DOWNTOWN PARKING IN AUTO CITY[*]

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Abstract

This paper develops and calibrates a model of downtown parking in a city without mass
transit, and applies it to investigate downtown parking policy. There is curbside and
garage parking and traffic congestion. Spatial competition between private parking
garages determines the equilibrium garage parking fee and spacing between parking
garages. Curbside parking is priced below its social opportunity cost. Cruising for
parking adjusts to equalize the full prices of on- and off-street parking, and contributes
to traffic congestion. The central result is that raising curbside parking fees appears to
be a very attractive policy since it generates efficiency gains that may be several times
as large as the increased revenues raised.

Keywords: parking, traffic congestion, parking garages, parking policy

JEL Classification: R40

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1 Introduction

Anyone who drives in a major city will attest to the high cost of parking. Parking in a convenient private parking garage or lot is expensive, while finding cheaper parking typically entails cruising for parking and walking some distance. To our knowledge, there are no reliable estimates of the proportion of the average downtown auto full trip price associated with parking. Informal estimates of one half seem too high. It seems warranted to say however that the attention paid by economists to parking is far less than its importance in urban travel merits. There is a sizeable literature in economics on urban auto congestion but only a few recent papers on the economics of parking[1].

This paper considers some aspects of parking policy in the downtown areas of major cities where mass transit plays only a minor role, which include most US cities whose major growth occurred during the post-World-War II automobile era. How much curbside should be allocated to parking? And how should the parking meter rate be set? A simple model of downtown parking in such auto-oriented cities is constructed. The model incorporates the principal features that distinguish downtown parking from parking elsewhere – that a considerable proportion of parking capacity is provided by parking garages and that during the business day curbside (alternatively, on-street) parking is fully saturated. To simplify, the model assumes that trip demand is perfectly inelastic, drivers are identical, and space is isotropic, and examines steady states. Curbside parking is underpriced while garage (alternatively, off-street) parking is overpriced due to the exercise of market power. The consequent excess demand for curbside parking is rationed through cruising for parking. The model is calibrated in order to get preliminary estimates of the quantitative importance of the various effects identified.

In the first best, on- and off-street parking should be priced at their opportunity costs. The shadow price of an on-street parking spot is the cost of the increased congestion caused by

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[1] Small and Verhoef (forthcoming) provides an excellent and up-to-date review of both bodies of literature. This paper focuses on downtown parking, which is saturated for most of the business day. Here we briefly review papers in economics that concentrate on other aspects of parking. Arnott, de Palma, and Lindsey (1992) and Anderson and de Palma (2004) focus on the temporo-spatial equilibrium of parking when all drivers have a common destination and desired arrival time, such as for a special event or the morning commute to the central business district. Arnott and Rowse (1999) examines steady-state equilibria of cars cruising for parking on a circle when parking is unsaturated. And Arnott and Inci (2006) presents a model similar to this paper's but without garage parking.

using the space for parking rather than for traffic, and the shadow price of an off-street parking spot equals its marginal cost. The split between on- and off-street parking should be chosen such that their opportunity costs are equalized. When, however, parking pricing is distorted, the optimal split is more difficult to determine. When curbside parking is priced below garage parking, drivers cruise for on-street parking. The stock of cars cruising for parking adjusts until the full price of curbside parking, including the private cost of cruising for parking, equals that for garage parking. The second-best optimal split between on- and off-street parking then depends on how the split affects cruising for parking. The cars cruising for parking also add to traffic congestion.

Previous literature (Calthrop, 2001, Calthrop and Proost, 2006 and Shoup[2], 2005, Ch. 13)) has recognized that the stock of cars cruising for parking adjusts to equilibrate the full prices of on- and off-street parking. Calthrop (2001) and Arnott (2006) consider the potential importance of garage market power, Calthrop by assuming a monopoly supplier, and Arnott by assuming spatial competition between parking garages. And Arnott and Inci (2006) take into account the effects of cruising for parking on traffic congestion, but without garage parking. This paper innovates in combining the three elements and in developing calibrated examples quantifying the various effects.

In terms of policy insights, our principal finding is that, under conditions of even moderate traffic congestion, the social benefits from raising on-street parking rates may be several times the additional meter revenue generated. This is, of course, a double dividend result. Another important finding is that normally less space should be allocated to curbside parking the larger is the wedge between the curbside and garage parking rates.

Section 2 considers a simple model in which off-street parking is supplied at constant unit cost. Section 3 presents and analyzes the central model that takes into account the technology of garage construction and spatial competition between parking garages. Section 4 presents the calibrated numerical examples. And Section 5 discusses directions for future research and concludes.

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[2]Shoup, Table 11-5, displays the results of 16 studies of cruising for parking in 11 cities. The mean share of traffic cruising was 30% and the average search time was 8.1 minutes. While the study locations were not chosen randomly, the results do indicate the potential importance of cruising for parking.

2   A Simple Model

Understanding the central model of the paper will be facilitated by starting with a simplified variant. A broad-brush description is followed by a precise statement.

2.1   Informal model description

The model describes the equilibrium of traffic flow and parking in the downtown area of a major city[3]. To simplify, it is assumed that the downtown area is spatially homogeneous (isotropic) and in steady state, and also that drivers are homogeneous. Drivers enter the downtown area at an exogenous uniform rate per unit area-time, and have destinations that are uniformly distributed over it. Each driver travels a fixed distance over the downtown streets to his destination. Once he reaches his destination, he decides whether to park on street or off street in a parking garage[4]. Both on- and off-street parking are provided continuously over space. If he parks on street, he may have to cruise for parking, circling the block until a space opens up. After he has parked, he visits his destination for a fixed period of time, and then exits the system. Garage parking is assumed to be provided competitively by the private sector at constant cost, with the city parking department deciding on the curbside meter rate and the proportion of curbside to allocate to parking. The on-street parking fee (the meter rate) is less than the garage fee. Consequently, all drivers would like to park on street but the demand inflow is sufficiently high that this is impossible. On-street parking is saturated (the occupancy rate is 100%) and the excess demand for curbside parking spaces is rationed through cruising for parking. In particular, the stock of cars cruising for parking adjusts such that the full price of curbside parking, which is the sum of the meter payment and the cost of time cruising for parking, equals the garage parking payment. The downtown streets are congested by cars in transit and cruising for parking. In particular, travel time per unit distance driven increases with the density of traffic and the proportion of curbside allocated to parking.

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[3]The model differs from that in Arnott and Inci (2006) in two respects. Arnott and Inci consider a situation where all parking is on street and the demand for trips is sensitive to the full price for a trip. Here, in contrast, the demand for trips is completely inelastic, and there is parking both on and off street.

The model specification is independent of the form of the street network, but for concreteness one may imagine that there is a Manhattan network of one-way streets.

[4]The paper does not consider parking lots. Parking lots are difficult to treat because most are transitional land uses between the demolition of one building on a site and the construction of the next.

2.2　Formal model

Consider a spatially homogeneous downtown area to which the demand for travel per unit area-time is constant, D. Each driver travels a distance δ over the downtown streets to his destination, parks there for a period of time λ, and then exits. A driver has a choice between parking on street, where the meter rate is f per unit time, and parking off street in a parking garage, at a rate c per unit time, equal to the resource cost of providing a garage parking space. Both curbside and garage parking are continuously provided over space. By assumption, $f < c$, and the excess demand for curbside parking is rationed through cruising for parking. The stock of curbside parking is P per unit area, so that the number of garage parking spaces per unit area needed to accommodate the exogenous demand is $D\lambda - P$. The technology of traffic congestion is described by the function $t = t(T, C, P)$, where t is travel time per unit distance, T is the stock of cars in-transit per unit area, and C the stock of cars cruising for parking per unit area. The larger are C and T, the higher the density of traffic on the city streets, so that t_T and t_C (subscripts denote partial derivatives) are positive, and the larger is P, the lower the proportion of street space available for traffic, so that t_P is positive too. It is assumed as well that t is a convex function of T, C, and P.

D is sufficiently high that, even if all curbside is allocated to parking (so that $P = P_{max}$), there is still a need for garage parking (i.e., $D\lambda < P_{max}$). Due to the underpricing of curbside parking, the stock of cars cruising for parking adjusts such that the full price of curbside parking, the sum of the meter payment and the value of time lost cruising for parking, equals the full price of garage parking. For the moment, it is assumed that, even when curbside parking is provided free and all curbside is allocated to parking, the street system can still accommodate the exogenous demand[5].

The density of cars per unit area is $T + C$, their velocity, v, is $1/t$, and since flow equals density times velocity[6], the flow in terms of car-distance per unit area-time is $(T + C)/t$. If there are M distance units of one-way streets per unit area, then a person standing on a sidewalk would observe a flow of $(T + C)/(Mt)$ cars per unit time. *Throughput* is defined

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[5]Primitive conditions for this assumption to hold are given in section 2.6, which examines the congestion technology in detail.

[6]Flow equals density times velocity is known as the Fundamental Identity of Traffic Flow. Applying this identity in this context requires some care. Ordinarily, density is measured in cars per unit distance, so that, with velocity measured as distance per unit time, the dimension of flow is cars per unit time. Here, however, density is measured as cars per unit area, so that application of the formula gives flow in units of car-distance per unit area-time.

Now, suppose that there are M miles of street per unit area. Then the density in terms of cars per unit distance on a street is $(T+C)/M$, and application of the formula gives flow as cars per unit time on a street, $(T+C)/(Mt)$.

analogously to flow but includes only cars in transit[7]; thus, the throughput in terms of car-distance per unit area-time is T/t.

Steady-state equilibrium is described by two conditions. The first, the *steady-state equilibrium condition*, is that the input rate into the in-transit pool, D, equals the output rate, which equals the stock of cars in the in-transit pool divided by the length of time each car stays in the pool, $T/(\delta t(T, C, P))$:

$$D = \frac{T}{\delta t(T, C, P)} \quad . \tag{1}$$

This may be written alternatively as $D\delta = T/t(T, C, P)$. $D\delta$ is the input in terms of car-distance per unit area-time, and $T/t(T, C, P)$ is the throughput. The second equilibrium condition, the *parking equilibrium condition*, is that the stock of cars cruising for parking adjusts to equilibrate the full prices of garage and curbside parking:

$$c\lambda = f\lambda + \frac{\rho C\lambda}{P} \quad . \tag{2}$$

The full price of garage parking is $c\lambda$. The full price of curbside parking is $f\lambda$ plus the (expected) cost of cruising for parking. The expected time cruising for parking equals the stock of cars cruising for parking, C, divided by the rate at which curbside parking spots are vacated, P/λ. Thus, holding fixed the expected time cruising for parking, the stock of cars cruising for parking increases with the number of curbside parking spaces available. The cost of cruising for parking equals the expected time cruising for parking times the value of time, ρ. Eq. (2) may be rewritten as

$$C = \frac{(c - f)P}{\rho} \quad , \tag{3}$$

indicating the equilibrium stock of cars cruising for parking as a function of c, f, P, and ρ.

This simple model has two equations in two unknowns, T and C. The equations are recursive. Eq. (2) determines C and then eq. (1) determines T. Resource costs per unit area-time, RC, are simply $\rho(T + C)$, the stock of cars in transit and cruising for parking, times the value of time, plus the resource cost of garage parking, $c(D\lambda - P)$.

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[7]Since the transportation engineering literature has not analyzed situations in which cars circle the block, it does not make a terminological distinction between flow and throughput. It seems intuitive to define flow as the number of cars that a bystander would count passing by per unit time. Throughput too seems an appropriate choice of term.

2.3 Full social optimum

The full social optimum entails no cruising for parking. The density of in-transit traffic is then determined by (1) with $C = 0$. The social welfare optimization problem is to choose T and P to minimize resource costs per unit area-time, subject to the steady-state equilibrium condition, given by (1):

$$\min_{T,P} RC = \rho T + c(D\lambda - P) \quad s.t. \quad T - \delta t(T, 0, P)D = 0 \quad . \tag{4}$$

The shadow cost of on-street parking is the increase in in-transit travel time cost per unit area-time from having one more on-street parking spot, which, from (1), is $\rho dT/dP = \rho \delta t_P D/(1 - \delta t_T D))$. If, with $P = 0$, the shadow cost of on-street parking exceeds c, it is optimal to allocate no curbside to parking. And if, with $P = P_{max}$, the shadow cost of on-street parking falls short of c, it is optimal to allocate all curbside to parking. Otherwise, the optimal P solves

$$\frac{\rho \delta t_P D}{1 - \delta t_T D} - c = 0 \quad ; \tag{5}$$

the level of on-street parking should be chosen to equalize the shadow costs of on- and off-street parking[8].

Let $*$ denote the value of a variable at the social optimum. The social optimum can be decentralized by setting $P = P^*$, $T = T^*$, and $f = f^* = c$.

2.4 Constrained (second-best) social optimum

The constrained social optimum is now considered, where the constraint is that the on-street parking fee is set below c, with the stock of cars cruising for parking adjusting so as to satisfy the parking equilibrium condition. The second-best optimal allocation of curbside to parking minimizes resource costs per unit area, subject to both the steady-state equilibrium condition, (1), and the parking equilibrium condition, (2). Resource costs per unit area-time are given by $\rho(T+C)+c(D\lambda-P)$. Thus, the constrained social welfare optimization problem

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[8]Recall that the constraint is that the inflow to the in-transit pool equal the outflow. In conventional traffic flow theory, there are two densities corresponding to a level of flow. The specification of the minimization problems implies the choice of the lower density. This complication is addressed in section 2.5. The convexity of the congestion function ensures that there is a unique minimum corresponding to the lower density.

is to choose T, C, and P to

$$\min_{T,C,P} RC = \rho(T+C) + c(D\lambda - P) \quad s.t. \quad \begin{array}{ll} i) & D\delta t(T,C,P) - T = 0 \ , \quad \phi \\ ii) & \frac{P}{\rho}(c-f) - C = 0 \ , \quad \varphi \end{array} \tag{6}$$

where ϕ is the Lagrange multiplier on constraint i) and φ that on constraint ii). The second-best optimum may entail no curbside allocated to parking, in which case there is no cruising for parking, or all the curbside allocated to parking. An interior optimum is characterized by the first-order conditions:

$$T \ : \quad \rho + \phi(D\delta t_T - 1) = 0 \tag{7a}$$

$$C \ : \quad \rho + \phi D\delta t_C - \varphi = 0 \tag{7b}$$

$$P \ : \quad -c + \phi D\delta t_P + \frac{\varphi(c-f)}{\rho} = 0 \tag{7c}$$

Substituting out the Lagrange multipliers yields

$$\frac{\rho D\delta t_P}{1 - D\delta t_T} + (c-f) + \frac{D\delta t_C}{1 - D\delta t_T}(c-f) - c = 0 \quad . \tag{8}$$

A heuristic derivation is as follows: P should be chosen such that $dRC/dP = 0$. From the objective function, $dRC/dP = \rho dT/dP + \rho dC/dP - c$; from constraint ii), $dC/dP = (c-f)/\rho$; and from constraint i), $(dT/dP)(1 - D\delta t_T) = D\delta t_C dC/dP + D\delta t_P$. Thus, an increase in P has three effects on travel costs. First, it has a direct effect on aggregate in-transit costs by reducing the road space available for travel; this *capacity reduction effect* is the first term in (8). Second, since the stock of cars cruising for parking is proportional to the amount of on-street parking, the increase in P has a direct effect on aggregate cruising-for-parking time costs; this *cruising-for-parking stock effect* is the second term. Third, by increasing the stock of cars cruising for parking, it increases the congestion experienced by cars in transit; this *cruising-for-parking congestion effect* is the third term.

Let $**$ denote values at the constrained social optimum. With the on-street meter rate set at the exogenous level, the constrained social optimum can be decentralized by setting $T = T^{**}$, $C = C^{**}$, and $P = P^{**}$.

Unless both allocations entail the same corner solution, the optimal amount of curbside to allocate to parking is greater in the full social optimum than in the second-best social optimum with underpriced curbside parking, i.e. $P^* > P^{**}$. In both allocations, the marginal social benefit of increasing P by one unit is the saving in garage resource costs, c. But the

marginal social cost of increasing P is lower in the full social optimum than in the second-best optimum since the costs deriving from the cruising-for-parking stock and congestion effects are absent.

2.5 Revenue multiplier: the effects of raising the on-street parking fee

A principal theme of the paper is that the underpricing of on-street parking is wasteful. To formalize this point, this subsection examines the resource savings from increasing the on-street parking fee by a small amount when it is below c, holding P fixed. From the expression for resource costs:

$$
\begin{aligned}
\frac{dRC}{df} &= \rho\frac{dT}{df} + \rho\frac{dC}{df} \\
&= \rho[\frac{dT}{dC}\Big|_{(1)} + 1]\frac{dC}{df} \quad .
\end{aligned}
\tag{9}
$$

where $dT/dC|_{(1)}$ denotes the change in T associated with a unit change in C along (1). Now, the revenue raised from the parking fee, R, is Pf, so that $dR/df = P$. From (2), $\rho dC/df = -P$. Thus,

$$
-\frac{dRC}{df} = [\frac{dT}{dC}\Big|_{(1)} + 1]\frac{dR}{df} \quad .
\tag{10}
$$

Hence, the resource cost saving per unit area-time from raising the on-street parking fee equals a multiple of the increase in the parking fee revenue raised. Since the full price of parking is c, whether a driver parks on street or off, cruising-for-parking costs fall by exactly the amount of the increase in the parking fee revenue, and there is the added benefit that in-transit travel costs fall due to the reduction in the stock of cars cruising for parking. The resource costs savings would be even larger if account were taken of the distortion generated by the collection of government tax revenue.

Define

$$
\mu \equiv -\frac{\frac{dRC}{df}}{\frac{dR}{df}} = 1 + \frac{dT}{dC}\Big|_{(1)}
\tag{11}
$$

to be the (marginal) *revenue multiplier*. What determines the size of the revenue multiplier? Or put alternatively, by how much does a unit reduction in the stock of cars cruising for parking reduce the total stock of cars on the road? The answer depends on the technology of congestion, as well as its level. The revenue multiplier is even larger if account is taken of

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the marginal cost of public funds exceeding 1.

2.6   The congestion technology

The steady-state equilibrium condition, (1), can be written implicitly as $C = C(T; P, D)$. Holding fixed P and D, for each level of T the function gives the stock of cars cruising for parking consistent with steady-state equilibrium. Under the assumption that the function $t(.)$ is convex in T, C, and P, holding P fixed the function C is concave in T. In the absence of cruising for parking, with realistic congestion functions there are normally two densities consistent with a given level of (feasible) flow, i.e. $T = \delta t(T, 0, P)D$ has two solutions, one corresponding to regular traffic flow, the other to traffic jam conditions. Figure 1 displays the graph of the function C, termed the *steady-state locus*, with these two properties. An increase in P causes the locus to shift down; holding T fixed, (1) determines an equilibrium travel time, and to offset the increase in travel time due to the increase in P requires a decrease in C. An increase in D causes the locus to shift down; holding P and T fixed, for (1) to continue to be satisfied the increase in D must be offset by a decrease in t, and hence a decrease in C. If D increases sufficiently, there is no (T, C) satisfying (1) and a steady-state equilibrium does not exist.

It is assumed furthermore that the function $t(.)$ is weakly separable, specifically that $t = t(T, C, P) = t(V(T, C), P)$, with V defined as the *effective density* of cars on the road[9] and normalized in terms of in-transit car-equivalents in the absence of cruising for parking, i.e. $V(T, 0) = T$.

Define $\kappa(C, P) = \max_T T/t(V(T, C), P)$ to be the *throughput capacity* of the street system as a function of C and P, so that $\kappa(0, P)$ is capacity as conventionally defined. It indicates the maximum entry rate, in terms of car-miles, the street system can accommodate in steady-state equilibrium for a given C and P. If $D\delta$ exceeds $\kappa(0, P)$, then the entry rate exceeds throughput capacity even in the absence of cruising for parking, and no steady-state equilibrium exists. If $D\delta$ is less than $\kappa(0, P)$, then the steady-state locus lies in the positive

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[9]In the numerical examples presented later, it will be assumed that congestion takes the form of a negative linear relationship between velocity and effective density: $v = v_f(1 - V/V_j)$, where v_f is free-flow speed and V_j is effective jam density, which is a slight generalization of Greenshield's relation. Travel time per unit distance is the reciprocal of velocity. Then letting t_0 denote free-flow travel time, this relationship can be rewritten as $t = t_0/(1 - V/V_j)$. It will also be assumed that a car cruising for parking creates 1.5 times as much congestion as a car in transit, i.e. $V = T + 1.5C$. Eq. (1) then becomes $T(V_j - T - 1.5C) = \delta Dt_0V_j$. Thus, capacity is $V_j/(4t_0)$, the effective density corresponding to this capacity is $V_j/2$, $(dC/dT)_{(1)} = (V_j - 2T - 1.5C)/(1.5T)$, and the revenue multiplier is $\mu = (V_j - 0.5T - 1.5C)/(V_j - 2T - 1.5C)$. We shall assume furthermore that effective jam density is linearly decreasing in the proportion of curbside allocated to parking: $V_j = \Omega(1 - P/P_{max})$, where Ω is the effective jam density with no curbside parking.

Figure 1: Steady-state equilibrium
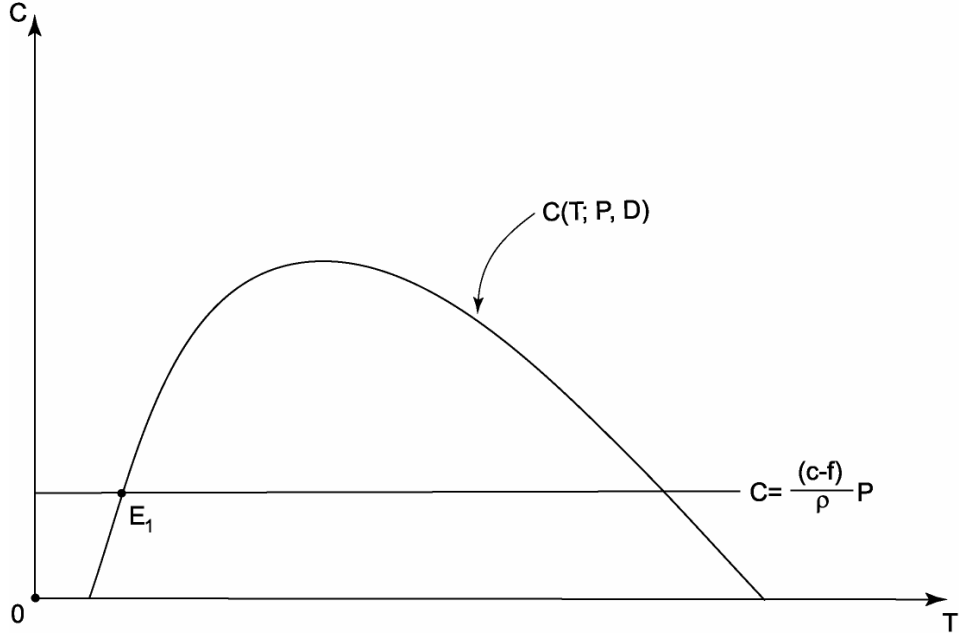
Notes :

1. $C(T; P, D)$ is the steady-state locus, eq.(1).
2. $C = \frac{c-f}{\rho} P$ is the parking equilibrium locus, eq.(2).

quadrant. The parking equilibrium condition, (2), can be written as $C = (c - f)P/\rho$, giving the equilibrium stock of cars cruising for parking. Since the condition is independent of T, its graph in Figure 1, the *parking equilibrium locus*, is a horizontal line. If (2) lies everywhere above (1), which occurs if $D\delta > \kappa((c-f)P/\rho, P)$, the entry rate exceeds throughput capacity at the equilibrium level of cruising for parking, and no equilibrium exists[10]. If (1) and (2) intersect, they do so twice. Following Vickrey's terminology, the intersection point with the lower level of T entails *congested* travel, and that with the higher level of T *hypercongested* travel. It is assumed, as is conventionally done, that travel is congested rather than hyper-congested, and hence that the former intersection point is the *equilibrium* for a given f and P.

An initial equilibrium is indicated by E_1 in Figure 1. The revenue multiplier equals one plus the reciprocal of the slope of the steady-state locus at the equilibrium point. If the parking fee is lowered, the parking equilibrium locus shifts up, causing the equilibrium to

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[10]Earlier, to avoid considering non-existence of a solution to the resource cost minimization problems, it was assumed an equilibrium exists even when traffic is as congested as possible, which occurs when $f = 0$ and $P = P_{max}$. This condition is that $D\delta < \kappa(cP_{max}/\rho, P_{max})$. With a positive parking fee and/or less curbside allocated to parking, a solution to the resource costs minimization problems (and the corresponding equilibria) may exist when this condition is not satisfied.

move up along the steady-state locus. Due to the convexity of the congestion technology, the slope of the steady-state locus at the equilibrium point falls, and hence the revenue multiplier increases. Now consider the effect of increasing the amount of curbside allocated to parking. The steady-state locus shifts down and the parking equilibrium locus shifts up. The equilibrium T and C increase; effective density increases, which, along with the decrease in road capacity, causes traffic congestion to worsen; and the revenue multiplier can be shown to increase[11].

2.7 Complications caused by garage construction technology

The above analysis laid out the economics of equilibrium when there is curbside and garage parking, when garage parking is priced at constant unit cost and when curbside parking is priced below this level. Unfortunately, the model is unrealistically simple in assuming that garage spaces are supplied uniformly over space at constant unit cost. The technology of garage construction and other factors result in parking garages being discretely spaced[12]. To reduce his walking costs, a driver is willing to pay a premium to park in the parking garage closest to his destination. Parking garages therefore have market power and may exercise it by pricing above marginal cost[13]. Furthermore, spatial competition between parking garages may result in their being inefficiently spaced. Taking these considerations into account complicates the economics, since there may then be three distortions that need to be taken into account, not only the underpricing of curbside parking but also the overpricing and

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[11]Substituting (2) into (1) gives $T = \delta t(T, (c-f)P/\rho, P) \Rightarrow \hat{T}(P)$. Also, from (2), $C = (c-f)P/\rho$. Then, $dT/dC = (d\hat{T}/dP) \div (dC/dP) = [\rho/(c-f)]d\hat{T}/dP$, and so $d(dT/dC)/dP = [\rho/(c-f)]d^2\hat{T}/dP^2$, which can be shown to be positive.

[12]Suppose, for the sake of argument, that parking garages are continuously distributed over space. It would be cheapest to construct garage parking on the ground floor of every building, but this space is especially valuable for retail purposes. Constructing below-ground parking may be cost effective at the time the building is constructed, but is expensive for buildings that were originally constructed without underground parking. Constructing above-ground parking in multi-use buildings raises structural issues. In most situations, the cost of constructing garage space is minimized with structures specifically designed as parking garages. Even if parking were distributed continuously over space, parking garage entrances would not be. Because of the fixed width required for a garage entrance and the fixed area required for a central ramp, parking garage entrances would be discretely spaced. Consideration of aesthetics and traffic circulation may play a role as well. Zoning may require that parking garages be located away from major traffic arteries to reduce the congestion and visual nuisance they cause.

The web version of the paper provides a map of downtown Boston, indicating the capacity and location of parking garages.

[13]The web version of the paper provides a figure showing the regular parking rate schedules, as a function of parking duration, for six Boston-area parking garages. The rate schedules are all concave. A parking garage incurs a constant marginal cost associated with a parked car (and a fixed cost as well, but this shall be ignored in the analysis). The concavity of the rate schedules therefore suggests price discrimination with respect to visit duration.

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inefficient spacing of garage parking.

In the next section, the model of this section is extended to take into account the exercise of market power caused by the discrete spacing of parking garages. The exact reason for the discrete spacing of parking garages is secondary. It is assumed that the discrete spacing arises from the fixed land area required for a central ramp, which generates horizontal economies of scale. The optimal spacing minimizes overall resources costs. The equilibrium spacing is the outcome of spatial competition between parking garages.

3   The Central Model

The primitives of the model differ from those of the simple model of the previous section in three respects. First, the garage cost function incorporates horizontal economies of scale, reflecting the fixed costs associated with the central ramp. Second, to avoid dealing with price discrimination based on parking duration, parking duration rather than visit duration is taken to be exogenous. And third, a grid street network is assumed.

In many cities, there are both public and private parking garages. To keep the analysis manageable, however, it is assumed that all parking garages are private[14]. The social optimum is solved first, then the spatial competition equilibrium is solved when the government intervenes only through its curbside parking policy.[15]

3.1   Social optimum

Since travel demand is perfectly inelastic, the social optimum entails minimizing resource costs per unit area-time. There are three components to resource costs per unit area-time: garage costs per unit area-time (GC), walking costs per unit area-time (WC), and in-transit travel costs per unit area-time (TT):

$$RC = GC + WC + TT \quad . \tag{12}$$

It is assumed that the presence of parking garages does not alter the distance drivers travel over city streets, or the optimality of on-street parkers parking at their destinations, or the

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[14]An obvious direction for future research is to investigate the situations where all garage parking is provided by the public sector, and where some is provided publicly and some privately.

[15]Less formal derivation of the results is provided in Arnott (2006).

spatial homogeneity of traffic flow[16]. These assumptions together imply that the steady-state equilibrium condition for the simple model, $T = \delta t(T, 0, P)D$, continues to hold. Denoting the corresponding congested equilibrium in-transit density as a function of P by $T^*(P)$ gives $TT = \rho T^*(P)$.

Efficiency entails identical parking garages being symmetrically arrayed over space, with diamond-shaped market areas. Let s be the grid or Manhattan distance between parking garages, x the capacity of each parking garage, and $K(x)$ the minimum cost per unit time of a garage as a function of capacity. Each garage services an area of $s^2/2$. With demand inflow D per unit area-time and parking duration λ, the total number of parking spaces in a garage's service area is $D\lambda s^2/2$. Since $Ps^2/2$ curbside parking spaces are provided in the service area, garage capacity is $x = (D\lambda - P)s^2/2$ and $GC = K((D\lambda - P)s^2/2) \div s^2/2$. Since the demand for garage parking is uniformly distributed over space, the average distance walked by a garage parker is $2s/3$ so that average walking time is $2s/(3w)$, where w is walking speed, and $WC = 2\rho s(D - P/\lambda)/(3w)$. Combining the above results gives

$$RC = \frac{K((D\lambda - P)\frac{s^2}{2})}{\frac{s^2}{2}} + 2\rho s \frac{D - \frac{P}{\lambda}}{3w} + \rho T^*(P) \quad . \tag{13}$$

Solution of the social optimum entails minimizing (13) with respect to P and s. The optimum may entail no curbside allocated to parking, all curbside allocated to parking, or only a fraction of curbside being allocated to parking. In the last case, the first-order condition with respect to P is

$$-K' - \frac{2\rho s}{3\lambda w} + \rho \frac{dT^*}{dP} = 0 \quad . \tag{14}$$

Expanding curbside parking capacity by one unit per unit area results in garage capacity being reduced by one unit per unit area, leading to a saving per unit area-time in garage costs of K' and in walking costs of $2\rho s/(3\lambda w)$, but in less curbside being allocated to traffic, resulting in an increase in in-transit travel costs of $\rho dT^*/dP$ (an expression for which is provided below (4)). With a realistic garage construction technology, the optimal spacing between parking garages solves the first-order condition of (13) with respect to s:

$$\frac{2(D\lambda - P)K'}{s} - \frac{4K}{s^3} + \frac{2\rho(D - \frac{P}{\lambda})}{3w} = 0 \quad .$$

---

[16]Solving for optimal traffic flow over space, taking into account the spatial inhomogeneity introduced by parking garages, would be formidably difficult.

Dividing through by $2(D - P/\lambda)/s$ and using $x = (D\lambda - P)s^2/2$ yields

$$\lambda K' - \frac{\lambda K}{x} + \frac{\rho s}{3w} = 0 \quad . \tag{15}$$

The social optimum minimizes resource costs per unit area-time. Since the input rate per unit area is constant, it also minimizes resource costs per driver. And since, in the choice of s, P and hence the ratio of garage parkers to drivers, is fixed, the optimal choice of s minimizes resource costs per garage parker. Since in-transit travel costs are independent of the spacing between parking garages, the optimal spacing between parking garages minimizes garage plus walking costs per garage parker. Let aGC and mGC be the average and marginal garage costs per garage parker, and define aWC and mWC accordingly. Since an average is minimized where the marginal equals the average, the optimal spacing between parking garages solves

$$aGC + aWC = mGC + mWC \quad , \tag{16}$$

which coincides with (15) since $aGC = \lambda K/x$, $mGC = \lambda K'$, $aWC = 2\rho s/(3w)$, and $mWC = \rho s/w$ because the marginal garage parker walks to the boundary of the garage service area.

3.2   Equilibrium

Parking garage structures are prohibitively costly to relocate and very costly to expand. Thus, the natural way to model spatial competition between parking garages is as a dynamic two-stage game with growing market demand. In each period's second stage, garages compete in fee schedules, taking the location and capacity of other garages as fixed. In each period's first stage, potential entrants decide simultaneously on entry, capacity, and location, anticipating the future evolution of market equilibrium. Since this game is intractable, a familiar spatial competition game that Tirole (1988) ascribes to Salop (1979), but which is discussed in Vickrey (1964) and probably has more distant origins, is adapted instead. The model is at least tractable and generates an equilibrium that seems reasonable. In the second stage, garages play a Bertrand-Nash game in mill prices, taking as given the location of their neighbors, who are by assumption arrayed symmetrically, and ignore the effects of their actions on traffic conditions. In the first stage, the number of garages, and hence the spacing between them, adjusts such that garage profits are zero.

Assume that the on-street parking fee is set below the marginal cost of a garage parking space, so that parking garages would find it unprofitable to undercut on-street parking[17].

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[17]Calthrop (2001) considers the profit-maximizing pricing of a monopoly parking garage, taking into

Consider a particular garage, garage 0. It has eight nearest neighbors, each of which is located a grid distance s away and charges the same amount S for a car to park for the period λ. Reasoning that each garage parker will choose to park in the garage with the lowest full parking price, which includes the garage charge and the cost of walking from the garage to the destination and back again, it calculates the distance of the boundary of its market area, b, to be related to its own parking charge, S_0, according to

$$b(S_0; S) = \frac{w(S - S_0)}{4\rho} + \frac{s}{2} \quad , \tag{17}$$

and its market area to be $2b(S_0; S)^2$. Since the on-street parking fee is below garage marginal cost undercutting on-street parking is unprofitable. Garage 0 then reasons that its per-period profits are related to its parking charge according to

$$\Pi = 2S_0(D - \frac{P}{\lambda})b(S_0; S)^2 - K(2(D\lambda - P)b(S_0; S)^2) \quad . \tag{18}$$

In its choice of S_0, garage 0 trades off a larger service area against a larger profit per inframarginal parker. Maximizing profits with respect to S_0 yields

$$S_0 = \lambda K' + \frac{2\rho b(S_0; S)}{w} \quad . \tag{19}$$

The Bertrand-Nash equilibrium is solved by setting $S = S_0$ in (19), yielding

$$S^e = \lambda K' + \frac{\rho s}{w} \quad . \tag{20}$$

Thus, equilibrium in the second-stage price game entails garages charging a markup over marginal garage cost equal to the walking cost incurred by a driver at the boundary between service areas. In the first stage of the game, entry and exit occur, driving profits to zero:

$$\Pi = (D - \frac{P}{\lambda})(\lambda K' + \frac{\rho s}{w})\frac{s^2}{2} - K = 0 \quad . \tag{21}$$

Dividing (21) through by $(D - P/\lambda)s^2/2$ yields

$$aGC = mGC + mWC \quad . \tag{22}$$

Comparing (16) and (22) implies that, under this form of spatial competition, parking garage market areas are inefficiently small.

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account that the profit-maximizing strategy may entail undercutting on-street parking.

It is assumed that, with discrete parking garages, curbside parkers drive to their destination block and then circle that block cruising for parking, as was the case with continuous parking garages, and that at each location (indexed by grid distance from the closest parking garage, m) the in-transit travel time is the same for curbside parkers as for garage parkers[18]. Under these assumptions, the stock of cars cruising for parking at location m adjusts to equilibrate the full prices of garage and curbside parking there. In contrast to the simple model, the full price of garage parking now includes walking costs. The parking equilibrium condition at location m is then

$$S^e + \frac{2\rho m}{w} = f\lambda + \frac{\rho C(m)\lambda}{P} \quad , \tag{23}$$

where $C(m)$ is the density of cars cruising for parking at location m, and S^e is given by (20), so that

$$C(m) = (S^e + \frac{2\rho m}{w} - f\lambda)\frac{P}{\rho\lambda} \quad . \tag{24}$$

To close the model, $T(m)$, the equilibrium density of cars in transit at each location, must be derived. Since there is no nice way to do this, it is assumed that $T(m)$ is the congested solution to the analog of the steady-state equilibrium condition at location m:

$$T(m) - \delta t(T(m), C(m), P)D = 0 \quad .$$

In-transit travel costs per unit area are obtained by averaging $\rho T(m)$ over the garage market area, and cruising-for-parking costs per unit area (CP) are obtained analogously.

3.3   Second-best parking policy

Regulation of private garage pricing, capacity, and location is not considered. Government intervention is restricted to curbside parking policy. Since modeling the full game between a local parking authority, with strategy variables f and P, and private parking garages, with strategy variables S and x, would be complex[19], the only policy to be investigated will be the local parking authority's second-best optimal choice of P on the assumption that the government behaves as a Stackleberg leader and that the meter rate is set sufficiently

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[18]Properly, the equilibrium spatial pattern of traffic flow with drivers optimizing over route and cruising-for-parking strategy should be determined, but this problem is intractable.

[19]Calthrop (2001) considers the profit-maximizing pricing of a garage monopolist in the face of an exogenous on-street meter rate, ignoring the discreteness of parking garages. If the meter rate is set above a critical level, it is profitable for the garage monopolist to undercut the meter rate. In the context of the model of this section, the full game between a local parking authority and private parking garages would need to take this undercutting possibility into account, and the equilibrium might involve garages undercutting on-street parking close to the parking garage but not further away.

low that parking garages do not have an incentive to undercut it. Relative to the social optimum, there are three sources of distortion. The underpricing of curbside parking and the overpricing of garage parking induce cruising for parking, which generates cruising-for-parking costs and increases in-transit travel costs. Also, spatial competition between parking garages results in parking garages being inefficiently closely spaced.

Let $GC^*(P)$ denote garage costs per unit area-time, as a function of P in the social optimum, etc., and $GC^e(P)$ the spatial competition equilibrium, etc. The second-best optimal level of P, P^{**}, minimizes $RC^e(P) = GC^e(P) + WC^e(P) + TT^e(P) + CP^e(P)$. If $RC^e(P)$ has a unique local, interior minimum, then $[dRC^e(P)/dP]_{P*} > 0$ is a necessary and sufficient condition for $P^{**} < P^*$. Letting $DWL(P)$ denote the deadweight loss in the spatial competition equilibrium relative to the social optimum, as a function of P, we have that

$$
\begin{aligned}
DWL(P) &= RC^e(P) - RC^*(P) \\
&= [GC^e(P) + WC^e(P) - GC^*(P) - WC^*(P)] \\
&\quad + [TT^e(P) + CP^e(P) - TT^*(P)] \\
&\equiv \widehat{DWL}(P) + \widehat{\widehat{DWL}}(P) \quad .
\end{aligned}
\tag{25}
$$

where $\widehat{DWL}(P)$ is the deadweight loss related to garages being inefficiently close in the spatial competition equilibrium, and $\widehat{\widehat{DWL}}(P)$ is that related to cruising for parking. Since $[dRC^*(P)/dP]_{P*} = 0$, $[dDWL(P)/dP]_{P*} = [dRC^e(P)/dP]_{P*}$. Thus, if $RC^e(P)$ has a unique local, interior minimum, a necessary and sufficient condition for $P^{**} < P^*$ is that deadweight loss be increasing in P at P^*. It can be shown that $\widehat{\widehat{DWL}}(P)$ is increasing[20] in P, but the sign of the derivative of $\widehat{DWL}(P)$ is ambiguous, depending in a complicated way on the properties of the garage cost function. The next section provides several numerical examples in which the second-best level of P is less than the first-best level but also one in which the second-best level exceeds the first-best level.

The analysis of this section's model was fairly complete but two thorny game-theoretic issues were sidestepped. The first concerns the modeling of spatial competition between parking garages. The game form analyzed was chosen on the basis of tractability and not because it is the most compelling. The second concerns the modeling of the game between private garage operators and the local authority. The paper considered the choice of the authority *qua* Stackleberg leader concerning how much curbside to allocate to parking but took the on-

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[20]It can be shown that in the spatial competition equilibrium, $dC/dP > 0$. It then follows from the convexity of the congestion function in T, C, and P that $\widehat{\widehat{DWL}}(P)$ is increasing in P.

street meter rate as fixed. Modeling a complete game between private garage operators and the local parking authority will be difficult. And this difficulty will be compounded when the parking authority's policy instruments are expanded to include regulation of parking garage fees, locations, and capacity, and when the political economy considerations that affect the authority's policy choices are taken into account.

4 Numerical Examples

4.1 Calibration

Arnott and Inci (2006) present numerical examples for a model similar to the one employed here, except that all parking is on street and demand is price-sensitive. This paper adopts all their values for the common parameters and functions, and adds a parameterized garage cost function and the level of demand as well.

The following parameters are employed. The units of measurement are hours for time, miles for distance, and dollars for value.

$$\delta = 2.0 \qquad \lambda = 2.0 \qquad f = 1.0 \qquad \rho = 20.0 \qquad P = 3712$$

The in-transit travel distance is 2.0 miles; the parking duration is 2.0 hours; the on-street parking fee is \$1.00 per hour; the value of time is \$20.00 per hour; and the number of curbside parking spaces is 3712 per square mile in the base case. We do not know of data on mean non-residential parking duration over the entire downtown area, but two hours seems reasonable when account is taken of non-work trips and auto trips taken by downtown employees during the working day. Since the model ignores downtown residents, the ratio of one mile traveled on downtown streets per hour parked seems reasonable too. The hourly meter rate for curbside parking is that employed in Boston. The value of time of \$20.00 per hour might seem high, but the average downtown parker is more highly paid and busier than the typical traveler[21]. The value of P chosen is for the base case, for which parking is on one side of the street, and requires explanation. Assuming 8 city blocks per mile on a Manhattan grid, a street width of 33 feet, a parking space length of 21 feet, and allowance

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[21]Small, Winston, and Yan (2005) find slightly higher mean values of travel time on the California State Route 91 Freeway in Orange County. The standard rule of thumb, based on many empirical studies, is that the value of travel time is half the wage rate, and the mean effective wage rate in the downtowns being considered must be around \$40.00 per hour, corresponding to an annual salary (48 weeks a year and 35 hours per week) of \$67200.

for crosswalks, 29 cars can be parked on one side of a block. With parking on one side of the street, there are 58 curbside parking spaces around each block. And with 64 blocks per square mile, there are 3712 curbside parking spaces per square mile.

The value of D is taken to be 7424 per ml.^{2}-hr. Since parking duration is two hours, this implies that the stock of parking spaces needed to accommodate the exogenous demand is 14848 per square mile. Thus, in the base case, one-quarter of the cars park on street and three-quarters off street.

The form of the congestion function employed was described earlier, in fn. 9. It is

$$t = \frac{t_0}{1 - \frac{V}{V_j}} \quad \text{with} \quad V_j = \Omega(1 - \frac{P}{P_{max}}) \quad \text{and} \quad V = T + \theta C \quad . \tag{26}$$

This congestion function has four parameters, all of which are the same as in Arnott and Inci (2006):

$$t_0 = 0.05 \qquad \Omega = 5932.38 \qquad P_{max} = 11136 \qquad \theta = 1.5$$

A value of t_0, free-flow travel time per mile, of 0.05 corresponds to a free-flow travel speed of 20 m.p.h. Ω is jam density in the absence of on-street parking. V_j is jam density with on-street parking, which is assumed to equal jam density in the absence of on-street parking times the proportion of street space available for traffic, $1 - P/P_{max}$. P_{max} is calculated on the basis that a 33-foot-wide, one-way road can accommodate three lanes of traffic and that parking on one side of the street reduces this to two lanes of traffic. In Arnott and Inci (2006), the above value of Ω was calculated to obtain cruising-for-parking results similar to those reported in Shoup (2005), but is also consistent with more basic traffic engineering reasoning.[22] θ is the in-transit PCE's (passenger-car equivalents) of a car cruising for parking. Since we know of no studies of its value, our estimate is a guess.

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[22]With the street geometry assumed, excluding intersections there are about 15.2 street-miles per square mile. $\Omega = 5932.38$ then corresponds to one car every 13.5 feet of street. Since, in the absence of on-street parking, a street has three lanes of traffic, this corresponds to one car every forty feet in a lane or 132 cars per lane-mile. In the absence of cruising for parking, with the assumed congestion function capacity throughput is achieved at one-half jam density or 66 cars per lane-mile, and the associated velocity and flow are 10 m.p.h. and 660 cars per lane-hour, respectively. These figures accord with those given in Table 16-17 of the *Transportation and Traffic Engineering Handbook* (Institute of Transportation Engineers, 1982), "Maximum Lane Service Volumes on Urban Arterials Based on 50% Cycle Split and Average Density and Speed Criteria."

The garage cost function is assumed to have the form[23]

$$\hat{K}(x, h) = R(A_0 + \frac{ax}{h}) + (k_0 + k_1 h)x + F_0 + F_1 h \quad ,$$

where $\hat{K}$ is the amortized (per hour) cost, x garage capacity, h garage height (number of floors of parking), R land rent, A_0 the area used for the central ramp, a the floor area needed for each additional car, $F_0 + F_1 h$ the cost of constructing the central ramp in a parking garage with h floors of parking, and $k_0 + k_1 h$ the unit cost of an additional parking space in a garage of h floors. We assume that an additional car needs 400 square feet of floor space, 200 square feet of parking space *per se* and 200 square feet of added space for traffic circulation, yielding $a = 1.44 \times 10^{-5}$ square miles, and that the parking ramp has a radius of 20 feet and therefore an area of 400π square feet, yielding $A_0 = 4.52 \times 10^{-5}$. In choosing cost parameter values, we draw heavily on Shoup (2005), Chapter 6. Table 6-1 gives an average cost in 2002 dollars per space added for garages constructed on the UCLA Campus between 1977 and 2002 of about[24] \$28000. Amortizing this cost, assuming that the garage is used for 8 hours a day, 200 days a year and that the annual user cost of capital is 0.05, which is consistent with a real interest rate of 4% and a 40-year life[25], yields a figure for $(k_0 + k_1 h) + (F_0 + F_1 h)/x$ of \$0.875/hr. It is assumed that $k_1 = 0.125 k_0$, $F_1 = 0.125 F_0$, and $F_0 = 10 k_0$. That leaves two parameters, k_0 and R. The values $k_0 = 0.5$ and[26] $R = \$2.5 \times 10^5$ are chosen. For a parking structure with 1000 spaces, which corresponds to the average at UCLA, the cost-minimizing height is 7.55 floors, so that the average amortized construction cost computed according to the above formula is \$0.982/hr. The amortized cost of land per garage space is \$0.488/hr. The ratio of land to construction costs seems reasonable. Average garage cost (corresponding to K/x in the theory) is therefore \$1.470/hr., and marginal garage cost (corresponding to K' in the theory) \$1.449/hr[27]. Though the parameters chosen for the garage cost function

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[23] Chapter 14 of the *Traffic Engineering Handbook* (Institute of Transportation Engineers, 2004), "Parking and Terminals", discusses the design of parking garages but does not provide engineering cost data. The form of the cost function in (27) was chosen for its ease of interpretation and analysis, and not on the basis of engineering data.

[24] In calculating this number, Shoup divided construction costs by the number of *added* parking spaces, on the assumption that the land was previously used as an on-ground parking lot.

[25] The real interest rate and the amortization period are those chosen by Shoup, Table 6-3, and he judged these to probably underestimate the user cost of capital.

[26] This is the amortized cost per ml.²-hr. It has been assumed that a garage space is utilized for 1600 hours per year. Thus, the land rent per ml.²-yr. is \$4.0 $\times$ 10^8. Since there are 640 acres per square mile, this corresponds to a land rent of \$6.25 $\times$ 10^5 per acre-year. Since land does not depreciate, an interest rate of 4% should be applied. Assume that land rent grows at the rate of 1% per year and that the property tax rate is 1% per year. Under these assumptions, the value of land per acre is \$15.625 million.

[27] Cost-minimizing building height increases as capacity increases. If building height were held fixed, marginal cost would be constant. The flexibility of building height causes marginal cost to fall. Average cost falls too, due to this effect and the fixed cost of the central ramp.

yield reasonable results, there is evidently considerable scope for improvement[28].

What kind of city does this parameterization correspond to? *Parking in Boston* (Boston Transportation Department, 2001) reports that in 1997/8 per square mile the number of employees in Downtown Boston was 160000 and the number of off-street, non-residential, non-hotel parking spaces was 29000. *Downtown Boston Transportation Plan* (Boston Transportation Department, 1995) reports that, in 1990, 36% of Downtown workers drove alone and 11% carpooled or vanpooled. On the assumption of no on-street parking, if all downtown workers had commuted by car, the required number of off-street parking spaces would have been about 62000 per square mile. This figure indicates that the calibrated city has a considerably lower employment density than Boston. Furthermore, applying the ratio for Boston of the off-street parking spaces if all downtown workers had commuted by car to the number of employees, suggests that an off-street parking density of 11136 per square mile corresponds to an employment density of around 29000 per square mile. According to Demographia (2007), such downtown employment densities are found in Winnipeg, Perth, San Diego, Sacramento, and Phoenix.

4.2 Numerical results

4.2.1 Base case outcomes

All the numerical exercises are for the central model. Table 1 presents the numerical results with the base case set of parameter values. Each of the columns corresponds to a different exercise. Each row gives the value for a particular variable across the various exercises. Column 1 describes the social optimum with the base case allocation of curbside to parking space of $P = 3712$, corresponding to curbside parking on one side of the street. The social optimum is defined to have no cruising for parking. Column 2 provides the numbers for the social optimum with the first-best allocation of curbside to parking. Column 3 presents the base case equilibrium. Column 4 displays results for the same case as column 3 but with the allocation of curbside to parking optimized. Column 5 gives the results for the base case equilibrium, but with the meter rate raised from $1.00 to $1.50 per hour. Finally, column 6 shows the equilibrium for the same case as column 5 but with the allocation of curbside to parking optimized.

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[28]Since Shoup's assumptions on the real interest rate and the amortization period likely underestimate the user cost of capital, and since administrative, operating, and maintenance costs (which for the UCLA parking garages add about 35% to costs – see Shoup, Table 6-3) have been ignored, it seems likely that the base case parameterization underestimates garage costs. In the numerical examples, the case in which garage

Table 1: Numerical results with base case parameter values

	1	2	3	4	5	6
	SO $P = 3712$	$SO(P^*)$	E $f = 1$ $P = 3712$	$E(P^{**})$ $f = 1$	E $f = 1.5$ $P = 3712$	$E(P^{**})$ $f = 1.5$
s	0.150	0.152	0.104	0.096	0.104	0.115
x	125	124	60	65	60	54
h	7.30	7.30	7.03	7.06	7.03	6.97
S			3.59	3.54	3.59	3.67
P		4144		718		6694
v	15.0	14.5	12.6	14.6	13.8	12.5
$CP/(TT+CP)$	0	0	0.14	0.033	0.084	0.14
GC/D	2.42	2.33	2.70	3.37	2.70	2.02
WC/D	0.50	0.49	0.35	0.41	0.35	0.28
TT/D	2.67	2.76	3.18	2.74	2.89	3.21
CP/D	0	0	0.51	0.095	0.26	0.53
RC/D	5.59	5.58	6.73	6.61	6.20	6.04
F			7.23	6.71	6.95	7.39

Notes:

1. The unit of time is an hour, of distance a mile, and of value a dollar.

2. GC/D is the garage cost per driver (including those who park on street) or average garage cost. Similarly, WC/D is average walking cost, CP/D average cruising-for-parking cost, TT/D average in-transit travel cost, and RC/D average resource cost.

3. A driver's in-transit travel time per mile is calculated as his in-transit travel cost (TT/D), divided by $\rho\delta$, and v, velocity, as the reciprocal of in-transit travel time. $CP/(TT + CP)$ measures the mean proportion of traffic flow that is cruising for parking. And F, the average full price of a trip, is calculated as average resource cost per driver plus curbside parking revenue per driver.

4. Blank cells correspond to variables that are not relevant for the exercise.

Consider first the social optimum, described in column 1, in which curbside parking is permitted on one side of every street, so that one-quarter of drivers park on street. The Manhattan spacing between parking garages is 0.15 miles, and each garage holds 125 cars and has 7.30

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costs are 40% higher than those of the base case is treated and seems to yield more reasonable results.

floors. All traffic is in transit and travel speed is 15.0 m.p.h. GC is garage cost per unit area-time, and D is throughput in cars per unit area-time, so that GC/D is garage cost per driver (or, equivalently, per trip), including those who park on street, or average garage cost. Similarly, WC/D is average walking cost, TT/D average in-transit travel cost, CP/D average cruising for parking cost, and RC/D average resource cost. Average garage cost is \$2.42, average walking cost \$0.50, average in-transit travel cost \$2.67, average cruising-for-parking cost zero since there is no cruising for parking in the social optimum, and average resource cost \$5.59. The cells for S (the garage parking charge) and F (the average full price of a trip) are blank since the social optimum allocation does not entail prices, and P is blank since its value is exogenous. The relative importance of average garage parking cost $(GC/D + WC/D)$ to average driving cost $(TT/D + CP/D)$ reflects the ratio of travel distance to parking duration, which in the example is set at 1.0.

Column 2 describes the social optimum in which the amount of curbside parking is chosen to minimize resource costs. Comparing columns 1 and 2 indicates that the optimal amount of curbside to allocate to parking is not very different from that assumed in the base case, 4144 parking spaces per square mile (55.8% of curbside) rather than 3712. Not surprisingly, therefore, optimizing the amount of curbside parking results in only small resource savings. Because more drivers park on street, average garage parking costs fall but average in-transit travel cost increases by almost the same amount, resulting in an average resource savings of only about \$0.01.

Column 3 describes the equilibrium in which the meter rate (the hourly on-street parking fee) is \$1.00/hr. and curbside parking is on one side of the street. Comparison of columns 3 and 1 is particularly interesting since it indicates the effects of moving from the social optimum to the equilibrium, holding constant the proportion of curbside allocated to parking. There are two qualitative differences between the equilibrium and the social optimum. First, spatial competition results in suboptimal spacing between parking garages. Second, since curbside parking is underpriced and garage parking overpriced, there is cruising for parking in the equilibrium, with the stock of cars cruising for parking adjusting such that the full prices of on- and off-street parking are equalized, and the cars cruising for parking slow down cars in transit.

In this equilibrium, the capacity of each parking garage is about half that in the social optimum, while the spacing between them is about two-thirds that in the social optimum. Since average garage parking cost is \$2.92 in the social optimum and \$3.05 in the equilibrium, average social cost associated with this distortion is relatively small, \$0.13. The distortion

generated by cruising for parking is considerably larger[29]. The distortion has two components. The first, average cruising-for-parking cost is, $0.51. The second, the increase in average in-transit travel cost due to the increased congestion caused by the cars cruising for parking, is $0.51 (that these two numbers are identical is a coincidence). The average deadweight loss caused by cruising for parking is therefore $1.02, which is an order of magnitude larger than that generated by the suboptimal spacing between parking garages. Cars cruising for parking constitute 14% of the traffic density and slow down traffic from 15.0 to 12.6 m.p.h. The results indicate that even a relatively small (compared to the numbers presented in Shoup, 2005, Table 11-5) proportion of cars cruising for parking can cause a substantial increase in congestion. Since free-flow travel speed is 20.0 m.p.h., congestion causes travel speed to fall by 5.0 m.p.h. in the social optimum and by 7.4 m.p.h. in the equilibrium. Thus, even though they constitute only 14% of cars on the road, cars cruising for parking generate an almost 50% increase in the time loss due to congestion. This result is due to the convexity of the congestion function. The combined effect of the two distortions is to raise average resource cost by $1.14, slightly more than 20% relative to the social optimum. The full price of travel exceeds average resource cost because of the curbside parking fee, which is a transfer from curbside parkers to the government. One-quarter of drivers pay $2.00 for curbside parking, causing the full price of travel to exceed average resource cost by $0.50.

Column 4 gives the second-best equilibrium, in which the meter rate remains at $1.00 and the proportion of curbside allocated to parking is optimized conditional on the distorted meter rate. Comparing columns 3 and 4 indicates by how much the deadweight loss due to the two distortions is reduced by optimizing the amount of curbside allocated to parking. It is second-best efficient to substantially reduce the amount of curbside allocated to parking – to 9.7% of curbside – in order to reduce the stock of cars cruising for parking. Since a larger proportion of drivers then park off street, the average garage parking cost increases from $3.05 to $3.78, but this is more than offset by the decrease in average driving costs, with average cruising-for-parking cost decreasing from $0.51 to $0.10 and average in-transit travel cost from $3.18 to $2.74. Average resource cost falls from $6.73 to $6.61. Thus, optimizing the amount of curbside parking reduces the deadweight loss from the two distortions by about ten percent.

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[29]In section 3.3, the deadweight loss was decomposed into that associated with cruising for parking and that associated with the inefficient spacing of parking garages. This decomposition might give the misleading impression that the deadweight loss associated with private provision of parking garages is small. Cruising for parking derives from not only the underpricing of curbside parking but also the overpricing of garage parking. In the example being considered, for the two-hour visit, the curbside parking charge is $2.00, the marginal cost of garage parking is $2.90, and the garage parking charge is $3.59. Thus, 43% of cruising-for-parking costs are attributable to the overpricing of garage parking. In the example of column 5, all cruising-for-parking costs are attributable to the overpricing of garage parking.

Column 5 shows the equilibrium when one side of the street is allocated to curbside parking, as in the base case, and the parking fee is raised from \$1.00/hr. to \$1.50/hr. Raising the parking fee has no effect on average garage or walking cost, but, by reducing the difference between the on- and off-street parking prices, almost halves the stock of cars cruising for parking, which reduces congestion and hence average in-transit travel time cost. It is of particular interest to examine the "revenue multiplier" – the ratio of the increase in social benefit to the increase in parking fee revenue due to the rise in the meter rate, holding fixed the curbside allocated to parking. Average meter fee revenue rises by \$0.25 and average social benefit by \$0.53. The revenue multiplier is therefore about 2.1; for every extra dollar of revenue raised from the increase in the meter rate, social benefit rises by about \$2.10. This seems almost too good to be true, but reflects how distortionary is the wedge between the on- and off-street parking rates.

Comparing columns 5 and 6 shows how the equilibrium changes when the allocation of curbside to parking is optimized conditional on the higher on-street meter rate rather than being set at its base level. Comparing columns 4 and 6 shows how the equilibrium changes when the meter rate increases, with the allocation of curbside to parking being optimized conditional on the meter rate. The most notable feature of the results is the almost ten-fold increase in the optimal allocation of curbside to parking with the increase in the meter rate. Holding P fixed at 718, the increase in the meter rate would cause the difference between the garage parking fee and the meter price of on-street parking, $S - fl$, to fall from \$1.54 to \$0.54. Since the increase in the stock of cars cruising for parking induced by an increase in curbside parking would then be reduced by almost two-thirds, it is efficient to allocate more curbside to parking. When the curbside allocated to parking is optimized conditional on the parking fee, raising the parking fee causes average resource cost to fall by \$0.57.

It is also noteworthy that the second-best amount of curbside parking with f = 1.5 exceeds the first-best level. Since there is cruising for parking in the second-best equilibrium but not in the social optimum, this must (recall the discussion of section 3.3) derive from the increase in curbside parking reducing the deadweight loss from the inefficient spacing of parking garages.

Now consider particular rows across the six allocations. First, the differences for s, x, and h are greater between the optimal and equilibrium allocations than among the equilibrium allocations. Second, the differences for s, x, and h across the equilibrium allocations follow directly from the amounts of curbside parking and hence the amounts of garage space needed. Third, recalling that the garage parking charge entails a markup over the marginal cost of a garage space equal to the cost of walking the boundary of the garage market area, the small

differences in the garage parking charge across equilibrium allocations can be explained by differences in the marginal cost and the markup. Fourth and obviously, across the equilibrium allocations, average garage parking cost is strongly and positively affected by the proportion of drivers who park off street. Fifth, across the equilibrium allocations, travel time (the reciprocal of v), the proportion of cars cruising for parking, cruising for parking costs per capita, and in-transit travel costs per capita move together and are directly related to the stock of cars cruising for parking. Sixth, across the equilibrium allocations, traffic congestion is moderate. In the least congested equilibrium allocation, 3% of cars in traffic are cruising for parking and travel speed is 14.6 m.p.h.; in the most congested equilibrium allocation, 14% of cars in traffic are cruising for parking and travel speed is 12.5 m.p.h.

Examining all six allocations simultaneously, what is most striking is the dominant importance of cruising for parking, even though the proportion of cars cruising for parking is less than 15% in all the equilibrium allocations. Almost 90% of the higher resource costs in the base-case equilibrium compared to the base-case social optimum are due to the cruising for parking induced by the wedge between on- and off-street parking charges. Also, the sensitivity of the second-best amount of curbside parking to the meter rate is driven by cruising for parking. As well, under even the moderate traffic congestion of the examples, the social cost of the increased congestion caused by cruising for parking can be more than double the direct cruising-for-parking costs.

4.2.2 Outcomes with higher garage construction costs

The aim of the numerical examples is to come up with reasonable numbers in order to provide insight into the absolute and relative magnitudes of various policy changes. When we looked at the results of the base case, we judged the garage parking fees to be unrealistically low. To correct this, all the garage cost parameters, R, F_0, F_1, k_0, and k_1 were first doubled. With a meter rate of $1.00 per hour, an equilibrium does not exist. More expensive garage construction (and land) results in a higher garage parking fee, increasing the price differential between on- and off-street parking and inducing more cruising for parking. With the increased cruising for parking caused by doubling the garage cost parameters, the exogenous level of throughput D could not be supported by the street system[30]. The garage construction costs parameters were then lowered to 40% above their base case levels[31], for

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[30] In terms of Figure 1, at least at some locations, the steady-state locus (the graph of (1) in $T - C$ space) and the parking equilibrium locus (the graph of (2)) did not intersect.

[31] The garage parking fee is still lower than that observed in major downtown areas. To obtain an allocation for which equilibrium exists with a realistic garage parking fee would require either price-sensitive demand or mass transit.

which equilibrium did exist for all cases. The results are displayed in Table 2.

Table 2: Numerical results with garage construction costs 40 percent higher than in base case

	1	2	3	4	5	6
	SO $P = 3712$	$SO(P^*)$	E $f = 1$ $P = 3712$	$E(P^{**})$ $f = 1$	E $f = 1.5$ $P = 3712$	$E(P^{**})$ $f = 1.5$
s	0.168	0.172	0.116	0.106	0.116	0.111
x	157	153	75	83	75	78
h	7.36	7.35	7.13	7.17	7.13	7.15
S			4.84	4.76	4.84	4.80
P		4506		0		2188
v	15.0	14.0	10.4	15.0	12.2	13.5
$C/(T+C)$	0	0	0.18	0	0.15	0.10
GC/D	3.32	3.09	3.63	4.76	3.63	4.10
WC/D	0.56	0.53	0.39	0.47	0.39	0.42
TT/D	2.67	2.86	3.84	2.67	3.29	2.97
CP/D	0	0	0.84	0	0.59	0.34
RC/D	6.55	6.48	8.69	7.90	7.89	7.82
F			9.19	7.90	8.64	8.27

Notes: See previous table.

How the increased garage costs alter the social optimal allocations is straightforward. Since unit transport costs remain unchanged while unit garage costs increase, the social optimum entails a substitution away from garage costs towards transport costs, which is achieved by allocating more curbside to parking. Increased garage costs have two conflicting effects on the qualitative properties of the equilibrium allocations. On one hand, there is the same substitution away from garage costs towards transport costs. On the other hand, the increased garage costs cause an increase in the marginal cost of a garage space, hence on the garage parking charge, hence on the price differential between on- and off-street parking, and hence on cruising-for-parking time costs. The latter effect dominates. Holding the proportion of curbside allocated to parking fixed, cruising for parking increases, and to offset this it is efficient to reduce the number of on-street parking spaces. This effect is so strong that with $f = 1$ it is second-best efficient to have no on-street parking. With one-half the curbside allocated to parking, the revenue multiplier associated with raising the parking

fee from \$1.00/hr. to \$1.50/hr. is 3.2. Parking fee revenue rises by \$0.25 per capita while resource costs fall by \$0.80. The higher garage costs result in the base case equilibrium being more congested. The increase in the meter rate causes the same reduction in the stock of cars cruising for parking but, because the congestion technology is convex, causes a greater reduction in in-transit travel time costs. The increase in garage costs causes the various equilibria to change in the ways that would be expected from earlier discussion. There is, however, one noteworthy qualitative difference between Tables 1 and 2. When $f = 1.5$, with the base case garage costs the first-best amount of curbside parking falls short of the second-best level, but with the higher garage costs the first-best amount of curbside parking exceeds the second-best level. The higher garage construction costs more than double the equilibrium price differential between on- and off-street parking, which causes cruising for parking to be more of a problem. With cruising-for-parking more of a problem, it is second-best optimal to devote less curbside to parking.

The numerical results for the equilibria are unrealistic in one important respect – travel speeds are too high. Under the assumptions that the demand inflow is inelastic and that travel is congested rather than hypercongested, and with the assumed form of the congestion function, equilibrium travel speed is never below 10 m.p.h. But in heavily congested downtown areas, average travel speeds of 6 m.p.h. and even lower are not uncommon. Realistic travel speeds can be achieved by making trip demand sensitive to the full price of a trip[32], as was done in Arnott and Inci (2006).

Another feature of the numerical analysis is that in none of our exercises was the proportion of cars in traffic that are cruising for parking as high as the average 30% that the cruising-for-parking studies cited in Shoup (2005) found. This derives from the choice of parameter values. It can be shown[33] that, in the base case, the maximum value of C/T that can be achieved is 22%. To achieve higher values of C/T, the parameter values would have to be adjusted.

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[32] The equilibria associated with low travel speeds would then correspond to hypercongestion. The parking equilibrium condition ties down C. There are then two levels of T corresponding to a given level of flow or throughput, with the higher level corresponding to hypercongestion.

Whether steady-state hypercongested equilibria can be stable remains a matter of dispute. Verhoef (2001) proves for one model that steady-state hypercongested equilibria are unstable. Arnott and Inci believe that they have proved that the equilibria they identify as hypercongested in Arnott and Inci (2006) are stable.

[33] With P fixed, the maximized value of C/T consistent with (1) can be shown to be $V_g/(4\delta D t_0 \theta) - 1/\theta$. The maximized value of C/T is therefore increasing in V_g and decreasing in δ, D, t_0, and θ. Note too that, in this maximized value, δ, D, and t_0 enter as their product, which is aggregate free-flow travel time per unit area-time. In the simple model, with P variable, the maximized value of C/T consistent with both (1) and (3) can be calculated by maximizing C/T subject to (1), (3), and $V_g = \Omega(1 - P/P_{max})$, with respect to C, T, P, and f.

Cruising for parking stems from the underpricing of curbside parking and the overpricing of garage parking. The most extreme case considered (Table 2, column 3) is not extreme compared to actual traffic conditions in the downtown areas of major cities. The price differential between on- and off-street parking for the two-hour stay was $2.84 (considerably less than that in downtown Boston, for example), the proportion of cars cruising for parking was 18%, and travel speed was 10.4 m.p.h. The per-trip resource cost was $2.14 higher in the equilibrium than in the corresponding social optimum, with $0.84 of the cost increase being cruising-for-parking time cost, $1.17 higher in-transit cost due to the increased congestion caused by the cars cruising for parking, and $0.13 higher garage parking costs deriving from inefficient spacing of parking garages. Raising the meter rate from $1.00 to $1.50 per hour resulted in social savings of $0.80 per driver. Eliminating curbside parking entirely with the $1.00 meter rate had almost exactly the same benefit. Since they are expressed in per driver terms, these numbers might appear small. But under the assumptions that there is an entry rate to downtown of 7424 drivers per square mile and that downtown operates at capacity for 1600 hours per year, a $0.80 social saving per trip translates into almost $10 million per square mile every year.

Why do local governments almost everywhere persist in setting the curbside parking rate so low? We have posed this question to several seminar audiences and have received two related answers. The first is that the downtown merchants' association lobbies city hall to set the meter rate low because they fear loss of customers to suburban shopping centers. But in the model of this paper lowering the meter rate *increases* the full price of a trip downtown. It has no effect on the full price of parking and increases traffic congestion. Do downtown merchants simply not understand this or is some essential consideration missing from our model?[34] The second answer we have received is that downtown merchants favor a combination of low meter rates and curbside parking time limits to facilitate short shopping visits. We shall examine these issues in the sequel to this paper that considers heterogeneous drivers.

A broader question is why local governments do not simply manage all garage parking themselves, since they would then have direct control over both curbside and garage parking and capacity.

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[34]Downtown merchants often pay for shoppers' garage parking by validating their parking tickets. Perhaps they view low curbside meter rates as a complementary policy, for shorter shopping trips.

5　Directions for Future Research

This paper is the fourth of an integrated series that investigates the steady-state equilibrium of downtown parking and traffic congestion when the underpricing of on-street parking leads to cruising for parking. Chapter 2 of Arnott, Rave, and Schöb (2005) is a preliminary essay that presents the basic model framework, discusses some aspects of the economics, and puts forward a tentative research agenda. Arnott and Inci (2006) looks at a variant of the model with price-sensitive demand for downtown travel and only on-street parking. Arnott (2006) provides a detailed derivation of the spatial competition equilibrium among parking garages, and discusses how the model might be extended to analyze the economic effects of Boston's downtown parking policy. The next paper in the series will extend the model of this paper to treat driver heterogeneity and the paper after that will add mass transit. The stage will then almost[35] be set to apply the model to simulate the effects of various downtown parking policies in an actual city, Boston, which is the ultimate goal of the project.

Incorporating driver heterogeneity in the value of time and visit duration will add richness to the model. First, heterogeneous drivers distribute themselves across on- and off-street parking in a systematic way. Those with higher values of time are willing to pay a higher premium to avoid cruising for parking and therefore choose garage parking. In the absence of time restrictions, those with longer visit lengths choose to park curbside since they amortize the fixed cost of cruising for parking over a longer period. Second, parking garage operators choose their parking fee schedules accounting for driver heterogeneity. Third, since the stock of cars cruising for parking adjusts to equalize the full prices of on- and off-street parking for the marginal driver, the positive effects of policies will depend on the characteristics of marginal drivers. Fourth, equity considerations come into play. And fifth, on-street parking time limits are an additional policy tool.

Adding mass transit will be essential in policy application to all cities outside the southern and western United States. Buses interact directly with cars on city streets, and light rail does too but to a lesser extent, while with subways there is no congestion interaction. All forms of mass transit exhibit economies of scale. Fares are policy instruments, and even with fixed networks capacity can be varied too, by altering schedule frequency and rolling stock.

In policy application, four general issues related to downtown transportation modeling will come to the fore.

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[35]The assumptions of temporal and spatial homogeneity will be retained but some other ingredients will be added: downtown residence, resident and hotel parking, subsidized employer-provided parking, downtown freight delivery, and perhaps taxis and pedestrians.

1. How should *downtown* traffic congestion be modeled? The inadequacy of applying models of freeway traffic to downtown traffic is becoming increasingly apparent. Microsimulation models that follow individual cars through the downtown road network are an improvement, but is there not an aggregate representation of downtown traffic congestion that provides a suitable approximation?

2. There is an ebb and flow to downtown traffic over the course of the day but steady-state models are much easier to deal with than intra-day dynamic models. Is there a steady-state model that in reduced form incorporates intra-day dynamics satisfactorily?

3. Even streets in Manhattan exhibit considerable variation in their capacity and geometry. What is an appropriate method of aggregation?

4. Trip chaining and non-work trips are becoming increasingly important. How should they be modeled?

This paper has presented and utilized a model of downtown parking and traffic congestion. The garage parking fee and the full price of parking are determined by spatial competition between private parking garages. Curbside parking is priced below its social opportunity cost. The combination of overpriced garage parking and underpriced curbside parking generates cruising for parking, with the cost of cruising for parking adjusting to equalize the full prices of on-and off-street parking. Because cars cruising for parking further clog city streets, the deadweight loss associated with the price wedge between on- and off-street parking can be several times the cruising-for-parking costs it induces. Since the stock of cars cruising for parking is proportional to the amount of curbside parking, a second-best policy response is to reduce or eliminate curbside parking.

The paper points to very substantial efficiency gains from raising curbside parking fees (meter rates). Are these potential gains real or is some essential factor missing from the model?

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