate the discounted present value of rent received over a tenancy:

$$Z(q_s, L) = \int_0^L [e\hat{q}(t; q_s) - \frac{f}{2}\hat{q}(t; q_s)^2] e^{-rt} dt,$$

with

$$\hat{q}(t; q_s) = \frac{q_L(q_s) - q_s e^{-\delta L}}{e^{(r+\delta)L} - e^{-\delta L}} \left( e^{(r+\delta)t} - e^{-\delta t} \right) + q_s e^{-\delta t}$$

where a final quality of a cycle,  $q_L(q_s)$ , is optimally chosen and is a function of an initial quality of a cycle,  $q_s$ . The functional form of  $q(t)$  implies that

$$Z(q_s, L) = B_1 q_s^2 + B_2 q_L(q_s)^2 + B_3 q_s q_L(q_s) + B_4 q_s + B_5 q_L(q_s).$$

Again,  $\{B_i\}_{i=1}^5$  are some known functions of the parameters.

### A.2.1 Program $\widehat{\mathcal{S}}$

In case of the  $\widehat{\mathcal{S}}$  program, the problem of the landlord boils down to an infinite horizon dynamic programming problem, in which the state variable is the initial quality while the control variable is the terminal quality of a unit. Thus, we have the following Bellman equation:

$$\begin{aligned}
\widehat{V}(q_s^i) &= Z(q_s^i) + \max_{q_L^i} [J(q_s^i, q_L^i, L) + \widehat{V}(q_s^{i+1})e^{-rL}] \\
s.t. \quad q_s^{i+1} &= q_L^i, \quad i = 1, 2, \dots \text{ is the number of the tenancy cycle.}
\end{aligned} \tag{A-8}$$

We apply the ‘guess-and-verify’ method. Notice that  $J$  is quadratic in  $q_L^i$ . If  $q_L^i$  is a linear function of  $q_s^i$ , then  $Z$  is also quadratic in  $q_L^i$ . Notice also that  $q_L^i$  is a linear function of  $q_s^i$  if  $\widehat{V}$  is quadratic. Thus, we make a guess that  $\widehat{V}$  is quadratic:

$$\widehat{V}(q) = A_0 + A_1 q + A_2 q^2. \tag{A-9}$$

We find  $A_0$ ,  $A_1$  and  $A_2$  by the method of undetermined coefficients. First we need to find  $q_L^i$  as a function of  $q_s^i$ . Assuming that  $J(q_s^i, q_L^i, L) + \widehat{V}(q_s^{i+1})e^{-rL}$  is concave, we use the first order condition:

$$\begin{aligned} \frac{\partial}{\partial q_L^i} \left[ J(q_s^i, q_L^i, L) + \widehat{V}(q_s^{i+1})e^{-rL} \right] &= 2G_2q_L^i + G_3q_s^i + e^{-rL}A_1 + 2e^{-rL}A_2q_L^i = 0, \\ q_L^i &= -\frac{G_3q_s^i + e^{-rL}A_1}{2(G_2 + e^{-rL}A_2)} \equiv K_1q_s^i + K_2. \end{aligned} \quad (\text{A-10})$$

Substituting (A-9) and (A-10) into the Bellman equation (A-8) and suppressing the index for the cycle  $i$ , we obtain

$$\begin{aligned} A_0 + A_1q_s + A_2(q_s)^2 &= B_1q_s^2 + B_2(K_1q_s + K_2)^2 + B_3q_s(K_1q_s + K_2) \\ &+ B_4q_s + B_5(K_1q_s + K_2) \\ &+ G_1q_s^2 + G_2(K_1q_s + K_2)^2 + G_3q_s(K_1q_s + K_2) \\ &+ e^{-rL}[A_0 + A_1(K_1q_s + K_2) + A_2(K_1q_s + K_2)^2]. \end{aligned} \quad (\text{A-11})$$

One can see that (A-11) is quadratic in  $q_s$ . We find the unknown constants  $A_0$ ,  $A_1$  and  $A_2$  by rewriting (A-11) in the form

$$W_0 + W_1q_s + W_2q_s^2 = 0$$

and solving the system

$$\begin{aligned} W_0 &= 0 \\ W_1 &= 0 \\ W_2 &= 0 \end{aligned} \quad (\text{A-12})$$

for  $A_0$ ,  $A_1$  and  $A_2$ .

After some simplification and a bit of rounding, the system (A-12) can be rewritten as

$$\begin{aligned} A_2 + 0.336 + \frac{0.779}{1.375A_2 - 1.195} + \frac{0.473 - 0.539A_2}{(1.375A_2 - 1.195)^2} &= 0 \\ A_1 - 0.254 + \frac{0.175 + 1.213A_1}{1.375A_2 - 1.195} + \frac{0.734A_1 - 0.837A_1A_2}{(1.375A_2 - 1.195)^2} &= 0 \\ 0.313A_0 + \frac{0.136A_1 + 0.472A_1^2}{1.375A_2 - 1.195} + \frac{0.285A_1^2 + 0.325A_2A_1^2}{(1.375A_2 - 1.195)^2} &= 0. \end{aligned}$$

This system of (cubic) equations has three solutions:

$$\begin{aligned} A_0 &= 3.012, A_1 = -1.634, A_2 = 0.567, \\ A_0 &= 34.012, A_1 = -12.235, A_2 = 0.861, \\ A_0 &= 0.635, A_1 = 0.772, A_2 = -0.025. \end{aligned}$$

Only the third solution results in a concave value function while other solutions have  $A_2 > 0$ . Indeed, one can check that the first and second solutions are spurious, since they result in convex  $J(q_s^i, q_L^i, L) + \widehat{V}(q_s^{i+1})e^{-rL}$ . We proceed further with the third solution

$$\widehat{V}(q) = 0.635 + 0.772q - 0.025q^2.$$

To complete the solution of the problem, we use the the first-order condition for the maximization of the value of the program:

$$\frac{d}{dq_0} \left( -\alpha q_0 + \widehat{V}(q_0) \right) = -\alpha + A_1 - 2A_2q_0 = 0.$$

Therefore,

$$q_0 = \frac{A_1 - \alpha}{2A_2}.$$

### A.2.2 Program $\widehat{\mathcal{D}}$

Recall that program  $\widehat{\mathcal{D}}$  entails an infinite number of repetitions of a construction-demolition cycle, each of which comprises of  $n$  tenancy cycles. Conditional on pursuing program  $\widehat{\mathcal{D}}$ , the problem of the landlord is not only to choose an optimal trajectory for each tenancy cycle but also to choose optimal  $n$ . Given  $n$ , the problem of the landlord is to find the optimal maintenance path and optimal construction quality. To find optimal maintenance, the landlord solves a finite-horizon dynamic programming problem similar to (A-8):

$$\begin{aligned} v(q_s^i) &= Z(q_s^i) + \max_{q_L^i} [J(q_s^i, q_L^i, L) + v(q_s^{i+1})e^{-rL}], \\ \text{s.t. } q_s^{i+1} &= q_L^i, \quad i = 1, 2, \dots, n \text{ is the number of the tenancy cycle,} \\ v(q_s^{n+1}) &= 0. \end{aligned} \tag{A-13}$$

Given our particular functional form, we show the solution for  $n = 1$ .  $v(q_s^2) = 0$ , so  $q_L^1$  is a solution to the first-order condition:

$$\frac{d}{dq_L^1} [-0.328(q_0^1)^2 + 0.886q_L^1q_0^1 - 0.598(q_L^1)^2] = 0.$$

Thus,

$$q_L^1 = 0.741q_0^1.$$

Given  $q_L^1$ ,

$$v(q_s^1) = 0.4q_s^1 - 0.016(q_s^1)^2.$$

Knowing  $v(q_s^1)$ , the landlord optimizes with respect to  $q_s^1$ :

$$\max_{q_s^1} (-\alpha q_s^1 + v(q_s^1)), \tag{A-14}$$

which gives

$$q_s^1 = -31.31\alpha + 12.522. \tag{A-15}$$

The value of program  $\widehat{\mathcal{D}}$  for  $n = 1$  is

$$\begin{aligned} \widehat{V}^1(q_s^1) &= \frac{1}{1 - e^{-rL}} (v^1(q_s^1, 1) - \alpha q_s^1) \\ &= -40.044\alpha + 16.016 - (0.051 + 3.198\alpha)(-31.31\alpha + 12.522). \end{aligned}$$

Following the same strategy we solve for  $\widehat{V}^n(q_s^1)$  for  $n$  from 1 to 20.

One more complication we encounter is that for sufficiently high values of  $\alpha$  (for  $\alpha > 0.55$ ) the non-negativity condition  $q \geq 0$  binds for an optimal  $\widehat{\mathcal{D}}$  program. We say that the demolition program under rent control does not exist for  $\alpha > 0.55$  given our choice of functional forms and parameters.

### A.2.3 Program $\widehat{\mathcal{R}}$

The profit-maximizing rehabilitation program under tenancy rent control requires finding the sequence of initial and terminal qualities in each tenancy cycle that maximizes the landlord's net income stream and



solves:

$$\begin{aligned} & \max_{\{q_s^i, q_L^i\}_{i=1}^{\infty}} [-\alpha q_s^1 + Z(q_s^1) + J(q_s^1, q_L^1, L)] \\ & + \sum_{i=2}^{\infty} e^{-(i-1)rL} (-(R(q_s^i, q_L^i))^+ + Z(q_s^i) + J(q_s^i, q_L^i, L)) \end{aligned}$$

where  $(x)^+ = x$  if  $x > 0$  and  $(x)^+ = 0$  if  $x \leq 0$ . The superscripts on  $q$  stand for the number of the tenancy cycle. In this case the main problem is to guess the solution. We make two conjectures. First, consider the following value function:

$$\begin{aligned} W(q_s^1) &= \max_{q_L^1, \{q_s^i, q_L^i\}_{i=2}^{\infty}} [-\alpha q_s^1 + Z(q_s^1) + J(q_s^1, q_L^1, L)] \\ &+ \sum_{i=2}^{\infty} e^{-(i-1)rL} (-(R(q_s^i, q_L^i))^+ + Z(q_s^i) + J(q_s^i, q_L^i, L)). \end{aligned}$$

Our first conjecture is that  $W(\cdot)$  is quadratic. This is suggested by the functional form of  $Z(\cdot)$  and  $J(\cdot, \cdot)$  which are quadratic. But even knowing that  $W(\cdot)$  is quadratic is not sufficient to get the complete solution as there is another issue: when does the landlord rehabilitate and when not? We look for the program that has the following form:

$$-\alpha q_s^1 + \sum_{i=1}^M e^{-(i-1)rL} [Z(q_s^i) + J(q_s^i, q_L^i, L)] + e^{-MrL} \widehat{V}^K(q_L^M)$$

where

$$\widehat{V}^K(q_L^M) = \sum_{j=1}^{\infty} e^{-(j-1)rKL} Y(q_L^M; K)$$

and

$$Y(q_L^M; K) = -R(q_s^{M+1}, q_L^M) + \sum_{i=1}^K e^{-(i-1)rL} [Z(q_s^{M+i}) + J(q_s^{M+i}, q_L^{M+i})].$$

Thus we are looking for programs that have two parts, a ‘non-stationary’ and a ‘stationary’ one. A stationary part  $\widehat{V}(q_L^M; K)$  consists of infinite repetition of the same cycle  $Y(q_L^M; K)$ , which starts with rehabilitation followed by  $K$  tenancy cycles without rehabilitation. The non-stationary part of the program is the initial part, which comprises  $M$  tenancy cycles without rehabilitation. We do not consider other conceivable programs but intuition and some properties of the solution for  $\widehat{V}(q; K)$  suggest that we do not omit anything substantial.

To find  $V^K(q_L^M)$ , we consider the following system:

$$V_1(q_L^0; K) = \max_{q_s^1} [-R(q_s^1, q_L^0) + V_2(q_s^1; K)], \quad (\text{A-16})$$

$$V_2(q_s^1; K) = Z(q_s^1) + \max_{q_L^1} [J(q_s^1, q_L^1) + e^{-rL} V_3(q_L^1; K)], \quad (\text{A-17})$$

...

$$V_{K+1}(q_s^K; K) = Z(q_s^K) + \max_{q_L^K} [J(q_s^K, q_L^K) + e^{-rL} V_1(q_L^K; K)], \quad (\text{A-18})$$

$$q_s^i = q_L^{i-1}, \quad i = 2, \dots, K.$$

Assuming that  $V_i$ ,  $i = 1, \dots, K+1$  is quadratic ( $V_i(x; K) = A_i + B_i x + C_i x^2$ ), one can notice that we have two types of equations. Let us examine the optimal choices for each equation type. First, we consider

maximization in equation (A-16).

$$\max_{q_s^1} [-\beta_1 q_s^1 + \beta_2 q_L^0 + A_2 + B_2 q_s^1 + C_2 (q_s^1)^2].$$

Provided that  $C_2 < 0$ ,

$$q_s^1 = \frac{\beta_1 - B_2}{2C_2},$$

i.e.  $q_s^1$  is just a constant. Note that  $q_s^1$  would be a constant when the rehabilitation function is additively separable in its two arguments. Additive separability of the rehabilitation function implies that as soon as the landlord finds it profitable to rehabilitate for the first time, the system loses memory about its history. This fact suggests that the solution indeed should contain a stationary cycle of the kind described above. Also, the loss of memory after rehabilitation implies that the non-stationary part of the solution may not contain rehabilitation and, therefore, necessarily consists of a sequence of tenancy cycles without rehabilitation.

Given that  $V_i(x) = A_i + B_i x + C_i x^2$ , it is straightforward to obtain the solutions to (A-17)-(A-18). We do not present the explicit solutions as they involve quite cumbersome expressions. Having obtained the solutions for optimal choices of the  $q$ 's, we substitute them into the system (A-16)-(A-18) and construct a new system that has  $3(K+1)$  equations in the coefficients on  $V_i(\cdot)$ ,  $i = 1, \dots, K+1$ . The properties of the system that we obtain are described in Table 1.

	Equation for coefficient on					
	$q^0$		$q$		$q^2$	
	Variables that enter the equation					
	linearly	nonlinearly	linearly	nonlinearly	linearly	nonlinearly
Eq. 1	$A_1, A_{K+1}$	$B_2, C_2$	$B_1$	–	$C_1$	–
Eq. 2	$A_2, A_3$	$B_3, C_3$	$B_2$	$B_3, C_3$	$C_2$	$C_3$
...	...	...	...	...	...	...
Eq. $K+1$	$A_{K+1}, A_1$	$B_{K+1}, C_{K+1}$	$B_{K+1}$	$B_1, C_1$	$C_{K+1}$	$C_1$

Table 1: Properties of the system of equations for the coefficients of value functions

Fortunately, this system can be solved and never involves anything more complicated than linear equations. First, solve for  $C_1$ , then for  $C_{K+1}, C_K, \dots, C_2$ . Then we are able to solve for  $B_1$  and combining this solution with solutions for  $C_1$  we solve for  $B_{K+1}, B_K, \dots, B_2$ . Substituting all these solutions into the the rest of equations involving  $A_i$ 's, we obtain a system of linear equations that (as it appears) has exactly one solution. Having the solution for the stationary part of the problem, it is easy to solve the problem completely by working backwards starting from the stationary part.

To find the optimal program, the programs with  $K, M = 1, 2, \dots, 20$  were considered. It appears that, under the chosen values of parameters, the stationary part of the program has two tenancies in one rehabilitation cycle. The optimal number of tenancies in the non-stationary part depends on  $\alpha$  and can be 2 or 3.

Depending on  $\alpha$ , the value function for this program has the following form:

$$\hat{V} = \begin{cases} 6.887 - 15.103\alpha + 10.554\alpha^2, & \text{if } 0.003 < \alpha \leq 0.168, \text{ (3 tenancies in a non-stationary cycle)} \\ 6.851 - 15.056\alpha + 11.527\alpha^2, & \text{if } 0.168 < \alpha \leq 0.25 \text{ (2 tenancies in a non-stationary cycle)} \end{cases}$$

(we do not consider  $\alpha > 0.24$ ). It is clear why we have more non-stationary cycles for lower  $\alpha$ : the lower the cost of construction, the higher the initial quality the landlord chooses and the longer it takes to downgrade to the quality where it is profitable to rehabilitate. Figure 6 shows the optimal trajectories for  $\alpha = 0.2$ .