The Valuation Channel of External Adjustment*

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Abstract

International financial integration has greatly increased the scope for changes in a country's net foreign asset position through the "valuation channel" of external adjustment, namely capital gains and losses on the country's external assets and liabilities. We examine this valuation channel theoretically in a dynamic equilibrium portfolio model with international trade in equity that encompasses complete and incomplete asset market scenarios. By separating asset prices and quantities in the definition of net foreign assets, we can characterize the first-order dynamics of both valuation effects and net foreign equity holdings. First-order excess returns are unanticipated and i.i.d. in our model, but capital gains and losses on equity positions feature persistent, anticipated dynamics in response to productivity shocks. The separation of prices and quantities in net foreign assets also enables us to characterize fully the role of capital gains and losses versus the current account in the dynamics of macroeconomic aggregates. Specifically, we disentangle the roles of excess returns, capital gains, and portfolio adjustment for consumption risk sharing when financial markets are incomplete, showing how these different channels contribute to dampening (or amplifying) the impact response of the cross-country consumption differential to shocks and to keeping it constant in subsequent periods.

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1 Introduction

The experience of the United States over the past decades shows that external adjustment, measured by changes in a country's net foreign asset position, can take place not only through changes in quantities and prices of goods and services – the so-called "trade channel" of adjustment – but also through changes in asset prices and returns – the so-called "financial channel" of adjustment. Although the precise magnitude, composition, and working of the financial channel of adjustment are the subject of an ongoing debate (e.g., Gourinchas and Rey, 2007a; Curcuru, Dvorak, and Warnock, 2008; Lane and Milesi-Ferretti, 2009), there is consensus that this channel is quantitatively important in the case of the United States. For instance, Gourinchas and Rey (2007b) estimate that the financial channel contributed on average about 30 percent of the (cyclical) external adjustment of the United States since the 1950s.[1]

This paper focuses on a specific component of the financial channel of external adjustment that works through valuation effects only, which we call the "valuation channel" of external adjustment. This valuation channel works solely through a country's capital gains and losses on the stock of gross foreign assets and liabilities due to expected or unexpected asset price changes.[2] We study the valuation channel of external adjustment theoretically in a two-country, dynamic, stochastic, general equilibrium (DSGE) portfolio model with international trade in equity that encompasses complete and incomplete asset market scenarios. We study the determinants of the valuation channel, its relative importance in external adjustment, and we illustrate its working and its implications for macroeconomic dynamics and risk sharing.

We introduce international equity trading in a two-country, DSGE model with production under monopolistic competition. Households in our model supply labor, consume a basket that aggregates sub-baskets of differentiated domestic and foreign goods in C.E.S. fashion, and hold shares in domestic and foreign firms.[3] To preserve the ability to obtain a set of analytical results, we consider a simple production structure in which output is produced using only labor subject to country-wide productivity shocks. Monopolistic competition, based on product differentiation within countries, generates non-zero profits and firm values, essential for the asset dynamics we

[1] Lane and Milesi-Ferretti (2001) provide an early discussion of the financial channel for industrial and emerging market economies. See also Obstfeld (2004).

[2] Gourinchas and Rey (2007a,b) use the terms "financial adjustment" and "valuation effects" interchangeably and refer to the role of total asset returns in external adjustment. Our definition of valuation effects is limited to the capital gain component of total asset returns. In this respect, our approach to valuation is closer to that of Lane and Milesi-Ferretti (2007), who distinguish capital gains from the income balance in defining valuation, and Kollmann (2006), who focuses only on capital gains. As we shall see, the distinction is important in our analysis.

[3] Our choice to focus on international trade in equity is motivated by the existence of a wider set of established results for economies with international trade in bonds. See, for instance, Benigno (2009) and Tille (2008). Lane and Milesi-Ferretti (2009) document the importance of equity price movements for the dynamics of U.S. net foreign assets. See also Coeurdacier, Kollmann, and Martin (2009).

focus on. Uncertainty arises as a consequence of productivity and government spending shocks, and asset markets are incomplete when both types of shocks are present.

The main contribution of our paper follows from the separation of asset prices and asset quantities in the definition of net foreign assets. We show that when this separation is taken into account, it is possible to characterize the first-order dynamics of valuation effects (changes in relative, cross-country equity prices, interchangeably referred to as "valuation" below) and portfolio adjustment (changes in quantities of net foreign equity holdings, or the current account of balance of payments statistics in our model) and their relative contributions to net foreign asset and macroeconomic dynamics.[4]

We solve the model by combining a second-order approximation of the portfolio optimality conditions with a first-order approximation of the rest of the model, according to the technique developed by Devereux and Sutherland (2009a) and Tille and van Wincoop (2008). Consistent with these and other studies, the excess return on foreign assets is a fully unanticipated, i.i.d. variable in our model (i.e., it is unanticipated and unpredictable). However, this is generally not the case for capital gains or losses on equity positions – our definition of the valuation channel. The initial response of valuation to a shock at time t is unanticipated as of time $t-1$, but the dynamics in all following periods are fully anticipated as of the time of the shock. For instance, we show that the response of the valuation channel to relative productivity shocks is generally described by an $ARMA(1,1)$ process, while the response to relative government spending is i.i.d. These results stem from the fact that the cross-country dividend differential, which determines relative equity values in our model, is proportional to the contemporaneous productivity and consumption differentials. The i.i.d. nature of valuation effects in response to government spending shocks then follows because the consumption differential obeys a random walk process in our model.[5] The proportionality of relative dividends to productivity (in addition to relative consumption) results in richer $ARMA$ dynamics of valuation.

We characterize the dynamics of international equity adjustment in response to productivity

[4] Importantly, this does not depend on the assumption that only two assets (home and foreign equity) are traded in our model. If we allowed for trade in multiple assets, our approach would still make it possible to solve for the first-order adjustment of a net foreign portfolio composite and a composite of cross-country, relative asset prices.

[5] Given the stylized nature of the model, we focus on impulse responses for numerical illustration. For this reason – and the objective to obtain transparent analytical results in a benchmark environment –, we do not introduce adjustment costs or other frictions that would ensure stationarity of responses to non-permanent shocks and make it possible to compute well-defined second moments. (See the discussions in Ghironi, 2006, and Schmitt-Grohé and Uribe, 2003.) The implied random walk property of relative consumption has consequences for the analytical solutions that we obtain. For instance, a stationarity inducing device would imply that valuation following government spending shocks would no longer be exactly i.i.d. Nevertheless, plausible calibrations of such stationarity inducing devices would result in quantitatively small departures from the results we obtain, other than ensuring stationary responses over a long horizon. We reserve the introduction of frictions to generate stationarity and the computation of second moments to be compared to business cycle data for extensions of this paper to richer, quantitative models.

and government spending, and we provide analytical solutions for the shares of valuation and the current account in net foreign asset adjustment. In particular, we show that the share of valuation is positive and constant in all periods after the impact of a productivity shock, thus playing a distinct role in the adjustment of external accounts. In contrast, the share of valuation in the adjustment to government spending shocks is zero in all periods but the impact one, with portfolio adjustment responsible for all changes in net foreign assets in subsequent periods.

The first-order decomposition of valuation and portfolio adjustment and the characterization of anticipated capital gain effects that we obtain cannot be accomplished when the definition of asset positions does not treat prices and quantities separately.[6] In our numerical illustrations, we show that the difference between our measure of the valuation channel and the excess-return-based measure used in Devereux and Sutherland (2009b) is non-negligible in response to productivity shocks, i.e., that our approach yields non-negligible predictable valuation effects along the dynamics that follow these shocks.[7] Plausible parameter values imply that valuation represents a significantly larger share of net foreign asset movements than portfolio adjustment in response to productivity shocks in an incomplete markets scenario, consistent with an equilibrium allocation that remains close to the complete markets outcome. Instead, analytical results and numerical illustration show that portfolio adjustment is the most important determinant of net foreign asset movements following government spending shocks.

Separation of prices and quantities in net foreign assets also enables us to fully characterize the role of capital gains and losses versus the current account in the dynamics of macroeconomic aggregates, which we explore analytically and by means of numerical examples. We disentangle the roles of excess returns, capital gains, and portfolio adjustment for consumption risk sharing when financial markets are incomplete, showing how these different channels contribute to dampening or amplifying the impact response of the cross-country consumption differential to shocks and to keeping it constant in subsequent periods. To the best of our knowledge, this paper is the first to provide such an analysis of the impact of valuation effects on macroeconomic dynamics during external adjustment.

Our contribution to the literature on the financial channel of external adjustment is thus twofold. On the methodological side, we show the importance of distinguishing quantities and prices in the definition of asset positions. On the substantive side, we obtain and illustrate a set of results that shed light on the mechanics of valuation effects and portfolio adjustment that can be

[6] In this case, it is possible to obtain results on international portfolio adjustment and anticipated valuation effects (based on excess returns) only by combining (at least) a third-order approximation of portfolio optimality conditions with a second-order approximation of the rest of the model. See Devereux and Sutherland (2007, 2009b) and Tille and van Wincoop (2008).

[7] It is also important to note that Devereux and Sutherland (2009b) find that predictable excess returns obtained by applying higher-order approximations to the model are quantitatively negligible for plausible parameter values.

at work in richer, quantitative models of international portfolio and business cycle dynamics.[8]

Besides the papers already mentioned, our work is related to a set of other studies that explore the role of financial adjustment in the international transmission of shocks. An early contribution is Kim (2002), who focuses on the consequences of revaluation of nominal asset prices, without modeling the portfolio choice. Blanchard, Giavazzi, and Sa (2005) set up a traditional portfolio balance model with imperfect asset substitutability along the lines of Kouri (1982) and discuss valuation effects caused by exchange rate movements, with no emphasis on equity prices. Tille (2008) studies the welfare implications of valuation effects from both equity and bonds, and Benigno (2009) provides a normative analysis of valuation effects, focusing on economies in which trade is restricted to nominal bonds. Both these papers assume perfect foresight and exogenously determined initial asset positions, and they focus on unanticipated valuation effects in the impact period of shocks. Pavlova and Rigobon (2009) study the role of capital gains and losses on foreign assets and liabilities in a continuous time model of external adjustment and valuation effects. Their analysis is based on an exact, closed-form solution of the model (as opposed to the approximation technique used here), but they focus on an endowment economy rather than a production one. Some numerical results on the role of capital gains and losses in net foreign asset dynamics can be found in Coeurdacier, Kollmann, and Martin (2009) and Tille and van Wincoop (2008). Kollmann (2006) highlights the effects of changes in equity prices on net foreign asset dynamics in his numerical exercises, but he does so in an endowment economy model. In contrast to this paper, his definition of the current account departs from the conventional definition of balance of payments statistics by including changes in equity prices. Nguyen (2008) studies the consequences of growth shocks for valuation and the current account using an approach similar to ours in an endowment economy model.

While we do not address the issue of international portfolio home bias, our model and solution technique are related also to several recent papers that study home bias and financial integration in models with international trade in equity and bonds. Home bias – the fact that households hold a disproportionate share of portfolios in domestic assets relative to what would be consistent with optimal risk sharing – has been the subject of extensive research since the influential analyses in Adler and Dumas (1983), Baxter and Jermann (1997), and French and Poterba (1991).[9] Our choice

[8] In the process of obtaining our main results, we also demonstrate analytically the importance of labor supply elasticity for optimal international portfolios and risk sharing. Regardless of labor supply elasticity, households in our model achieve perfect insurance against country-specific productivity shocks when there is no government spending uncertainty or when steady-state government spending is zero. But households achieve the same outcome also when labor supply is inelastic (regardless of government spending). In this case, output and firm profits are determined independently of government spending, and thus international trade in equities provides no hedge against government spending shocks. The optimal portfolio strategy is to insure fully against idiosyncratic productivity shocks, while remaining exposed to government spending shocks.

[9] See also Bottazzi, Pesenti, and van Wincoop (1996) and the survey in Lewis (1999). The recent literature

not to address home bias is motivated by the existence of established results in a literature that uses DSGE modeling and the same solution technique adopted here. For instance, Coeurdacier, Kollmann, and Martin (2009) show that a combination of trade in equities and bonds in a DSGE model with capital accumulation generates realistic home equity bias and (for low enough elasticity of substitution between home and foreign goods) external borrowing in home bonds. Rather than replicating these results, we choose to focus on the consequences of explicitly separating asset prices and quantities for the analysis of valuation versus first-order portfolio adjustment in a simpler setup that yields a set of benchmark, analytical results. Our methodology of separating asset prices and quantities has no implication for the home bias problem, because it does not affect the agents' incentive to hedge the assumed sources of risk in the economy.[10]

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 studies the anatomy of portfolio adjustment and valuation. It presents our analytical results on the roles of valuation and the current account in net foreign asset and macroeconomic dynamics. Section 4 illustrates our results by means of numerical examples, presenting impulse responses to relative productivity and government spending shocks for a plausible parametrization of the model. Section 5 concludes. Details are in a Technical Appendix available on request.

2 The Model

We present the most important ingredients of our model in this section and relegate details to the Technical Appendix.

We assume that the world economy consists of two countries, home and foreign, populated by infinitely lived, atomistic households. World population equals the continuum $[0, 1]$, with home households between $[0, a)$ and foreign households between $[a, 1]$. The world economy also features a continuum of monopolistically competitive firms on $[0, 1]$, each producing a differentiated good. Home firms are indexed by $z \in [0, a)$; foreign firms are indexed by $z^* \in [a, 1]$.

includes, among others, Benigno and Küçük-Tuğer (2008), Coeurdacier (2009), Coeurdacier and Gourinchas (2008), Coeurdacier, Kollmann, and Martin (2008, 2009), Engel and Matsumoto (2009), Hnatkovska (2009), Heathcote and Perri (2007), Kollmann (2006), and Matsumoto (2007). With the exception of numerical results in Kollmann (2006) and Coeurdacier, Kollmann, and Martin (2009), none of these papers focuses on the role of valuation in international adjustment. Obstfeld (2006) provides a survey and discussion of the recent literature on international risk sharing and portfolio models.

[10] Realistic home bias would be an important feature of a quantitative extension of our exercise that we leave for future research.

2.1 Households and Governments

The representative home household maximizes an expected intertemporal utility function that depends on consumption, C_t, and labor effort, L_t, supplied in a competitive, home labor market:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{C_s^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \chi \frac{L_s^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right),$$

with $1 > \beta > 0$ and $\sigma, \chi, \varphi > 0$. The representative foreign household maximizes a similar utility function and supplies labor in the foreign labor market.

The consumption basket C aggregates sub-baskets of individual home and foreign goods in CES fashion:

$$C_t = \left[a^{\frac{1}{\omega}} C_{Ht}^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} C_{Ft}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}},$$

where $\omega > 0$ is the elasticity of substitution between home and foreign goods. The consumption sub-baskets C_H and C_F aggregate individual home and foreign goods, respectively, in Dixit-Stiglitz fashion with elasticity of substitution $\theta > 1$:

$$C_{Ht} = \left[\left(\frac{1}{a} \right)^{\frac{1}{\theta}} \int_0^a c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad C_{Ft} = \left[\left(\frac{1}{1-a} \right)^{\frac{1}{\theta}} \int_a^1 c_t(z^*)^{\frac{\theta-1}{\theta}} dz^* \right]^{\frac{\theta}{\theta-1}}.$$

This structure of consumption preferences implies CPI and price sub-indexes in the home economy:

$$P_t = \left[a P_{Ht}^{1-\omega} + (1-a) P_{Ft}^{1-\omega} \right]^{\frac{1}{1-\omega}},$$

$$P_{Ht} = \left(\frac{1}{a} \int_0^a p_t(z)^{1-\theta} dz \right)^{\frac{1}{1-\theta}}, \quad P_{Ft} = \left(\frac{1}{1-a} \int_a^1 p_t(z^*)^{1-\theta} dz^* \right)^{\frac{1}{1-\theta}},$$

where $p_t(z)$ and $p_t(z^*)$ are the home prices of individual home and foreign goods, respectively. For convenience, we assume that all prices in the home (foreign) country are denominated in units of home (foreign) currency. In our model, currency serves the sole role of unit of account, and we adopt a cashless specification following Woodford (2003). Since we assume that nominal prices and wages are perfectly flexible, we focus on real variables in the model presentation and solution below.[11]

[11] Note that it would not be possible to focus only on real variables if we allowed for international trade in nominal bonds as in Tille (2008) and Benigno (2009). Since the nominal interest rate between $t-1$ and t is predetermined relative to the price level at t, real valuation effects arise with flexible prices and wages as a consequence of nominal price and exchange rate movements that induce unexpected movements in *ex post* real returns on outstanding bond positions. Fully forward-looking nominal equity prices are determined at the same time as nominal price levels, making changes in these and the exchange rate irrelevant for real valuation effects when only equities are traded.

We assume that there are no impediments to trade, so that the law of one price holds for each individual good. Assuming further that consumption preferences are identical across countries, consumption-based PPP holds.

Households in each country can hold shares in domestic and foreign firms. Denote with x_{t+1} the aggregate per capita home holdings of shares in home firms entering period $t+1$. With the same timing convention, let:

$$
\begin{aligned}
x^*_{t+1} &\equiv \text{aggregate per capita home holdings of shares in foreign firms,} \\
x_{*t+1} &\equiv \text{aggregate per capita foreign holdings of shares in home firms,} \\
x^*_{*t+1} &\equiv \text{aggregate per capita foreign holdings of shares in foreign firms.}
\end{aligned}
$$

In each country, there is a government that consumes the same consumption basket as households in exogenous, wasteful fashion. Governments run balanced budgets, such that government spending is equal to lump-sum taxation of household income. Then, *equilibrium* versions of the budget constraints for home and foreign households can be written as:

$$
v_t x_{t+1} + v^*_t x^*_{t+1} + C_t + G_t = (v_t + d_t)\, x_t + (v^*_t + d^*_t)\, x^*_t + w_t L_t, \tag{1}
$$

$$
v_t x_{*t+1} + v^*_t x^*_{*t+1} + C^*_t + G^*_t = (v_t + d_t)\, x_{*t} + (v^*_t + d^*_t)\, x^*_{*t} + w^*_t L^*_t, \tag{2}
$$

where v_t and v^*_t are the prices of shares in home and foreign firms, d_t and d^*_t are dividends, w_t and w^*_t are real wages – all in units of the common consumption basket –, and G_t and G^*_t denote exogenous government spending (following $AR(1)$ processes in logs).

Define the gross returns from holding home and foreign equity as:

$$
R_t \equiv \frac{v_t + d_t}{v_{t-1}}, \quad R^*_t \equiv \frac{v^*_t + d^*_t}{v^*_{t-1}}.
$$

Optimal equity holding behavior by home households requires:

$$
C_t^{-\frac{1}{\sigma}} = \beta E_t \left(C_{t+1}^{-\frac{1}{\sigma}} R_{t+1} \right), \tag{3}
$$

$$
E_t \left(C_{t+1}^{-\frac{1}{\sigma}} R_{t+1} \right) = E_t \left(C_{t+1}^{-\frac{1}{\sigma}} R^*_{t+1} \right), \tag{4}
$$

where equation (3) is the Euler equation for optimal holdings of home equity, and equation (4) implies indifference between home and foreign equity at the optimum.[12] In addition to these conditions, optimal labor supply requires:

$$
C_t^{-\frac{1}{\sigma}} w_t = \chi L_t^{\frac{1}{\varphi}}.
$$

Similar optimality conditions hold for foreign households.

[12] We omit transversality conditions.

2.1.1 From the Budget Constraints to the Law of Motion for Net Foreign Assets

Equity market clearing requires:

$$a x_{t+1} + (1-a)\, x_{*t+1} = a, \tag{5}$$

$$a x^*_{t+1} + (1-a)\, x^*_{*t+1} = 1-a. \tag{6}$$

Home aggregate per capital assets entering period $t+1$ are given by $v_t x_{t+1} + v^*_t x^*_{t+1}$. Home aggregate per capita net foreign assets entering $t+1$ (nfa_{t+1}) are obtained by netting out the values of home holdings of home shares ($v_t x_{t+1}$) and foreign holdings of home shares ($v_t x_{*t+1}$) adjusted for the population ratio:

$$nfa_{t+1} \equiv v^*_t x^*_{t+1} - \frac{1-a}{a} v_t x_{*t+1}. \tag{7}$$

Net foreign assets are the population-adjusted difference between home holdings of foreign equity and foreign holdings of home equity.

Using the definition (7) and the market clearing conditions (5)-(6), we can rewrite the home budget constraint (1) as:

$$nfa_{t+1} = R^*_t v^*_{t-1} x^*_t - \frac{1-a}{a} R_t v_{t-1} x_{*t-1} + y_t - C_t - G_t,$$

where y_t is home GDP, distributed as labor income and dividend income ($y_t \equiv w_t L_t + d_t$).

Next, define the excess return from holding foreign equity $R^D_t \equiv R^*_t - R_t$ and the portfolio holding $\alpha_t \equiv v^*_{t-1} x^*_t$. Note that this is a composite of foreign asset price last period and foreign asset quantity chosen during that period. Using these definitions, the home budget constraint becomes:

$$nfa_{t+1} = R^D_t \alpha_t + R_t nfa_t + y_t - C_t - G_t. \tag{8}$$

Proceeding similarly with the foreign budget constraint (2) yields:

$$nfa^*_{t+1} = R^D_t \alpha^*_t + R_t nfa^*_t + y^*_t - C^*_t - G^*_t, \tag{9}$$

where foreign net foreign assets nfa^*_{t+1} and portfolio holding α^*_t satisfy the market clearing conditions:

$$nfa^*_{t+1} = -\frac{a}{1-a} nfa_{t+1} \quad \text{and} \quad \alpha^*_t = -\frac{a}{1-a} v^*_{t-1} x^*_t = -\frac{a}{1-a} \alpha_t.$$

Note that the net foreign asset equations (8) and (9) correspond to the *starting* formulation of net foreign asset equations in Devereux and Sutherland (2009a, 2007) and other related work on international portfolios and valuation effects – a point that will become important below.

Finally, subtracting (9) from (8) and imposing asset market clearing yields the following law of motion for net foreign assets:

$$nfa_{t+1} = R^D_t \alpha_t + R_t nfa_t + (1-a)\left(y^D_t - C^D_t - G^D_t\right), \tag{10}$$

where: $y^D_t \equiv y_t - y^*_t$, $C^D_t \equiv C_t - C^*_t$, and $G^D_t \equiv G_t - G^*_t$.

2.2 Firms

Firms in both countries produce using only labor according to linear production functions subject to aggregate productivity shocks Z_t at home and Z_t^* abroad (following $AR(1)$ processes in logs). Optimal price setting implies that prices of individual home and foreign goods (and, by symmetry of equilibrium firm behavior, prices of home and foreign sub-baskets) are given by constant markups over marginal cost. In units of the common consumption basket, the real prices of home and foreign goods are, respectively:

$$RP_t = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t} \quad \text{and} \quad RP_t^* = \frac{\theta}{\theta - 1} \frac{w_t^*}{Z_t^*}. \tag{11}$$

From the production side, GDP in each country aggregates outputs of individual firms after converting them into units of consumption:

$$y_t = RP_t Z_t L_t \quad \text{and} \quad y_t^* = RP_t^* Z_t^* L_t^*. \tag{12}$$

Finally, defining aggregate per capita world demand of the consumption basket $y_t^W \equiv aC_t + (1-a)\,C_t^* + aG_t + (1-a)\,G_t^*$, aggregate per capita labor demand in each country is:

$$L_t = \frac{RP_t^{-\omega} y_t^W}{Z_t} \quad \text{and} \quad L_t^* = \frac{RP_t^{*-\omega} y_t^W}{Z_t^*}. \tag{13}$$

2.3 Some Useful Properties

We summarize here some properties of our model that will be useful for the interpretation of results below.

Using the optimality conditions for firms and households above, it is possible to verify that:

$$\frac{y_t}{y_t^*} = \left(\frac{Z_t}{Z_t^*}\right)^{\frac{(1+\varphi)(\omega-1)}{\omega+\varphi}} \left(\frac{C_t}{C_t^*}\right)^{-\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)}}. \tag{14}$$

It follows immediately from this expression that home and foreign GDPs are equal ($y_t = y_t^*$) if the elasticity of substitution between home and foreign goods, ω, is equal to 1. Also, if the Frisch elasticity of labor supply, φ, is equal to 0 (i.e., if labor supply is inelastic), the GDP differential reduces to $y_t/y_t^* = (Z_t/Z_t^*)^{\frac{\omega-1}{\omega}}$, so that relative GDP depends only on relative productivity and is completely insulated from government spending shocks.

Observe now that world GDP y_t^W (from the production side) is such that $y_t^W \equiv ay_t + (1-a)\,y_t^* = aRP_t Z_t L_t + (1-a)RP_t^* Z_t^* L_t^*$. If labor supply is inelastic (implying $L_t = L_t^* = 1$), it follows that $y_t^W = aRP_t Z_t + (1-a)RP_t^* Z_t^*$. Labor market clearing at home and abroad then requires:

$$1 = RP_t^{-\omega}\frac{aRP_tZ_t + (1-a)RP_t^*Z_t^*}{Z_t}, \tag{15}$$

$$1 = RP_t^{*-\omega}\frac{aRP_tZ_t + (1-a)RP_t^*Z_t^*}{Z_t^*}. \tag{16}$$

This is a system of two equations that determines the prices RP_t and RP_t^* uniquely as functions of home and foreign productivity Z_t and Z_t^*. Given the solutions for RP_t and RP_t^*, home and foreign GDPs are then pinned down by $y_t = RP_tZ_t$ and $y_t^* = RP_t^*Z_t^*$, implying that home and foreign GDPs do not respond to government spending shocks if labor supply is inelastic.

In general, the terms of trade between the two countries, $TOT_t \equiv RP_t/RP_t^*$, are such that:

$$TOT_t = \left(\frac{Z_t}{Z_t^*}\right)^{-\frac{1+\varphi}{\omega+\varphi}}\left(\frac{C_t}{C_t^*}\right)^{\frac{\varphi}{\sigma(\omega+\varphi)}}. \tag{17}$$

If labor supply is inelastic, this reduces to $TOT_t = (Z_t/Z_t^*)^{-\frac{1}{\omega}}$, which implies that the terms of trade move one-for-one with the productivity differential if, additionally, $\omega = 1$. This is the central mechanism for the results in Cole and Obstfeld (1991) and Corsetti and Pesenti (2001): When $\omega = 1$, terms of trade adjustment transfers purchasing power across countries so as to replicate the perfect risk sharing outcome around an initial position with zero net foreign assets without need of any adjustment in this position. Importantly, this property – particularly transparent with inelastic labor supply – holds also with $\varphi \neq 0$.[13]

Wage and labor effort differentials are determined by:

$$\frac{w_t}{w_t^*} = \left(\frac{Z_t}{Z_t^*}\right)^{\frac{\omega-1}{\omega+\varphi}}\left(\frac{C_t}{C_t^*}\right)^{\frac{\varphi}{\sigma(\omega+\varphi)}}, \tag{18}$$

$$\frac{L_t}{L_t^*} = \left(\frac{Z_t}{Z_t^*}\right)^{\frac{\varphi(\omega-1)}{\omega+\varphi}}\left(\frac{C_t}{C_t^*}\right)^{-\frac{\varphi\omega}{\sigma(\omega+\varphi)}}. \tag{19}$$

Finally, observe that optimal pricing by firms, the expression for GDP from the production side ($y_t = RP_tZ_tL_t$), and the definition of GDP from the income side imply that income distribution in each country is determined by constant proportions:

$$w_tL_t = \frac{(\theta-1)\,y_t}{\theta} \quad \text{and} \quad d_t = y_t - w_tL_t = \frac{1}{\theta}y_t. \tag{20}$$

[13] When $\omega = 1$ and $\varphi \neq 0$,

$$TOT_t = \left(\frac{Z_t}{Z_t^*}\right)^{-1}\left(\frac{C_t}{C_t^*}\right)^{\frac{\varphi}{\sigma(1+\varphi)}}.$$

However, it is possible to verify that, as with $\varphi = 0$, the consumption differential does not respond to productivity shocks, and the equilibrium terms of trade respond in one-for-one fashion, if the initial position features zero net foreign assets. Key for this result is that the GDP ratio in equation (14) is one regardless of φ if $\omega = 1$. Since the consumption value of each country's output responds to shocks by the same percentage, so does each country's consumption.

In a perfectly competitive environment in which $\theta \to \infty$, all GDP per capita would be distributed to domestic labor in the form of wage income. Conversely, absolute monopoly power by firms ($\theta \to 1$) would lead to all income being distributed as dividends to shareholders. Importantly for the discussion below, since GDP is independent from government spending shocks when labor supply is inelastic, the income distribution in (20) implies that dividends would also be independent from government spending if $\varphi = 0$.

2.4 The Steady-State Portfolio and the Role of Labor Supply Elasticity

We denote steady-state levels of variables by dropping the time subscript and assume that steady state productivity and government spending are such that $Z = Z^* = 1$ and $G = G^*$. Assuming a symmetric steady state with zero net foreign assets and applying the technique developed in Devereux and Sutherland (2009a) and Tille and van Wincoop (2008) as illustrated in the Technical Appendix yields the steady-state portfolio:

$$\alpha = \frac{\beta(1-a)}{1-\beta}\left[1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}\right], \tag{21}$$

where ϕ_Z (ϕ_G) is the persistence of relative productivity (government spending) shocks (in percentage deviation from the steady state), and $\sigma^2_{\varepsilon GD}$ ($\sigma^2_{\varepsilon ZD}$) is the variance of i.i.d., zero-mean innovations to relative government spending (productivity) with bounded support, assumed uncorrelated with each other.[14]

Suppose $G = 0$. Then, $\alpha = \beta(1-a)/(1-\beta)$, and it is easy to verify that this is the constant portfolio allocation that fully insures home and foreign households against idiosyncratic productivity shocks (but not government spending shocks).[15] As in Devereux and Sutherland (2009a), when government spending is zero on average, households optimally select the portfolio that fully insulates their welfare against idiosyncratic productivity shocks, leaving welfare fully exposed to the consequences of market incompleteness in case of random variation of government spending around zero. If $G > 0$, households hold a portfolio that is less diversified than the portfolio that optimally insures against idiosyncratic productivity. This happens because home equity now provides a valuable hedge against the effect of government spending on consumption, as the profits of home firms (and therefore home dividends) increase when government spending increases.

[14] We assume that $y = y^* = L = L^* = 1$ in the symmetric steady state by appropriate choice of the weight of the disutility from labor effort χ. This implies that G denotes both the absolute level of government spending and its ratio to GDP in equation (21). In turn, this imposes the constraint $1 > G \geq 0$.

[15] By using the definition of α and the steady-state equity price, $\alpha = \beta(1-a)/(1-\beta)$ returns the planner's international allocation of equity in the model without government spending shocks. This allocation of equity fully insures households against movements in relative productivity. See the Technical Appendix for details.

Importantly, $\alpha = \beta(1-a)/(1-\beta)$ and perfect insurance against idiosyncratic productivity arise also when $\varphi = 0$, i.e., when labor supply is inelastic, regardless of the value of G. In other words, even if $G > 0$, households choose a portfolio that provides less-than-perfect pooling of productivity risk only if labor supply is elastic. The intuition is simple and follows from properties of the model that we summarized above. When $\varphi = 0$, government spending shocks do not affect firm profits and hence equity holding does not provide a hedge against uncertainty in government spending: $d_t = y_t/\theta$, and we established above that y_t does not depend on G_t and G_t^* if $\varphi = 0$. In this case, a government spending shock simply crowds out consumption, leaving output and profits unchanged. Since trade in equities cannot provide a hedge against government spending shocks, the best households can do is to hold the equity portfolio that perfectly insures against productivity shocks, while again fully absorbing the consequences of market incompleteness if government spending shocks happen. When $\varphi > 0$, equilibrium profits become a function of government spending, and the risk diversification motive becomes a determinant of the steady-state portfolio if $G > 0$.[16,17]

3 The Anatomy of Portfolio Adjustment and Valuation

In this section, we analyze the anatomy of portfolio adjustment and valuation as determinants of net foreign asset dynamics in a log-linear approximation of the model. We show that the explicit separation of asset prices and quantities in the definition of net foreign assets in our model yields a set of novel results on the role of first-order portfolio adjustment and valuation, and we study the role of valuation in macroeconomic dynamics.

3.1 First-Order Portfolio Adjustment and Valuation

Log-linearizing the definition of net foreign assets (7) (and normalizing the percent deviation of net foreign assets from the steady state by steady-state consumption, $1-G$) yields:

$$n\hat{f}a_{t+1} = \frac{\alpha}{1-G}\left(\hat{x}_{t+1}^D - \hat{v}_t^D\right), \tag{22}$$

[16] The case in which disutility of labor is linear (i.e., labor supply is infinitely elastic, $\varphi \to \infty$) is often studied in models with endogenous production (see, for instance, Devereux and Sutherland, 2007, and 2009a - working paper version). In this case,

$$\alpha = \frac{\beta(1-a)}{1-\beta}\left[1 - \frac{G^2(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}\right],$$

and whether or not $G > 0$ becomes the only determinant of whether or not households fully insure against idiosyncratic productivity.

[17] Inspection of equation (21) shows that (assuming $\omega > 1$) $\partial\alpha/\partial a < 0$, $\partial\alpha/\partial\sigma > 0$, $\partial\alpha/\partial\sigma^2_{\varepsilon GD} < 0$, $\partial\alpha/\partial\phi_G < 0$, $\partial\alpha/\partial\sigma^2_{\varepsilon ZD} > 0$, and $\partial\alpha/\partial\phi_Z > 0$. We show in the Technical Appendix that $\partial\alpha/\partial G < 0$ and, for plausible parameter values, $\partial\alpha/\partial\beta > 0$, $\partial\alpha/\partial\varphi < 0$, and $\partial\alpha/\partial\omega > 0$.

where hats denote percentage deviations from the steady state, $\hat{x}^D_{t+1} \equiv \hat{x}^*_{t+1} - \hat{x}_{*t+1}$, and $\hat{v}^D_t \equiv \hat{v}_t - \hat{v}^*_t$. We thus have:

$$\Delta n\hat{f}a_{t+1} = \frac{\alpha}{1-G}\Delta\hat{x}^D_{t+1} - \frac{\alpha}{1-G}\Delta\hat{v}^D_t = c\hat{a}_t + v\hat{a}l_t, \tag{23}$$

where Δ denotes first difference (the change over time), and we defined $c\hat{a}_t \equiv \alpha\Delta\hat{x}^D_{t+1}/(1-G)$ and $v\hat{a}l_t \equiv \alpha\Delta\hat{v}^D_t/(1-G)$. The change in net foreign assets between t and $t+1$ depends on portfolio adjustment (the current account – given by the change in net foreign equity holdings in our model) and valuation (the change in relative equity prices) in period t. Home's net foreign asset position improves when the share of foreign equity held by home households increases relative to the share of home equity held by foreign households (an asset quantity effect). The net foreign asset position worsens when the price of home equity increases relative to the price of foreign equity (an asset price effect). As we show below, our measure of the current account as portfolio quantity adjustment corresponds to the traditional current account in balance of payments data, comprising the income balance and the trade balance.

To determine the roles of valuation and portfolio adjustment in net foreign asset dynamics, we must find the solution for growth in relative equity prices, $\Delta\hat{v}^D_t$ (and thus $v\hat{a}l_t$), and the change in the net foreign equity position, $\Delta\hat{x}^D_{t+1}$ (and thus $c\hat{a}_t$). To accomplish this purpose, we proceed by log-linearizing the law of motion for net foreign assets (10) to obtain:

$$n\hat{f}a_{t+1} = \frac{1}{\beta}n\hat{f}a_t + \frac{\alpha}{\beta(1-G)}\hat{R}^D_t + \frac{1-a}{1-G}\hat{y}^D_t - (1-a)\hat{C}^D_t - \frac{(1-a)G}{1-G}\hat{G}^D_t, \tag{24}$$

where we used $\mathsf{d}R^D_t = \mathsf{d}R^*_t - \mathsf{d}R_t = \left(\hat{R}^*_t - \hat{R}_t\right)/\beta = \hat{R}^D_t/\beta$ and d is the differentiation operator.

Next, observe that the definitions of the gross returns R_t and R^*_t imply:

$$\hat{R}^D_t = -\beta\hat{v}^D_t - (1-\beta)\hat{d}^D_t + \hat{v}^D_{t-1}.$$

Therefore, substituting this and (22) into (24), we have:

$$\Delta\hat{x}^D_{t+1} = \frac{1-G}{\alpha}c\hat{a}_t = \frac{1-\beta}{\beta}\left(\hat{x}^D_t - \hat{d}^D_t\right) + \frac{1-a}{\alpha}\left[\hat{y}^D_t - (1-G)\hat{C}^D_t - G\hat{G}^D_t\right]. \tag{25}$$

First-order international portfolio adjustment (the change in net foreign equity holdings, or the current account balance in (23), scaled by $(1-G)/\alpha$) is the sum of the income balance from the net foreign equity position entering the current period plus the trade balance. For given net equity position entering period t, an improvement in the profitability of home firms relative to foreign firms ($\hat{d}^D_t$) worsens the income balance, as it increases net payments to foreigners. As a result, it reduces home's accumulation of net foreign equity and, through this, net foreign assets. A trade surplus ($\hat{y}_t^D > (1-G)\,\hat{C}_t^D + G\hat{G}_t^D$) induces home agents to increase their relative holdings of foreign equity and improves the net foreign asset position.

We assume that relative productivity and government spending follow the $AR(1)$ processes:

$$\hat{Z}_t^D = \phi_Z \hat{Z}_{t-1}^D + \varepsilon_t^{Z^D}, \quad 1 \geq \phi_Z \geq 0, \tag{26}$$

$$\hat{G}_t^D = \phi_G \hat{G}_{t-1}^D + \varepsilon_t^{G^D}, \quad 1 \geq \phi_G \geq 0, \tag{27}$$

where $\varepsilon_t^{Z^D}$ and $\varepsilon_t^{G^D}$ are i.i.d., zero-mean innovations with bounded support.[18]

We show in the Technical Appendix that the solution for the first-order portfolio adjustment $\Delta\hat{x}_{t+1}^D$ is:

$$\Delta\hat{x}_{t+1}^D = \eta_{\Delta x^D Z^D}\hat{Z}_t^D + \eta_{\Delta x^D G^D}\hat{G}_t^D, \tag{28}$$

with elasticities $\eta_{\Delta x^D Z^D}$ and $\eta_{\Delta x^D G^D}$ determined by:

$$\eta_{\Delta x^D Z^D} = \frac{\beta(1-a)(1-\phi_Z)(\omega-1)(1+\varphi)}{\alpha(1-\beta\phi_Z)(\omega+\varphi)}\left[1-\frac{(1-\beta)\alpha}{\beta(1-a)}\right],$$

$$\eta_{\Delta x^D G^D} = -\frac{G(1-\phi_G)\beta(1-a)}{\alpha(1-\beta\phi_G)}.$$

Assume $\omega > 1$, $\phi_Z < 1$, and $\phi_G < 1$ unless otherwise noted.[19] Substituting the solution for α (21) in the expression for $\eta_{\Delta x^D Z^D}$ and rearranging shows that the elasticity of net equity adjustment to relative productivity shocks is (strictly) positive if and only if:[20]

$$\frac{\sigma(\omega-1)(1+\varphi)^2}{(\omega+\varphi)\varphi} > \frac{G^2(1-\beta\phi_Z)^2\sigma^2_{\varepsilon^{GD}}}{(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon^{ZD}}}. \tag{29}$$

This is the same condition that ensures that $\alpha > 0$, i.e., that home agents hold a positive steady-state foreign equity balance in their portfolio. Unless otherwise noted, we assume that parameter

[18] Given a symmetric, bivariate $AR(1)$ process for Z_t and Z_t^* of the form:

$$\begin{bmatrix} Z_t \\ Z_t^* \end{bmatrix} = \begin{bmatrix} \phi_Z & \phi_{ZZ^*} \\ \phi_{ZZ^*} & \phi_Z \end{bmatrix}\begin{bmatrix} Z_{t-1} \\ Z_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t^Z \\ \varepsilon_t^{Z^*} \end{bmatrix},$$

relative productivity Z_t^D can be written as:

$$Z_t^D = (\phi_Z - \phi_{ZZ^*})\,Z_{t-1}^D + \varepsilon_t^{Z^D}.$$

Thus, we implicitly assume that the spillover parameter ϕ_{ZZ^*} is equal to 0 (or that the ϕ_Z in (26) is actually a mixture of persistence and spillover parameters). Similarly for G_t^D.

[19] Some recent studies highlight the consequences of $\omega < 1$ (for instance, Bodenstein, 2006, and Corsetti, Dedola, and Leduc, 2008). However, the combined trade and macroeconomic evidence still leans toward $\omega > 1$ as the most empirically relevant scenario, with $\omega = 1$ a limiting case. Consistent with standard intuition, there is no change in net foreign equity holdings if shocks to productivity and/or government spending are permanent ($\phi_Z = 1$ and/or $\phi_G = 1$). In this case, changes in equity prices are the sole source of net foreign asset adjustment.

[20] See the Technical Appendix for details.

values are such that this condition is satisfied. This is the case under a wide range of empirically plausible parameter values. Under condition (29), an increase in home productivity above foreign induces home agents to increase their holdings of foreign equity (relative to foreign holdings of home equity) to smooth consumption. In contrast, an increase in home government spending above foreign causes a negative change in $\hat{x}_{t+1}^D$. *Ceteris paribus*, higher government spending causes a trade deficit, which reduces net equity accumulation in equation (25). In general equilibrium, higher government spending causes home consumption to fall relative to foreign (the standard crowding-out effect of consumption in the presence of Ricardian households demonstrated by Baxter and King, 1993) and induces home's terms of trade to deteriorate (equation (17)). This boosts the demand for home output and labor effort above foreign (equation (19)), and relative GDP rises (equation (14)). The profitability of home firms improves, making home equity relatively more attractive to foreign households for their own consumption smoothing than foreign equity is for home households.[21,22]

Turning to first-order valuation effects, the solution of the model yields:

$$\hat{v}_t^D = \eta_{v^D a} n\hat{f}a_t + \eta_{v^D Z^D} \hat{Z}_t^D + \eta_{v^D G^D} \hat{G}_t^D + \eta_{v^D \xi} \hat{\xi}_t, \tag{30}$$

where the elasticities η are obtained with the method of undetermined coefficients and $\hat{\xi}_t \equiv \alpha \hat{R}_t^D / [\beta (1 - G)]$ is an excess return shock, as in Devereux and Sutherland (2009a).[23] Expressions for the elasticities in equation (30) are in the Technical Appendix. The condition $\omega > 1$ is necessary and sufficient for $\eta_{v^D a} < 0$, $\eta_{v^D G^D} > 0$, and $\eta_{v^D \xi} < 0$. A negative elasticity of $\hat{v}_t^D$ to $n\hat{f}a_t$ means that a larger net foreign asset position entering the current period causes a lower value of home equity relative to foreign. The intuition for this is that a larger net foreign asset position allows home households to sustain a given level of consumption with reduced labor effort. The positive effect of net foreign assets on consumption reduces equilibrium employment (equation (19)) and GDP (14) relative to foreign. Therefore, the relative profitability of home firms declines, and so does their relative stock market price. An increase in relative government spending boosts relative home GDP and home firms' profits, thus causing the value of home equity to rise relative

[21] The profitability of home firms improves also in response to a favorable shock to relative home productivity. The key for the different behavior of equity holdings is the different response of relative consumption, which increases after the productivity shock. Put differently, after a relative government spending shock, the prevailing effect is the incentive of foreign household's to smooth the increase in their consumption relative to home by saving in the form of increased home equity.

[22] Sufficiently high (low) persistence of fiscal (productivity) shocks, ϕ_G (ϕ_Z), implies that the condition (29) does not hold. In this case, an increase in home government spending raises GDP and lowers home relative consumption enough to result in a trade surplus and a positive change in net foreign equity holdings. An increase in home productivity raises consumption relative to GDP enough to result in a trade deficit and a negative adjustment of net foreign equity.

[23] As shown below, $\hat{\xi}_t$ (and therefore $\hat{R}_t^D$) turns out to be a linear function of the innovations to relative productivity and government spending with elasticities that can also be determined with the method of undetermined coefficients.

to foreign. A higher than expected return on the net foreign portfolio allows home households to increase their consumption. Therefore, it causes the relative price of home equity to fall for the same reasons as a larger net foreign asset position at the beginning of the period. Finally, assuming $\omega > 1$, the elasticity of relative equity values to the productivity differential is positive, $\eta_{v^D Z^D} > 0$, if and only if:

$$\frac{\sigma(\omega+\varphi)}{\varphi(\omega-1)} > \frac{1-\phi_Z}{\phi_Z(1-G)}.$$

This condition is satisfied for most reasonable parameter values, and it implies that an increase in home productivity relative to foreign causes the relative value of home equity to rise by generating higher profits for home firms.

Given the solution for relative equity price (30), it is straightforward to recover the solution for the valuation effect on net foreign asset changes from $\hat{val}_t \equiv -\alpha\Delta\hat{v}_t^D/(1-G)$. We show in the Technical Appendix that the change in relative equity valuation $\Delta\hat{v}_t^D$ that determines this valuation effect can be written as:

$$\Delta\hat{v}_t^D = \eta_{\Delta v^D \varepsilon^{ZD}}\varepsilon_t^{Z^D} - \frac{(1-\beta)(1+\varphi)(\omega-1)\phi_Z(1-\phi_Z)}{(\omega+\varphi)(1-\beta\phi_Z)}\hat{Z}_{t-1}^D + \eta_{\Delta v^D \varepsilon^{GD}}\varepsilon_t^{G^D}, \tag{31}$$

where:

$$\eta_{\Delta v^D \varepsilon^{ZD}} \equiv \eta_{v^D Z^D} + \eta_{v^D \xi}\eta_{R^D \varepsilon^{ZD}}\frac{\alpha}{\beta(1-G)}, \quad \eta_{\Delta v^D \varepsilon^{GD}} \equiv \eta_{v^D G^D} + \eta_{v^D \xi}\eta_{R^D \varepsilon^{GD}}\frac{\alpha}{\beta(1-G)},$$

and $\eta_{R^D \varepsilon^{ZD}}$ and $\eta_{R^D \varepsilon^{GD}}$ are the elasticities of the excess return $\hat{R}_t^D$ to relative productivity and government spending innovations, respectively.[24]

Equation (31) shows that valuation effects display persistence in response to relative productivity shocks, owing to the presence of $\hat{Z}_{t-1}^D$ in the equation, but not in response to government spending shocks. In particular, suppose we focus on the consequences of innovations to relative productivity only. It is straightforward to verify that equation (31) implies:

$$\Delta\hat{v}_t^D = \phi_Z\Delta\hat{v}_{t-1}^D + \eta_{\Delta v^D \varepsilon^{ZD}}\varepsilon_t^{Z^D} + \eta_{\Delta v^D \varepsilon_{-1}^{ZD}}\varepsilon_{t-1}^{Z^D}, \tag{32}$$

where:

$$\eta_{\Delta v^D \varepsilon_{-1}^{ZD}} \equiv -\left[\phi_Z\eta_{\Delta v^D \varepsilon^{ZD}} + \frac{(1-\beta)(1+\varphi)(\omega-1)\phi_Z(1-\phi_Z)}{(\omega+\varphi)(1-\beta\phi_Z)}\right].$$

[24] The expressions for the elasticities $\eta_{\Delta v^D \varepsilon^{ZD}}$, $\eta_{R^D \varepsilon^{ZD}}$, $\eta_{\Delta v^D \varepsilon^{GD}}$, and $\eta_{R^D \varepsilon^{GD}}$ as functions of structural parameters are in the Technical Appendix. Standard parameter values imply $\eta_{R^D \varepsilon^{ZD}} < 0$ and $\eta_{R^D \varepsilon^{GD}} < 0$. Since both innovations to relative home productivity and government spending cause the profits of home firms to rise relative to foreign, this amounts to a negative shock to the excess return on the foreign equity portfolio. The same parameter values yield $\eta_{\Delta v^D \varepsilon^{ZD}} > 0$ and $\eta_{\Delta v^D \varepsilon^{GD}} > 0$. Positive innovations to relative productivity and government spending cause growth in the relative value of home equity by inducing higher profits for home firms.

Equation (32) shows that valuation effects follow an $ARMA(1,1)$ process in response to relative productivity shocks.[25,26] In contrast, when we focus on the consequences of relative government spending shocks, it is apparent from (31) that valuation effects are tied to i.i.d. innovations and thus display no persistence. The intuition for these results is straightforward from equation (14) for the cross-country GDP ratio. Since dividends are a constant fraction of GDP in each country, the same equation determines relative dividends. Therefore, in response to government spending shocks, the response of the dividend ratio is proportional to that of relative consumption. Since relative consumption is such that $\hat{C}_t^D = E_t \hat{C}_{t+1}^D$, it follows that relative dividends respond to a government spending shock in similar random walk fashion – with a permanent upward or downward jump, depending on the direction of the shock. With relative equity prices determined by the expected, discounted path of relative dividends, it follows that the valuation effect in response to a relative government spending shock is limited to the impact of the i.i.d. innovation $\varepsilon_t^{G^D}$. Instead, productivity shocks that alter the effectiveness of labor induce adjustment of relative labor effort and GDP over time, resulting in dynamics of valuation that unfold beyond the impact of the initial innovation.[27]

The results above on net foreign equity adjustment and valuation dynamics contribute to a growing literature on international portfolios and valuation effects, some of which we reviewed in the Introduction. Most recently, Devereux and Sutherland (2009b) apply the solution technique developed in their work and in Tille and van Wincoop (2008) – and used also in this paper – to the analysis of valuation effects and portfolio adjustment in net foreign asset dynamics. In their paper, as in other studies, equation (24) is the starting point for theoretical analysis. Since log-linearization of (10) implies that there is no role for time-variation of the portfolio α_t in equation (24), a conclusion of this work is that portfolio adjustment operates only at the level of second- (and higher-) order approximation of the law of motion for net foreign assets. Moreover, Devereux and Sutherland define the valuation channel as the effect of the excess return term $\hat{R}_t^D$ in equation (24)

[25] Standard parameter values imply $\eta_{\Delta v^D \varepsilon_{-1}^{Z^D}} < 0$. A positive innovation to relative productivity at $t-1$ results in negative equity price growth at t as the relative value of home equity returns toward the steady state.

[26] Note that, to obtain equation (32), we are not assuming that $\varepsilon_t^{G^D} = 0$ in *all* periods, or that $\sigma^2_{\varepsilon^{G^D}} = 0$. This would imply $\alpha = \beta(1-a)/(1-\beta)$ in equation (21) and therefore the complete markets allocation. Our assumption is that there is no *realization* of relative government spending innovations over the time horizon to which equation (32) applies. Under this assumption, equation (32) correctly determines the impulse responses to relative productivity innovations.

[27] As noted in the Introduction, the precise analytical form of these results (and those below) hinges on the absence of a stationarity inducing device, such as a cost of adjusting net foreign assets, from the model. Introducing such a device would remove the random walk property of relative consumption, implying that – for instance – valuation effects in response to government spending would no longer be tied only to i.i.d. innovations. However, the presence of a stationarity inducing mechanism of the type commonly used in the literature would complicate the analysis without adding significant content to the main points of this paper. Moreover, plausible calibrations of such devices would produce impulse responses that would not differ significantly from those of the benchmark scenario with no device, except for reversion to the initial steady state over the very long horizon.

rather than the relative equity price growth in (23).[28] A conclusion that follows is that first-order valuation effects are purely unanticipated, as it is possible to verify that:

$$\hat{R}_t^D = \frac{\beta(1-G)}{\alpha}\hat{\xi}_t = \eta_{R^D\varepsilon^{Z^D}}\varepsilon_t^{Z^D} + \eta_{R^D\varepsilon^{G^D}}\varepsilon_t^{G^D}. \tag{33}$$

Since the excess return on the foreign equity portfolio is a function only of innovations to relative productivity and government spending, it follows from this approach that first-order valuation is purely temporary and unanticipated.

However, these conclusions rest on taking equation (10) (or its log-linear counterpart (24)) as the starting point for theoretical analysis without separating asset prices and quantities in the definition of net foreign assets. As our results above show, when this separation is introduced – and the portfolio is taken to be the *quantity* of net foreign equity holdings –, it is possible to solve for the first-order dynamics of portfolio adjustment – the term $\Delta\hat{x}_{t+1}^D$ in (23) – and the valuation term in the same equation without restricting attention to $\hat{R}_t^D$ as the source of first-order valuation effects. It is thus possible to characterize portfolio adjustment effects on net foreign asset dynamics without necessarily having to combine a third-order approximation of portfolio optimality conditions with a second-order approximation of the rest of the model. Moreover, as shown in equation (25), our definition of portfolio adjustment corresponds to the current account as conventionally measured in balance of payments statistics, thereby implying that our definition of valuation also corresponds to the conventional measure of valuation as difference between net foreign asset changes and the current account. In contrast, as noted by Devereux and Sutherland (2009b), their definitions of portfolio adjustment and valuation are better suited for exploring the role of excess returns in higher orders of approximation.

With our definition of valuation as capital gains or losses on asset positions, it is only in response to relative government spending shocks that the optimizing behavior of Ricardian households restricts the first-order valuation channel to an i.i.d., fully unanticipated effect (with the implication – explored below – that all of net foreign asset dynamics in the periods after impact are generated by portfolio adjustment). If a relative productivity shock takes place at time t, its contemporaneous impact on $\hat{val}_t$ is unanticipated (from the perspective of $t-1$), but the entire path of $\hat{val}_t$ generated by the $ARMA(1,1)$ process obtained above in all the following periods is fully anticipated from the perspective of time t.[29] Importantly, our conclusions do not hinge on

[28] Note, however, that Devereux and Sutherland measure valuation effects empirically by using an equation of the form $\Delta nfa_{t+1} = ca_t + val_t$, using data on net foreign assets and the current account to back out valuation.

[29] If valuation effects are identified with excess returns, predictable effects arise only when we solve the model with higher-order approximation (at least third-order approximation of the portfolio optimality conditions and second-order approximation of the rest of the model). However, Devereux and Sutherland (2009b) show that the contribution of these higher-order terms to valuation effects is minuscule for reasonable calibrations of risk aversion and shock processes – another manifestation of equity-premium-type problems in this class of models, since higher-order approximation is precisely intended to capture the effects of risk premia.

the restriction that only two assets (home and foreign equity) are traded in our model. If a larger menu of assets were traded, our approach would still make it possible to solve for the first-order adjustment of the combined net foreign portfolio position and a valuation term that comprises the cross-country, relative prices of the multiple assets that are traded.

To evaluate the relative importance of valuation and the current account in net foreign asset dynamics in our model, we can define the shares of valuation and the current account in net foreign asset changes as:

$$v\hat{a}l_t^S \equiv \frac{v\hat{a}l_t}{\Delta n\hat{f}a_{t+1}} = \left(1 - \frac{\Delta\hat{x}_{t+1}^D}{\Delta\hat{v}_t^D}\right)^{-1}, \quad c\hat{a}_t^S \equiv \frac{c\hat{a}_t}{\Delta n\hat{f}a_{t+1}} = \left[1 - \left(\frac{\Delta\hat{x}_{t+1}^D}{\Delta\hat{v}_t^D}\right)^{-1}\right]^{-1}. \tag{34}$$

Note that $v\hat{a}l_t^S + c\hat{a}_t^S = 1$, but $v\hat{a}l_t^S$ and $c\hat{a}_t^S$ are not constrained to being between 0 and 1. For instance, a more than proportional contribution of valuation can offset a negative share of the current account in a given increase in net foreign assets.[30] Equations (28) and (31) can be used in conjunction with (34) to find analytical solutions for the shares of valuation and portfolio adjustment in net foreign asset changes following innovations to relative productivity and government spending.

Focus first on the case of relative productivity. Assume that the innovation $\varepsilon_0^{Z^D} = 1$ takes place at time $t = 0$, with no other innovation in the following periods. Then, it is possible to verify that:

$$v\hat{a}l_0^S = \left(1 - \frac{\eta_{\Delta x^D Z^D}}{\eta_{\Delta v^D \varepsilon^{Z^D}}}\right)^{-1} \quad \text{and} \quad v\hat{a}l_{t\geq 1}^S = \left[1 + \frac{\eta_{\Delta x^D Z^D}(\omega + \varphi)(1 - \beta\phi_Z)}{(1-\beta)(1+\varphi)(\omega - 1)(1 - \phi_Z)}\right]^{-1},$$

with $c\hat{a}_t^S = 1 - v\hat{a}l_t^S$. Importantly, the share of valuation is constant, and thus fully anticipated from the perspective of time 0, in all periods following the impact period. To understand this result, note that the solution for portfolio adjustment in equation (28) and our assumption on the relative productivity process imply:

$$\Delta\hat{x}_{t+1}^D - \Delta\hat{x}_t^D = -\eta_{\Delta x^D Z^D}(1 - \phi_Z)\hat{Z}_{t-1}^D + \eta_{\Delta x^D Z^D}\varepsilon_t^{Z^D}$$

in response to relative productivity innovations. Given an innovation $\varepsilon_0^{Z^D} = 1$ at time $t = 0$, followed by no other innovation, this equation and the lagged version of (28) imply $\Delta\hat{x}_{t+1}^D = \phi_Z \Delta\hat{x}_t^D$ in all periods $t \geq 1$. International portfolio adjustment is such that growth in net foreign equity

[30] Absent government spending shocks, the asset market is complete, and households achieve perfect insurance against idiosyncratic productivity shocks with $\Delta\hat{x}_{t+1}^D = 0$, and $v\hat{a}l_t^S = 1$. (It is easy to see that $\eta_{\Delta x^D Z^D} = 0$ when $\alpha = \beta(1-a)/(1-\beta)$, implying $\Delta\hat{x}_{t+1}^D = 0$, or $\hat{x}_{t+1}^D = \hat{x}_t^D = 0$, given the initial condition $\hat{x}_t^D = 0$ at the time of a shock.) The same result arises if there are government spending shocks but steady-state government spending is zero or if labor supply is inelastic (regardless of government spending), for the reasons discussed above.

is returning to the steady state at a rate ϕ_Z in all periods following the impact realization of a relative productivity shock. The $ARMA$ $(1,1)$ equation for relative equity price growth in response to productivity shocks, equation (32), implies $\Delta\hat{v}_t^D = \phi_Z \Delta\hat{v}_{t-1}^D$ in all periods $t \geq 2$. Given the definition of valuation and current account shares in (34), it follows that these shares remain constant in all periods $t > 1$ at the level they reach in period $t = 1$.[31]

When we focus on innovations to government spending, the process for valuation reduces to $\Delta\hat{v}_t^D = \eta_{\Delta v^D \varepsilon^{GD}} \varepsilon_t^{GD}$. In this case, the valuation channel reduces to the fully unanticipated, impact effect of government spending innovations. Given an innovation $\varepsilon_0^{GD} = 1$ at time $t = 0$, with no other innovation in the following periods, $v\hat{a}l_0^S = (1 - \eta_{\Delta x^D G^D}/\eta_{\Delta v^D \varepsilon^{GD}})^{-1}$, and $v\hat{a}l_{t\geq 1}^S = 0$:[32] Following government spending innovations, portfolio adjustment is fully responsible for net foreign asset changes in all periods $t \geq 1$. As for the result on the valuation process above, the intuition for these results hinges on the determination of relative GDP in equation (14). Absent productivity innovations, relative GDP is proportional to the consumption differential. Since the model is such that $\hat{C}_t^D = E_t \hat{C}_{t+1}^D$, it follows that relative GDP remains constant in all periods $t \geq 0$ at the level it reaches on impact when there are innovations to relative government spending – regardless of the persistence ϕ_G of the government spending process.[33] Since dividends are proportional to GDP, this implies that relative dividends immediately jump to a new, constant level upon realization of $\varepsilon_0^{GD} = 1$, and so does relative equity valuation $\hat{v}_t^D$. By implication, $\Delta\hat{v}_t^D$ responds only to the innovation on impact, and it is zero in all the following periods, so that net foreign asset adjustment for $t \geq 1$ is performed entirely by portfolio adjustment. Regardless of the persistence of government spending, Ricardian agents that fully anticipate the path of spending (and therefore taxation) upon realization of an innovation, immediately adjust both consumption and labor effort to fully smooth the country-specific component of their dynamics over time. The instantaneous and permanent adjustment of relative labor effort does not happen in response to relative productivity innovations that alter the effectiveness of labor over time, and thus result in persistent dynamics of relative equity valuation.

3.2 Valuation, Portfolio Adjustment, and Macroeconomic Dynamics

Much of the debate on the importance of valuation effects in the literature has focused on their role for net foreign asset dynamics, with comparatively less attention in academic research to the

[31] We show in the Technical Appendix that $\partial v\hat{a}l_0^S/\partial\omega \leq 0$ for plausible parameter values. Moreover, it is straightforward to verify that $v\hat{a}l_{t\geq 1}^S = (1-\beta)\alpha/[\beta(1-a)]$. Hence, assuming $\omega > 1$ and using the results on the steady-state portfolio α, we have: $\partial v\hat{a}l_{t\geq 1}^S/\partial\sigma > 0$, $\partial v\hat{a}l_{t\geq 1}^S/\partial\sigma^2_{\varepsilon^{GD}} < 0$, $\partial v\hat{a}l_{t\geq 1}^S/\partial\phi_G < 0$, $\partial v\hat{a}l_{t\geq 1}^S/\partial\sigma^2_{\varepsilon^{ZD}} > 0$, $\partial v\hat{a}l_{t\geq 1}^S/\partial\phi_Z > 0$, $\partial v\hat{a}l_{t\geq 1}^S/\partial G < 0$, $\partial v\hat{a}l_{t\geq 1}^S/\partial\beta \geq 0$, $\partial v\hat{a}l_{t\geq 1}^S/\partial\varphi < 0$, and $\partial v\hat{a}l_{t\geq 1}^S/\partial\omega > 0$, where the last three results hold for plausible parameter values.

[32] We show in the Technical Appendix that $\partial v\hat{a}l_0^S/\partial\omega > 0$ under plausible assumptions.

[33] Equations (17)-(19) imply that the same is true for the terms of trade, relative wage, and relative labor effort.

role of valuation for macroeconomic dynamics.[34] Nevertheless, understanding the role of valuation effects for macroeconomic dynamics during external adjustment is necessary to answer questions with potentially important policy implications: How do changes in international asset valuation affect macroeconomic aggregates such as consumption, employment, and output? What is the role of valuation in international risk sharing? Our approach allows us to address these questions.

For instance, consider the solution for the cross-country consumption differential:

$$\hat{C}_t^D = \eta_{C^D a} n\hat{f}a_t + \eta_{C^D Z^D} \hat{Z}_t^D + \eta_{C^D G^D} \hat{G}_t^D + \eta_{C^D \xi} \hat{\xi}_t. \tag{35}$$

The condition $\omega > 1$ is sufficient for $\eta_{C^D a} > 0$, $\eta_{C^D G^D} < 0$, and $\eta_{C^D \xi} > 0$; it is necessary and sufficient for $\eta_{C^D Z^D} > 0$.[35] This equation makes it possible to disentangle the first-order contributions of valuation and portfolio rebalancing to relative consumption via their effects on net foreign assets, and the direct contribution of the excess return from holding foreign equity. In particular, first-differencing equation (35) yields:

$$\begin{aligned} \Delta\hat{C}_t^D &= \eta_{C^D a} \Delta n\hat{f}a_t + \eta_{C^D Z^D} \Delta\hat{Z}_t^D + \eta_{C^D G^D} \Delta\hat{G}_t^D + \eta_{C^D \xi} \Delta\hat{\xi}_t \\ &= \eta_{C^D a} c\hat{a}_{t-1} + \eta_{C^D a} v\hat{a}l_{t-1} + \eta_{C^D Z^D} \Delta\hat{Z}_t^D + \eta_{C^D G^D} \Delta\hat{G}_t^D + \eta_{C^D \xi} \Delta\hat{\xi}_t, \end{aligned}$$

where we used (23). Consumption growth between periods $t-1$ and t depends on the current account and valuation at $t-1$, as well as on growth in productivity, government spending, and excess returns in period t.[36] Note that, given $c\hat{a}_t \equiv \alpha\Delta\hat{x}_{t+1}^D / (1-G)$ and equation (25), we can further decompose the contribution of portfolio adjustment at $t-1$ to consumption growth in period t between the contributions of income and trade balances. Similar decompositions can be performed for relative labor effort, GDP, and other endogenous variables.[37]

Continuing to focus on the consumption differential, note that home and foreign households in our model economy trade assets to share risks and accomplish their desired degree of consumption smoothing. The extent to which households are successful in their risk sharing is measured by the extent to which the consumption differential deviates from zero in response to idiosyncratic shocks. Moreover, optimal consumption smoothing in our model implies that the consumption differential will remain constant in all periods after the impact one at the level it reaches on impact, as implied

[34] Benigno (2009) and Tille (2008) address the role of unanticipated valuation effects for welfare in perfect foresight models with exogenous portfolios. Some numerical results on the role of valuation in risk sharing are in Devereux and Sutherland (2009b). Gourinchas and Rey (2007a) relate valuation effects and exchange rate predictability.

[35] See the Technical Appendix for details.

[36] Since we are decomposing the state-space solution for consumption growth, the items at the right-hand side of the equation are exogenous to the left-hand side, allowing us to determine their contributions to consumption growth.

[37] The solution for country-level variables is a linear combination of the solutions for the world aggregate and the cross-country differential. World aggregates are unaffected by the current account and valuation effects when steady-state net foreign assets are zero. Hence, focusing on cross country differentials is sufficient to evaluate the roles of valuation and the current account for macroeconomic dynamics.

by the condition $\hat{C}_t^D = E_t \hat{C}_{t+1}^D$. It follows that, if there is an unexpected shock at time $t = 0$, the extent to which risk sharing is successful is measured by $\hat{C}_0^D = \eta_{C^D Z^D} \hat{Z}_0^D + \eta_{C^D G^D} \hat{G}_0^D + \eta_{C^D \xi} \hat{\xi}_0$, and the contributions of valuation and the current account to consumption dynamics and risk sharing in all subsequent periods are measured by how these channels contribute to keeping relative consumption constant at that level – or relative consumption growth at zero – for $t \geq 1$.

Consider for instance the consequences of a positive innovation to relative productivity at time $t = 0$: $\varepsilon_0^{Z^D} = 1$ and $\varepsilon_{t\geq 1}^{Z^D} = 0$. It is $\Delta \hat{Z}_0^D = 1$. If $\omega > 1$ (and thus $\eta_{C^D Z^D} > 0$), the favorable innovation directly causes home consumption to increase above foreign in period 0.[38] The positive productivity innovation results in a negative shock to the excess return on foreign equity $\hat{R}_t^D$ by improving the profitability of home firms and increasing their share price relative to foreign firms (recall that $\eta_{R^D \varepsilon^{ZD}} < 0$ in equation (33) for plausible parameter values). This negative excess return effect dampens the increase in relative consumption on impact (since $\eta_{C^D \xi} > 0$), positively contributing to risk sharing.

In all subsequent periods, since $\Delta \hat{Z}_t^D = -(1 - \phi_Z) Z_{t-1}^D$ for all $t \geq 1$, the path of relative productivity would generate negative relative consumption growth (as long as $\phi_Z < 1$). Consumption growth in period 1 is kept at zero by the effects of $\Delta \hat{\xi}_1 = -\hat{\xi}_0 > 0$, the current account in period 0 (a surplus, as home households used part of their increased income to buy additional shares in foreign firms), and the valuation effect in that period (which was negative as the price of home shares rose relative to foreign). Given $\eta_{C^D a} > 0$ (an improved foreign asset position makes it possible to sustain higher consumption), valuation contributes negatively to consumption growth in period 1, reinforcing the effect of the productivity differential. It is the combination of time-0 current account surplus and the reversal of the excess return shock that keeps consumption growth at zero at $t = 1$. However, in all the following periods, there is no more excess return effect ($\Delta \hat{\xi}_{t\geq 2} = 0$), and the effect of the valuation term becomes positive as the relative equity price $\hat{v}_t^D$ decreases toward the steady state after the impact increase at time 0. Current account surplus and valuation combine to offset the direct effect of productivity growth, keeping consumption growth at zero in all periods from $t = 2$ on.[39]

Focus next on the effects of a positive innovation to relative government spending at time $t = 0$: $\varepsilon_0^{G^D} = 1$ and $\varepsilon_{t\geq 1}^{G^D} = 0$. The optimal behavior of Ricardian households implies a negative direct effect of the shock on consumption growth on impact (if $\omega > 1$, $\eta_{C^D G^D} < 0$). This is amplified by the negative excess return shock generated by the government spending innovation (since $\eta_{C^D \xi} > 0$

[38] We are implicitly assuming a scenario in which market incompleteness "bites."

[39] For plausible parameter values, a positive relative productivity shock causes trade surplus, but income balance deficit by increasing relative dividend payments to foreigners. Thus, the positive contribution of the current account to consumption growth is the result of a larger contribution of the trade balance than the income balance. If the productivity shock is permanent ($\phi_Z = 1$), positive trade balance and negative income balance exactly offset each other to deliver a zero current account.

and $\eta_{R^D{}_\xi G^D} < 0$ in equation (33)). Relative consumption growth is zero in all periods $t \geq 1$. In period 1, as relative government spending is returning to the steady state (assuming $\phi_G < 1$), the direct contribution of government spending growth to consumption growth is positive, and so is the contribution from the reversal of the excess return shock. Relative consumption growth is kept at zero by the contributions of current account deficit and negative valuation effect in period 0. In particular, it is possible to verify that $\eta_{C^D a} v\hat{a}l_0 = -\eta_{C^D \xi} \Delta\hat{\xi}_1$, so that the contribution of the valuation channel in period 0 to consumption growth in period 1 fully offsets that of excess returns. It follows that $\eta_{C^D a} c\hat{a}_0 = -\eta_{C^D G^D} \Delta\hat{G}_1^D$: Given a zero impact on consumption growth of the combined valuation and excess return effects, it is the portfolio adjustment performed in period 0 that fully offsets the direct impact of government spending on consumption growth in period 1. This role of portfolio adjustment continues in the following periods ($t \geq 2$), when there is no excess return or valuation effect (for the reasons discussed above), and it is $\eta_{C^D a} c\hat{a}_{t-1} = -\eta_{C^D G^D} \Delta\hat{G}_t^D$.[40]

In the next section, we substantiate the intuitions and results of this section, and evaluate the relative importance of different effects, by means of numerical examples.

4 The Valuation Channel at Work

In this section, we present impulse responses to productivity and government spending shocks that illustrate the functioning of the channels explored analytically in Section 3. We begin by presenting our choice of parameter values for the exercise.

4.1 Calibration

We use standard parameter values from the literature. We assume that home and foreign have equal size ($a = .5$). We interpret periods as quarters and set the households' discount factor β to the standard value of .99. The elasticity of intertemporal substitution in utility from consumption is $\sigma = .5$, implying the conventional value of 2 for relative risk aversion. The Frisch elasticity of labor supply is $\varphi = 4$ to mimic King and Rebelo's (1999) benchmark calibration.[41] We set the elasticity of substitution between individual goods produced in each country to $\theta = 6$, following Rotemberg and Woodford (1992) and several studies since. This implies a 20 percent markup of price over marginal cost. The elasticity of substitution between home and foreign goods is $\omega = 2$. Studies

[40] Different from the case of productivity shocks, plausible parameter values imply that a positive shock to relative government spending subject to condition (29) pushes both the trade balance and the income balance in the same (negative) direction. In the case of a permanent fiscal shock ($\phi_G = 1$), the current account is zero in all periods, and the trade balance is positive.

[41] The period utility function is defined over leisure $(1 - L_t)$ in King and Rebelo (1999), where the endowment of time in each period is normalized to 1. The elasticity of labor supply is then the risk aversion to variations in leisure (set to 1 in their benchmark calibration) multiplied by $(1 - L)/L$, where L is steady-state effort, calibrated to 1/5. This yields $\varphi = 4$ in our specification.

based on macroeconomic data since Backus, Kehoe, and Kydland (1994) calibrate or estimate this parameter in the neighborhood of 1.5. The trade evidence from disaggregated data points to values as high as 12 (for instance, see Lai and Trefler, 2002, and references therein). Ravn, Schmitt-Grohé, and Uribe (2007) report a 20 percent mean value of the observed share of government spending in GDP for the U.S., UK, Canada, and Australia between 1975:Q1-2005:Q4. Thus, we set $G = .2$.[42] For the relative productivity process, we assume persistence $\phi_Z = .95$, well within the range of values in the international real business cycle literature.[43] The variance of relative productivity innovations is $\sigma^2_{\varepsilon^{ZD}} = 2\left(\sigma^2_{\varepsilon^Z} - \sigma_{\varepsilon^Z \varepsilon^{Z*}}\right)$, where $\sigma^2_{\varepsilon^Z}$ is the variance of home and foreign productivity innovations, assumed equal across countries, and $\sigma_{\varepsilon^Z \varepsilon^{Z*}}$ is the covariance. Since Backus, Kehoe, and Kydland (1992), $\sigma^2_{\varepsilon^Z} = .73$ and $\sigma_{\varepsilon^Z \varepsilon^{Z*}} = .19$ (both in percentage terms) are standard values in the literature, implying $\sigma^2_{\varepsilon^{ZD}} = 1.08$. For government spending, Schmitt-Grohé and Uribe (2006) estimate $\phi_G = .87$ and $\sigma^2_{\varepsilon^G} = 2.56$ percent for the U.S., 1947:Q1-2004:Q3. For simplicity, we use their estimate of ϕ_G for the persistence of relative government spending and set $\sigma^2_{\varepsilon^{GD}} = 5.12$ (thus assuming zero spillovers in government spending and zero covariance of government spending innovations across countries).

4.2 A Productivity Shock

Figure 1 presents impulse responses to a one-percent relative productivity innovation at time $t = 0$. In each panel (and in the figures that follow), time is on the horizontal axis and the responses of variables (percentage deviations from the steady state) are measured on the primary vertical axis unless otherwise noted below. Responses are scaled so that, for instance, .3 denotes .3 percent rather than 30 percent. Panel (a) shows the responses of standard macroeconomic aggregates and the terms of trade: The shock raises domestic GDP above foreign, improves the relative profitability of home firms, and causes the terms of trade to deteriorate by increasing the supply of home goods. The net foreign asset position worsens. Faced with lower relative purchasing power, increased demand for home output, and a deteriorated asset position for several periods, home households increase their supply of labor relative to foreign households, and higher relative income results in a positive consumption differential relative to the foreign country (measured on the secondary vertical axis). Note that the net foreign asset position worsens in the short to medium run, but it improves in the long run.[44]

[42] Recall that, in our model, appropriate normalization of the weight of labor disutility χ ensures that steady-state GDP is 1, so that G is both steady-state government spending and the steady-state share of government spending in GDP.

[43] We follow Baxter (1995) and assume zero productivity spillovers.

[44] The improved long-run net foreign asset position causes home households to reduce their labor supply relative to foreign households in the long run. In turn, this implies that the GDP and dividend differentials converge to slightly negative levels, and so does the equity price differential (panel (b)). If we introduced a stationarity inducing device

To understand the response of net foreign assets, panel (b) decomposes the response of net foreign assets between the responses of relative equity prices and net foreign equity holdings, and it presents the response of the excess return shock $\hat{\xi}_t$ (*csi* in the figure). Improved relative profitability of home firms causes relative equity prices (measured on the secondary vertical axis) to rise above the steady state, but higher relative home dividends and equity prices translate into a negative excess return shock. Higher income induces home households to increase their net holdings of foreign equity (measured on the secondary vertical axis) to smooth consumption. It is the change in relative equity valuation that is therefore responsible for the initial deterioration in home's net foreign assets.

Panel (c) further clarifies this point by decomposing the change in nfa_{t+1} into its valuation and current account components. The current account (measured on the secondary vertical axis) improves as the home country runs a surplus and home households purchase shares in foreign firms, but holdings of foreign equity lose value relative to home equity, and this ultimately determines the negative change in net foreign assets between periods 0 and 1. As the relative equity price differential shrinks (panel (b)), val_t becomes positive, combining with ca_t to deliver positive (but progressively smaller) net foreign asset changes that bring the net foreign asset position to its new, improved long-run level.

Finally, panel (d) decomposes the response of portfolio adjustment, $\Delta \hat{x}^D_{t+1}$ (proportional to the response of the current account), between those of trade balance and income balance. The home country runs a trade surplus, consistent with its output having become relatively cheaper, but the income balance worsens, as home is receiving relatively less income from its net holdings of foreign equity due to the increase in the relative profitability of home firms. Consistent with a standard consumption smoothing argument, the trade balance more than offsets the income balance to determine first-order portfolio adjustment in favor of increased holdings of foreign equity.

What are the shares of valuation and the current account in net foreign asset adjustment? Using the solution obtained in Section 3, in the impact period of the shock, $v\hat{a}l^S_0 = 1.0056$ and $c\hat{a}^S_0 = -.0056$. In all following period, $v\hat{a}l^S_{t\geq 1} = .9162$ and $c\hat{a}^S_{t\geq 1} = .0838$. Capital gains or losses represent the main portion of net foreign asset movements. This is not surprising, since the equilibrium allocation comes very close to the complete markets outcome (the consumption differential is very small), and changes in asset prices would be the sole source of net foreign asset movements in response to productivity shocks in that case.

It is also possible to quantify the difference between the first-order valuation effect identified by Devereux and Sutherland (2009b) with $\hat{\xi}_t$ and the valuation effect $v\hat{a}l_t \equiv -\alpha \Delta \hat{v}^D_t / (1 - G)$ by considering the difference between these two expressions. Using $\hat{R}^D_t = -\beta \hat{v}^D_t - (1 - \beta) \hat{d}^D_t + \hat{v}^D_{t-1}$,

in the model, net foreign assets would return to zero in the long run.

this difference is equal to $\alpha(1-\beta)\left(\hat{d}_t^D - \hat{v}_{t-1}^D\right)/(1-G)$. For comparison, Figure 2 plots the responses of $v\hat{a}l_t$, $\hat{\xi}_t$, and their difference ($val\hat{D}iff_t \equiv v\hat{a}l_t - \hat{\xi}_t$, measured on the secondary vertical axis). A non-negligible difference is evident at least for four years after the shock, and it captures the effect of persistent relative equity price dynamics on net foreign assets that is not captured by a first-order measure of valuation based on excess returns.[45]

Figure 3 illustrates our results on the contributions of excess returns, valuation, and portfolio adjustment to macroeconomic dynamics and risk sharing. Panel (a) decomposes relative consumption growth (DCD), which is positive in the impact period and zero in all following periods, between its determinants.[46] As explained in Section 3, the impact response of relative consumption growth directly caused by productivity growth ($DCDZD_0 = \eta_{C^D Z^D}\Delta\hat{Z}_0^D > 0$) is dampened by the effect of the excess return movement ($DCDcsi_0 = \eta_{C^D \xi}\Delta\hat{\xi}_0 < 0$). In period 1, $DCDval_1 = \eta_{C^D a} v\hat{a}l_0 < 0$ would reinforce the direct negative effect of a now negative relative productivity growth on relative consumption growth, but these effects are offset by positive contributions from the current account ($DCDca_1 = \eta_{C^D a} c\hat{a}_0 > 0$, measured on the secondary vertical axis) and the reversal of excess return growth ($DCDcsi_1 = -\eta_{C^D \xi}\Delta\hat{\xi}_0 > 0$). In all following periods, portfolio adjustment and valuation contribute in the same direction to offsetting the negative direct impact of the change in relative productivity and to keeping the consumption differential constant. Quantitatively, the contribution of valuation is roughly an order of magnitude larger than that of the current account in all periods that follow the impact period. As in the case of the determinants of net foreign asset changes, this is in line with the economy's small departure from the complete markets outcome.

For completeness of illustration, panel (b) decomposes the contribution of the current account between the income balance and the trade balance. Consistent with the fact that home runs a trade balance surplus but faces a worsened income balance, the former contributes positively to relative consumption growth, and the latter contributes negatively.

[45] Devereux and Sutherland (2009b) document little persistence of valuation in the data, which $\hat{\xi}_t$ obviously matches. Even if $v\hat{a}l_t$ displays dynamics that unfold over time, we conjecture that the reversal of its movement between the impact period of a shock and the following period, and the difference in amplitude between impact response and the remainder of the path, would be consistent with a small first-order autocorrelation if we were to introduce a stationarity inducing device and compute the second moment properties of our model. Devereux and Sutherland also document a negative correlation of valuation with GDP and the current account, matched by the excess return $\hat{\xi}_t$. This is reproduced by $v\hat{a}l_t$ on impact, which suggests negative contemporaneous correlation also for this measure of valuation. Valuation, the current account, and GDP are all above the steady state in subsequent periods, but it is not clear that this would more than offset the impact response in the determination of unconditional correlations.

[46] Similar exercises can be performed for all endogenous variables in the model, disentangling the contributions of excess returns, valuation, and portfolio adjustment to their dynamics. Results are available on request.

4.3 A Government Spending Shock

Figure 4 replicates Figure 1 for the case of a one-percent relative government spending innovation. In panel (a), relative GDP, labor effort, and the terms of trade all jump immediately to their new long-run levels, and net foreign assets deteriorate in the short and in the long run. Negative wealth effects and larger output demand induce home households to increase their labor supply above foreign, but relative consumption (measured on the secondary vertical axis) falls, as implied by Ricardian household behavior. The deterioration of net foreign assets is the outcome of both a negative valuation effect and a reduction in net foreign equity holdings. Permanently higher relative equity prices (panel (b), secondary vertical axis) and dividends (panel (a)) imply a negative excess return shock (panel (b)) and a negative time-0 valuation effect on the change in net foreign assets between periods 0 and 1 (panel (c)). Since relative equity prices are constant, valuation does not contribute to net foreign asset dynamics in the following periods. Home households reduce their relative holdings of foreign equity in an effort to sustain consumption, while foreign households increase their relative holdings of more profitable home firms, and home runs a current account deficit (panels (b) and (c), secondary vertical axis). The change in net foreign assets is therefore negative, though progressively smaller, throughout the transition to the new steady state. Trade deficit and a worsened income balance initially combine to determine portfolio adjustment away from net foreign equity. Trade turns into surplus in the latter portion of the transition to offset a permanently worse income balance and bring the current account to zero. Consistent with the analytical results in Section 3, $v\hat{a}l_0^S = .9116$ and $c\hat{a}_0^S = .0884$, $v\hat{a}l_{t\geq 1}^S = 0$ and $c\hat{a}_{t\geq 1}^S = 1$. Although valuation is responsible for the majority of the initial movement in net foreign assets, this is entirely determined by portfolio adjustment in all the following periods. Intuitively, the difference between $v\hat{a}l_t$ and $\hat{\xi}_t$ is zero for all $t > 0$, and it turns out to be positive but very small (.012) on impact.[47]

Panel (a) of Figure 5 illustrates the determinants of consumption growth and risk sharing in response to government spending shocks. The initial, negative consumption growth is the result of unfavorable excess return compounding the direct effect of government spending. In period 1, the reversal of excess return growth would compound the direct government spending growth effect to generate positive consumption growth, but this is offset by negative consumption growth effects of both valuation and the current account (measured on the secondary vertical axis). As explained in Section 3, valuation fully offsets the effect of the excess return reversal, and portfolio adjustment absorbs the direct effect of relative government spending. In all subsequent periods, there is no more valuation or excess return effect, and the current account is fully responsible for offsetting the impact of government spending growth. As in Figure 3, panel (b) decomposes the contribution

[47] We omit the figure.

of the current account between trade balance and income balance. Consistent with the dynamics of these variables in Figure 1, the contribution of trade balance growth to consumption growth becomes positive after approximately four years to bring the overall contribution of the current account back toward zero.[48]

5 Conclusions

We studied the role of valuation effects – capital gains or losses on international asset positions – and the current account for net foreign asset dynamics and consumption risk sharing in response to country-specific shocks in a two-country, dynamic, stochastic, general equilibrium, portfolio model with international trade in equity. We showed that separation of asset prices and quantities in the definition of net foreign assets makes it possible to characterize the first-order dynamics of both the valuation channel and international equity holdings by means of a first-order approximation of the non-portfolio equilibrium conditions of the model and a second-order approximation of the portfolio optimality conditions. Consistent with the literature, excess returns on foreign assets are i.i.d. and unanticipated in our model. However, capital gains or losses in response to shocks feature persistent, anticipated dynamics that feed into net foreign asset dynamics over time depending on parameter values and the nature of the shocks. While valuation is the most important determinant of net foreign asset changes in response to productivity shocks in our model, portfolio adjustment is responsible for all net foreign asset movements but the initial one in response to government spending shocks.

The separation of prices and quantities in net foreign assets also enables us to fully characterize the role of capital gains and losses versus first-order adjustment of international portfolios in the dynamics of macroeconomic aggregates. We showed analytically how excess returns, capital gains, and portfolio adjustment contribute to consumption risk sharing when asset markets are incomplete by dampening (or amplifying) the impact response of the cross-country consumption differential to shocks and keeping it constant in subsequent periods.

By focusing on a relatively stylized model, we obtained a set of analytical results and intuitions that provide guidance for the analysis of richer, quantitative models with a wider array of shocks, assets, and frictions. We view the extension of our analysis to such models as a fruitful area for future research.

[48] The results for both productivity and government spending shocks are similar for several alternative plausible combinations of values for the parameters of the model. Figures for alternative parametrizations are available on request.

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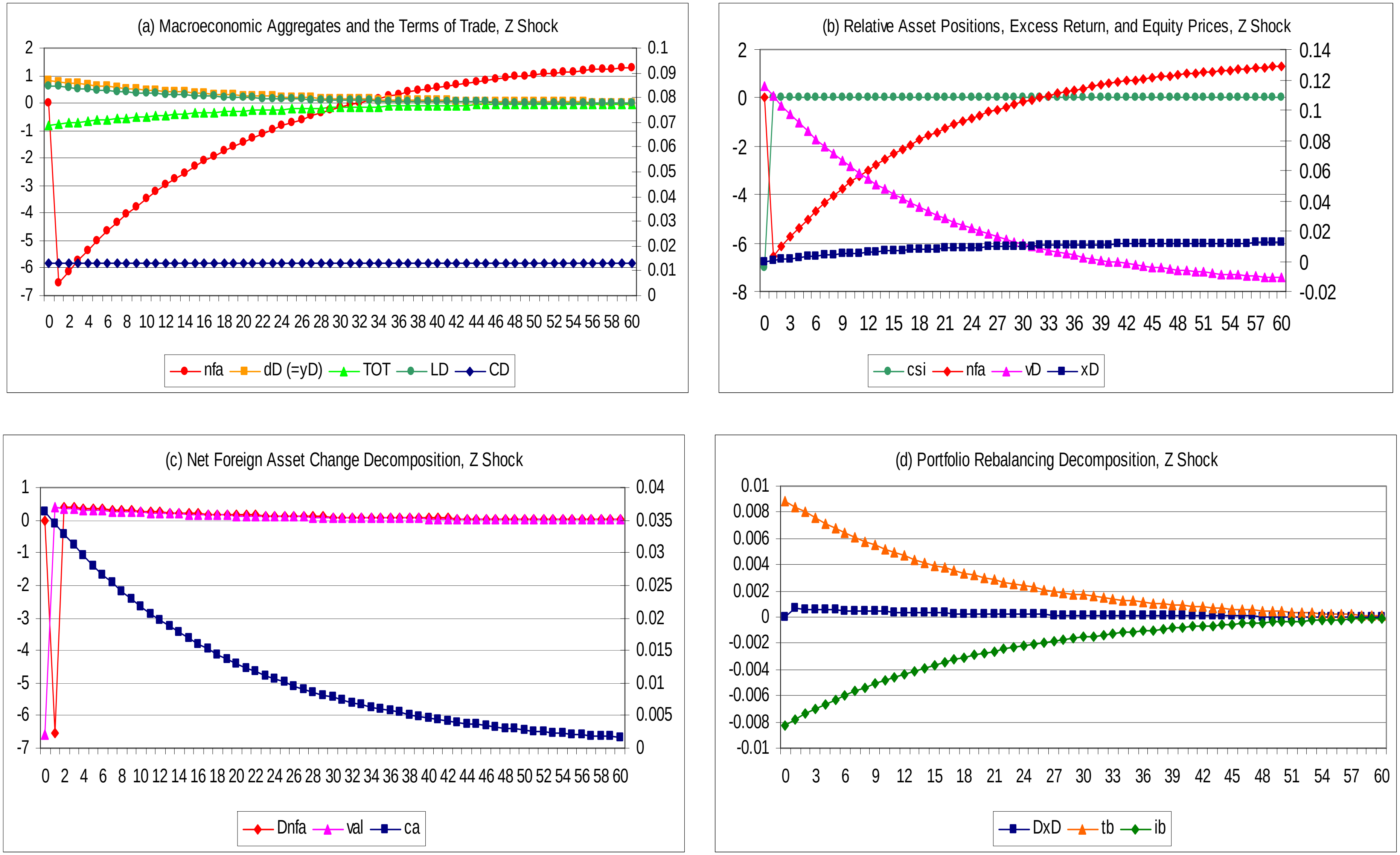

Figure 1. Impulse Responses, Productivity Shock

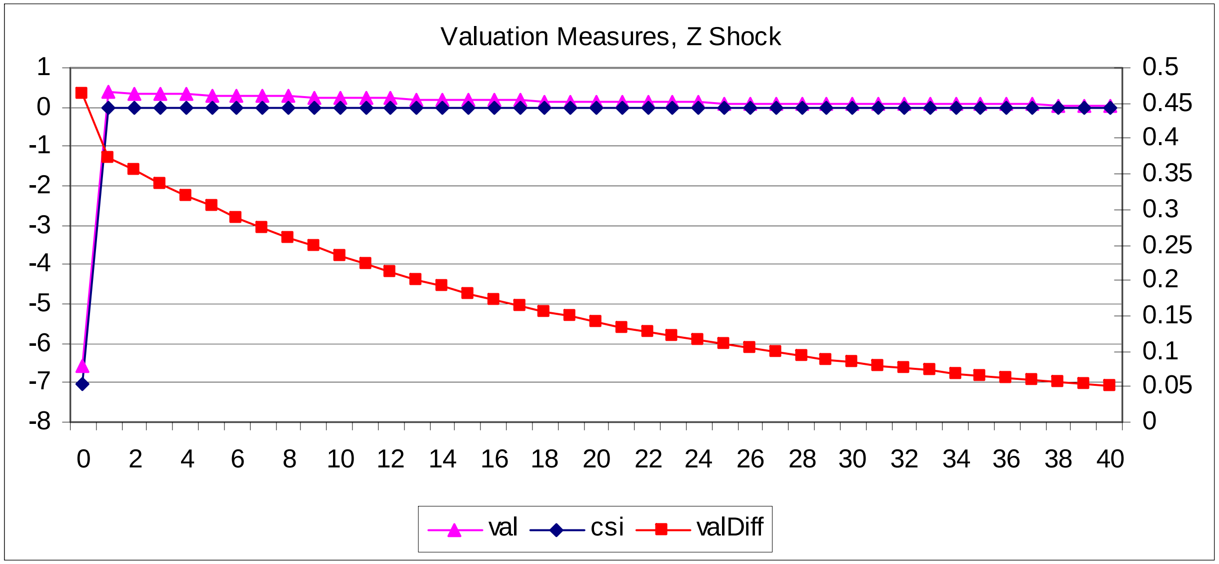

Figure 2. Comparison of Valuation Measures, Productivity Shock

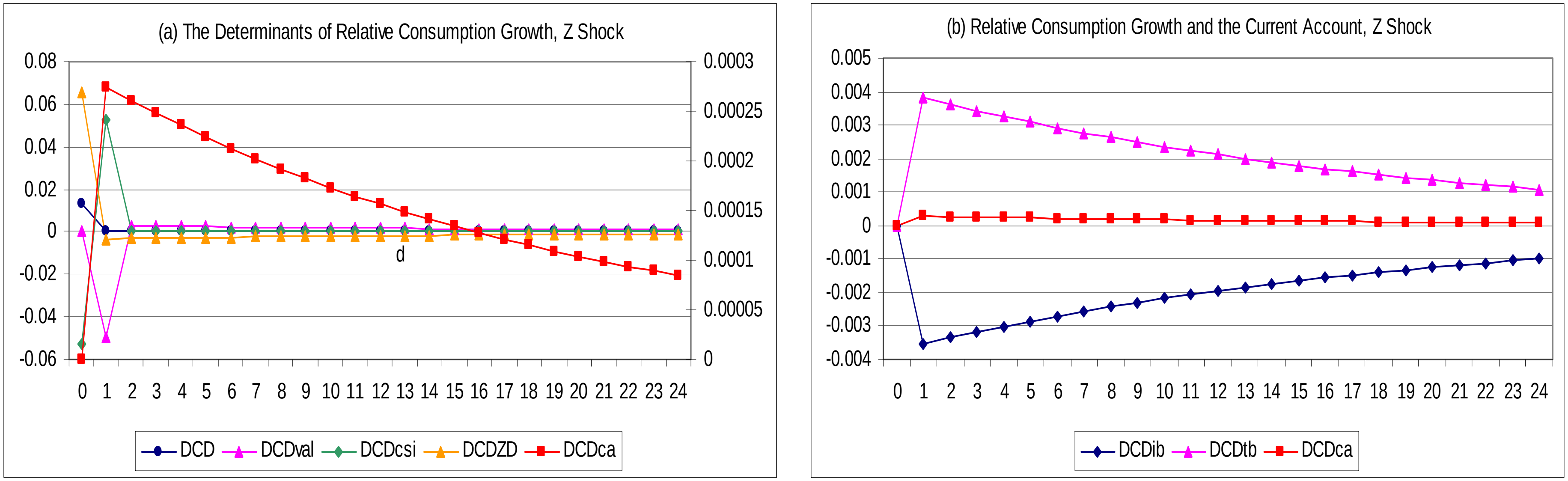

Figure 3. Valuation, the Current Account, and Risk Sharing, Productivity Shock

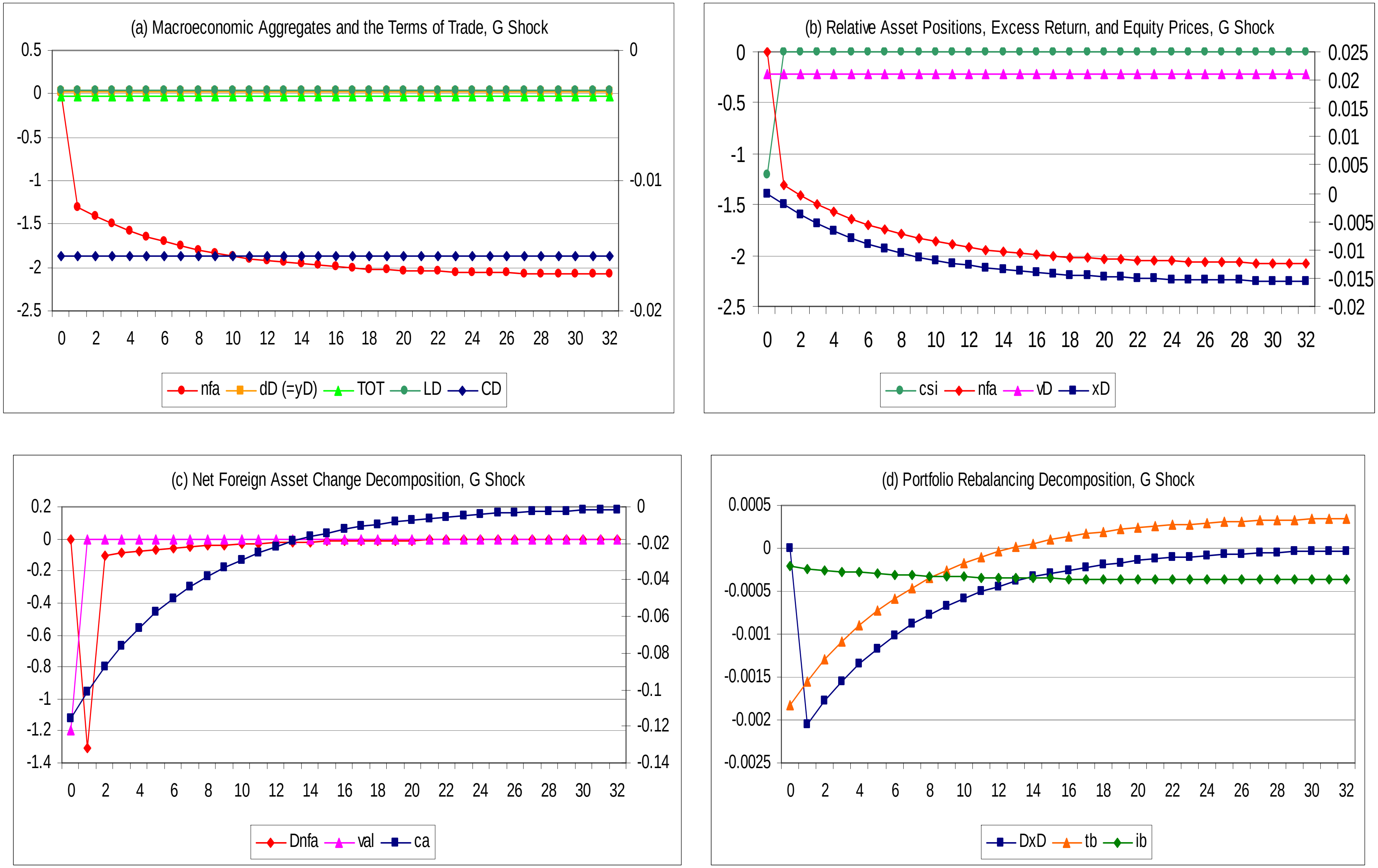

Figure 4. Impulse Responses, Government Spending Shock

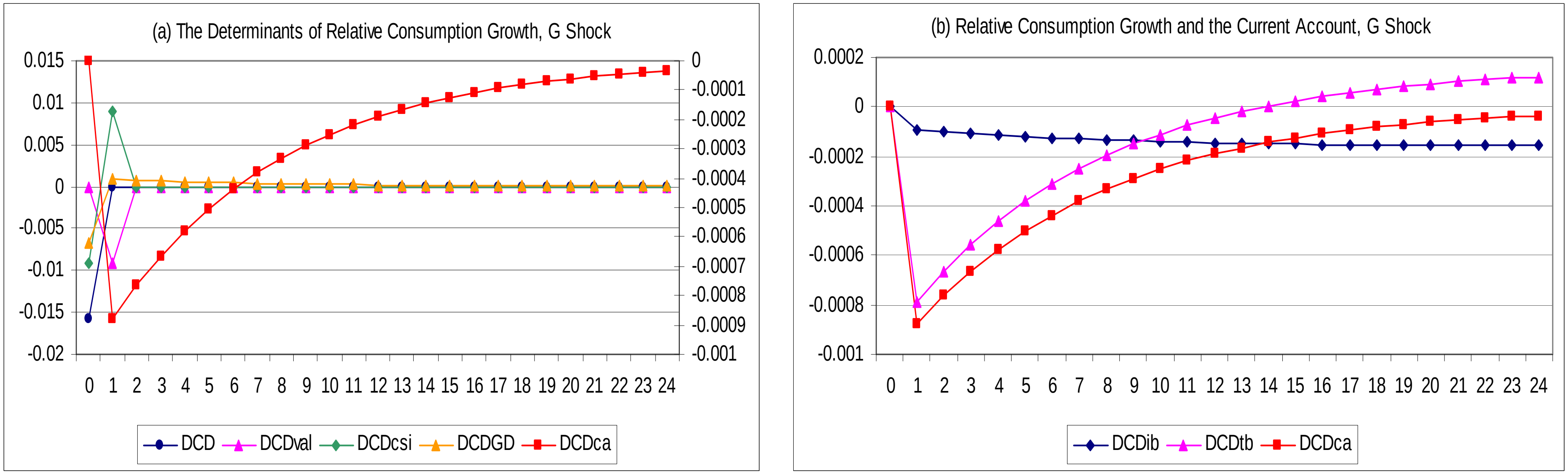

Figure 5. Valuation, the Current Account, and Risk Sharing, Government Spending Shock

The Valuation Channel of External Adjustment
Technical Appendix

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A Model Details

A.1 Households and Governments

Let V_t^z $(V_t^{z^*})$ denote the nominal price of shares in home (foreign) firm z (z^*) during period t, D_t^z $(D_t^{z^*})$ denote nominal dividends issued by the firm, and x_{t+1}^z $(x_{t+1}^{z^*})$ denote the representative household's holdings of shares in home firm z (foreign firm z^*) entering period $t+1$. The budget constraint of the representative home household is:

$$\begin{aligned} & \int_0^a V_t^z x_{t+1}^z dz + \mathcal{E}_t \int_a^1 V_t^{z^*} x_{t+1}^{z^*} dz^* + P_t C_t + P_t T_t \\ = & \int_0^a \left(V_t^z + D_t^z\right) x_t^z dz + \mathcal{E}_t \int_a^1 \left(V_t^{z^*} + D_t^{z^*}\right) x_t^{z^*} dz^* + W_t L_t, \end{aligned} \tag{36}$$

where T_t is lump-sum taxation, $\mathcal{E}_t$ is the nominal exchange rate (units of home currency per unit of foreign), and W_t is the nominal wage. The foreign household's budget constraint is similar. Equation (1) follows from (36) by imposing the balanced budget constraint of the government ($T_t = G_t$, where G_t is aggregate per capita home government spending), symmetry of firm behavior in equilibrium (implying equal share prices and dividends across firms in each country), dividing by the price level and denoting real variables by lower case letters, and using PPP and the following definitions:

$$\begin{aligned} \int_0^a x_{t+1}^z dz &= a x_{t+1}^z \equiv x_{t+1} = \text{ share of home equity held by the representative home household,} \\ \int_a^1 x_{t+1}^{z^*} dz^* &= (1-a)\, x_{t+1}^{z^*} \equiv x_{t+1}^* = \text{ share of foreign equity held by the representative home household.} \end{aligned}$$

Equation (2) follows similarly from the foreign household's budget constraint, defining:

$$\begin{aligned} \int_0^a x_{*t+1}^z dz &= a x_{*t+1}^z = x_{*t+1} = \text{ share of home equity held by the representative foreign household,} \\ \int_a^1 x_{*t+1}^{z^*} dz^* &= (1-a)\, x_{*t+1}^{z^*} = x_{*t+1}^* = \text{ share of foreign equity held by the representative foreign household.} \end{aligned}$$

A.2 Firms

Home firm z produces output with linear technology using labor as the only input:

$$Y_t^{Sz} = Z_t L_t^z,$$

where Z_t is aggregate home productivity.

Home firm z faces demand for its output given by:

$$Y_t^{Dz} = \left(\frac{p_t(z)}{P_{Ht}}\right)^{-\theta} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} Y_t^W = \left(RP_t^z\right)^{-\theta} \left(RP_t\right)^{\theta-\omega} Y_t^W,$$

where $RP_t^z \equiv p_t(z)/P_t$ is the price of good z in units of the world consumption basket, $RP_t \equiv P_{Ht}/P_t$ is the price of the home sub-basket of goods in units of the world consumption basket, and Y_t^W is aggregate world demand of the consumption basket.

Firm profit maximization results in the pricing equation:

$$RP_t^z = \frac{\theta}{\theta-1}\frac{w_t}{Z_t}.$$

Since $RP_t^z = RP_t$ at an optimum, labor demand is determined by

$$L_t^z = L_t = RP_t^{-\omega}\frac{Y_t^W}{Z_t}.$$

B Model Solution

We solve the model by using the technique developed by Devereux and Sutherland (2009) and Tille and van Wincoop (2008). The technique combines a second-order approximation of the portfolio optimality conditions with first-order approximation of the rest of the model to obtain the optimal steady-state portfolio composition. Since we impose no cost of adjusting the net foreign asset position or other stationarity inducing device for transparency of results, there is no restriction in the model to pin down endogenously the steady-state level of overall net foreign assets. As customary in this situation, we assume an initial, symmetric steady state with zero net foreign assets. In this steady state, $RP = RP^* = 1$, and $y = y^* = L = L^* = 1$ by appropriate choice of χ. With $nfa = 0$ and $R^D = 0$, it follows immediately that steady-state consumption is $C = C^* = 1-G$. Income distribution is such that: $d = d^* = 1/\theta$ and $wL = w^*L^* = w = w^* = (\theta-1)/\theta$. Hence, Euler equations for equity holdings imply $v = v^* = \beta/\left[(1-\beta)\theta\right]$.

Now, log-linearizing (10) around the symmetric steady state with zero net foreign assets, we have:

$$n\hat{f}a_{t+1} = \frac{1}{\beta}n\hat{f}a_t + \hat{\xi}_t + \frac{1-a}{1-G}\hat{y}_t^D - (1-a)\hat{C}_t^D - \frac{(1-a)G}{1-G}\hat{G}_t^D, \tag{37}$$

where:

$$\hat{\xi}_t \equiv \frac{\alpha}{\beta(1-G)}\hat{R}_t^D.$$

From equation (14), it follows that GDP and consumption differentials are related by:

$$\hat{y}_t^D = \frac{(1+\varphi)(\omega-1)}{\omega+\varphi}\hat{Z}_t^D - \frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)}\hat{C}_t^D. \tag{38}$$

Hence, (37) becomes:

$$\begin{aligned} n\hat{f}a_{t+1} &= \frac{1}{\beta}n\hat{f}a_t + \hat{\xi}_t + \frac{(1-a)(1+\varphi)(\omega-1)}{(1-G)(\omega+\varphi)}\hat{Z}_t^D \\ &\quad -(1-a)\left[1+\frac{\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)}\right]\hat{C}_t^D - \frac{(1-a)G}{1-G}\hat{G}_t^D. \end{aligned} \tag{39}$$

Log-linear versions of Euler equations for home and foreign consumption imply that the consumption differential is such that:

$$E_t\hat{C}_{t+1}^D = \hat{C}_t^D. \tag{40}$$

Since we are introducing no adjustment cost in net foreign assets or other stationarity-inducing device, the consumption differential is subject to familiar random walk behavior.

Equations (39) and (40) have solution:

$$\hat{C}_t^D = \eta_{C^Da}n\hat{f}a_t + \eta_{C^DZ^D}\hat{Z}_t^D + \eta_{C^DG^D}\hat{G}_t^D + \eta_{C^D\xi}\hat{\xi}_t, \tag{41}$$

$$n\hat{f}a_{t+1} = n\hat{f}a_t + \eta_{aZ^D}\hat{Z}_t^D + \eta_{aG^D}\hat{G}_t^D + \eta_{a\xi}\hat{\xi}_t, \tag{42}$$

where we guess that the elasticity of net foreign assets entering $t+1$ to net foreign assets at the start of period t is $\eta_{aa} = 1$ because the model features no mechanism to generate stationarity of net foreign assets. (A convex cost of adjusting net foreign assets, or other stationarity-inducing devices, would pin down a unique, deterministic steady-state level of net foreign assets by making expected growth of the marginal utility of consumption a function of net foreign assets in the Euler equations. This would also imply $0 < \eta_{aa} < 1$, ensuring stationary net foreign asset dynamics in response to temporary shocks. See Ghironi, 2006, for more details.)

The elasticities η in (41) and (42) can be obtained with the method of undetermined coefficients. This yields:

$$\begin{aligned}
\eta_{C^D a} &= \frac{\sigma(1-\beta)(1-G)(\omega+\varphi)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{C^D Z^D} &= \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{C^D G^D} &= -\frac{\sigma(1-\beta)G(\omega+\varphi)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{C^D \xi} &= \frac{\sigma(1-\beta)(1-G)(\omega+\varphi)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{a Z^D} &= \frac{\beta(1-\phi_Z)(1-a)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)(1-G)(\omega+\varphi)}, \\
\eta_{a G^D} &= -\frac{\beta(1-\phi_G)G(1-a)}{(1-\beta\phi_G)(1-G)}, \\
\eta_{a\xi} &= \beta.
\end{aligned}$$

Now, log-linearizing the Euler equations for holdings of home and foreign shares in each country and taking the difference yields:

$$\hat{v}_t^D = E_t\left[\beta\hat{v}_{t+1}^D + (1-\beta)\hat{d}_{t+1}^D\right]. \tag{43}$$

Recall that income distribution with Dixit-Stiglitz preferences is such that dividends are a proportion $1/\theta$ of GDP in each country. Therefore, $d_t/d_t^* = y_t/y_t^*$, and equation (38) determines also the dividend differential in log-linear form. It follows that:

$$\hat{v}_t^D = \beta E_t\hat{v}_{t+1}^D + \frac{\phi_Z(1-\beta)(1+\varphi)(\omega-1)}{\omega+\varphi}\hat{Z}_t^D - \frac{\varphi(\omega-1)(1-\beta)}{\sigma(\omega+\varphi)}\hat{C}_t^D. \tag{44}$$

The solution for $\hat{v}_t^D$ then takes the form in equation (30). Substituting the guess (30) and its $t+1$ version into (44), using the solutions for $\hat{C}_t^D$ and $n\hat{f}a_{t+1}$, and applying the method of undetermined coefficients yields the elasticities in (30):

$$\begin{aligned}
\eta_{v^D a} &= -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{v^D Z^D} &= \frac{(1-\beta)(1+\varphi)(\omega-1)[\sigma\phi_Z(1-G)(\omega+\varphi)-\varphi(\omega-1)(1-\phi_Z)]}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{v^D G^D} &= \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{v^D \xi} &= -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}.
\end{aligned}$$

Given the solution for $\hat{C}_t^D$ and the log-linear equation for $\hat{d}_t^D$ implied by (38) and $\hat{d}_t^D = \hat{y}_t^D$, we also have the solution for $\hat{d}_t^D$:

$$\hat{d}_t^D = \eta_{d^D a} n\hat{f}a_t + \eta_{d^D Z^D}\hat{Z}_t^D + \eta_{d^D G^D}\hat{G}_t^D + \eta_{d^D\xi}\hat{\xi}_t, \tag{45}$$

with:

$$\begin{aligned}
\eta_{d^D a} &= -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]} = \eta_{v^D a}, \\
\eta_{d^D Z^D} &= \frac{(1+\varphi)(\omega-1)\left[\beta\varphi(\omega-1)(1-\phi_Z)+\sigma(1-\beta\phi_Z)(1-G)(\omega+\varphi)\right]}{(1-\beta\phi_Z)(\omega+\varphi)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}, \\
\eta_{d^D G^D} &= \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]} = \eta_{v^D G^D}, \\
\eta_{d^D \xi} &= -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]} = \eta_{v^D \xi}.
\end{aligned}$$

The next step in solving for the steady-state portfolio consists of showing that the excess return $\hat{R}_t^D$ is a linear function of innovations to relative productivity and government spending. For this purpose, recall that

$$\hat{R}_t^D = -\beta\hat{v}_t^D - (1-\beta)\hat{d}_t^D + \hat{v}_{t-1}^D = -\left[\beta\hat{v}_t^D + (1-\beta)\hat{d}_t^D\right] + \hat{v}_{t-1}^D.$$

However, the following results hold:

$$\begin{aligned}
\beta\eta_{v^D a} + (1-\beta)\eta_{d^D a} &= \eta_{v^D a} = -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}, \\
\beta\eta_{v^D Z^D} + (1-\beta)\eta_{d^D Z^D} &= \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}, \\
\beta\eta_{v^D G^D} + (1-\beta)\eta_{d^D G^D} &= \eta_{v^D G^D} = \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}, \\
\beta\eta_{v^D \xi} + (1-\beta)\eta_{d^D \xi} &= \eta_{v^D \xi} = -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}.
\end{aligned}$$

Hence,

$$\hat{R}_t^D = -\eta_{v^D a} n\hat{f}a_t - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}\hat{Z}_t^D - \eta_{v^D G^D}\hat{G}_t^D - \eta_{v^D \xi}\hat{\xi}_t + \hat{v}_{t-1}^D.$$

Or:

$$\hat{R}_{t+1}^D = -\eta_{v^D a} n\hat{f}a_{t+1} - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}\hat{Z}_{t+1}^D - \eta_{v^D G^D}\hat{G}_{t+1}^D - \eta_{v^D \xi}\hat{\xi}_{t+1} + \hat{v}_t^D.$$

Taking (30) into account, it follows that:

$$\begin{aligned}
\hat{R}_{t+1}^D &= -\eta_{v^D a} n\hat{f}a_{t+1} + \eta_{v^D a} n\hat{f}a_t - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}\hat{Z}_{t+1}^D + \eta_{v^D Z^D}\hat{Z}_t^D \\
&\quad -\eta_{v^D G^D}\hat{G}_{t+1}^D + \eta_{v^D G^D}\hat{G}_t^D - \eta_{v^D \xi}\hat{\xi}_{t+1} + \eta_{v^D \xi}\hat{\xi}_t \\
&= -\eta_{v^D a}\left(n\hat{f}a_{t+1} - n\hat{f}a_t\right) - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}\hat{Z}_{t+1}^D + \eta_{v^D Z^D}\hat{Z}_t^D \\
&\quad -\eta_{v^D G^D}\left(\hat{G}_{t+1}^D - \hat{G}_t^D\right) - \eta_{v^D \xi}\left(\hat{\xi}_{t+1} - \hat{\xi}_t\right).
\end{aligned}$$

Now, use the solution for net foreign assets (42) and the assumptions

$$\hat{Z}^D_{t+1} = \phi_Z \hat{Z}^D_t + \varepsilon^{Z^D}_{t+1}, \quad \hat{G}^D_{t+1} = \phi_G \hat{G}^D_t + \varepsilon^{G^D}_{t+1}.$$

Then:

$$\begin{aligned}
\hat{R}^D_{t+1} &= -\eta_{v^D a}(\eta_{aZ^D}\hat{Z}^D_t + \eta_{aG^D}\hat{G}^D_t + \eta_{a\xi}\hat{\xi}_t) - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\varepsilon^{Z^D}_{t+1} \\
&\quad -\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)\phi_Z}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\hat{Z}^D_t + \eta_{v^D Z^D}\hat{Z}^D_t \\
&\quad -\eta_{v^D G^D}\left[\varepsilon^{G^D}_{t+1} - (1-\phi_G)\hat{G}^D_t\right] - \eta_{v^D\xi}\left(\hat{\xi}_{t+1} - \hat{\xi}_t\right).
\end{aligned}$$

Next, straightforward algebra shows that:

$$\begin{aligned}
-\eta_{v^D a}\eta_{aZ^D} + \eta_{v^D Z^D} - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)\phi_Z}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]} &= 0, \\
-\eta_{v^D a}\eta_{aG^D} + \eta_{v^D G^D}(1-\phi_G) &= 0, \\
-\eta_{v^D a}\eta_{a\xi} + \eta_{v^D\xi} &= 0,
\end{aligned}$$

leaving us with:

$$\hat{R}^D_{t+1} = -\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\varepsilon^{Z^D}_{t+1} - \eta_{v^D G^D}\varepsilon^{G^D}_{t+1} - \eta_{v^D\xi}\hat{\xi}_{t+1},$$

or:

$$\begin{aligned}
\hat{R}^D_{t+1} &= -\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\varepsilon^{Z^D}_{t+1} \\
&\quad -\frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\varepsilon^{G^D}_{t+1} \\
&\quad +\frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\hat{\xi}_{t+1}.
\end{aligned}$$

But now recall that

$$\hat{\xi}_{t+1} \equiv \frac{\alpha}{\beta(1-G)}\hat{R}^D_{t+1}.$$

Hence, we can solve for the excess return $\hat{R}^D_{t+1}$ as:

$$\hat{R}^D_{t+1} = \eta_{R^D\varepsilon^{ZD}}\varepsilon^{Z^D}_{t+1} + \eta_{R^D\varepsilon^{GD}}\varepsilon^{G^D}_{t+1}, \tag{46}$$

with

$$\begin{aligned}
\eta_{R^D\varepsilon^{ZD}} &= -\frac{\beta\sigma(1-a)(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)] - \alpha\varphi(\omega-1)(1-\beta)\}}, \\
\eta_{R^D\varepsilon^{GD}} &= -\frac{\beta G\varphi(1-a)(1-\beta)(\omega-1)}{(1-\beta\phi_G)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)] - \alpha\varphi(\omega-1)(1-\beta)\}}.
\end{aligned}$$

Note:

1. The solution for $\hat{R}_{t+1}^{D}$ is such that $E_t \hat{R}_{t+1}^{D} = 0$, as expected from Devereux and Sutherland (2009).

2. The elasticities $\eta_{R^D \varepsilon^{ZD}}$ and $\eta_{R^D \varepsilon^{GD}}$ depend on the steady-state portfolio holding α, which we aim to solve for.

3. If labor supply is inelastic ($\varphi = 0$), $\eta_{R^D \varepsilon^{GD}} = 0$ and

$$\eta_{R^D \varepsilon^{ZD}} = -\frac{(1-\beta)(\omega-1)}{(1-\beta\phi_Z)\,\omega}.$$

The excess return does not depend on government spending when labor supply is inelastic because government spending does not affect equilibrium profits in this case.

Now recall the no-arbitrage condition between home and foreign equity for home households:

$$E_t\left(C_{t+1}^{-\frac{1}{\sigma}} R_{t+1}\right) = E_t\left(C_{t+1}^{-\frac{1}{\sigma}} R_{t+1}^{*}\right).$$

A similar condition holds for foreign households:

$$E_t\left(C_{t+1}^{*-\frac{1}{\sigma}} R_{t+1}\right) = E_t\left(C_{t+1}^{*-\frac{1}{\sigma}} R_{t+1}^{*}\right).$$

Taking second-order approximations to these conditions and considering the difference of the resulting equations yields:

$$E_t\left(\hat{C}_{t+1}\hat{R}_{t+1}^{D}\right) - E_t\left(\hat{C}_{t+1}^{*}\hat{R}_{t+1}^{D}\right) = 0,$$

or:

$$E_t\left(\hat{C}_{t+1}^{D}\hat{R}_{t+1}^{D}\right) = 0. \tag{47}$$

Using (41) at $t+1$ and (46), this becomes:

$$E_t\left[\left(\eta_{C^D a}\hat{nfa}_{t+1} + \eta_{C^D Z^D}\hat{Z}_{t+1}^{D} + \eta_{C^D G^D}\hat{G}_{t+1}^{D} + \eta_{C^D \xi}\hat{\xi}_{t+1}\right)\left(\eta_{R^D \varepsilon^{ZD}}\varepsilon_{t+1}^{Z^D} + \eta_{R^D \varepsilon^{GD}}\varepsilon_{t+1}^{G^D}\right)\right] = 0,$$

or, using (42) and the assumptions on $\hat{Z}_{t+1}^{D}$ and $\hat{G}_{t+1}^{D}$:

$$E_t\left[\begin{array}{c} \eta_{C^D Z^D}\varepsilon_{t+1}^{Z^D}(\eta_{R^D \varepsilon^{ZD}}\varepsilon_{t+1}^{Z^D}) + \eta_{C^D G^D}\varepsilon_{t+1}^{G^D}(\eta_{R^D \varepsilon^{GD}}\varepsilon_{t+1}^{G^D}) \\ +\eta_{C^D \xi}\frac{\alpha}{\beta(1-G)}\left(\eta_{R^D \varepsilon^{ZD}}\varepsilon_{t+1}^{Z^D} + \eta_{R^D \varepsilon^{GD}}\varepsilon_{t+1}^{G^D}\right)^2 \end{array}\right] =$$

$$\eta_{C^D Z^D}\eta_{R^D \varepsilon^{ZD}}\sigma_{\varepsilon^{ZD}}^2 + \eta_{C^D G^D}\eta_{R^D \varepsilon^{GD}}\sigma_{\varepsilon^{GD}}^2 + \frac{\alpha}{\beta(1-G)}\eta_{C^D \xi}\eta_{R^D \varepsilon^{ZD}}^2\sigma_{\varepsilon^{ZD}}^2 + \frac{\alpha}{\beta(1-G)}\eta_{C^D \xi}\eta_{R^D \varepsilon^{GD}}^2\sigma_{\varepsilon^{GD}}^2 = 0, \tag{48}$$

where we used the assumption that $\varepsilon_{t+1}^{Z^D}$ is distributed independently from $\varepsilon_{t+1}^{G^D}$.

Substituting the solutions for the elasticities obtained above into (48) and rearranging yields the solution for the steady-state portfolio discussed in the main text:

$$\alpha = \frac{\beta(1-a)}{1-\beta}\left[1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}\right].$$

B.1 The Determinants of the Steady-State Portfolio

Define

$$\Omega \equiv \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}.$$

Then,

$$\alpha = \frac{\beta(1-a)}{1-\beta}(1-\Omega).$$

Assume $\omega > 1$ unless otherwise noted. The results below follow immediately from inspection of Ω and α:

$$\frac{\partial\alpha}{\partial a} < 0, \quad \frac{\partial\alpha}{\partial\sigma} > 0, \quad \frac{\partial\alpha}{\partial\sigma^2_{\varepsilon GD}} < 0, \quad \frac{\partial\alpha}{\partial\phi_G} < 0, \quad \frac{\partial\alpha}{\partial\sigma^2_{\varepsilon ZD}} > 0, \quad \frac{\partial\alpha}{\partial\phi_Z} > 0.$$

The derivative of α with respect to G is negatively proportional to the derivative of $G^2/(1-G)$. It is:

$$\frac{\partial G^2/(1-G)}{\partial G} = \frac{G(2-G)}{(1-G)^2} > 0,$$

since $0 \leq G < 1$. Hence, $\partial\alpha/\partial G < 0$.

The derivative of α with respect to β is determined by:

$$\begin{aligned}\frac{\partial\alpha}{\partial\beta} &= (1-a)\left[\frac{\partial\beta/(1-\beta)}{\partial\beta}(1-\Omega) - \frac{\beta}{1-\beta}\frac{\partial\Omega}{\partial\beta}\right] \\ &= (1-a)\left[\frac{1-\Omega}{(1-\beta)^2} - \frac{\beta}{1-\beta}\frac{\partial\Omega}{\partial\beta}\right].\end{aligned}$$

Plausible parameter values imply $\Omega < 1$. Note also that the derivative of Ω with respect to β is proportional to the derivative of $(1-\beta\phi_Z)^2/(1-\beta\phi_G)^2$. It is:

$$\frac{\partial(1-\beta\phi_Z)^2/(1-\beta\phi_G)^2}{\partial\beta} = -\frac{2(\phi_Z-\phi_G)[1-\beta\phi_Z(1-\beta\phi_G)]}{(1-\beta\phi_G)^4} \leq 0$$

under the plausible assumption $\phi_Z \geq \phi_G$. Therefore, under plausible assumptions on parameter values, $\partial\Omega/\partial\beta \leq 0$, and $\partial\alpha/\partial\beta > 0$.

The derivative of α with respect to φ is negatively proportional to the derivative of $(\omega+\varphi)\varphi/(1+\varphi)^2$. It is:

$$\frac{\partial(\omega+\varphi)\varphi/(1+\varphi)^2}{\partial\varphi} = \frac{2\varphi-\omega(\varphi-1)}{(1+\varphi)^3},$$

which is positive if $2\varphi/(\varphi-1) > \omega$. This restriction is satisfied for plausible parameter values, implying that $\partial\alpha/\partial\varphi < 0$ for values of φ that do not violate the restriction.

Finally, the derivative of α with respect to ω is negatively proportional to the derivative of $(\omega+\varphi)/(\omega-1)$. It is:

$$\frac{\partial(\omega+\varphi)/(\omega-1)}{\partial\omega} = -\frac{(1+\varphi)}{(\omega-1)^2} < 0.$$

Hence, $\partial\alpha/\partial\omega > 0$.

B.2 The Labor Effort Differential

The solution for relative labor effort can be recovered easily from $\hat{L}_t^D = \hat{y}_t^D - T\hat{O}T_t - \hat{Z}_t^D$ by using $\hat{y}_t^D = \hat{d}_t^D$, the solution for $\hat{d}_t^D$ in (45), the log-linear version of (17):

$$T\hat{O}T_t = -\frac{1+\varphi}{\omega+\varphi}\hat{Z}_t^D + \frac{\varphi}{\sigma(\omega+\varphi)}\hat{C}_t^D,$$

and the solution for $\hat{C}_t^D$ in (41). Tedious but straightforward algebra yields:

$$\hat{L}_t^D = \eta_{L^D a} n\hat{f}a_t + \eta_{L^D Z^D}\hat{Z}_t^D + \eta_{L^D G^D}\hat{G}_t^D + \eta_{L^D\xi}\hat{\xi}_t,$$

with:

$$\begin{aligned}
\eta_{L^D a} &= -\frac{\omega\varphi(1-\beta)(1-G)}{\beta(1-a)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]},\\
\eta_{L^D Z^D} &= \frac{\varphi(\omega-1)\left\{\begin{array}{c}\sigma(1-G)(\omega+\varphi)(1-\beta\phi_Z)\\ +(\omega-1)\left[\beta(1+\varphi)(1-\phi_Z)-(1-\beta\phi_Z)\right]-(1+\varphi)(1-\beta)\end{array}\right\}}{(1-\beta\phi_Z)(\omega+\varphi)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]},\\
\eta_{L^D G^D} &= \frac{G\omega\varphi(1-\beta)}{(1-\beta\phi_G)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]},\\
\eta_{L^D\xi} &= -\frac{\omega\varphi(1-\beta)(1-G)}{(1-a)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}.
\end{aligned}$$

C Steady-State Equity Holdings

The symmetric steady state described above is such that $v = v^* = \beta/[(1-\beta)\theta]$. Given $\alpha \equiv v^* x^*$ and the solution for α, it follows that $x^* = \alpha(1-\beta)\theta/\beta$, and x_*^* is determined by $ax^* + (1-a)x_*^* = 1-a$. Next, combining $nfa = 0$ in steady state with $nfa \equiv v^* x^* - [(1-a)/a] v x_*$, $\alpha \equiv v^* x^*$, and the solution for $v = v^*$, we have $x_* = a\alpha(1-\beta)\theta/[(1-a)\beta]$, and x is determined by $ax + (1-a)x_* = a$.

If $G = 0$ or $\varphi = 0$, these steady-state equity holdings reduce to:

$$\begin{aligned}
x^* &= (1-a)\theta, \quad x = a-(1-a)(\theta-1),\\
x_* &= a\theta, \quad x_*^* = 1-a\theta.
\end{aligned} \tag{49}$$

The smaller the share of income distributed as profit (the higher θ), the smaller the share of home equity that home households should hold under this allocation, and the larger the share of foreign equity. Given $\theta > 1$, $x > 0$ if and only if $\theta < 1 + a/(1-a)$. If $a = 1/2$ (symmetric country size), the planner's equity allocation implies going short in domestic equity whenever $\theta > 2$ (i.e., whenever less than half of income is distributed as profit). The planner's allocation always requires holding a positive amount of foreign equity ($\theta/2$ if $a = 1/2$).

D Productivity Insurance

To verify that the constant portfolio $\alpha = \beta(1-a)/(1-\beta)$ (or the constant equity holdings in (49)) provide perfect insurance against productivity shocks, observe that, using equity market equilibrium and the proportionality of dividends and labor incomes to GDP, we can write the difference between the home and foreign budget constraints (1) and (2) as:

$$\begin{aligned}
&\frac{v_t}{1-a}(x_{t+1}-x_t) + \frac{v_t^*}{1-a}(x_{t+1}^*-x_t^*) + C_t^D + G_t^D \\
= \; & \left[\left(\frac{x_t}{1-a} - \frac{a}{1-a}\right)\frac{1}{\theta} + \frac{\theta-1}{\theta}\right] y_t + \left[\left(\frac{x_t^*}{1-a} - 1\right)\frac{1}{\theta} - \frac{\theta-1}{\theta}\right] y_t^*.
\end{aligned}$$

Straightforward substitutions show that $x_{t+1} = x_t = a - (1-a)(\theta-1)$ and $x_{t+1}^* = x_t^* = (1-a)\theta$ (i.e., the equity allocation in (49)) imply $C_t^D = 0$ for every possible realization of y_t and y_t^* (i.e., for every possible realization of Z_t and Z_t^*) if $G_t^D = 0$. Thus, (49) is the allocation of equity that ensures perfect risk sharing in response to productivity shocks. As we showed in the main text, this is the allocation of equity chosen by households if labor supply is inelastic and/or steady-state government spending is zero.

E Obtaining Equation (31)

First-differencing (30) and using the lagged version of (42) yields:

$$\begin{aligned}
\Delta\hat{v}_t^D &= \eta_{v^D a}\left(\eta_{aZ^D}\hat{Z}_{t-1}^D + \eta_{aG^D}\hat{G}_{t-1}^D + \eta_{a\xi}\hat{\xi}_{t-1}\right) + \eta_{v^D Z^D}\Delta\hat{Z}_t^D + \eta_{v^D G^D}\Delta\hat{G}_t^D + \eta_{v^D \xi}\Delta\hat{\xi}_t \\
&= \eta_{v^D Z^D}\hat{Z}_t^D + \eta_{v^D G^D}\hat{G}_t^D + \eta_{v^D\xi}\hat{\xi}_t - \left(\eta_{v^D Z^D} - \eta_{v^D a}\eta_{aZ^D}\right)\hat{Z}_{t-1}^D \\
&\quad - \left(\eta_{v^D G^D} - \eta_{v^D a}\eta_{aG^D}\right)\hat{G}_{t-1}^D - \left(\eta_{v^D\xi} - \eta_{v^D a}\eta_{a\xi}\right)\hat{\xi}_{t-1}.
\end{aligned} \tag{50}$$

It is possible to verify that the following equalities hold:

$$\begin{aligned}
\eta_{v^D\xi} &= \eta_{v^D a}\eta_{a\xi}, \\
\eta_{v^D G^D}(1-\phi_G) &= \eta_{v^D a}\eta_{aG^D}, \\
\eta_{v^D Z^D} - \eta_{v^D a}\eta_{aZ^D} &= \frac{(1-\beta)(\omega-1)(1+\varphi)(1-G)\sigma\phi_Z}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}.
\end{aligned}$$

Using these results and our assumption on the relative government spending process, we can rewrite (50) as:

$$\Delta \hat{v}_t^D = \eta_{v^D Z^D} \hat{Z}_t^D - \frac{(1-\beta)(\omega-1)(1+\varphi)(1-G)\sigma\phi_Z}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]} \hat{Z}_{t-1}^D + \eta_{v^D G^D} \varepsilon_t^{G^D} + \eta_{v^D \xi} \hat{\xi}_t. \quad (51)$$

Next, note that (33) implies:

$$\hat{\xi}_t = \frac{\eta_{R^D \varepsilon^{ZD}} \alpha}{\beta(1-G)} \varepsilon_t^{Z^D} + \frac{\eta_{R^D \varepsilon^{GD}} \alpha}{\beta(1-G)} \varepsilon_t^{G^D}. \quad (52)$$

Using this result and the assumption on the relative productivity process, equation (51) becomes:

$$\begin{aligned} \Delta \hat{v}_t^D = & \left[\eta_{v^D Z^D} + \eta_{v^D \xi} \eta_{R^D \varepsilon^{ZD}} \frac{\alpha}{\beta(1-G)}\right] \varepsilon_t^{Z^D} \\ & - \frac{(1-\beta)(1+\varphi)(\omega-1)\phi_Z(1-\phi_Z)}{(\omega+\varphi)(1-\beta\phi_Z)} \hat{Z}_{t-1}^D + \left[\eta_{v^D G^D} + \eta_{v^D \xi} \eta_{R^D \varepsilon^{GD}} \frac{\alpha}{\beta(1-G)}\right] \varepsilon_t^{G^D}, \end{aligned}$$

i.e., equation (31), where

$$\eta_{\Delta v^D \varepsilon^{ZD}} \equiv \eta_{v^D Z^D} + \eta_{v^D \xi} \eta_{R^D \varepsilon^{ZD}} \frac{\alpha}{\beta(1-G)} \quad \text{and} \quad \eta_{\Delta v^D \varepsilon^{GD}} \equiv \eta_{v^D G^D} + \eta_{v^D \xi} \eta_{R^D \varepsilon^{GD}} \frac{\alpha}{\beta(1-G)}.$$

Finally, substituting the expressions for α and the elasticities η's obtained above in the definitions of $\eta_{\Delta v^D \varepsilon^{ZD}}$ and $\eta_{\Delta v^D \varepsilon^{GD}}$ yields:

$$\eta_{\Delta v^D \varepsilon^{ZD}} = \frac{(1-\beta)(1+\varphi)(\omega-1)\left[\sigma\phi_Z(1-G)(\omega+\varphi) - \frac{\varphi^2 G^2(\omega+\varphi)(1-\phi_Z)(1-\beta\phi_Z)^2 \sigma^2_{\varepsilon GD}}{\sigma(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}\right]}{(\omega+\varphi)(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi) + \frac{\varphi^2 G^2(\omega+\varphi)(1-\beta\phi_Z)^2 \sigma^2_{\varepsilon GD}}{\sigma(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}\right]}$$

and

$$\eta_{\Delta v^D \varepsilon^{GD}} = \frac{\sigma\varphi(1-\beta)(\omega-1)}{(1-\beta\phi_G)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]} \left\{ 1 - \frac{1}{1 - \frac{\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)}{\varphi(\omega-1)\left[1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}\right]}} \right\}.$$

F Obtaining Equation (28)

Given equations (23) and (31), and the solution for net foreign assets in (42), we can obtain the solution for portfolio rebalancing from:

$$\Delta \hat{x}_{t+1}^D = \Delta \hat{v}_t^D + \frac{1-G}{\alpha} \Delta n\hat{f}a_{t+1}.$$

It is:

$$
\begin{aligned}
\Delta\hat{x}_{t+1}^{D} &= \left[\eta_{v^D Z^D} + \eta_{v^D \xi}\eta_{R^D \varepsilon^{ZD}}\frac{\alpha}{\beta(1-G)}\right]\varepsilon_t^{Z^D} - \frac{(1-\beta)(1+\varphi)(\omega-1)\phi_Z(1-\phi_Z)}{(\omega+\varphi)(1-\beta\phi_Z)}\hat{Z}_{t-1}^{D} \\
&+ \left[\eta_{v^D G^D} + \eta_{v^D \xi}\eta_{R^D \varepsilon^{GD}}\frac{\alpha}{\beta(1-G)}\right]\varepsilon_t^{G^D} \\
&+ \frac{1-G}{\alpha}\left[\eta_{aZ^D}\hat{Z}_t^D + \eta_{aG^D}\hat{G}_t^D + \eta_{a\xi}\left(\frac{\eta_{R^D\varepsilon^{ZD}}\alpha}{\beta(1-G)}\varepsilon_t^{Z^D} + \frac{\eta_{R^D\varepsilon^{GD}}\alpha}{\beta(1-G)}\varepsilon_t^{G^D}\right)\right],
\end{aligned}
$$

where we used (52). Using $\eta_{a\xi} = \beta$ and rearranging this equation yields:

$$
\Delta\hat{x}_{t+1}^{D} = \eta_{\Delta x^D \varepsilon^{ZD}}\varepsilon_t^{Z^D} + \eta_{\Delta x^D Z_{-1}^D}\hat{Z}_{t-1}^D + \eta_{\Delta x^D \varepsilon^{GD}}\varepsilon_t^{G^D} + \eta_{\Delta x^D G_{-1}^D}\hat{G}_{t-1}^D,
$$

where:

$$
\begin{aligned}
\eta_{\Delta x^D \varepsilon^{ZD}} &\equiv \eta_{v^D Z^D} + \eta_{v^D\xi}\eta_{R^D\varepsilon^{ZD}}\frac{\alpha}{\beta(1-G)} + \frac{(1-G)\eta_{aZ^D}}{\alpha} + \eta_{R^D\varepsilon^{ZD}}, \\
\eta_{\Delta x^D Z_{-1}^D} &\equiv \frac{(1-G)\eta_{aZ^D}\phi_Z}{\alpha} - \frac{(1-\beta)(1+\varphi)(\omega-1)\phi_Z(1-\phi_Z)}{(\omega+\varphi)(1-\beta\phi_Z)}, \\
\eta_{\Delta x^D \varepsilon^{GD}} &\equiv \eta_{v^D G^D} + \eta_{v^D\xi}\eta_{R^D\varepsilon^{GD}}\frac{\alpha}{\beta(1-G)} + \frac{(1-G)\eta_{aG^D}}{\alpha} + \eta_{R^D\varepsilon^{GD}}, \\
\eta_{\Delta x^D G_{-1}^D} &\equiv \frac{(1-G)\eta_{aG^D}\phi_G}{\alpha}.
\end{aligned}
$$

Tedious algebra shows that:

$$
\eta_{\Delta x^D Z_{-1}^D} = \phi_Z\eta_{\Delta x^D\varepsilon^{ZD}} \quad \text{and} \quad \eta_{\Delta x^D G_{-1}^D} = \phi_G\eta_{\Delta x^D\varepsilon^{GD}},
$$

with:

$$
\begin{aligned}
\eta_{\Delta x^D\varepsilon^{ZD}} &= \frac{\beta(1-a)(1-\phi_Z)(\omega-1)(1+\varphi)}{\alpha(1-\beta\phi_Z)(\omega+\varphi)}\left[1 - \frac{(1-\beta)\alpha}{\beta(1-a)}\right], \\
\eta_{\Delta x^D\varepsilon^{GD}} &= -\frac{G(1-\phi_G)\beta(1-a)}{\alpha(1-\beta\phi_G)}.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\Delta\hat{x}_{t+1}^{D} &= \eta_{\Delta x^D\varepsilon^{ZD}}\left(\varepsilon_t^{Z^D} + \phi_Z\hat{Z}_{t-1}^D\right) + \eta_{\Delta x^D\varepsilon^{GD}}\left(\varepsilon_t^{G^D} + \phi_G\hat{G}_{t-1}^D\right) \\
&= \eta_{\Delta x^D Z^D}\hat{Z}_t^D + \eta_{\Delta x^D G^D}\hat{G}_t^D,
\end{aligned}
$$

i.e., equation (28), where we conveniently redefined $\eta_{\Delta x^D Z^D} \equiv \eta_{\Delta x^D\varepsilon^{ZD}}$ and $\eta_{\Delta x^D G^D} \equiv \eta_{\Delta x^D\varepsilon^{GD}}$.

Finally, substituting the expressions for α and the elasticities η's obtained above in the definitions of $\eta_{\Delta x^D Z^D}$ and $\eta_{\Delta x^D G^D}$ yields:

$$
\eta_{\Delta x^D Z^D} = \frac{(1-\phi_Z)(1-\beta)(1+\varphi)(\omega-1)}{(\omega+\varphi)(1-\beta\phi_Z)}\left[\frac{\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}}{1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}}\right]
$$

and

$$\eta_{\Delta x^D G^D} = -\frac{G\left(1-\phi_G\right)\left(1-\beta\right)}{\left(1-\beta\phi_G\right)\left[1-\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}\right]}.$$

G The Determinants of the Valuation Share

G.1 Productivity Shocks

We begin by studying the determinants of the valuation share for periods that follow the impact period of a shock, i.e., $v\hat{a}l^S_{t\geq 1}$.

Recall the definition of Ω in the steady-state portfolio α obtained in Appendix B:

$$\Omega \equiv \frac{G^2\left(\omega+\varphi\right)\varphi\left(1-\beta\phi_Z\right)^2\sigma^2_{\varepsilon GD}}{\sigma\left(\omega-1\right)\left(1+\varphi\right)^2\left(1-G\right)\left(1-\beta\phi_G\right)^2\sigma^2_{\varepsilon ZD}}.$$

Then,

$$\eta_{\Delta x^D Z^D} = \frac{\left(1-\phi_Z\right)\left(1-\beta\right)\left(1+\varphi\right)\left(\omega-1\right)\Omega}{\left(\omega+\varphi\right)\left(1-\beta\phi_Z\right)\left(1-\Omega\right)},$$

and it is straightforward to verify that:

$$v\hat{a}l^S_{t\geq 1} = 1-\Omega = \frac{\left(1-\beta\right)\alpha}{\beta\left(1-a\right)}.$$

Assuming $\omega > 1$ throughout, the results on the steady-state portfolio in Appendix B imply that:

$$\frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\sigma} > 0, \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\sigma^2_{\varepsilon GD}} < 0, \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\phi_G} < 0, \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\sigma^2_{\varepsilon ZD}} > 0, \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\phi_Z} > 0.$$

$$\frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial G} < 0, \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\beta} \geq 0, \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\varphi} < 0, \quad \text{and} \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\omega} > 0,$$

where the last three results hold for plausible parameter values.

Next, we prove that the share of valuation in net foreign asset adjustment in the impact period ($v\hat{a}l^S_0$) is smaller if substitutability between home and foreign goods (ω) rises.

Recall that $v\hat{a}l^S_0 = \left(1-\eta_{\Delta x^D Z^D}/\eta_{\Delta v^D \varepsilon^{ZD}}\right)^{-1}$. Using the definition of Ω and the expressions for $\eta_{\Delta x^D Z^D}$ and $\eta_{\Delta v^D \varepsilon^{ZD}}$, we can write:

$$\frac{\eta_{\Delta x^D Z^D}}{\eta_{\Delta v^D \varepsilon^{ZD}}} = \Gamma\Lambda,$$

where:

$$\begin{aligned}
\Gamma &\equiv \frac{\Omega\left(1-\phi_Z\right)}{\left(1-\Omega\right)\phi_Z}, \\
\Lambda &\equiv \frac{\sigma\left(1-G\right)\left(\omega+\varphi\right)+\varphi\left(\omega-1\right)\Omega}{\sigma\left(1-G\right)\left(\omega+\varphi\right)-\varphi\left(\omega-1\right)\Omega\frac{1-\phi_Z}{\phi_Z}}.
\end{aligned}$$

Tedious algebra shows that $\partial\Lambda/\partial\omega = 0$. (To verify this, we use the result from Appendix B that the derivative of Ω with respect to ω is proportional to $-(1+\varphi)/(\omega-1)^2$, and, in particular, $\partial\Omega/\partial\omega = -\Omega(1+\varphi)/[(\omega-1)(\omega+\varphi)]$.) It follows that:

$$\begin{aligned}\frac{\partial\frac{\eta_{\Delta x^D Z^D}}{\eta_{\Delta v^D \varepsilon Z D}}}{\partial\omega} &= \Lambda\frac{\partial\Gamma}{\partial\omega} = \Lambda\frac{(1-\phi_Z)}{\phi_Z(1-\Omega)^2}\frac{\partial\Omega}{\partial\omega} \\ &= -\frac{\Lambda(1-\phi_Z)\Omega(1+\varphi)}{\phi_Z(1-\Omega)^2(\omega-1)(\omega+\varphi)}.\end{aligned}$$

Hence, assuming $\omega > 1$ and $\Omega \neq 1$, $\Lambda > 0$ is necessary and sufficient for $\partial\left(\eta_{\Delta x^D Z^D}/\eta_{\Delta v^D \varepsilon Z D}\right)/\partial\omega \leq 0$. Given $\omega > 1$, the condition $\Lambda > 0$ is satisfied if and only if:

$$\frac{\sigma(1-G)(\omega+\varphi)\phi_Z}{\varphi(\omega-1)(1-\phi_Z)} > \Omega.$$

This holds for plausible parameter values (for instance, with the parameters in our numerical exercise, $\Omega = .084$ and the left-hand side of the inequality is equal to 11.4). Hence, for parameters in a plausible range, $\partial\left(\eta_{\Delta x^D Z^D}/\eta_{\Delta v^D \varepsilon Z D}\right)/\partial\omega \leq 0$. Therefore, $\partial v\hat{a}l_0^S/\partial\omega \leq 0$.

G.2 Government Spending Shocks

The share of valuation in net foreign adjustment to government spending shocks is zero in all periods but the impact one. We verify here that the share in the impact period is an increasing function of substitutability between home and foreign goods (ω).

Recall that, in response to a relative government spending shock, $v\hat{a}l_0^S = \left(1 - \eta_{\Delta x^D G^D}/\eta_{\Delta v^D \varepsilon G D}\right)^{-1}$. Using the definition of Ω and the expressions for $\eta_{\Delta x^D G^D}$ and $\eta_{\Delta v^D \varepsilon G D}$, we can write:

$$v\hat{a}l_0^S = \frac{G(1-\phi_G)(1-\Psi)}{\sigma},$$

where:

$$\Psi \equiv \frac{\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)}{\varphi(1-\Omega)(\omega-1)}.$$

Tedious but straightforward algebra shows that:

$$\frac{\partial\Psi}{\partial\omega} = -\frac{\sigma(1-G)\varphi(1-\Omega)(1+\varphi)+\frac{\varphi\Omega(1+\varphi)}{\omega+\varphi}\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}{\left[\varphi(1-\Omega)(\omega-1)\right]^2}.$$

The assumptions $\omega > 1$ and $\Omega < 1$ (satisfied for all parameter values we experimented with) are sufficient for $\partial\Psi/\partial\omega < 0$. It follows that $\partial\left(\eta_{\Delta x^D G^D}/\eta_{\Delta v^D \varepsilon G D}\right)/\partial\omega > 0$ and, therefore, $\partial v\hat{a}l_0^S/\partial\omega > 0$ when we consider adjustment to relative government spending shocks.

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