

The Barnett Critique After Three Decades: A New Keynesian Analysis*

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Abstract

This paper extends a New Keynesian model to include roles for currency and deposits as competing sources of liquidity services demanded by households. It shows that, both qualitatively and quantitatively, the Barnett critique applies: While a Divisia aggregate of monetary services tracks the true monetary aggregate almost perfectly, a simple-sum measure often behaves quite differently. The model also shows that movements in both quantity and price indices for monetary services correlate strongly with movements in output following a variety of real and nominal shocks. Finally, the analysis characterizes the optimal monetary policy response to shocks that originate in an explicitly-modeled financial sector.

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1 Introduction

Thirty years ago, William Barnett (1980) demonstrated how economic aggregation theory can be used to construct coherent and consistent measures of money in economies, like the United States', where liquidity services are provided through an entire spectrum of assets including various types of interest-bearing deposits as well as noninterest-bearing currency. During the three decades that have followed, Barnett's work has influenced profoundly the research conducted by a number of economists, including the two authors of this paper. Indeed, if pressed on this issue, virtually all monetary economists today would no doubt concede that the Divisia aggregates proposed by Barnett are both theoretically and empirically superior to their simple-sum counterparts. The reason is that the latter simply add up the nominal value of all monetary assets in circulation and ignore the fact that these different assets yield different flows of liquidity services and, in equilibrium, also differ in the opportunity, or user, costs that households and firms incur when they demand those liquidity services.¹ Because the necessary condition for simple-sum aggregation is that all component assets are perfect substitutes, the only question about simple-sum aggregates is the magnitude of their measurement error.

Yet, despite this widespread appreciation of the advantages of Divisia monetary aggregation, Barnett's article has had a surprisingly small impact on empirical work in monetary economics, where throughout the past three decades and down to the present day, analysts have continued, overwhelmingly, to rely on the readily available but conceptually flawed simple-sum measures. Likewise, central banks around the world, including those like the Federal Reserve that possess the resources to construct high quality monetary statistics and to disseminate them widely, have continued to prepare and release data on the simple-sum aggregates alone even as, for example, national income accountants have gradually adopted theoretically-consistent economic aggregation methods in producing data on gross domestic

¹Simple-sum and superlative indexes of money also differ when examined from the perspective of the production of monetary services as in Barnett, Hinich, and Weber (1986).

product and its components. And in what is perhaps the biggest irony of all, while the various contributions celebrating the thirtieth anniversary of Barnett's article on monetary aggregation all deal with important issues, the vast majority focus on problems relating to economic aggregation more broadly: Almost none focus on the Divisia monetary aggregates! Evidently, the "Barnett critique," to borrow the phrase coined by Chrystal and MacDonald (1994) to summarize the basic message of the arguments articulated in Barnett (1980), applies with equal force today as it did three decades ago.

Building on this last point, this paper revisits the issues regarding the appropriate measurement of money studied by Barnett (1980) using a state-of-the-art model of the monetary transmission mechanism: The microfounded, dynamic and stochastic New Keynesian model, which features prominently in widely-read surveys such as Clarida, Gali, and Gertler (1999) and in leading graduate-level textbooks like Woodford (2003) and which, partly as a consequence, has become in recent years become something of a canonical model for monetary policy evaluation and the analysis of the monetary business cycle. As discussed by Ireland (2004*b*), virtually all standard presentations of the New Keynesian model focus on the behavior of output, inflation, and interest rates, with little or no attention paid to measures of the money supply. Hence, the analysis begins here, in section 2 below, by extending the New Keynesian model to incorporate roles for both noninterest-bearing currency and interest-bearing deposits as alternative sources of liquidity services consumed by households.

In this extended New Keynesian environment, Barnett's (1980) critique applies: Simple-sum measures of the money supply that merely add the nominal value of currency and deposits are theoretically flawed. Moreover, unlike other studies which have attempted to resolve the merits of simple-sum versus superlative indexes of money on the basis of goodness-of-fit criteria or have judged money not to be an important variable when a simple-sum measure alone was used in empirical work, the current paper shows that, with a model fully-specified at the level of tastes and technologies an *exact* aggregate of monetary services is well-defined and observable. Hence, the results presented in section 3 can confirm that Barnett's

critique applies *quantitatively* as well as qualitatively. In particular, those results show that the Divisia approximations proposed by Barnett (1980) track movements in the true monetary aggregate almost perfectly, despite the fact that the theoretical framework used here extends Barnett's along several dimensions by being fully stochastic as well as dynamic and by explicitly accounting for the forward-looking and optimizing behavior of households, firms, and financial institutions in general equilibrium. The simple-sum aggregates, by sharp contrast, often behave quite differently, echoing with New Keynesian theory the points made empirically by Belongia (1996): that "measurement matters" when considering, for instance, the behavior of the money supply in the aftermath of a monetary policy disturbance.

Extending previous work by Barnett (1978, 1980), the results from section 3 below also confirm that movements in an exact *price* index for monetary services are mirrored almost perfectly by a Divisia approximation. These same results also complement those derived by Belongia and Ireland (2006) in a real business cycle setting by showing that important movements in price as well as quantity indices for money occur when a variety of monetary and financial-sector shocks hit the economy and that movements in the price as well as the quantity of money often correlate strongly with movements in output.

Indeed, by developing an extended New Keynesian framework in which private financial institutions create deposits as imperfect substitutes for government-issued currency, this paper can go on to examine, quantitatively, the macroeconomic effects of a range of shocks impacting on the economy through the banking sector. Motivated in particular by events from the US economy over the past several years, the results from section 3 trace out the aggregate consequences of financial-sector disturbances that give rise to a sharp increase in banks' demand for reserves or that raise the costs that those same banks must incur in creating highly-liquid deposits that substitute for government-issued currency. Finally, the results from section 3 begin to shed light on the optimal policy response to such financial-sector shocks, suggesting in particular that a standard Taylor (1993) rule must be expanded to allow for a monetary policy response to changes in a welfare-theoretic measure of the

output gap, in addition to changes in inflation and instead of changes in output itself, to insulate the macroeconomy fully from a wider range of disturbances, real and nominal alike. And interpreted more broadly, those results also indicate that there may be an expanded role for the Divisia aggregates as indicators of monetary conditions during times of financial crises, along the lines suggested by Barnett and Chauvet (2010).

2 The New Keynesian Model

2.1 Overview

The model developed here extends the standard New Keynesian framework, with its key features of monopolistic competition and nominal goods price rigidity, in the same way that Belongia and Ireland (2006) extend the standard real business cycle model, with its key features of frictionless markets and technology shocks, by introducing a role for money through a shopping-time specification and by allowing private banks to produce interest-bearing deposits that compete with government-issued currency as a source of liquidity services that households consume to economize on their time spent purchasing other goods and services. As noted above, this model provides an analytic framework that is fully stochastic and dynamic and that is based on microfoundations and general equilibrium reasoning, which can be used to revisit a range of issues associated with the Barnett (1980) critique as it continues to apply to work in monetary economics at the state-of-the-art today.

Also as noted above, the model provides an analytic framework for examining how the economy responds to a variety of disturbances that originate in the financial sector and for considering the appropriate monetary policy response to those same shocks. In particular, with its focus on reserves and deposits – items on the liability side of the banking system’s balance sheet – this model complements Goodfriend and McCallum’s (2007), with its focus on securities and loans – items on the asset side of the same balance sheet.

The model economy consists of a representative household, a representative finished

goods-producing firm, a continuum of intermediate goods-producing firms indexed by $i \in [0, 1]$, a representative bank, and a monetary authority. During each period $t = 0, 1, 2, \dots$, each intermediate goods-producing firm produces a distinct, perishable intermediate good. Hence, intermediate goods also may be indexed by $i \in [0, 1]$, where firm i produces good i . The model features enough symmetry, however, to allow the analysis to focus on the behavior of a representative intermediate goods-producing firm that produces the generic intermediate good i . The activities of each of these agents now will be described in turn.

2.2 The Representative Household

The representative household enters each period $t = 0, 1, 2, \dots$ with M_{t-1} units of currency, B_{t-1} bonds, and $s_{t-1}(i)$ shares in each intermediate goods-producing firm $i \in [0, 1]$. At the beginning of the period, the household receives T_t additional units of currency in the form of a lump-sum transfer from the monetary authority. Next, the household's bonds mature, providing B_{t-1} more units of currency. The household uses some of this currency to purchase B_t new bonds at the price of $1/r_t$ dollars per bond, where r_t denotes the gross nominal interest rate between t and $t+1$, and $s_t(i)$ shares in each intermediate goods-producing firm $i \in [0, 1]$ at the price of $Q_t(i)$ dollars per share.

After this initial session of securities trading, the household sets aside N_t dollars of currency to be used in purchasing goods and services and deposits the rest in the bank. At the same time, the household also borrows L_t dollars from the bank, bringing the total nominal value of its deposits to

$$D_t = M_{t-1} + T_t + B_{t-1} + \int_0^1 Q_t(i)s_{t-1}(i) di - B_t/r_t - \int_0^1 Q_t(i)s_t(i) di - N_t + L_t. \quad (1)$$

During period t , the household supplies $h_t(i)$ units of labor to each intermediate goods-

producing firm $i \in [0, 1]$, for a total of

$$h_t = \int_0^1 h_t(i) di,$$

receiving labor income $W_t h_t$ in return, where W_t denotes the nominal wage rate. In addition, the household purchases C_t units of the finished good at the nominal price P_t from the representative finished goods-producing firm; making these transactions also requires

$$h_t^s = \left(\frac{1}{\chi}\right) \left(\frac{v_t P_t C_t}{M_t^A}\right)^\chi \quad (2)$$

units of shopping time, where M_t^A is an aggregate of monetary services provided from currency N_t and deposits D_t according to

$$M_t^A = [\nu^{1/\omega} N_t^{(\omega-1)/\omega} + (1-\nu)^{1/\omega} D_t^{(\omega-1)/\omega}]^{\omega/(\omega-1)}. \quad (3)$$

In the shopping-time specification (2), the parameter $\chi > 1$ governs the rate at which the effort required to purchase goods and services increases as the household economizes on its holdings of monetary assets. Extending the specification from Ireland (2004*b*), the shock, v_t , that impacts the total demand for monetary services follows an autoregressive process of the form

$$\ln(v_t) = (1 - \rho_v) \ln(v) + \rho_v \ln(v_{t-1}) + \varepsilon_{vt}, \quad (4)$$

where $v > 0$ determines the steady-state level of real monetary services demanded relative to consumption, the persistence parameter satisfies $0 \leq \rho_v < 1$, and the serially uncorrelated innovation ε_{vt} has mean zero and standard deviation σ_v . In the monetary aggregation specification (3), the parameter ν , satisfying $0 < \nu < 1$, governs the steady-state expenditure shares on currency versus deposits and the parameter $\omega > 0$ governs the elasticity of substitution between currency and deposits in producing the monetary aggregate M_t^A .

At the end of period t , the household owes the bank $r_t^L L_t$ dollars, where r_t^L is the gross

nominal interest rate on loans. At the same time, however, the bank owes the household $r_t^D D_t$ dollars, where r_t^D is the gross nominal interest rate on deposits. The household also receives a nominal dividend payment of $F_t(i)$ for each share that it owns in each intermediate goods-producing firm $i \in [0, 1]$. After all these payments are made and received, the household carries M_t units of currency into period $t + 1$, where

$$M_t = N_t + W_t h_t + \int_0^1 F_t(i) s_t(i) di + r_t^D D_t - P_t C_t - r_t^L L_t. \quad (5)$$

The household, therefore, chooses sequences for B_t , $s_t(i)$ for all $i \in [0, 1]$, N_t , D_t , L_t , h_t , C_t , h_t^s , M_t^A , and M_t for all $t = 0, 1, 2, \dots$ to maximize the expected utility function

$$E \sum_{t=0}^{\infty} \beta^t a_t [\ln(C_t) - \eta(h_t + h_t^s)], \quad (6)$$

where the discount factor and the weight on leisure relative to consumption satisfy $0 < \beta < 1$ and $\eta > 0$. The preference shock a_t in (6) follows the autoregressive process

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at}, \quad (7)$$

where $0 \leq \rho_a < 1$ and the serially uncorrelated innovation ε_{at} has mean zero and standard deviation σ_a ; as shown by Ireland (2004b) a preference shock of this kind translates, in equilibrium, into a disturbance to the New Keynesian model's forward-looking IS curve, linking expected consumption growth to the real interest rate. The household makes its optimal choices subject to the constraints (1)-(3) and (5), each of which must hold for all $t = 0, 1, 2, \dots$, taking as given the behavior of the exogenous shocks described by (4) and (7) for all $t = 0, 1, 2, \dots$. Part 1 of the appendix displays the first-order conditions for the household's problem.

2.3 The Representative Finished Goods-Producing Firm

During each period $t = 0, 1, 2, \dots$, the representative finished goods-producing firm uses $Y_t(i)$ units of each intermediate good $i \in [0, 1]$, purchased at the nominal price $P_t(i)$, to manufacture Y_t units of the finished good according to the constant-returns-to-scale technology described by

$$Y_t = \left[\int_0^1 Y_t(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)}, \quad (8)$$

where $\theta > 1$ governs the elasticity of substitution between the various intermediate goods in producing the finished good. Thus, the finished goods-producing firm chooses $Y_t(i)$ for all $i \in [0, 1]$ to maximize its profits, given by

$$P_t \left[\int_0^1 Y_t(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} - \int_0^1 P_t(i) Y_t(i) di,$$

for all $t = 0, 1, 2, \dots$. The first-order conditions for this problem are

$$Y_t(i) = [P_t(i)/P_t]^{-\theta} Y_t \quad (9)$$

for all $i \in [0, 1]$ and $t = 0, 1, 2, \dots$.

Competition drives the finished goods-producing firm's profits to zero. This zero-profit condition implies that

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{1/(1-\theta)}$$

in equilibrium for all $t = 0, 1, 2, \dots$.

2.4 The Representative Intermediate Goods-Producing Firm

During each period $t = 0, 1, 2, \dots$, the representative intermediate goods-producing firm hires $h_t(i)$ units of labor from the representative household to manufacture $Y_t(i)$ units of interme-

intermediate good i according to the constant-returns-to-scale technology described by

$$Y_t(i) = Z_t h_t(i). \quad (10)$$

The aggregate technology shock Z_t follows a random walk with positive drift:

$$\ln(Z_t) = \ln(z) + \ln(Z_{t-1}) + \varepsilon_{zt}, \quad (11)$$

where $z > 1$ and the serially uncorrelated innovation ε_{zt} has mean zero and standard deviation σ_z .

Since the intermediate goods substitute imperfectly for one another in producing the finished good, the representative intermediate goods-producing firm sells its output in a monopolistically competitive market; during each period $t = 0, 1, 2, \dots$, the intermediate goods-producing firm sets the nominal price for its output, subject to the requirement that it satisfy the representative finished goods-producing firm's demand, described by (9). In addition, following a specification first proposed by Rotemberg (1982), the intermediate goods-producing firm faces a quadratic cost of adjusting its nominal price, measured in units of the finished good and given by

$$\frac{\phi}{2} \left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right]^2 Y_t,$$

where the parameter $\phi \geq 0$ governs the magnitude of the price adjustment costs and where $\pi > 1$ denotes the gross, steady-state inflation rate.

This costly price adjustment makes the intermediate goods-producing firm's problem dynamic. As described in part 2 of the appendix, the firm chooses a sequence for $P_t(i)$ for all $t = 0, 1, 2, \dots$ to maximize its total, real market value. When log-linearized, the first-order conditions for this problem, also shown in part 2 of the appendix, combine to take the form of a forward-looking, New Keynesian Phillips curve, which links inflation during period t

to expected inflation during period $t + 1$ and real marginal cost; because labor is the only factor of production in this model, real marginal cost is measured by dividing the real wage (W_t/P_t) by the aggregate technology shock Z_t during period t .

2.5 The Representative Bank

During each period $t = 0, 1, 2, \dots$, the representative bank accepts deposits worth D_t dollars from the representative household and makes loans worth L_t dollars to the representative household. It pays interest on the deposits it accepts at the gross rate r_t^D and receives interest on the loans it makes at the gross rate r_t^L . Let τ_t denote the bank's reserve ratio during period t . Here, this reserve ratio is allowed to vary stochastically, but exogenously, to capture the macroeconomic effects of unexpected changes in banks' demand for reserves; it follows the autoregressive process

$$\ln(\tau_t) = (1 - \rho_\tau) \ln(\tau) + \rho_\tau \ln(\tau_{t-1}) + \varepsilon_{\tau t}, \quad (12)$$

where $0 < \tau < 1$, $0 \leq \rho_\tau < 1$, and the serially uncorrelated innovation ε_τ has zero mean and standard deviation σ_τ . Bank loans are then related to bank deposits according to the stochastic relationship

$$L_t = (1 - \tau_t) D_t \quad (13)$$

for all $t = 0, 1, 2, \dots$

Following the specification introduced into a real business cycle framework by Belongia and Ireland (2006), the bank creates deposits with total real value D_t/P_t during each period $t = 0, 1, 2, \dots$ according to a constant-returns-to-scale technology that requires $x_t(D_t/P_t)$ units of the finished good. The financial-sector cost shock follows the autoregressive process

$$\ln(x_t) = (1 - \rho_x) \ln(x) + \rho_x \ln(x_{t-1}) + \varepsilon_{xt}, \quad (14)$$

where $x > 0$, $0 \leq \rho_x < 1$, and the serially uncorrelated innovation ε_{xt} has mean zero and standard deviation σ_x . Hence, the bank's nominal profits during period t are

$$\Pi_t^b = (r_t - 1)L_t - (r_t^D - 1)D_t - P_t x_t (D_t/P_t).$$

Substituting (13), which links loans to deposits, into this last expression reveals that as competition drives profits in the banking industry to zero,

$$r_t^D = 1 + (r_t^L - 1)(1 - \tau_t) - x_t, \tag{15}$$

must hold in equilibrium during each period $t = 0, 1, 2, \dots$. This implies that the spread between the loan and deposit rates is affected by the financial-sector cost shock as well as the stochastically-varying reserve ratio.

2.6 Efficient Allocations and the Output Gap

As a preliminary step in describing the monetary authority's behavior, it is helpful to consider the problem faced by a benevolent social planner who can overcome both the shopping-time frictions that generate demands for the liquidity services provided by currency and deposits in equilibrium and the quadratic adjustment costs that give rise to sluggish price adjustment in equilibrium. During each period $t = 0, 1, 2, \dots$, this social planner allocates $h^*(i)$ units of the representative household's labor to produce $Y_t^*(i)$ units of each intermediate good $i \in [0, 1]$ according to the technology described by (10), then uses those various intermediate goods to produce Y_t^* units of the finished good according to the technology described by (8).

Part 3 of the appendix shows that the first-order conditions for the social planner's problem can be combined to yield the simple expression

$$Y_t^* = (1/\eta)Z_t \tag{16}$$

for the efficient level of output. This expression confirms a general applicability of Kydland and Prescott's (1982) insight: To the extent that business cycle fluctuations are driven by technology shocks, those fluctuations are efficient and therefore should not be the target of monetary or fiscal stabilization policies in this New Keynesian model. Based on this insight, a welfare-theoretic measure of the output gap can be defined for this economy as

$$g_t^* = Y_t/Y_t^* = \eta(Y_t/Z_t) \quad (17)$$

for all $t = 0, 1, 2, \dots$, indicating that the converse to Kydland and Prescott's observation is also true: To the extent that shocks other than those to productivity lead to fluctuations in output, there is a potential role for activist stabilization policies. Some of the results presented in section 3 below speak directly to this last point.

2.7 The Monetary Authority

In equilibrium, $M_t = M_{t-1} + T_t$, $B_t = B_{t-1} = 0$, and $s_t(i) = s_{t-1}(i) = 1$ for all $i \in [0, 1]$ must hold for all $t = 0, 1, 2, \dots$. Substituting these conditions, together with (13), into (1) confirms that the monetary base M_t equals currency in circulation N_t plus reserves N_t^v :

$$M_t = N_t + N_t^v,$$

where

$$N_t^v = \tau_t D_t \quad (18)$$

for all $t = 0, 1, 2, \dots$

In a real business cycle version of this model, Belongia and Ireland (2006) assume that the monetary authority conducts monetary policy as described by a rule for managing the monetary base. Here, in line with most New Keynesian analyses, the central bank is assumed instead to follow a Taylor (1993) rule for managing the interest rate instead. Thus,

the extended model developed here has, at its core, the three basic equations common to all New Keynesian models: a forward-looking IS curve describing the behavior of optimizing households, a forward-looking Phillips curve describing the behavior of optimizing firms, and a Taylor rule describing the conduct of monetary policy. The extended model developed here, however, adds to this New Keynesian core the shopping-time specification described by (2) and (3) that generates a demand for currency and deposits as sources of liquidity services consumed by households and the stochastic relations described by (12)-(15) describing the competitive behavior of the private financial institutions that produce interest-bearing deposits as imperfect substitutes for government-issued currency in producing those same liquidity services.

Accordingly, let

$$\pi_t = P_t/P_{t-1} \quad (19)$$

and

$$g_t^y = Y_t/Y_{t-1} \quad (20)$$

denote the gross rates of inflation and output growth between periods $t - 1$ and t , and let r , π , g^* , and g^y denote the average, or steady-state, values of the nominal interest rate, the inflation rate, the output gap, and the output growth rate in this model where, as explained below, all of these variables turn out to be stationary. Then the modified Taylor (1993) rule

$$\ln(r_t/r) = \rho_r \ln(r_{t-1}/r) + \rho_\pi \ln(\pi_{t-1}/\pi) + \rho_{g^*} \ln(g_{t-1}^*/g^*) + \rho_{g^y} \ln(g_{t-1}^y/g^y) + \varepsilon_{rt} \quad (21)$$

allows the monetary authority to adjust the interest rate in response to changes in two stationary real variables, the output gap and output growth, in addition to the inflation rate, and also allows for interest rate smoothing through the introduction of the lagged interest rate on the right-hand-side, all depending on specific settings for the policy reaction coefficients ρ_r , ρ_π , ρ_{g^*} , and ρ_{g^y} . In (21), the serially uncorrelated innovation ε_{rt} has mean zero and standard deviation σ_r .

2.8 Monetary Aggregation

In this model with currency and deposits, the variable M_t^A represents the true aggregate of monetary or liquidity services demanded by the representative household during each period $t = 0, 1, 2, \dots$. Part 4 of the appendix demonstrates that the own rate of return r_t^A on this true monetary aggregate and the associated opportunity cost $r_t - r_t^A$ incurred by households when they hold this monetary aggregate instead of bonds can be defined with reference to the price dual of the quantity aggregation formula (3),

$$r_t - r_t^A = [\nu(r_t - 1)^{1-\omega} + (1 - \nu)(r_t - r_t^D)^{1-\omega}]^{1/(1-\omega)}, \quad (22)$$

which also recognizes that $r_t - 1$ measures the opportunity cost of holding noninterest-bearing currency and $r_t - r_t^D$ measures the opportunity cost of holding interest-bearing deposits. Part 4 of the appendix also confirms that Barnett's (1978) formula for the user cost of the monetary aggregate M_t^A , namely

$$u_t^A = (r_t - r_t^A)/r_t, \quad (23)$$

applies here as well.

As shown in (3), (22), and (23), however, the monetary services aggregate M_t^A and its user cost u_t^A depend on the share and elasticity parameters ν and ω , which though observable to private agents within this model economy may well be unobservable to outsiders, including analysts working for the monetary authority and applied econometricians more generally. In this model economy, as in the US economy, therefore, some observers may construct and monitor alternative measures of money that do not depend on unknown parameters or functional forms. One such measure is the simple-sum aggregate

$$M_t^S = N_t + D_t, \quad (24)$$

computed in the usual way by adding the nominal value of currency plus deposits.

Barnett's (1980) critique states, however, that simple-sum aggregates like (24) are flawed measures of money, except under the extreme assumption – violated both within this model and in the US economy – that all monetary assets are perfect substitutes in terms of the liquidity services they provide. But, Barnett (1980) goes on to show how Divisia indices can be used to approximate the true monetary quantity and price aggregates, even when the parameters and more basic functional forms of (3) and (22) are unknown.

Start by defining the user costs of currency and deposits in a manner analogous to (23), that is, using Barnett's (1978) formula:

$$u_t^N = (r_t - 1)/r_t \quad (25)$$

and

$$u_t^D = (r_t^A - r_t^D)/r_t. \quad (26)$$

Next, compute total expenditures on monetary services as

$$E_t = u_t^N N_t + u_t^D D_t. \quad (27)$$

Part 4 of the appendix demonstrates that

$$E_t = u_t^A M_t^A,$$

indicating that while the true quantity and price indices cannot be observed separately without knowledge of the functional forms and parameters in (3) and (22), their product, equal to total expenditures on monetary services, can be measured directly from the data.

Next, compute the associated expenditure shares as

$$s_t^N = u_t^N N_t / E_t \quad (28)$$

and

$$s_t^D = u_t^D D_t / E_t. \quad (29)$$

Now the growth rate of the Divisia quantity index for monetary services can be computed as

$$\mu_t^Q = (\mu_t^N)^{(s_t^N + s_{t-1}^N)/2} (\mu_t^D)^{(s_t^D + s_{t-1}^D)/2}, \quad (30)$$

where

$$\mu_t^N = N_t / N_{t-1} \quad (31)$$

and

$$\mu_t^D = D_t / D_{t-1} \quad (32)$$

denote the growth rates of currency and deposits. Meanwhile, the growth rate of the Divisia price index for monetary services is

$$\pi_t^P = \left(\frac{u_t^N}{u_{t-1}^N} \right)^{(s_t^N + s_{t-1}^N)/2} \left(\frac{u_t^D}{u_{t-1}^D} \right)^{(s_t^D + s_{t-1}^D)/2}. \quad (33)$$

Theil (1967) shows that in discrete time, this Divisia price index is not dual to the Divisia quantity index in (30). The results below demonstrate, however, that across a wide range of circumstances, the Divisia quantity index defined in (30) tracks the true quantity index defined in (3) and the Divisia price index defined in (33) tracks the true price index defined in (23) almost perfectly, and (23) is the price dual associated with (3).

2.9 The Equilibrium System

Part 5 of the appendix collects equations describing private agents' optimizing behavior, the monetary authority's chosen policy, market clearing conditions, and the evolution of the exogenous shocks to form a system that determines the behavior of all of the model's endogenous variables in a symmetric equilibrium in which all intermediate goods-producing firms

make identical decisions. This system of equations implies that most of these variables will be nonstationary, with the real variables inheriting a unit root from the nonstationary process (11) for the technology shock and the nominal variables inheriting a unit root from the conduct of monetary policy via the Taylor rule (21), which makes the price level nonstationary. However, when the system is rewritten in terms of a set of appropriately-transformed variables, also identified in part 5 of the appendix, it implies that the model has a balanced growth path along which these transformed variables remain stationary.

The transformed system also implies that, in the absence of shocks, each of the stationary variables converges to a unique steady-state value. The transformed system can therefore be log-linearized around its unique steady state to form a set of linear expectational difference equations that can be solved using methods outlined by Klein (2000).

3 The Barnett Critique Revisited

3.1 Calibration

Computational implementation of Klein's (2000) solution method requires that numerical values be assigned to each of the model's parameters. Here, this task can be accomplished using the calibration methodology first proposed by Kydland and Prescott (1982), one used throughout the literature on dynamic, stochastic, general equilibrium macroeconomic theory.

Accordingly, setting $\beta = 0.99$ for the representative household's discount factor identifies one period in the model as one quarter year in real time. Given the linearity of utility in hours worked displayed in (6), the setting $\eta = 1$ turns out to be a normalization that pins down the steady-state value of hours worked without affecting any of the model's dynamics.

The setting $\theta = 6$, drawn from previous work by Ireland (2000, 2004*a*, 2004*b*), makes the steady-state markup of price over marginal cost resulting from monopolistic competition in the markets for the differentiated intermediate goods equal to 20 percent. And, as explained in Ireland (2004*b*), the setting $\phi = 50$ implies a speed of price adjustment in this model

with quadratic price adjustment costs that is the same as the speed of price adjustment in a model with staggered price setting following Calvo's (1983) popular specification in which individual goods prices are adjusted, on average, every 3.75 quarters, that is, slightly more than once per year.

The setting $\chi = 2$ makes the shopping time specification in (2) quadratic, and the setting $\omega = 1.5$ implies that there is more substitutability between currency and deposits in the general CES aggregator (3) than there would be in a more restrictive Cobb-Douglas specification. The setting $v = 3.1$ works to match the steady-state ratio of the simple-sum aggregate M_t^S to nominal consumption expenditures $P_t C_t$ in the model with the average, equal to approximately 10, as measured by the ratio of simple-sum M2 to personal consumption expenditures in US data running from 1959 through 2009. Likewise, the setting $\nu = 0.225$ works to match the steady-state ratio of currency N_t to the simple-sum aggregate M_t^S in the model with the average, equal to approximately 0.10, as measured by the ratio of currency in circulation to simple-sum M2 in the same sample of US data.

The calibrated value $z = 1.005$ implies growth of 2 percent per year on average for most of the model's real variables. The value $\tau = 0.03$ reflects the observation that, again in a sample of US data running from 1959 through 2009, the ratio of total reserves to the deposit (non-currency) components of M2 averages about 3 percent. The value $x = 0.01$ is similar to the one used by Belongia and Ireland (2006) and implies that the average spread between the interest rates on loans and deposits exceeds 100 basis points by just a slight amount, because that spread reflects, in addition to the 100-basis-point cost of creating deposits, the much smaller cost of holding 3 percent reserves against those deposits as well.

The inflation target $\pi = 1.005$ implies a steady-state inflation rate of 2 percent per year in the model. And, for a benchmark parameterization, the settings $\rho_r = 0.75$ and $\rho_\pi = 0.30$ provide for a substantial amount of interest rate smoothing together with a monetary policy response to inflation that is strong enough to make the linearized model's rational expectations equilibrium unique according to the criteria outlined by Blanchard and

Kahn (1980). While this benchmark parameterization sets the policy coefficients ρ_{g^*} and ρ_{g^v} on the output gap and output growth equal to zero, positive values for these parameters are considered as alternatives below, in characterizing the optimal policy response to the model's financial-sector shocks.

For the parameters determining the degree of persistence in the exogenous shocks, the settings $\rho_v = 0.95$ and $\rho_a = 0.90$ imply that the money demand shock is highly persistent and the preference shock just slightly less so. The settings $\rho_\tau = 0.50$ and $\rho_x = 0.50$ mean that the financial-sector shocks to reserves demand and the cost of creating liquid deposits retain some serial correlation but die out more quickly than the other macroeconomic disturbances.

In this linearized model, different numerical settings for the standard deviation of the innovations to each shock simply scale up or down the impulse response functions around which the quantitative analysis is organized below. Hence, the settings $\sigma_v = \sigma_a = \sigma_z = 0.01$ are used for illustrative purposes, describing the effects of money demand, IS, and technology shocks of a more-or-less typical size. The much larger values $\sigma_\tau = 1$ and $\sigma_x = 0.25$ are selected so that the impulse responses to the reserve demand and deposit cost shocks can be interpreted as describing what happens during extreme conditions accompanying a financial crisis in which banks' demand for reserves doubles and the cost of creating deposits rises by 25 percent. Finally, the setting $\sigma_r = 0.0025$ implies that a one-standard-deviation monetary policy shock raises the short-term nominal interest rate in the model by one quarter of one percent; but since the interest rates from the model are expressed in quarterly terms and not annualized, this shock should be interpreted as one that increases the federal funds rate, quoted as usual in annualized terms, by 100 basis points.

3.2 The Quantity and Price of Money: Measurement Matters

Figure 1 plots the impulse responses of the growth rates of three monetary aggregates, the true aggregate defined in (3), its Divisia approximation defined in (30), and the simple-sum aggregate defined in (24), to each of the model's six exogenous shocks. These impulse re-

sponses confirm that the Barnett (1980) critique and Belongia's (1996) related empirical findings that "measurement matters" when gauging the linkages between the money supply and other key macroeconomic variables carry over to this state-of-the-art theoretical specification.

In particular, the graphs presented in the first two columns of each row confirm that the Divisia quantity aggregate tracks the true monetary aggregate almost perfectly after each of the six shocks. Indeed, if the impulse responses for the true monetary aggregate and its Divisia approximation were plotted together in the same graphs, the two lines would appear indistinguishable. These findings are encouraging since, in terms of theory, they show that the Divisia monetary aggregate continues to be a trustworthy approximation even in this fully dynamic, stochastic, general equilibrium setting in which a range of real and nominal disturbances are accounted for. These findings also are encouraging since, in terms of practice, they show that specific knowledge of the true functional forms describing how households aggregate currency and deposits into a composite yielding liquidity services and specific knowledge of the parameters entering into those functional forms are not needed in constructing reliable monetary statistics. This last goal, of course, is one of the principle objectives of Barnett's (1980) original study: To use economic theory as the basis for defining "*parameter-free* approximations to aggregator functions," with the emphasis taken directly from page 12 of the original text.

On the other hand, the graphs in the third column of each row in figure 1 reveal that the more conventional simple-sum aggregate often behaves quite differently than its true and Divisia counterparts. This happens following the preference, or IS, shock and both of the financial-sector shocks, but especially in the aftermath of a monetary policy shock. Since, in particular, the decline in the growth rate of the simple-sum aggregate is much smaller than the decline in either the true or Divisia monetary aggregate, any econometrician looking for evidence of a liquidity effect as manifested by a decline in the money supply following a policy-induced increase in the interest rate would reach quite different conclusions depending

on whether he or she uses data on the Divisia or the simple-sum aggregate from this economy. Here, in theory, “measurement matters,” just as it does in Belongia’s (1996) empirical study.

Figure 2 extends the comparison by tracing out impulse responses of the growth rate of the user cost of the true monetary aggregate, defined in (23) using Barnett’s (1978) formula, and the growth rate of the Divisia price index for the aggregate of monetary services, defined in (33). Again, the measures move in virtual lockstep following each type of shock, confirming that reliable price, as well as quantity, indices for monetary aggregates can be compiled directly from observable data alone and without knowledge of the true form of the aggregator functions or the true values of the parameters entering into those functions.

Figure 3 shifts the focus back to the three macroeconomic variables that more typically hold center stage in New Keynesian analyses: output, inflation, and the short-term nominal interest rate. The top row, in particular, echoes the famous results from Poole (1970) and extended to a New Keynesian setting in Ireland (2000), showing that by holding the nominal interest rate fixed in the face of a money demand shock, the Taylor rule (23) requires the monetary authority to accommodate that shift in money demand with an increase in the monetary base, leaving output and inflation unchanged. Likewise, in this setting where the technology shock follows a random walk, the immediate and permanent shift in the level of productivity that follows a technological innovation leaves the natural real rate of interest unchanged. As a consequence, under a Taylor rule that also holds the market nominal rate of interest unchanged, output follows its efficient path, as dictated by (16), rising immediately and permanently without a change in inflation. Note, however, that bringing about this efficient response of output to the technology shock does require deliberate action on the part of the monetary authority, in the form of an increase in the monetary base that, again, helps accommodate the increased demand for liquidity services reflected in the third row of figure 1.

The last row of figure 3 shows how an unanticipated monetary tightening, brought about through a policy-induced rise in the short-term nominal interest, leads to a sizable decline

in output and a smaller decline in inflation, as the New Keynesian element of nominal price rigidity leads to monetary nonneutrality in the short run. Again, going back to figure 1, this monetary tightening also is associated with a liquidity effect, as the true monetary aggregate falls even more sharply than output.

Row two of figure 3 reveals that an exogenous, nonmonetary demand-side disturbance – the preference, or New Keynesian IS – shock generates a rise in both output and inflation. The benchmark Taylor rule, with its settings of $\rho_r = 0.75$ and $\rho_\pi = 0.30$, calls for a sustained monetary tightening to counteract what, according to the expression (16) for the efficient level of output, represents an inefficient overheating of the economy; as discussed in more detail below, a Taylor rule with a larger value for ρ_π , and hence a more aggressive policy response to inflation, would insulate more successfully the economy against the volatility set off by this IS disturbance.

Finally, rows four and five of figure 3 show the responses to both financial-sector disturbances: When banks' demand for reserves or when banks' cost of creating deposits rises unexpectedly. Whereas output declines in reaction to either disturbance, inflation remains unchanged after each of these shocks. These outcomes get shaped, to a large extent, by the workings of monetary policy under the benchmark Taylor rule, which again calls for the monetary authority to fully accommodate the increased demand for base money that arises as a consequence of each of the two shocks so as to stabilize the price level. Yet, this policy-induced increase in the monetary base does not suffice to offset the change in either the reserve ratio or the currency-deposit ratio – more generally, the change in the money multiplier – caused by these shocks. And so, returning again to figures 1 and 2, the true and Divisia price and quantity indices for the monetary aggregate respond sharply; in both cases, the quantity index falls and the price index rises, indicative of a liquidity crunch centered in the private financial sector that generates the loss in output shown in figure 3. Empirically, Belongia and Ireland (2006) isolate effects quite similar to these in US data using a vector autoregression to associate increases in the price index for monetary services with reductions

in aggregate output. Belongia and Ireland (2006) also show that these *price* effects of money on output arise, too, in a real business cycle framework, that is, even in the absence of other, New Keynesian, monetary nonneutralities.

3.3 The Optimal Monetary Policy Response to Financial-Sector Shocks

As noted above with reference to the expressions (16) and (17) for the efficient level of output and the associated welfare-theoretic measure of the output gap, the New Keynesian model shares with the real business cycle framework the implication that output fluctuations driven by technology shocks represent the economy's optimal response to shifts in production possibilities and therefore ought not to be stabilized by countercyclical monetary policies. On the other hand, (16) and (17) also imply that the output fluctuations exhibited in figure 3 as responses to the model's non-technology shock are inefficient and raise the question of whether alternatives to the benchmark settings of $\rho_r = 0.75$, $\rho_\pi = 0.30$, $\rho_{g^*} = 0$, and $\rho_{g^y} = 0$ for the parameters of the Taylor rule (21) can be found that might help the monetary authority better accomplish both goals: Insulating output from the effects of the other disturbances while, at the same time, allowing output to respond to the technology shock.

Starting with the preference shock, again as noted above, a more aggressive response to changes in inflation, brought about through the choice of a larger value for ρ_π , will produce the more vigorous monetary tightening that prevents the economy from overheating after a positive IS shock and, by the symmetry of this linearized model, also produce a monetary easing that prevents output from falling below the efficient level following a negative IS shock. Rows four and five of figure 3 show, however, that after either of the two financial-sector disturbances, inflation remains unchanged even under the benchmark setting of $\rho_\pi = 0.30$. A further increase in the value of that policy coefficient, therefore, will not help in bringing about the monetary easing that is required to stabilize output when the demand for reserves or the cost of creating deposits increases. Instead, a response to one of the measures of real

economy activity, either output growth g_t^y or the output gap g_t^* , becomes necessary.

Intuitively, compared to the scenarios that unfold under the benchmark policy illustrated in figure 3, setting the policy response coefficient ρ_{g^y} equal to a positive number will produce both the monetary tightening that works to stabilize output after a positive IS shock and the monetary easing that works to stabilize output after either of the adverse financial-sector shocks. That same modification to the output growth coefficient in the Taylor rule, however, will generate a monetary tightening after a positive technology shock, slowing down the economy's efficient adjustment to that disturbance.

Instead, the appropriate monetary policy response to all six of the model's shocks gets generated under a Taylor rule that calls for an aggressive reaction to changes in the welfare-theoretic measure of the output gap g_t^* defined in (17). Figure 4 confirms this by plotting impulse responses of output, inflation, and the nominal interest rate using the Taylor rule (21) that augments the benchmark settings of $\rho_r = 0.75$ and $\rho_\pi = 0.30$ with an alternative setting for $\rho_{g^*} = 100$, representing an extremely vigorous policy response to changes in the output gap. The first column of figure 4, in particular, shows that this alternative monetary policy succeeds in insulating output almost completely (note the changing scale of the y-axes when comparing the individual graphs presented in this first column) from the effects of shocks to money demand, preferences, reserves demand, deposit costs, and even to the Taylor rule itself, while still allowing the economy to respond efficiently to technology shocks. Moreover, the third column of figure 4 reveals that, despite the very large numerical value assigned to ρ_{g^*} , this policy gets implemented with only modest adjustments in the short-term interest rate, as the reduction in volatility in the output gap itself helps offset the larger value of the response coefficient in determining the total volatility of the interest rate.

Of course, these results lean quite heavily on the assumption that the monetary authority can identify successfully the various shocks that hit the economy and thereby measure accurately the output gap in real time. Orphanides' (2003) careful study of Federal Reserve policy during the 1970s raises strong doubts about whether this assumption is tenable in

practice. At a minimum, however, these results indicate that the optimal monetary response to financial-sector shocks may differ considerably from what is prescribed by a standard Taylor rule specification. And given that these shocks affect the economy through the money multiplier as opposed to the monetary base – a point that also lies, of course, at the heart of Friedman and Schwartz’s (1963) critique of Federal Reserve policy during the Great Depression – under more realistic assumptions about the information available to monetary policymakers in real time they may require a shift in focus back to the broader monetary aggregates as useful indicators of monetary conditions. Here again, the Barnett (1980) critique applies and, indeed, as emphasized by Barnett and Chauvet (2010), access to Divisia monetary aggregates that improve on their simple-sum counterparts may become especially useful, if not absolutely essential, during periods of financial crisis.²

4 Conclusion

Very few papers in economics – especially in monetary economics – remain as relevant thirty years after publication as they were on the day when they first appeared in print. But, both for better and for worse, William Barnett’s article from 1980 volume of the *Journal of Econometrics* surely counts as one among those elite few.

For better, the analysis conducted here demonstrates that the Divisia approach to monetary aggregation first proposed by Barnett (1980) works exactly as intended within a state-of-the-art, dynamic, stochastic, New Keynesian model of the monetary business cycle, extended to include roles for both currency and interest-bearing bank deposits as sources of the liquidity services demanded by households in general equilibrium. Within the confines of this theoretical model, the true aggregate of liquidity services becomes an observable variable.

²Bernanke and Blinder (1988), for example, offer a nice illustration of this point in practice. After constructing what they label a “CC curve” – a modified IS curve that embeds a credit sector such that it will shift in response to credit market shocks – they examine whether a central bank should use money or the funds rate as an intermediate target in response to financial sector shocks. Using simple-sum measures of money, their empirical tests find in favor of the funds rate. One wonders, however, what these same tests might reveal if Divisia data were available to re-examine the disturbance experienced by their CC curve in 2008.

The results of the analysis show, however, that a Divisia monetary aggregate, which can be measured without reference to the functional forms describing tastes and technologies and without knowledge of the values of the parameters entering into those functional forms, tracks the true aggregate almost perfectly following a wide variety of macroeconomic shocks, both real and nominal.

But for worse, the Federal Reserve, like most other central banks around the world, continues to assemble and report data on simple-sum monetary aggregates alone. And, in large part precisely because no official data on the Divisia monetary aggregates exist, virtually all empirical work in monetary economics relies on the simple-sum measures instead. Yet Barnett (1980) shows that these simple-sum measures are theoretically flawed, Belongia (1996) demonstrates that a wide range of apparently puzzling empirical results concerning the links between money and other key macroeconomic variables stem exclusively from the use of these simple-sum aggregates, and Barnett and Chauvet (2010) list numerous occasions throughout post-World War II US monetary history when Federal Reserve officials or outside observers have been led astray in making or evaluating monetary policy by signals mistakenly gleaned from the simple-sum aggregates. The results of the analysis conducted here confirm, with state-of-the-art monetary theory, that “measurement matters” in all of these ways.

Barnett (1980) concludes with reference to this quote from Irving Fisher’s (1922, pp.29-30) treatise on index number theory:

The simple arithmetic average is perhaps still the favorite one in use In fields other than index numbers it is often the best form of average to use. But we shall see that the simple arithmetic average produces one of the very worst of index numbers. And if this book has no other effect than to lead to the total abandonment of the simple arithmetic type of index number, it will have served a useful purpose.

In fact, Fisher’s arguments have served their useful purpose in the area of national income accounting and, indeed, the Federal Reserve itself now employs indices like those advocated by

Fisher in its monthly measurements of industrial production. But for monetary aggregation, even thirty years after its publication, Barnett’s work has yet to make its full impact felt: the Federal Reserve and the scientific community continue to operate without theoretically-coherent and empirically-reliable indices of monetary services. The “Barnett critique” still applies.

5 Appendix

5.1 Household Optimization

A convenient way to characterize the solution to the household’s problem is to substitute the shopping-time specification (2) into the utility function (6) to obtain

$$E \sum_{t=0}^{\infty} \left[\ln(C_t) - \eta h_t - \eta \left(\frac{1}{\chi} \right) \left(\frac{v_t P_t C_t}{M_t^A} \right)^{\chi} \right], \quad (34)$$

and to express the remaining constraints (1), (3), and (5) in real terms by dividing through by the nominal price level to obtain

$$\frac{M_{t-1} + T_t + B_{t-1} - B_t/r_t - N_t + L_t}{P_t} + \int_0^1 \left[\frac{Q_t(i)}{P_t} \right] [s_{t-1}(i) - s_t(i)] di \geq \frac{D_t}{P_t}, \quad (35)$$

$$\left[\nu^{1/\omega} \left(\frac{N_t}{P_t} \right)^{(\omega-1)/\omega} + (1-\nu)^{1/\omega} \left(\frac{D_t}{P_t} \right)^{(\omega-1)/\omega} \right]^{\omega/(\omega-1)} \geq \frac{M_t^A}{P_t}, \quad (36)$$

and

$$\frac{N_t + W_t h_t + r_t^D D_t}{P_t} + \int_0^1 \left[\frac{F_t(i)}{P_t} \right] s_t(i) di \geq C_t + \frac{r_t^L L_t + M_t}{P_t}, \quad (37)$$

after allowing for free disposal. Letting Λ_t^1 , Λ_t^2 , and Λ_t^3 denote the nonnegative Lagrange multipliers on these three constraints, the first-order conditions for the household’s problem can be written as

$$\frac{\Lambda_t^1}{r_t} = \beta E_t \left(\frac{\Lambda_{t+1}^1 P_t}{P_{t+1}} \right), \quad (38)$$

$$\Lambda_t^1 \left[\frac{Q_t(i)}{P_t} \right] = \Lambda_t^3 \left[\frac{F_t(i)}{P_t} \right] + \beta E_t \left\{ \Lambda_{t+1}^1 \left[\frac{Q_{t+1}(i)}{P_{t+1}} \right] \right\} \quad (39)$$

for all $i \in [0, 1]$,

$$\Lambda_t^1 - \Lambda_t^3 = \Lambda_t^2 \left[\nu^{1/\omega} \left(\frac{N_t}{P_t} \right)^{(\omega-1)/\omega} + (1-\nu)^{1/\omega} \left(\frac{D_t}{P_t} \right)^{(\omega-1)/\omega} \right]^{1/(\omega-1)} \nu^{1/\omega} \left(\frac{N_t}{P_t} \right)^{-1/\omega}, \quad (40)$$

$$\begin{aligned} & \Lambda_t^1 - r_t^D \Lambda_t^3 \\ &= \Lambda_t^2 \left[\nu^{1/\omega} \left(\frac{N_t}{P_t} \right)^{(\omega-1)/\omega} + (1-\nu)^{1/\omega} \left(\frac{D_t}{P_t} \right)^{(\omega-1)/\omega} \right]^{1/(\omega-1)} (1-\nu)^{1/\omega} \left(\frac{D_t}{P_t} \right)^{-1/\omega}, \end{aligned} \quad (41)$$

$$\Lambda_t^1 = r_t^L \Lambda_t^3, \quad (42)$$

$$\eta a_t = \Lambda_t^3 \left(\frac{W_t}{P_t} \right), \quad (43)$$

$$\frac{a_t}{C_t} \left[1 - \eta \left(\frac{v_t P_t C_t}{M_t^A} \right)^\chi \right] = \Lambda_t^3, \quad (44)$$

$$\eta a_t \left(\frac{v_t P_t C_t}{M_t^A} \right)^\chi = \Lambda_t^2 \left(\frac{M_t^A}{P_t} \right), \quad (45)$$

and

$$\Lambda_t^3 = \beta E_t \left(\frac{\Lambda_{t+1}^1 P_t}{P_{t+1}} \right), \quad (46)$$

together with (2) and (35)-(37) with equality for all $t = 0, 1, 2, \dots$

5.2 Intermediate Goods-Producing Firm Optimization

From the equity-pricing relationship (39), the representative intermediate goods-producing firm's total, real market value is proportional to

$$E \sum_{t=0}^{\infty} \beta^t \Lambda_t^3 \left[\frac{F_t(i)}{P_t} \right]$$

where

$$\frac{F_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t} \right]^{1-\theta} Y_t - \left[\frac{P_t(i)}{P_t} \right]^{-\theta} \left(\frac{W_t Y_t}{P_t Z_t} \right) - \frac{\phi}{2} \left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right]^2 Y_t \quad (47)$$

for all $t = 0, 1, 2, \dots$. The first-order conditions for the firm's problem of choosing a sequence for $P_t(i)$ to maximize this total market value are

$$\begin{aligned} 0 = & (1 - \theta) \Lambda_t^3 \left[\frac{P_t(i)}{P_t} \right]^{-\theta} Y_t + \theta \Lambda_t^3 \left[\frac{P_t(i)}{P_t} \right]^{-\theta-1} \left(\frac{W_t Y_t}{P_t Z_t} \right) \\ & - \phi \Lambda_t^3 \left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right] \left[\frac{Y_t P_t}{\pi P_{t-1}(i)} \right] \\ & + \beta \phi E_t \left\{ \Lambda_{t+1}^3 \left[\frac{P_{t+1}(i)}{\pi P_t(i)} - 1 \right] \left[\frac{Y_{t+1} P_{t+1}(i) P_t}{\pi P_t(i)^2} \right] \right\} \end{aligned} \quad (48)$$

for all $t = 0, 1, 2, \dots$. As noted above in the text, in a symmetric equilibrium where $P_t(i) = P_t$ for all $i \in [0, 1]$ and all $t = 0, 1, 2, \dots$, this optimality condition, when log-linearized, takes the form of the New Keynesian Phillips curve.

5.3 Efficient Allocations

The social planner chooses Y_t^* and $h_t^*(i)$ for all $i \in [0, 1]$ and $t = 0, 1, 2, \dots$ to maximize the household's welfare, as measured by

$$E \sum_{t=0}^{\infty} \beta^t a_t \left[\ln(Y_t^*) - \eta \left(\int_0^1 h_t^*(i) di \right) \right]$$

subject to the feasibility constraints

$$Z_t \left[\int_0^1 h_t^*(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \geq Y_t^*$$

for all $t = 0, 1, 2, \dots$, which combine the restrictions imposed by the underlying technologies (8) and (10) for the finished and intermediate goods. The first-order conditions for this

problem can be written as

$$h_t^*(i)^{-1/\theta} = \eta \left[\int_0^1 h_t^*(i)^{(\theta-1)/\theta} di \right]$$

for all $i \in [0, 1]$ and $t = 0, 1, 2, \dots$. Combining these first-order conditions with the binding constraints leads to the expression (16) for the efficient level of output.

5.4 Monetary Aggregation

Returning to the first-order conditions from the household's problem, note that (38), (42), and (46) imply that

$$r_t^L = r_t,$$

$$\Lambda_t^1 - \Lambda_t^3 = (r_t - 1)\Lambda_t^3,$$

and

$$\Lambda_t^1 - r_t^D \Lambda_t^3 = (r_t - r_t^D)\Lambda_t^3$$

for all $t = 0, 1, 2, \dots$. Using these relationships together with (36), (40) and (41) can be written more compactly as

$$\frac{N_t}{P_t} = \nu \left(\frac{M_t^A}{P_t} \right) \left[\frac{\Lambda_t^2}{(r_t - 1)\Lambda_t^3} \right]^\omega$$

and

$$\frac{D_t}{P_t} = (1 - \nu) \left(\frac{M_t^A}{P_t} \right) \left[\frac{\Lambda_t^2}{(r_t - r_t^D)\Lambda_t^3} \right]^\omega.$$

Substituting these last two conditions back into (36) then yields

$$\frac{\Lambda_t^2}{\Lambda_t^3} = [\nu(r_t - 1)^{1-\omega} + (1 - \nu)(r_t - r_t^D)^{1-\omega}]^{1/(1-\omega)} \quad (49)$$

Let the own-rate of return r_t^A on the monetary aggregate M_t^A be defined with reference to the right-hand side of this last equation:

$$r_t - r_t^A = [\nu(r_t - 1)^{1-\omega} + (1 - \nu)(r_t - r_t^D)^{1-\omega}]^{1/(1-\omega)}. \quad (50)$$

Then it can be verified that if the choices of currency and deposits are not of independent interest, the household's problem can be stated more simply as one of choosing sequences for B_t , $s_t(i)$ for all $i \in [0, 1]$, L_t , h_t , C_t , M_t^A and M_t to maximize the utility function as written in (34) subject to the constraints

$$\frac{M_{t-1} + T_t + B_{t-1} - B_t/r_t + L_t}{P_t} + \int_0^1 \left[\frac{Q_t(i)}{P_t} \right] [s_{t-1}(i) - s_t(i)] di \geq \frac{M_t^A}{P_t}$$

and

$$\frac{W_t h_t + r_t^A M_t^A}{P_t} + \int_0^1 \left[\frac{F_t(i)}{P_t} \right] s_t(i) di \geq C_t + \frac{r_t^L L_t + M_t}{P_t}$$

for all $t = 0, 1, 2, \dots$, confirming that M_t^A can be treated as a true aggregate of monetary services.

Next, note that (45) and (46), when rewritten in nominal terms, can be combined into the single budget constraint

$$\begin{aligned} & r_{t-1}^A M_{t-1}^A + B_{t-1} - r_{t-1} L_{t-1} + T_t + W_{t-1} h_{t-1} + \int_0^1 [Q_t(i) + F_{t-1}(i)] s_{t-1}(i) di \\ & \geq P_{t-1} C_{t-1} + \int_0^1 Q_t(i) s_t(i) di + M_t^A + B_t/r_t - L_t, \end{aligned}$$

where the equilibrium condition $r_t^L = r_t$ has also been used. Iterating forward to turn this single-period budget constraint into an infinite-horizon budget constraint, in a manner following Barnett (1978), requires rewriting the last three terms on the right-hand-side as

$$M_t^A + \frac{B_t}{r_t} - L_t = \frac{r_t^A M_t^A + B_t - r_t L_t}{r_t} + \left(\frac{r_t - r_t^A}{r_t} \right) M_t^A,$$

which confirms that Barnett's (1978) formula for the user cost u_t^A of the true monetary aggregate, restated in (23), applies here as well. Finally, note that (23), (25)-(27), (36), (38), (40), (41), (46), (49), and (50) imply that $E_t = u_t^A M_t^A$, as indicated above in the text.

5.5 The Equilibrium System

In a symmetric equilibrium, all intermediate goods-producing firms make identical decisions, so that $Y_t(i) = Y_t$, $h_t(i) = h_t$, $P_t(i) = P_t$, $F_t(i) = F_t$, and $Q_t(i) = Q_t$ for all $i \in [0, 1]$ and $t = 0, 1, 2, \dots$. In addition, the market-clearing conditions $M_t = M_{t-1} + T_t$, $B_t = B_{t-1} = 0$, and $s_t(i) = s_{t-1}(i) = 1$ for all $i \in [0, 1]$ must hold for all $t = 0, 1, 2, \dots$. After imposing these conditions, (2), (4), (7), (10)-(33), and (35)-(48) can be collected together to form a system of 41 equations that determine the equilibrium behavior of the 41 variables $C_t, Y_t, Y_t^*, g_t^y, g_t^*, h_t, h_t^s, F_t, \Lambda_t^1, \Lambda_t^2, \Lambda_t^3, M_t^A, N_t, D_t, M_t, L_t, N_t^v, M_t^S, E_t, \mu_t^N, \mu_t^D, \mu_t^Q, P_t, W_t, Q_t, \pi_t, r_t, r_t^D, r_t^L, r_t^A, u_t^A, u_t^N, u_t^D, s_t^N, s_t^D, \pi_t^P, v_t, a_t, Z_t, \tau_t$, and x_t .

As noted above in the text, most of the variables entering into this system will be non-stationary, with the real variables inheriting a unit root from the nonstationary process (11) for the technology shock and the nominal variables inheriting a unit root from the conduct of monetary policy via the Taylor rule (21). However, when a new set of variables is defined to include $c_t = C_t/Z_{t-1}$, $y_t = Y_t/Z_{t-1}$, $y_t^* = Y_t^*/Z_{t-1}$, $g_t^y, g_t^*, h_t, h_t^s, f_t = (F_t/P_t)/Z_{t-1}$, $\lambda_t^1 = Z_{t-1}\Lambda_t^1$, $\lambda_t^2 = Z_{t-1}\Lambda_t^2$, $\lambda_t^3 = Z_{t-1}\Lambda_t^3$, $m_t^A = (M_t^A/P_t)/Z_{t-1}$, $n_t = (N_t/P_t)/Z_{t-1}$, $d_t = (D_t/P_t)/Z_{t-1}$, $m_t = (M_t/P_t)/Z_{t-1}$, $l_t = (L_t/P_t)/Z_{t-1}$, $n_t^v = (N_t^v/P_t)/Z_{t-1}$, $m_t^S = (M_t^S/P_t)/Z_{t-1}$, $e_t = (E_t/P_t)/Z_{t-1}$, $\mu_t^N, \mu_t^D, \mu_t^Q, \pi_t = P_t/P_{t-1}$, $w_t = (W_t/P_t)/Z_{t-1}$, $q_t = (Q_t/P_t)/Z_{t-1}$, $r_t, r_t^D, r_t^L, r_t^A, u_t^A, u_t^N, u_t^D, s_t^N, s_t^D, \pi_t^P, v_t, a_t, z_t = Z_t/Z_{t-1}$, τ_t , and x_t , and the system is rewritten in terms of these transformed variables, it implies that each transformed variable will remain stationary along the model's balanced growth path.

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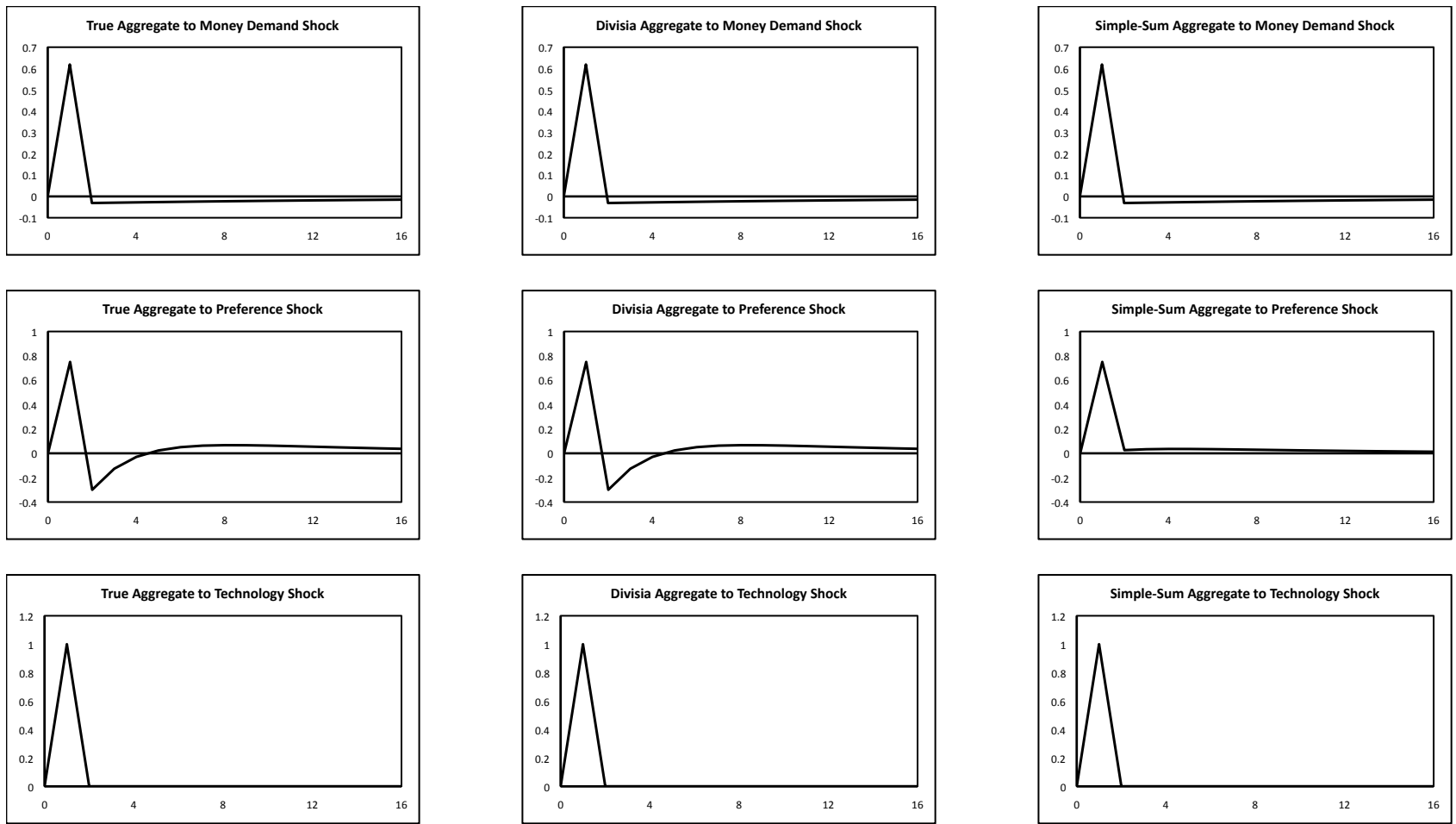


Figure 1a. Impulse Responses for Nominal Money Growth. Each panel shows the percentage-point response in the nominal growth rate of the true monetary aggregate, its Divisia approximation, or its simple-sum counterpart to a one-standard-deviation innovation in one of the model's six shocks.

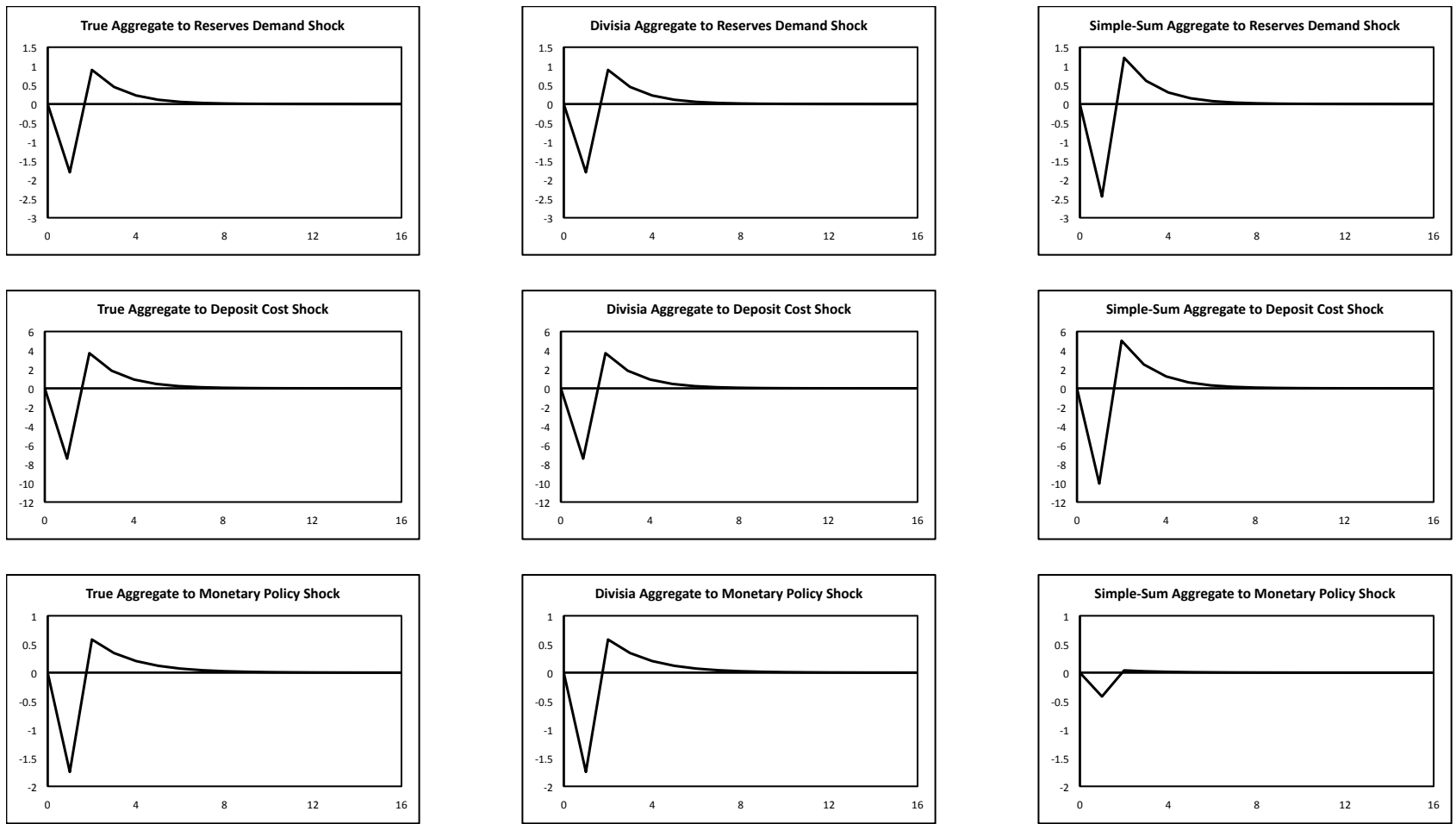


Figure 1b. Impulse Responses for Nominal Money Growth. Each panel shows the percentage-point response in the nominal growth rate of the true monetary aggregate, its Divisia approximation, or its simple-sum counterpart to a one-standard-deviation innovation in one of the model's six shocks.

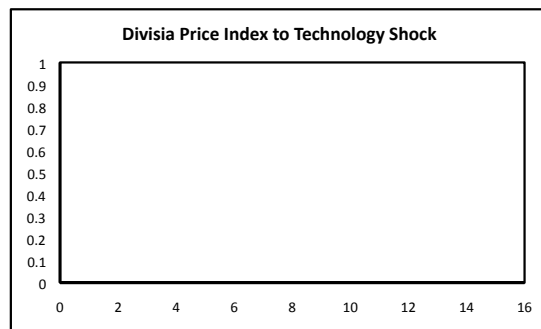
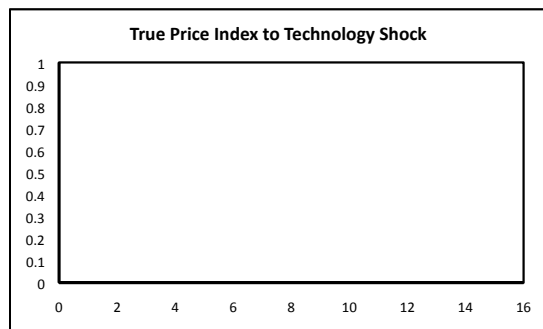
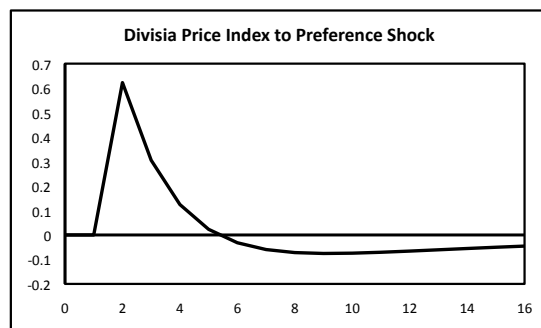
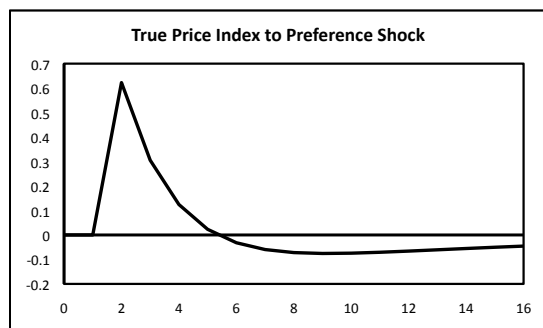
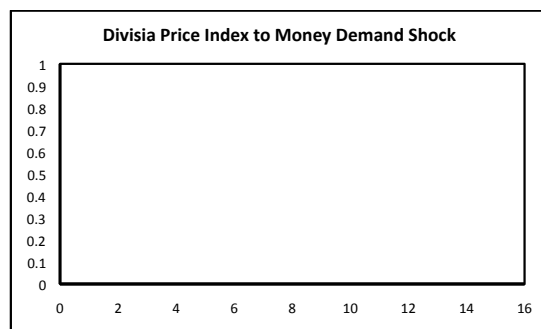
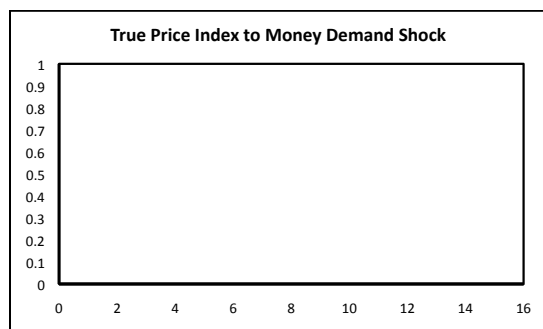


Figure 2a. Impulse Responses for Monetary Services Price Indices. Each panel shows the percentage-point response of the growth rate of the true price index for monetary services or its Divisia approximation to a one-standard-deviation innovation in one of the model's six shocks.

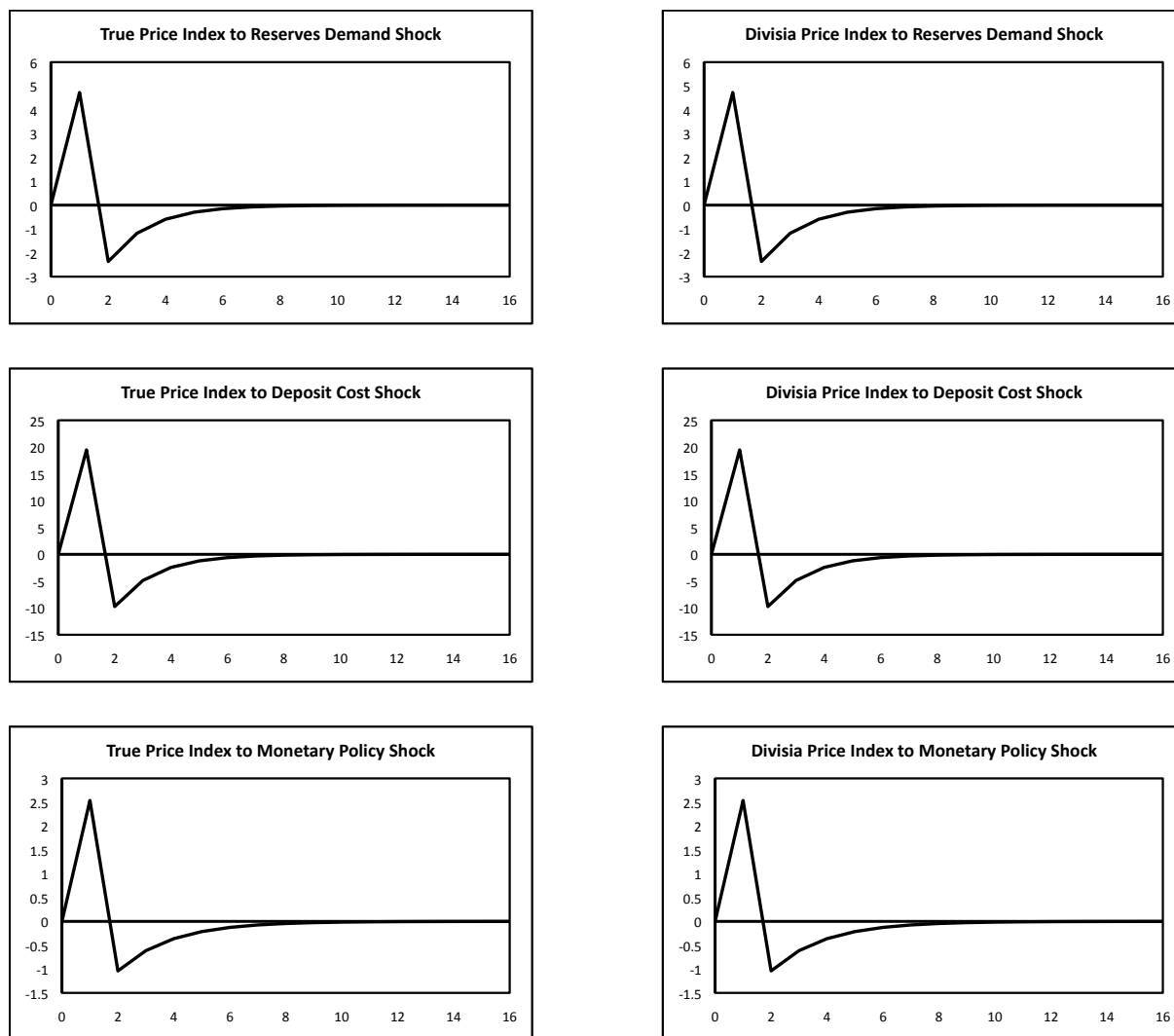


Figure 2b. Impulse Responses for Monetary Services Price Indices. Each panel shows the percentage-point response of the growth rate of the true price index for monetary services or its Divisia approximation to a one-standard-deviation innovation in one of the model's six shocks.

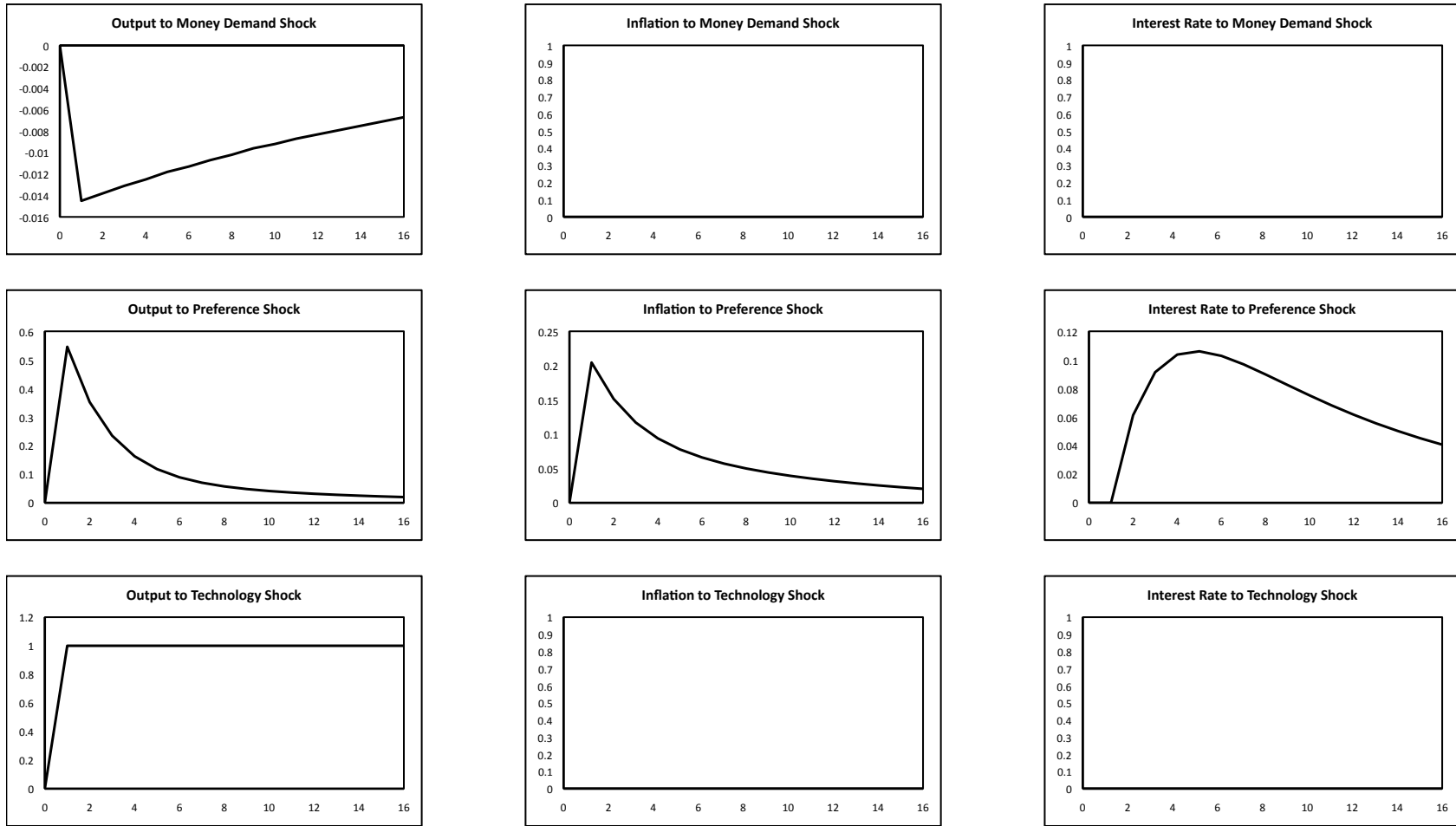


Figure 3a. Impulse Responses for Macroeconomic Variables. Each panel shows the percentage-point response in output, inflation, or the nominal interest rate to a one-standard-deviation innovation in one of the model's six shocks.

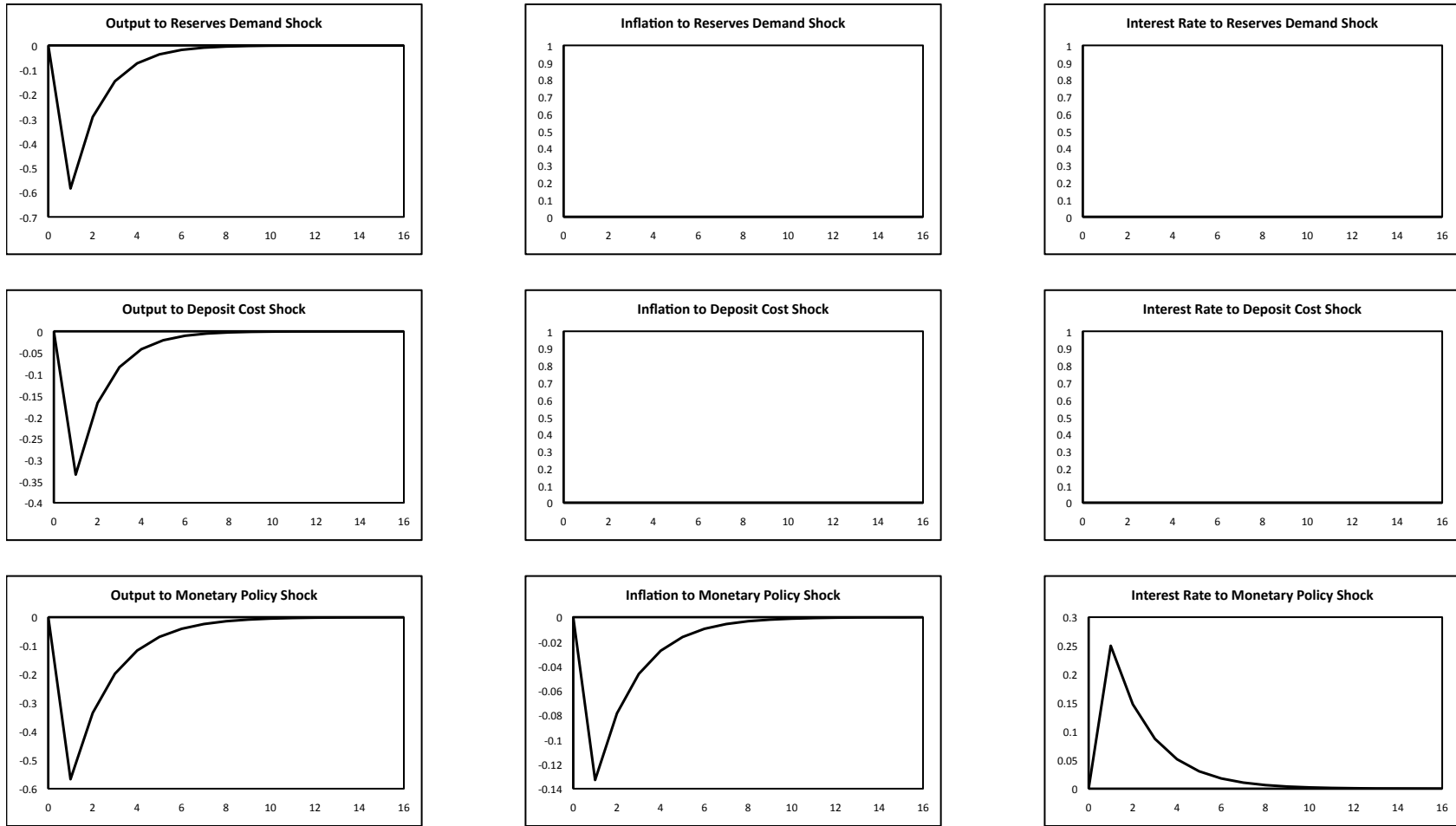


Figure 3b. Impulse Responses for Macroeconomic Variables. Each panel shows the percentage-point response in output, inflation, or the nominal interest rate to a one-standard-deviation innovation in one of the model's six shocks.

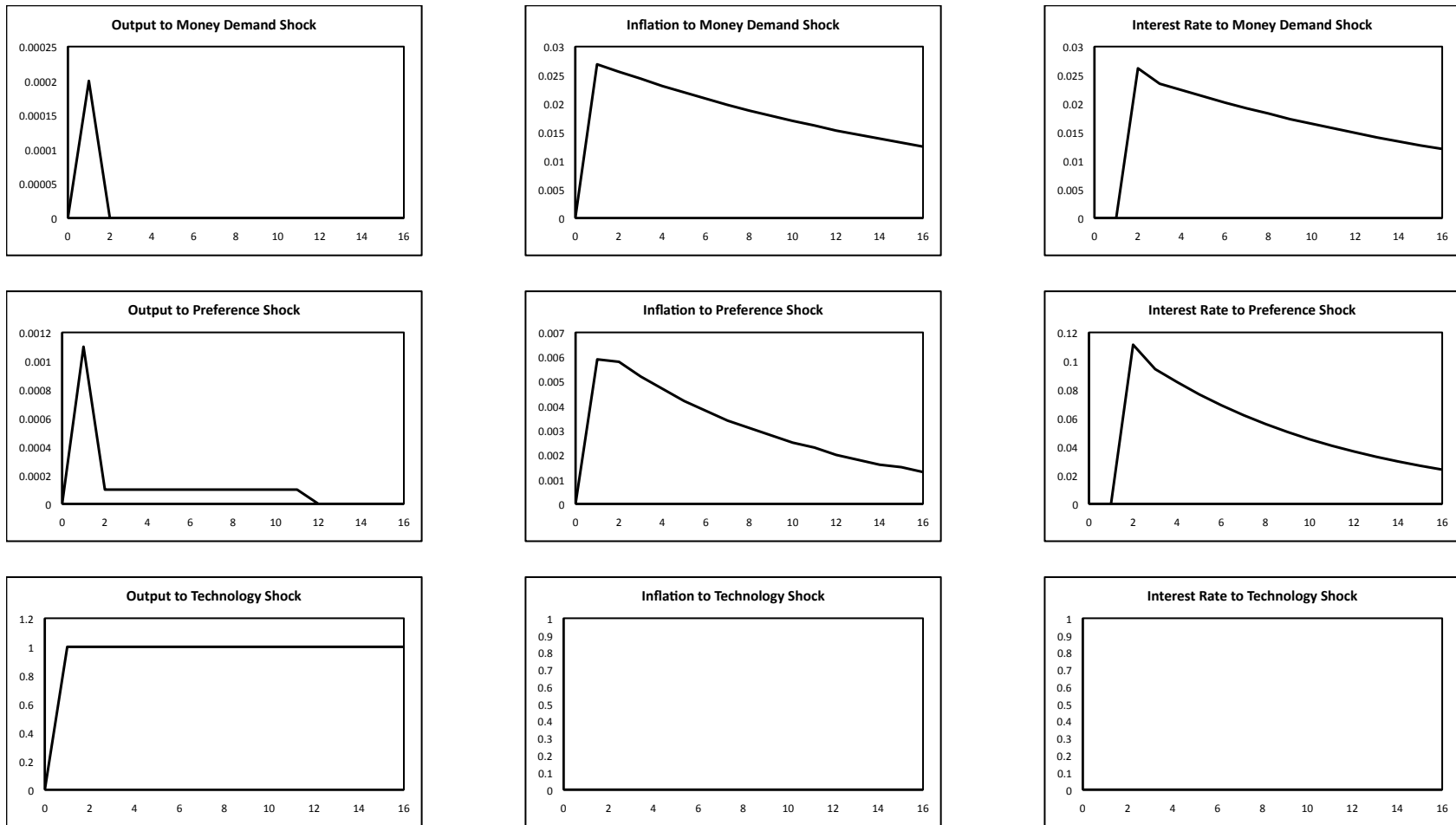


Figure 4a. Impulse Responses for Macroeconomic Variables When Monetary Policy Reacts to the Output Gap. Each panel shows the percentage-point response in output, inflation, or the nominal interest rate to a one-standard-deviation innovation in one of the model's six shocks.

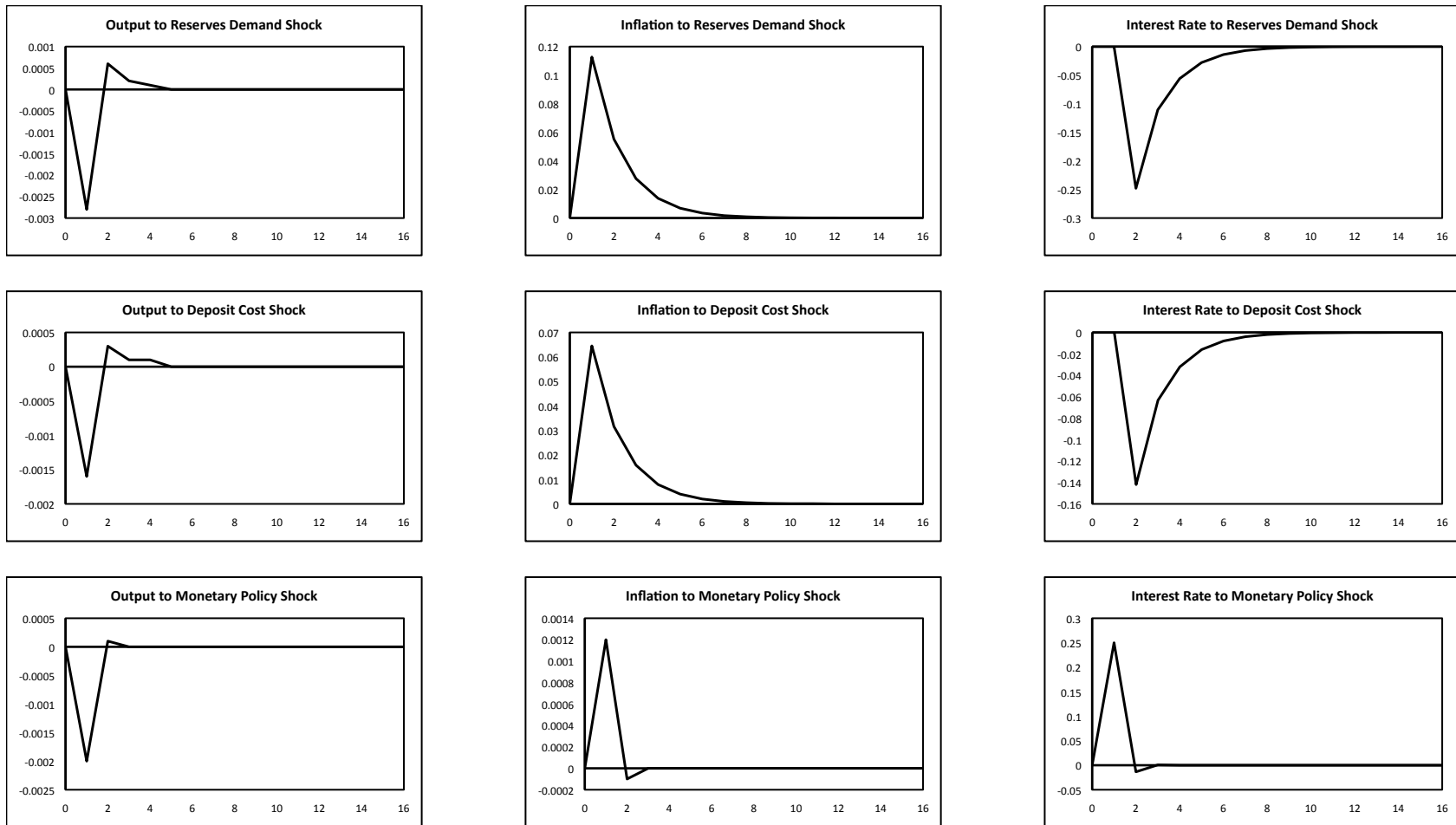


Figure 4b. Impulse Responses for Macroeconomic Variables When Monetary Policy Reacts to the Output Gap. Each panel shows the percentage-point response in output, inflation, or the nominal interest rate to a one-standard-deviation innovation in one of the model's six shocks.