On the Welfare Cost of Inflation and the Recent Behavior of Money Demand

Peter N. Ireland*
Boston College and NBER

April 2007

Abstract

Post-1980 U.S. data trace out a stable long-run money demand relationship of Cagan's semi-log form between the M1-income ratio and the nominal interest rate, with an interest semi-elasticity of 1.79. Integrating under this money demand curve yields estimates of the welfare cost of modest departures from Friedman's zero nominal interest rate rule for the optimum quantity of money that are quite small. The results suggest that the Federal Reserve's current policy, which generates low but still positive rates of inflation, provides an adequate approximation in welfare terms to the alternative of moving all the way to the Friedman rule.

JEL: E31, E41, E52.

*Please address correspondence to: Peter N. Ireland, Boston College, Department of Economics, 140 Commonwealth Avenue, Chestnut Hill, MA 02467-3859. Tel: (617) 552-3687. Fax: (617) 552-2308. Email: irelandp@bc.edu. http://www2.bc.edu/~irelandp. The opinions, findings, conclusions, and recommendations expressed herein are my own and do not reflect those of the National Bureau of Economic Research.

1 On the Welfare Cost of Inflation ...

Inflation, brought under control in the early eighties, remains subdued today. Still, the question remains: what cost does the Federal Reserve's well-established policy of low but positive inflation impose on the economy, when compared to the optimal monetary policy prescribed by Friedman (1969), which calls for a deflation that makes the nominal interest rate equal to zero?

Lucas (2000), working in the tradition of Bailey (1956) and Friedman (1969), addresses this question directly. Lucas' analysis juxtaposes two competing specifications for money demand. One, inspired by Meltzer (1963), relates the natural logarithm of m, the ratio of nominal money balances to nominal income, to the natural logarithm of r, the short-term nominal interest rate, according to

$$\ln(m) = \ln(A) - \eta \ln(r) \tag{1}$$

where $A > 0$ is a constant and $\eta > 0$ measures the absolute value of the interest elasticity of money demand. The other, adapted from Cagan (1956), links the log of m instead to the level of r via

$$\ln(m) = \ln(B) - \xi r, \tag{2}$$

where $B > 0$ is a constant and $\xi > 0$ measures the absolute value of the interest semi-elasticity of money demand.

Figure 1 plots the log-log demand curve (1) and the semi-log demand curve (2) on the same graph, where the axes measure both m and r in levels. Lucas' (2000) preferred specifications set $\eta = 0.5$ in (1) and $\xi = 7$ in (2), then pin down the constants $A = 0.0488$ and $B = 0.3548$ so that $\ln(A)$ equals the average value of $\ln(m) + \eta \ln(r)$ and $\ln(B)$ equals the average value of $\ln(m) + \xi r$ in annual U.S. data, 1900-1994. These same settings determine the curvature and horizontal placement of the two curves in Figure 1.

The graph highlights how (1) and (2) describe very different money demand behavior at low interest rates: as r approaches zero, (1) implies that real balances become arbitrarily large, while (2) implies that real balances reach the finite satiation point B when expressed as a fraction of real income. Hence, as emphasized by Lucas (2000), these competing money demand specifications also have very different implications for the welfare cost of modest departures from Friedman's (1969) zero nominal interest rate rule for the optimum quantity of money.

Bailey's (1956) traditional approach measures this welfare cost by integrating under the money demand curve as the interest rate rises from zero to $r > 0$ to find the lost consumer surplus then subtracting off the seigniorage revenue rm to isolate the deadweight loss. Let $w(r)$ denote this welfare-cost measure, expressed as a function of r. Lucas (2000) shows that

$$w(r) = A\left(\frac{\eta}{1-\eta}\right)r^{1-\eta} \tag{3}$$

when money demand takes the log-log form (1) and

$$w(r) = \frac{B}{\xi}[1-(1+\xi r)e^{-\xi r}] \tag{4}$$

when money demand takes the semi-log form (2). If, as assumed by Lucas, the steady-state real interest rate equals three percent, so that $r = 0.03$ prevails under a policy of zero inflation or price stability, then (3) and (4) imply that this policy costs the economy the equivalent of 0.85 percent of income when money demand is log-log, but only 0.10 percent of income when money demand has the semi-log form. Likewise, an ongoing two percent inflation costs the economy 1.09 percent of income under (1) and (3), but only 0.25 percent of income under (2) and (4).

These calculations underscore the importance of discerning the appropriate form of the money demand function before evaluating alternative monetary policies, including those that generate very low but still positive rates of inflation. Hence, Figure 1 also plots U.S. data on the money-income ratio and the nominal interest rate, from an annual sample extending from 1900 through 1994 that is constructed, as described below in the Appendix, to resemble closely the one used by Lucas (2000). Following Lucas, m is measured by dividing the M1 money stock by nominal GDP and r is measured by the six-month commercial paper rate. Based on the same comparison between these data and the plots of (1) and (2) shown in Figure 1, Lucas concludes that the log-log specification provides a better fit, and thereby argues implicitly that the Federal Reserve could secure a substantial welfare gain for American consumers by abandoning its current, low-but-positive inflation policy and adopting the Friedman rule instead.

An important caveat to Lucas' (2000) argument arises, however, once one recognizes that the log-log specification appears to deliver a significantly better fit in Figure 1 thanks in large part to its ability to track the five data points that lie farthest out along the x-axis, representing (m, r) pairs such that m exceeds 0.4 and r falls below 0.0015. Viewed in one way, these data points are quite informative, since they show how the demand for M1 in the United States did, in fact, increase sharply when the interest rate fell to very low levels. But, all the same, one might reasonably question the relevance of these particular data points to an exercise that evaluates Federal Reserve policy today, since they come from 1945 through 1949, a distant period when the U.S. financial system and indeed the U.S. economy as a whole looked very different from the way they appear now.

Fortunately, new data has accumulated since the mid-1990s that quite usefully complement those used in Lucas' (2000) study: importantly, those new data include observations from a much more recent episode from 2002 through 2004 that also features very low nominal interest rates. Hence, Figure 2 reproduces Figure 1 after updating Lucas' sample to run through 2006. The more recent data also cover a period when the development and proliferation of retail deposit sweep programs, involving banks' efforts to reclassify their checkable deposits as money market deposits and thereby avoid statutory reserve requirements, severely distort official measures of the M1 money stock. Since, as argued by Anderson (2003b), these sweep operations take place behind the scenes, invisible to the eyes of most account holders, Figure 2 uses data on the M1RS aggregate, defined and constructed by Dutkowsky and Cynamon (2003), Cynamon, Dutkowsky, and Jones (2006), and Dutkowsky, Cynamon, and Jones (2006) by adding the value of swept funds back into the standard M1 figures, to measure the money-income ratio since 1994.

To focus more clearly on the recent behavior of money demand, Figure 2 distinguishes between the data from 1980-2006 and the data from 1900-1979, the breakpoint coinciding with both the arrival of Paul Volcker at the Federal Reserve Board and the implementation of the Depository Institutions Deregulation and Monetary Control Act of 1980 as key events marking the start of a new chapter in U.S. monetary history. Strikingly, the data points from the post-1980 period also trace out what looks like a stable money demand relationship, but one that seems very different from the log-log specification preferred by Lucas (2000) based on his examination of the earlier data.

Even after correcting for the effects of retail sweep programs, money balances displayed only modest growth relative to income during the 2002-2004 episode of very low interest rates, suggesting that the semi-log specification (2) with its finite satiation point may now provide a more accurate description of money demand. Futhermore, the new data points appear to trace out a demand curve that is far less interest-elastic than either of the two curves drawn in to track the earlier data from Figure 1. Both of these shifts, in functional form and towards a lower elasticity or semi-elasticity, work to reduce Lucas' (2000) estimate of the welfare cost of inflation. But, to make sure that the patterns appearing in Figure 2 are real and not optical illusions and to sharpen the quantitative estimate of the welfare cost of inflation implied by the recent behavior of money demand, the next section presents some more formal statistical results.

2 ... and the Recent Behavior of Money Demand

While Lucas' (2000) focus on a long historical time series extending back to the start of previous century requires the use of annual data, the focus here on the post-1980 period allows for the use of readily-available quarterly figures, again as described below in the Appendix. The money-income ratio is measured by dividing the sweep-adjusted M1 money stock, the M1RS aggregate referred to above, by nominal GDP. And since the Federal Reserve discontinued its reported series for the six-month commercial paper rate in 1997, the three-month U.S. Treasury bill rate serves instead as the measure of r; in any case, U.S. Treasury bills come closer to matching the risk-free, nominally-denominated bonds that serve as an an alternative store of value in theoretical models of money demand.

Following most of the empirical literature on U.S. money demand since Hafer and Jansen (1991) and Hoffman and Rasche (1991), the econometric analysis of these data revolves around the ideas of nonstationarity and cointegration introduced by Engle and Granger (1987). Specifically, a finding that the semi-log specification (2) describes a cointegrating relationship linking two nonstationary variables, the money-income ratio and the nominal interest rate, coupled with a finding that the log-log specification (1) fails to describe the same sort of relationship, provides formal statistical evidence supporting the more casual impressions gleaned from visual inspection of Figure 2 that the semi-log form offers a better fit to the post-1980 data.

Note that these statistical tests, which check first for nonstationarity in and then for cointegration between the variables $\ln(m)$ and $\ln(r)$ in (1) and the variables $\ln(m)$ and r in (2), require one to adopt a somewhat schizophrenic view of those data since, in a linear statistical framework, the analysis of (1) requires $\ln(r)$ to follow an autoregressive process with a unit root, while the same analysis of (2) requires r to follow an autoregressive process with a unit root. Bae (2005) helps to resolve this schizophrenia by providing a more detailed discussion of the case in which both (1) and (2) can be estimated under the common assumption that r follows an autoregressive process with a unit root, with (1) viewed as a nonlinear relationship between $\ln(m)$ and r and (2) viewed as a linear relationship between the same two variables. The analysis here, by contrast, follows Anderson and Rasche (2001) by putting the two competing specifications on equal footing ex-ante, treating both as linear relationships linking $\ln(m)$ to $\ln(r)$ in one case and $\ln(m)$ to r in the other.

Table 1 displays results from applying the Phillips-Perron (1988) unit root test described by Hamilton (1994, Ch.17) to each of the three variable: $\ln(m)$, $\ln(r)$, and r. The table reports values for $\hat{\mu}$ and $\hat{\rho}$, the intercept and slope coefficients from an ordinary least squares regression of each variable on a constant and its own lagged value, together with t, the conventional OLS t-statistic associated with the null hypothesis of a unit root: $\rho = 1$. The Phillips-Perron test statistic Z_t corrects this conventional t-statistic for the presence of serial correlation in the regression error by using the Newey-West (1987) estimator of its variance; Table 1 reports Z_t as computed for values of the lag truncation parameter q, that is, the bound on the number of sample autocovariances used in computing the Newey-West estimate, ranging from 0 (no serial correlation) to 8 (positive autocorrelations running out to eight quarters or two years). Critical values for Z_t appear under the heading "Case 2" in Hamilton's (1994, p.763) Table B.6. None of these test statistics allows the null hypothesis of a unit root to be rejected, paving the way for tests of cointegration between pairs of these apparently nonstationary variables.

Intuitively, the Phillips-Ouliaris (1990) test for cointegration described by Hamilton (1994, Ch.19) uses ordinary least squares to estimate the intercept and slope coefficient in the linear relationship (1) linking the nonstationary variables $\ln(m)$ and $\ln(r)$ or (2) linking the nonstationary variables $\ln(m)$ and r, then applies a Phillips-Perron test to determine whether the regression error from the equation is stationary or nonstationary. In the case where the null hypothesis of a unit root in the error can be rejected, then either (1) or (2) represents a cointegrating relationship: a stationary linear combination of two nonstationary variables. Table 2 displays results associated with these Phillips-Ouliaris tests: the intercept and slope coefficients $\hat{\alpha}$ and $\hat{\beta}$ from a linear regression of the form (1) or (2), the slope coefficient $\hat{\rho}$ from a regression of the error term from (1) or (2) on its own lagged value (without a constant, since the error has mean zero), the conventional t-statistic for the null hypothesis of no cointegration ($\rho = 1$), and the Phillips-Perron statistic Z_t for values of the Newey-West (1987) lag truncation parameter q ranging again from 0 through 8. Critical values for Z_t so constructed appear under the heading "Case 2" in Hamilton's (1994, p.766) Table B.9.

Confirming the apparent breakdown from Figure 2 of Lucas' (2000) preferred log-log specification in the post-1980 data, none of the tests summarized in Table 2's top panel rejects the null hypothesis of no cointegration between $\ln(m)$ and $\ln(r)$. On the other hand, all of the tests in the table's bottom panel reject their null of no cointegration between $\ln(m)$ and r at the 90 or 95 percent confidence level. Taken together, these results provide statistical evidence of a tighter money demand relationship of the semi-log form for the post-1980 period. And again confirming the visual impressions from Figure 2, the estimated semi-elasticity of 1.79 for 1980-2006 stands far below Lucas' choice of 7 made to fit the data from 1900-1994.

Table 3 complements Table 2 by reporting results from Johansen's (1991) test for cointegration, again applied first to $\ln(m)$ and $\ln(r)$ and then to $\ln(m)$ and r. As described by Hamilton (1994, Ch.20), Johansen's approach assumes that each pair of variables follows a p-th order vector autoregression in levels and expresses this VAR(p) in error-correction form, with the stationary first difference of each pair of variables on the left-hand side and $p - 1$ lags of first-differenced variables together with one lag of the hypothesized stationary linear combination of the two variables in levels on the right. Johansen's technique simultaneously estimates via maximum likelihood the parameters of the cointegrating vector in the error-correction term and the coefficients on the lagged first-differenced variables describing the model's short-run dynamics. Table 3 shows, for each pair of variables and for each value of p ranging from 2 through 9 (implying 1 through 8 lags when the VAR(p) in levels is written in error-correction form in first differences), the maximum eigenvalue λ_1 computed when evaluating the model's likelihood function using Johansen's algorithm, and the associated maximum eigenvalue statistic $LR = -T\ln(1-\lambda_1)$, a likelihood ratio statistic for testing the null of no cointegration against the alternative of cointegration. Since the cointegrating vector in (1) or (2) has a constant term, the critical values for LR appear under the heading "Case 2" in Hamilton's (1994, p.768) Table B.11.

Again, the results point to the semi-log specification (2) as providing a better description of the post-1980 data. The test statistics shown in Table 3's top panel fail to reject the null of no cointegration between $\ln(m)$ and $\ln(r)$ in six out of eight cases. By contrast, all but one of the test statistics from the table's bottom panel reject the null of no cointegration at the 90 percent level, and for values of p greater than 5 the null can in fact be rejected at the 99 percent confidence level.

The OLS estimates of the intercept and slope coefficient reported for the semi-log specification in Table 2 imply values of $B = 0.1686$ and $\xi = 1.7944$ in (2). Plugging these values into the corresponding formula (4) for Bailey's (1956) welfare cost measure and assuming, as above, that the steady-state real interest rate equals three percent, so that $r = 0.03$ corresponds to zero inflation, $r = 0.05$ corresponds to two percent annual inflation, and $r = 0.13$ corresponds to ten percent annual inflation, yields an estimate of only 0.01 percent of income for the cost of pursuing a policy of price stability as opposed to the Friedman (1969) rule, an estimate of 0.04 percent of income for the cost of two percent inflation, and an estimate of 0.22 percent of income for the cost of ten percent inflation. These welfare cost estimates lie far below those computed by Lucas (2000) and bring the analysis fill circle, back to Figures 1 and 2 and the apparent steepening and leftward shift of the money demand function in the years since 1980. Interestingly, these figures also provide an estimate of the cost of ten percent inflation compared to price stability, $w(0.13) - w(0.03)$, equal to 0.21 percent of income, a number that is still smaller than, but resembles more closely, Fischer's (1981) estimate of 0.30 percent of income and Lucas' (1981) estimate of 0.45 percent of income.

These results suggest that the Federal Reserve's current policy, which generates low but still positive rates of inflation, provides an adequate approximation in welfare terms to the alternative of moving all the way to Friedman's (1969) deflationary rule for a zero nominal interest rate. Before closing, however, it should be emphasized that these welfare cost estimates account only for the money demand distortion brought about by positive nominal interest rates. Dotsey and Ireland (1996) demonstrate that, in general equilibrium, other marginal decisions can also be distorted when inflation rises, impacting on both the level and growth rate of aggregate output, while Feldstein (1997) argues that the interactions between inflation and a tax code that is not completely indexed can add substantially to the welfare cost of inflation. To the extent that these additional sources of inefficiency remain present in the post-1980 U.S. economy, there will of course be larger gains to reducing inflation below its current low level.

3 Appendix: Data Sources

The annual data displayed in Figures 1 and 2 come from sources identical or very closely comparable to those used by Lucas (2000). To measure money, figures on M1 for 1900-1914 are taken from the U.S. Bureau of the Census (1960, Series X-267). Figures on M1 for 1915-1958 are taken from Anderson (2003*a*, Table 3, Columns 3 and 10) and come, originally, from Friedman and Schwartz (1970) for 1915-1946 and Rasche (1987, 1990) for 1947-1958. Figures on M1 for 1959-2006 are taken from the Federal Reserve Bank of St. Louis FRED database and are adjusted from 1994 onward by adding back into M1 the funds removed by retail deposit sweep programs using estimates described by Cynamon, Dutkowsky, and Jones (2006).

To measure nominal income, figures on nominal GDP for 1900-1928 are constructed by taking Kendrick's (1961, Table A-III, Column 5) series for real GDP and multiplying it by a series for the deflator constructed by dividing nominal GNP (Table A-IIb, Column 11) by real GNP (Table A-III, Column 1). Lucas (2000), too, uses the deflator for GNP to translate Kendrick's figures for real GDP into a corresponding series for nominal GDP; though his source for the deflator is U.S. Bureau of the Census (1960, Series F-5), the numbers from that table resemble quite closely those that come directly from Kendrick's (1961) monograph. Figures on nominal GDP for 1929-2006 come from the FRED database.

Finally, to measure the nominal interest rate, data on the six-month commercial paper rate are taken from Friedman and Schwartz (1982, Table 4.8, Column 6) for 1900-1975 and from the *Economic Report of the President* (2003, Table B-73) for 1976-1997. The Federal Reserve stopped publishing the interest rate series reported in this last source in 1997; hence, the interest rate for 1998-2006 is the three-month AA nonfinancial commercial paper rate, drawn from the FRED database.

The quarterly, post-1980 data used in the econometric analysis summarized in Tables 1, 2, and 3 all come from the Federal Reserve Bank of St. Louis FRED database, except that the series for M1 is adjusted by adding back the funds removed by retail deposit sweep programs using estimates described by Cynamon, Dutkowsky, and Jones (2006): the money stock is therefore measured by their M1RS aggregate. Nominal GDP again measures income, and the three-month U.S. Treasury bill rate measures the nominal interest rate.

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Table 1. Phillips-Perron Unit Root Test Results

$\ln(m)$	$\hat{\mu}$	$\hat{\rho}$	t	q	Z_t
	−0.0410	0.9778	−1.1923	0	−1.1923
				1	−1.3826
				2	−1.5262
				3	−1.6488
				4	−1.7269
				5	−1.7792
				6	−1.8165
				7	−1.8313
				8	−1.8256
$\ln(r)$	$\hat{\mu}$	$\hat{\rho}$	t	q	Z_t
	−0.1049	0.9682	−1.6449	0	−1.6449
				1	−1.7672
				2	−1.8595
				3	−1.9385
				4	−1.9943
				5	−2.0366
				6	−2.0697
				7	−2.0836
				8	−2.0863
r	$\hat{\mu}$	$\hat{\rho}$	t	q	Z_t
	0.0029	0.9361	−2.4602	0	−2.4602
				1	−2.5036
				2	−2.5028
				3	−2.5100
				4	−2.4994
				5	−2.4939
				6	−2.4932
				7	−2.4757
				8	−2.4604

Notes: Each panel reports $\hat{\mu}$ and $\hat{\rho}$, the intercept and slope coefficient from an ordinary least squares regression of the variable on a constant and its own lag, together with t, the t-statistic associated with null hypothesis of $\rho = 1$, and Z_t, the Phillips-Perron statistic corrected for autocorrelation in the regression error, computed using the Newey-West standard error of its variance for various values of the lag truncation parameter q. The critical values for Z_t are reported by Hamilton (1994, Table B.6, p.763): −2.58 (10 percent), −2.89 (5 percent), and −3.51 (1 percent).

Table 2. Phillips-Ouliaris Cointegration Test Results

$\ln(m) = \alpha - \beta \ln(r)$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\rho}$	t	q	Z_t
	−2.1474	0.0873	0.9351	−1.8768	0	−1.8768
					1	−2.0501
					2	−2.1720
					3	−2.3457
					4	−2.4447
					5	−2.5277
					6	−2.6090
					7	−2.6401
					8	−2.6430
$\ln(m) = \alpha - \beta r$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\rho}$	t	q	Z_t
	−1.7800	1.7944	0.8575	−3.1065	0	−3.1065*
					1	−3.1926*
					2	−3.1612*
					3	−3.2526*
					4	−3.2694*
					5	−3.3238*
					6	−3.4075**
					7	−3.4134**
					8	−3.4083**

Notes: Each panel reports $\hat{\alpha}$ and $\hat{\beta}$, the intercept and slope coefficient from the ordinary least squares regression of $\ln(m)$ on $\ln(r)$ or r; $\hat{\rho}$, the slope coefficient from an ordinary least squares regression of the corresponding regression error on its own lagged value; t, the t-statistic associated with null hypothesis of $\rho = 1$; and Z_t, the Phillips-Perron statistic for $\rho = 1$, corrected for autocorrelation in the residual, computed using the Newey-West standard error of its variance for various values of the lag truncation parameter q. The critical values for Z_t are reported by Hamilton (1994, Table B.9, p.766): −3.07 (10 percent), −3.37 (5 percent), and −3.96 (1 percent). Hence * and ** indicate that the null hypothesis of no cointegration can be rejected at the 90 and 95 percent confidence levels.

Table 3. Johansen Cointegration Test Results

$\ln(m) = \alpha - \beta \ln(r)$	p	$LR = -T \ln(1 - \lambda_1)$	Cointegrating Vector
	2	6.6154	$31.0889 \ln m_t = -68.5210 - 3.3000 \ln r_t$
	3	11.1572	$33.6092 \ln m_t = -71.9444 - 2.8586 \ln r_t$
	4	11.3538	$35.2480 \ln m_t = -75.1127 - 2.8850 \ln r_t$
	5	11.4929	$37.6776 \ln m_t = -81.1610 - 3.3735 \ln r_t$
	6	19.0392***	$37.1250 \ln m_t = -81.9482 - 3.9818 \ln r_t$
	7	10.5451	$42.4132 \ln m_t = -92.6301 - 4.2193 \ln r_t$
	8	11.5461	$44.5248 \ln m_t = -97.2603 - 4.4355 \ln r_t$
	9	15.2944**	$41.7114 \ln m_t = -93.1040 - 4.8169 \ln r_t$
$\ln(m) = \alpha - \beta r$	p	$LR = -T \ln(1 - \lambda_1)$	Cointegrating Vector
	2	11.4511	$34.4628 \ln m_t = -60.4502 - 77.1440 r_t$
	3	17.4283**	$33.9142 \ln m_t = -61.0110 - 49.8548 r_t$
	4	13.8940*	$21.3753 \ln m_t = -39.7389 - 9.4334 r_t$
	5	15.5605**	$19.0282 \ln m_t = -35.8125 - 0.9179 r_t$
	6	31.0005***	$35.7493 \ln m_t = -62.0107 - 91.9355 r_t$
	7	19.9672***	$34.1291 \ln m_t = -58.5019 - 99.7169 r_t$
	8	25.0197***	$53.7868 \ln m_t = -95.2640 - 104.6905 r_t$
	9	19.3294***	$47.4775 \ln m_t = -82.1158 - 126.1795 r_t$

Notes: Each panel reports the cointegrating vector linking $\ln(m)$ and $\ln(r)$ or $\ln(m)$ and r, estimated using Johansen's maximum likelihood technique under the assumption that the two variables follow a VAR(p) in levels, together with the statistic $LR = -T \ln(1 - \lambda_1)$ for testing the null hypothesis of no cointegration against the alternative of cointegration, where $T = 108$ for quarterly data, 1980-2006, and λ_1 denotes the maximum eigenvalue computed when evaluating the model's likelihood function using Johansen's technique. The critical values for LR are reported by Hamilton (1994, Table B.11, p.768): 12.783 (10 percent), 14.595 (5 percent), and 18.782 (1 percent). Hence *, **, and *** indicate that the null hypothesis of no cointegration can be rejected at the 90, 95, and 99 percent confidence levels.

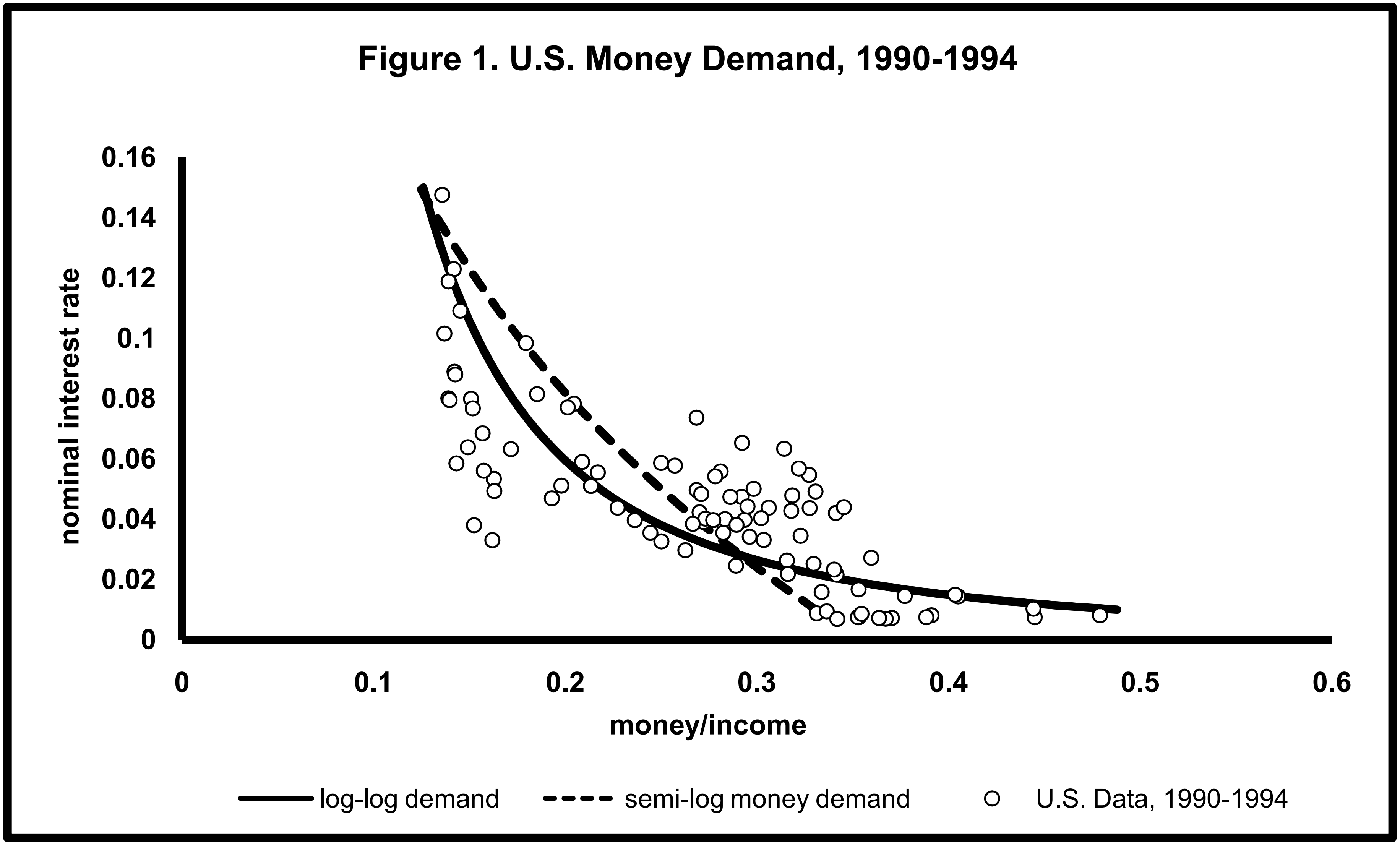
Figure 1. U.S. Money Demand, 1990-1994
nominal interest rate
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.1
0.2
0.3
0.4
0.5
0.6
money/income
log-log demand
semi-log money demand
U.S. Data, 1990-1994

Figure 2. U.S. Money Demand, 1900-2006