Monetary Policy, Bond Risk Premia, and the Economy

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Abstract

This paper develops an affine model of the term structure of interest rates in which bond yields are driven by observable and unobservable macroeconomic factors. It imposes restrictions to identify the effects of monetary policy and other structural disturbances on output, inflation, and interest rates and to decompose movements in long-term rates into terms attributable to changing expected future short rates versus risk premia. The estimated model highlights a broad range of channels through which monetary policy affects risk premia and the economy, risk premia affect monetary policy and the economy, and the economy affects monetary policy and risk premia.

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1 Introduction

With their traditional instrument of monetary policy, the short-term federal funds rate, locked up against its zero lower bound since 2008, Federal Reserve officials have resorted to other means for influencing long-term interest rates in order to provide further stimulus to a struggling US economy. Some of these non-traditional policy measures, such as the Federal Open Market Committee's provision of "forward guidance," aim to lower long-term interest rates by shaping expectations about the future path of short-term rates, in particular, by creating expectations that the federal funds rate will remain at or near zero even as the economic recovery continues to strengthen. Other new programs, including multiple rounds of "large-scale asset purchases," known more popularly as "quantitative easing," attempt to lower long-term interest rates more directly by reducing the term, or risk, premia that ordinarily cause long-term rates to exceed the average expected future value of the short-term policy rate and generate a yield curve with its most typical, upward slope. As former Federal Reserve Chairman Ben Bernanke explains:

> Federal Reserve actions have also affected term premiums in recent years, most prominently through a series of Large-Scale Asset Purchase (LSAP) programs. . . . To the extent that Treasury securities and agency-guaranteed securities are not perfect substitutes for other assets, Federal Reserve purchases of these assets should lower their term premiums, putting downward pressure on longer-term interest rates and easing financial conditions more broadly. . . . Of course, the Federal Reserve has used this unconventional approach to lowering longer-term rates because, with short-term rates near zero, it can no longer use its conventional approach of cutting the target for the federal funds rate. (Bernanke 2013, p.7)

In addition to the assumption, stated clearly by the Chairman, that Federal Reserve bond purchases work to lower long-term rates by reducing the size of term or risk premia, a second assumption, equally important but not as clearly stated, that provides the rationale for those policy actions is that these reductions in risk premia are effective at stimulating the private demand for goods and services and thereby work to increase aggregate output and inflation in much the same way that more traditional monetary policy actions do. Yet, as Rudebusch, Sack, and Swanson (2007) astutely note, although this "practitioner view" that smaller long-term bond risk premia help stimulate economic activity is quite widely held, surprisingly little support for the view can be found in existing theoretical or empirical work. In textbook New Keynesian models such as Woodford (2003) and Gali's (2008), for instance, the effects of monetary policy actions on aggregate output arise only to the extent that those actions have implications for current and future values of the short-term interest rate. Thus, as Eggertsson and Woodford (2003) show, these models offer a rationale for the FOMC's policy of forward guidance but not for its large-scale asset purchases. Andres, Lopez-Salido, and Nelson (2004) elaborate on the New Keynesian framework, introducing features that imply exactly the sort of imperfect substitutability referred to in Chairman Bernanke's comments from above, to demonstrate how downward movements in long-term yields can stimulate aggregate demand even holding the path of short rates fixed. More recently, however, Chen, Curdia, and Ferrero (2012) have estimated this model with US data from 1987 through 2009 and concluded that the extra effects running through the additional channel are of limited practical importance. In a similar exercise, Kiley (2012) finds somewhat stronger effects of changes in term premia on aggregate demand, but mainly when the long-term interest rates used in the estimation are those on corporate bonds instead of Treasury securities.

In the meantime, using a variety of empirical approaches, Ang, Piazzesi, and Wei (2006) and Dewachter, Iania, and Lyrio (forthcoming) find that changes in bond risk premia do not help forecast future output growth, while Hamilton and Kim (2002), Favero, Kaminiska, and Soderstrom (2005), and Wright (2006) obtain estimates associating larger bond risk premia with *faster* output growth in the future, exactly the opposite of what the practitioner view asserts. Jardet, Monfort, and Pegoraro (2013), by contrast, detect evidence of the expected, inverse relation between current risk premia and future output, but estimate the effect to be short-lived, reversing itself after less than one year. Rudebusch, Sack, and Swanson (2007) also find some evidence of an inverse relation between term premia and future output growth, although, as they also point out, this result appears quite sensitive to both the specification of the forecasting equation and choice of sample period used to estimate the model. Finally, Bekaert, Hoerova, and Lo Duca (2013) find stronger links between monetary policy actions, financial market measures of risk, and aggregate economic activity that are consistent with the practitioner view, but derive their risk measures from the stock-option-based VIX instead of from risk premia embedded into the prices of the government bonds that the Federal Reserve has been purchasing.

Motivated by the weak and often conflicting results reported in previous studies, this paper develops and estimates a model designed specifically to explore the interplay between monetary policy actions, bond risk premia, and the economy. Rather than imposing a strong set of theoretical assumptions about how these channels of transmission arise, as for example Andres, Lopez-Salido, and Nelson (2004) do in their extension of the tightly-parameterized New Keynesian model, the approach taken here uses a more flexible, multivariate time series model to assess the extent to which, operating through any sort of mechanism, changes in monetary policy may or may not have affected bond risk premia and the economy in the past, changes in bond risk premia may or may not have influenced aggregate output and inflation in the past, and changes in bond risk premia may or may not have led the Federal Reserve, in turn, to historically adjust its monetary policy stance relative to what the then-current levels of output and inflation might have otherwise dictated. The paper's goal, therefore, is to add to the existing empirical literature, cited above, in hopes of highlighting more clearly some stylized facts or regularities in the data that future theoretical work, perhaps along the same lines as Andres, Lopez-Salido, and Nelson (2004), might try to explain more fully. Partly for this reason, the model is estimated with quarterly US data from a sample that begins in 1959 but ends in 2007, before the onset of the financial crisis and most recent severe recession. Thus, while special interest in the channels linking monetary policy, bond risk premia, and the macroeconomy has been kindled by the more dramatic events from the past five or six years, this paper looks for evidence that characterizes these channels in data from more normal times, in sample periods similar to those considered in previous work, to see how, in the past, they have operated as regular features of the American economy and, by extension, to predict how they might continue to operate, event after the extreme conditions of recent years have faded away.

Of course, even with a more flexible empirical specification, *some* identifying assumptions must be drawn from theory in order to disentangle the effects that different fundamental shocks have on observable, endogenous variables. Here, those assumptions are borrowed from two sources. First, following Ang and Piazzesi (2003), cross-equation restrictions implied by no-arbitrage in an affine theory of the term structure of interest rates are used to identify the unobserved risk premia build into observable bond yields. But while Ang and Piazzesi's (2003) original model allows macroeconomic variables to affect the behavior of the yield curve, by design it omits channels through which changes in the yield curve can feed back on and affect their macroeconomic drivers. Here, as in Ang, Piazzesi, and Wei (2006), Dieblond, Rudebusch, and Aruoba (2006), and Pericoli and Taboga (2008), the model allows for such feedback effects. Going further than those previous studies, however, the model developed here also draws on identifying assumptions like those used in more conventional vector autoregressions for macroeconomic variables alone to isolate the effects of monetary policy shocks on bond risk premia and the effects of bond risk premia on output and inflation. Similar assumptions are also employed by Bekaert, Hoerova, and Lo Duca (2013), but, as noted above, using observed movements in the equity options-based VIX measure of stock market volatility rather than inferred movements of bond risk premia implied by no-arbitrage.

In addition to its three core macroeconomic variables – the short-term nominal interest rate, a measure of the output gap, and inflation – and five longer-term bond yields, the model developed here also includes two unobservable state variables. Inspired by Cochrane and Piazzesi (2008), time-variation in bond risk premia within the affine pricing framework is assumed to be driven by a single factor. Rather than measuring this factor using the observable combination of forward rates isolated by Cochrane and Piazzesi (2005) in their earlier work, however, the specification used here follows Dewahcter, Iania, and Lyrio (forthcoming) by treated this "risk" variable or factor as unobservable, to be identified through the comparison of long-term rates and the expected path of future short-term rates implied by the affine model's cross-equation restrictions. This more flexible approach leaves the model free to focus on the possible linkages between monetary policy actions, bond risk premia, and the economy, while still imposing enough structure to avoid the kind of overparameterization that, as Bauer (2011) explains, often blurs the view of bond risk premia provided by less highly-constrained term structure models.

The model features, in addition, an unobservable long-run trend component of inflation, interpreted as a time-varying target around which the Federal Reserve has used its interest rate policies to stabilize actual inflation. A fluctuating, but unobserved, long-run inflation target of this kind is introduced into the New Keynesian macroeconomic model by Ireland (2007) and into models that include both macroeconomic and term structure variables by Kozicki and Tinsley (2001*a*, 2001*b*), Dewachter and Lyrio (2006), Hordahl, Tristani, and Vestin (2006), Spencer (2008), Doh (2012), Hordahl and Tristani (2012), and Rudebusch and Swanson (2012). Implied time paths for these unobservable risk premium and inflation target variables, generated using the same Kalman filtering and smoothing algorithm used to estimate model's parameters via maximum likelihood, provide additional insights into the broader effects of monetary policy and other shocks to the US economy over the sample period. They are therefore examined and discussed below, together with the model's implications for the interplay between monetary policy, bond risk premia, aggregate output, and inflation.

2 Model

2.1 Macroeconomic Dynamics

As noted above, bond yields in this affine pricing model get driven by five state variables: two unobservables and three observables. The first unobservable, denoted v_t, is a "risk" variable, so called because, as explained below, it governs all time variation in bond risk premia and is normalized so as to associate higher values of this index with higher levels of bond-market risk. The second unobservable is the central bank's inflation target τ_t, which is assumed to follow the autoregressive process

$$\tau_t = (1 - \rho_\tau)\tau + \rho_\tau \tau_{t-1} + \sigma_\tau \varepsilon_{\tau t}, \tag{1}$$

where τ measures the average, or steady-state, value of the inflation target, the persistence and volatility parameters satisfy $0 \leq \rho_\tau < 1$ and $\sigma_\tau > 0$, and the serially uncorrelated innovation $\varepsilon_{\tau t}$ has the standard normal distribution. The observable state variables are the short-term (one-period) nominal interest rate r_t, the inflation rate π_t, and the output gap g_t^y.

Although the equations of the model could be specified directly in terms of r_t and π_t, it is more convenient to define the interest rate and inflation gap variables as

$$g_t^r = r_t - \tau_t$$

and

$$g_t^\pi = \pi_t - \tau_t.$$

In Ireland's (2007) extension of the New Keynesian macroeconomic model, a random walk specification for the inflation target τ_t generates nonstationary behavior in nominal interest rates and inflation, so that the transformations introduced in these definitions of the interest rate and inflation gaps are needed to obtain an empirical model cast in terms of stationary variables. Here, by contrast, the stationary law of motion (1) for the inflation target implies that all of the model's observable variables, including interest rates and inflation, remain stationary as well. This change in specification works to sidestep the technical problem, noted by Campbell, Lo, and MacKinlay (1997, p.433) and discussed further by Spencer (2008), that asymptotically long-term yields become undefined in models, like this one, with homoskedastic shocks when the short-term interest rate follows a process that contains a unit root. Of course, settings for the parameter ρ_τ that are arbitrarily close to the upper bound of one can – and will – allow the model to explain much of the persistence in nominal variables found in the US data.

The central bank can now be depicted as changing its inflation target according to (1) and then managing the interest rate gap according to the policy rule

$$g_t^r - g^r = \rho_r(g_{t-1}^r - g^r) + (1-\rho_r)[\rho_\pi g_t^\pi + \rho_y(g_t^y - g^y) + \rho_v v_t] + \sigma_r \varepsilon_{rt}. \tag{2}$$

In (2), the policy parameter ρ_r, satisfying $0 \leq \rho_r < 1$, governs the degree of interest rate smoothing, the parameters $\rho_\pi \geq 0$ and $\rho_y \geq 0$ measure the strength of the central bank's policy response to changes in the inflation and output gaps, the volatility parameter satisfies $\sigma_r > 0$, and the serially uncorrelated monetary policy shock ε_{rt} has the standard normal distribution. The rule in (2) also allows for a systematic response of monetary policy to changes in the risk variable v_t. While, in the estimation procedure described below, the parameters ρ_π and ρ_y are constrained to be nonnegative, as they would be in more conventional Taylor (1993) rule specifications, the response coefficient ρ_v attached to the index of bond-market risk is left unconstrained in sign. It may be negative, if over the sample period the Federal Reserve has attempted to offset the effects of larger bond risk premia on long-term interest rates by lowering the short-term interest rate. It may be positive if, as suggested by Goodfriend (1993) and McCallum (2005), the Federal Reserve has interpreted upward movements in long-term bond yields, given expectations of future short-term rates, as evidence of an upward movement in inflationary expectations and therefore increased the short-term rate in order to help quell those "inflation scares." Or the coefficient may be zero if the Federal Reserve has focused exclusively on movements in the inflation and output gaps in setting policy. Indeed, one purpose of this empirical study, beyond examining how changes in bond risk premia feed back on the economy to affect output and inflation, is to characterize how the Federal Reserve has itself reacted to these changes, as summarized by the sign and magnitude of the estimated value of ρ_v. Finally, in (2), g^r and g^y denote the steady-state values of the interest rate and output gaps. The inflation gap is assumed to have zero mean, so that actual inflation π_t equals the central bank's target τ_t on average, and the risk variable v_t is normalized to have zero mean as well.

Given (1) and (2), describing the conduct of monetary policy in both the long-run and the short-run, the inflation and output gaps and the risk variable v_t are allowed to depend on their own lagged values and lagged values of the model's other variables, much as they would in a more conventional macroeconomic vector autoregression. Specifically,

$$g_t^\pi = \rho_{\pi r}(g_{t-1}^r - g^r) + \rho_{\pi\pi} g_{t-1}^\pi + \rho_{\pi y}(g_{t-1}^y - g^y) + \rho_{\pi v} v_{t-1} + \sigma_{\pi\tau}\sigma_\tau\varepsilon_{\tau t} + \sigma_\pi\varepsilon_{\pi t}, \tag{3}$$

$$\begin{aligned} g_t^y - g^y = {} & \rho_{yr}(g_{t-1}^r - g^r) + \rho_{y\pi} g_{t-1}^\pi + \rho_{yy}(g_{t-1}^y - g^y) + \rho_{yv} v_{t-1} \\ & + \sigma_{y\pi}\sigma_\pi\varepsilon_{\pi t} + \sigma_{y\tau}\sigma_\tau\varepsilon_{\tau t} + \sigma_y\varepsilon_{yt}, \end{aligned} \tag{4}$$

and

$$\begin{aligned} v_t = {} & \rho_{vr}(g_{t-1}^r - g^r) + \rho_{v\pi} g_{t-1}^\pi + \rho_{vy}(g_{t-1}^y - g^y) + \rho_{v\tau}(\tau_{t-1} - \tau) + \rho_{vv} v_{t-1} \\ & + \sigma_{vr}\sigma_r\varepsilon_{rt} + \sigma_{v\pi}\sigma_\pi\varepsilon_{\pi t} + \sigma_{vy}\sigma_y\varepsilon_{yt} + \sigma_{v\tau}\sigma_\tau\varepsilon_{\tau t} + \sigma_v\varepsilon_{vt}, \end{aligned} \tag{5}$$

where the volatility parameters satisfy $\sigma_\pi > 0$, $\sigma_y > 0$, and $\sigma_v > 0$ and the serially and mutually uncorrelated innovations $\varepsilon_{\pi t}$, ε_{yt}, and ε_{vt} all have standard normal distributions. Although (3)-(5) allow for considerable flexibility in the behavior of the three state variables

g_t^π, g_t^y, and v_t, each does, nevertheless, impose some restrictions and identifying assumptions. In particular, (3) and (4) allow innovations in the inflation target to impact immediately on the inflation and output gaps, but allow for further effects of changes in the inflation target τ_t only to the extent that they are not met by proportional changes in the nominal interest rate and inflation rate and therefore affect the interest rate and inflation gaps; these restrictions are meant to impose a form of long-run monetary neutrality that limits the extent to which changes in the inflation target influence the other variables. Equations (3) and (4) also impose the timing restrictions typically incorporated into the specification of more conventional macroeconomic vector autoregressions: they assume, in particular, that the monetary policy shock ε_{rt} and the shock ε_{vt} that immediately increases bond risk premia affect have no contemporaneous effect on the inflation and output gaps and that the innovation ε_{yt} to the output gap has no contemporaneous effect on the inflation gap. These assumptions, similar to those invoked by Bekaert, Hoerova, and Lo Duca (2013), for example, help disentangle the effects of changes in monetary policy and bond risk premia on inflation and output from the effects of changes in inflation and output on monetary policy and bond risk premia. Equation (5), however, does allow the monetary policy shock – and all of the model's other shocks – to have immediate effects on bond risk premia, as they should if asset prices react immediately to all developments in the economy.

To write (1)-(5) more compactly, collect the five state variables into the vector

$$X_t = \begin{bmatrix} g_t^r & g_t^\pi & g_t^y & \tau_t & v_t \end{bmatrix}'$$

and the five innovations into the vector

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{rt} & \varepsilon_{\pi t} & \varepsilon_{yt} & \varepsilon_{\tau t} & \varepsilon_{vt} \end{bmatrix}'.$$

Then (1)-(5) can be stacked together as

$$P_0 X_t = \mu_0 + P_1 X_{t-1} + \Sigma_0 \varepsilon_t,$$

where

$$P_0 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & -(1-\rho_r)\rho_\pi & -(1-\rho_r)\rho_y & 0 & -(1-\rho_r)\rho_v \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mu_0 = \begin{bmatrix} (1-\rho_\tau)\tau \\ (1-\rho_r)(g^r - \rho_y g^y) \\ -\rho_{\pi r} g^r - \rho_{\pi y} g^y \\ (1-\rho_{yy}) g^y - \rho_{yr} g^r \\ -\rho_{vr} g^r - \rho_{vy} g^y - \rho_{v\tau}\tau \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 0 & 0 & 0 & \rho_\tau & 0 \\ \rho_r & 0 & 0 & 0 & 0 \\ \rho_{\pi r} & \rho_{\pi\pi} & \rho_{\pi y} & 0 & \rho_{\pi v} \\ \rho_{yr} & \rho_{y\pi} & \rho_{yy} & 0 & \rho_{yv} \\ \rho_{vr} & \rho_{v\pi} & \rho_{vy} & \rho_{v\tau} & \rho_{vv} \end{bmatrix},$$

and

$$\Sigma_0 = \begin{bmatrix} 0 & 0 & 0 & \sigma_\tau & 0 \\ \sigma_r & 0 & 0 & 0 & 0 \\ 0 & \sigma_\pi & 0 & \sigma_{\pi\tau}\sigma_\tau & 0 \\ 0 & \sigma_{y\pi}\sigma_\pi & \sigma_y & \sigma_{y\tau}\sigma_\tau & 0 \\ \sigma_{vr}\sigma_r & \sigma_{v\pi}\sigma_\pi & \sigma_{vy}\sigma_y & \sigma_{v\tau}\sigma_\tau & \sigma_v \end{bmatrix}.$$

Multiplying through by P_0^{-1} then puts the state equation in its most convenient form:

$$X_t = \mu + PX_{t-1} + \Sigma\varepsilon_t, \tag{6}$$

where

$$\mu = P_0^{-1}\mu_0,$$

$$P = P_0^{-1}P_1,$$

and

$$\Sigma = P_0^{-1}\Sigma_0.$$

2.2 Bond Pricing

The short-term nominal interest rate r_t can be expressed as a linear function of the state vector by inverting the transformation defining the interest rate gap:

$$r_t = \delta' X_t, \tag{7}$$

where

$$\delta = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \end{bmatrix}'.$$

Prices of risk are assigned to each of the state variables, but are allowed to vary over time only in response to movements in the single unobserved factor v_t. Inspired by the work of Cochrane and Piazzesi (2005, 2008), which attributes the bulk of all movements in long-term bond risk premia to variation in a single combination of forward rates, this assumption implies that all variation in risk premia implied by this model will, likewise, be driven by changes in v_t. Unlike the return forecasting factor that Cochrane and Piazzesi (2008) incorporate into their affine term structure model, but similar to the one used by Dewachter, Iania, and Lyrio (forthcoming) in theirs, the risk-driving variable v_t is treated here as being unobservable in the data. This specification, therefore, is designed to reflect the observation, made implicitly by Cochrane and Piazzesi (2005, 2008) and more explicitly by Bauer (2011), that the large number of parameters included in less highly constrained affine term structure models more frequently lead to overfitting that blurs, rather than sharpens, their interpretation of movements in bond risk premia. At the same time, however, treating the single risk factor v_t as unobservable permits it to move in line with Cochrane and Piazzesi's observable combination of forward rates, but also leaves the estimation procedure free to account for the links, if any, not only between this risk variable and long-term interest rates, but also between bond risk premia and the behavior of output and inflation.

Thus, in this specification, as in other members of Duffee's (2002) essentially affine class of dynamic term structure models, the log nominal asset pricing kernel takes the form

$$m_{t+1} = -r_t = \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}, \tag{8}$$

where the time-varying prices of risk

$$\lambda_t = \begin{bmatrix} \lambda_t^r & \lambda_t^\pi & \lambda_t^y & \lambda_t^\tau & \lambda_t^v \end{bmatrix}'$$

satisfy

$$\lambda_t = \lambda + \Lambda X_t. \tag{9}$$

But while the vector of constant terms in (8),

$$\lambda = \begin{bmatrix} \lambda^r & \lambda^\pi & \lambda^y & \lambda^\tau & \lambda^v \end{bmatrix}', \tag{10}$$

is left unconstrained, the assumption that the unobserved variable v_t is the exclusive source of time-variation in risk premia requires that all but the final column of the matrix

$$\Lambda = \begin{bmatrix} 0 & 0 & 0 & 0 & \Lambda^r \\ 0 & 0 & 0 & 0 & \Lambda^\pi \\ 0 & 0 & 0 & 0 & \Lambda^y \\ 0 & 0 & 0 & 0 & \Lambda^\tau \\ 0 & 0 & 0 & 0 & \Lambda^v \end{bmatrix} \tag{11}$$

consist entirely of zeros.

Equations (6)-(11) imply that the log price p_t^n of an n-period discount bond at time t is determined as an affine function

$$p_t^n = \bar{A}_n + \bar{B}_n' X_t \tag{12}$$

of the state vector by the no-arbitrage condition

$$\exp(p_t^{n+1}) = E_t[\exp(m_{t+1}) \exp(p_{t+1}^n)], \tag{13}$$

where the scalars $\bar{A}_n$ and 5×1 vectors $\bar{B}_n$ for $n = 1, 2, 3, \ldots$ can be generated recursively, starting from the initial conditions

$$\bar{A}_1 = 0 \tag{14}$$

and

$$\bar{B}_1' = \delta' \tag{15}$$

required to make (12) for $n = 1$ consistent with (7) for $r_t = -p_t^1$, using the difference equations

$$\bar{A}_{n+1} = \bar{A}_n + \bar{B}_n'(\mu - \Sigma\lambda) + \frac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n \tag{16}$$

and

$$\bar{B}_{n+1}' = \bar{B}_n'(P - \Sigma\Lambda) - \delta' \tag{17}$$

obtained, as shown in part one of the appendix, by substituting (7), (8), and (12) into the right-hand side of (13), taking expectations, and matching coefficients after substituting (12) into the left-hand side of the same expression. Once bond prices are found using (14)-(17), the yield y_t^n on an n-period discount bond at time t is easily computed as

$$y_t^n = -\frac{p_t^n}{n} = A_n + B_n' X_t, \tag{18}$$

where $A_n = \bar{A}_n/n$ and $B_n = -\bar{B}_n/n$ for all values of $n = 1, 2, 3, \ldots$.

2.3 Bond Risk Premia

Cochrane and Piazzesi (2008) define and discuss various measures of the risk premia incorporated into long-term interest rates. The most familiar, and the one preferred by Rudebusch, Sack, and Swanson (2007) as well, is given by the yield on a long-term bond, minus the average of the short-term rates expected to prevail over the lifetime of that long-term bond:

$$q_t^n = y_t^n - \frac{1}{n} E_t(r_t + r_{t+1} + \ldots + r_{t+n-1}). \tag{19}$$

The n-period bond yield implied by the model used here has already been found using (18). To compute the expected future short-term rates, rewrite the law of motion (6) for the state vector as

$$X_{t+1} - \bar{\mu} = P(X_t - \bar{\mu}) + \Sigma \varepsilon_{t+1}, \tag{20}$$

where $\bar{\mu} = (I - P)^{-1}\mu$. This last expression, together with (7), implies that

$$E_t r_{t+j} = \delta' E_t X_{t+j} = \delta' \bar{\mu} + \delta' P^j (X_t - \bar{\mu}). \tag{21}$$

Combining (19), (20), and (21) yields

$$q_t^n = A_n - \delta' \left(I - \frac{1}{n} \sum_{j=0}^{n-1} P^j \right) \bar{\mu} + \left(B_n' - \delta' \frac{1}{n} \sum_{j=0}^{n-1} P^j \right) X_t,$$

or, in a form more convenient for computational purposes,

$$q_t^n = A_n - \delta' \left[I - \frac{1}{n}(I - P^n)(I - P)^{-1} \right] \bar{\mu} + \left[B_n' - \frac{1}{n}\delta'(I - P^n)(I - P)^{-1} \right] X_t. \tag{22}$$

Note that when even the last column of (11) consists of zeros, so that $\Lambda = 0$, (15) and (17) imply that

$$B_n' = \frac{1}{n}\delta' \sum_{j=0}^{n-1} P^j = \frac{1}{n}\delta'(I - P^n)(I - P)^{-1},$$

so that the second term on the right-hand side of (22) vanishes and the bond risk premium is constant. Similarly, without variation in the risk variable v_t, the restricted form of Λ in (11) will imply that bond risk premia are constant. Thus, to the extent that evidence of time-variation in bond risk premia does appear in the data, this variation will be attributed by the estimated model to variation in the otherwise unobservable variable v_t.

3 Estimation

Interpreting each period in the model as a quarter year in real time, its parameters can be estimated with US data on the short-term interest rate r_t, the inflation rate π_t, the output gap g_t^y, and yields y_t^4, y_t^8, y_t^{12}, y_t^{16}, and y_t^{20} on discount bonds with one through five years to maturity. The figures for inflation and the output gap are drawn from the Federal Reserve bank of St. Louis' FRED database, with inflation measured by quarter-to-quarter changes in the GDP deflator as reported by the US Department of Commerce and the output gap as the percentage (logarithmic) deviation of the Commerce Department's measure of actual real GDP from the Congressional Budget Office's measure of potential real GDP. The interest rate data are those most commonly used in empirical studies of the term structure. The short-term interest rate is the three-month rate from the Center for Research in Security Prices' Monthly Treasury/Fama Risk Free Rate Files and the longer-term discount bond rates are from the CRSP Monthly Treasury/Fama-Bliss Discount Bond Yield Files. The dataset begins in 1959:1 and runs through 2007:4, so that the extreme readings on all series recorded during the most recent financial crisis and severe recession do not exert an undue influence on the parameter estimates. Thus, as explained above, the estimation exercise is meant to shed light on the interlinkages between monetary policy, bond risk premia, and the macroeconomy during more normal periods of expansion and recession, both in the past and when they return in the future.

With eight variables treated as observable and only five fundamental disturbances, at least three of the observables must be interpreted as being measured with error in order to avoid the problems of stochastic singularity discussed by Ireland (2004) for macroeconomic models and Piazzesi (2010, pp.726-727) for affine models of the term structure. Thus, the analysis here follows the general approach first used by Chen and Scott (1993), treating exactly three of the longer-term interest rates as being subject to measurement error, so as to obtain a variant of the model with the same number of observables as shocks. The choice of exactly which rates to view as error-ridden instead of perfectly observed is, admittedly, somewhat arbitrary, but attaching measurement errors to the one, two, and four-year rates forces the estimation procedure to track the three and five-year rates without error; since the short-term interest rate is also taken as perfectly measured, the model's fundamental shocks must then account for most broad movements along the yield curve.

Note that the steady-state values of g^r, τ, and g^y can be calibrated to match the average values of the model's three macroeconomic variables – the short-term nominal interest rate, inflation, and the output gap – under the assumption that actual inflation equals the central bank's inflation target on average. Part two of the appendix shows that, likewise, steady-state values for the five long-term bond yields can be pinned down through the appropriate choices of the five elements of the vector λ that appears in (9) and (10), so as to match the average yields in the data. Hence, the model can be made to match the average values of the macroeconomic variables together with the average slope of the yield curve, and the estimation exercise simplified by using de-meaned data and dropping the constant terms that appear in (6) and (9).

Thus, the empirical model consists of (6) without its mean,

$$X_t = PX_{t-1} + \Sigma\varepsilon_t, \tag{23}$$

for the state, together with

$$d_t = UX_t + V\eta_t, \tag{24}$$

for the observables, where

$$d_t = \begin{bmatrix} r_t & \pi_t & g^y & y_t^4 & y_t^8 & y_t^{12} & y_t^{16} & y_t^{20} \end{bmatrix}'$$

keeps track of the now de-meaned data,

$$U = \begin{bmatrix} U_r \\ U_\pi \\ U_y \\ B_4' \\ B_8' \\ B_{12}' \\ B_{16}' \\ B_{20}' \end{bmatrix}$$

with

$$U_r = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$U_\pi = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \end{bmatrix},$$

$$U_y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

and the remaining rows determined by (15) and (17) links the observables to the state,

$$V = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \sigma_4 & 0 & 0 \\ 0 & \sigma_8 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{16} \\ 0 & 0 & 0 \end{bmatrix} \tag{25}$$

with $\sigma_4 > 0$, $\sigma_8 > 0$, and $\sigma_{16} > 0$ picks out the three yields that are measured with error, and the three elements of the measurement error vector

$$\eta_t = \begin{bmatrix} \eta_t^4 & \eta_t^8 & \eta_t^{16} \end{bmatrix}'$$

are mutually and serially uncorrelated with standard normal distributions. Equations (23) and (24) are in state-space form, allowing the model's parameters to be estimated using the Kalman filtering methods outlined by Hamilton (1994, Ch.13).

Preliminary attempts to implement this maximum likelihood routine revealed the usefulness, and in some cases the necessity, of imposing two sets of additional restrictions on some of the model's parameters, especially so as to sharpen the identification of the two unobservable state variables, the inflation target τ_t and the risk variable v_t. First, for the inflation target, when the persistence parameter ρ_τ in (1) is left unconstrained, the estimation procedure pushes the value of this parameter very close to its upper bound of one, leading to convergence problems when numerically maximizing the log likelihood function; in practice, imposing the restriction $\rho_\tau = 0.999$ avoids these problems while remaining consistent with the observation that data strongly prefer an extremely high degree of persistence in the inflation target. Related, but more generally, the estimation procedure also constrains the eigenvalues of the matrix P in (6), governing the "physical" persistence of the state variables, and $P-\Sigma\Lambda$ in (17), governing the "risk neutral" dynamics and hence the pricing of long-term bonds, to be less than one in absolute value, so that the entire system of macroeconomic and bond-pricing equations remains dynamically stable.

Second, for the unobserved variable v_t that, as explained above, is responsible in the model for driving all fluctuations in bond risk premia, if the value of σ_v in its law of motion (5) is scaled up or down by multiplying by some number $\alpha > 0$, then multiplying the parameters ρ_{vr}, $\rho_{v\pi}$, ρ_{vy}, $\rho_{v\tau}$, σ_{vr}, $\sigma_{v\pi}$, σ_{vy}, and $\sigma_{v\tau}$ in (5) by α and dividing the parameters ρ_v, $\rho_{\pi v}$, ρ_{yv}, Λ^r, Λ^π, Λ^y, Λ^τ, and Λ^v in (2)-(4), (9), and (11) by α leaves the model's implications for the dynamic behavior of all observable variables unchanged. Hence, the constraint $\sigma_v = 0.01$ is imposed as a normalization, to pin down the scale of movements in v_t. Likewise, the sign restriction $\Lambda^\pi < 0$ is imposed during the estimation since no other feature of the model works to determine the direction, positive or negative, in which an increase in v_t changes bond risk premia and all other variables. And while this additional restriction is not needed for identification, imposing the constraint $\Lambda^v = 0$ implies the variable v_t works solely, as in Cochrane and Piazzesi (2008), to move prices of risk associated with the model's remaining four factors and is not itself a source of priced risk. Finally, preliminary attempts to estimate the model revealed difficulties in estimating the law of motion (5) for v_t in its most general form, with vastly different values of its parameters associated with very similar values of the log-likelihood function. In particular, it appears that there is simply not enough information in the data to distinguish the contemporaneous effects of the macroeconomic shocks on v_t from the effects of the lagged values of the corresponding macroeconomic variables themselves. More stable results obtained, however, once the constraints $\rho_{vr} = \rho_{v\pi} = \rho_{vy} = \rho_{v\tau} = 0$ are imposed. While these restrictions imply that the persistence in movements in v_t are governed by the single autoregressive parameter ρ_{vv}, they allow the model to be estimated when all shocks that hit the economy are permitted to have immediate effects on bond risk premia.

Hence, with these restrictions and normalizations imposed, the maximum likelihood estimates are obtained for the model's remaining 31 parameters: the coefficients ρ_r, ρ_π, ρ_y, and ρ_v from the monetary policy rule (2), the coefficients $\rho_{\pi r}$, $\rho_{\pi\pi}$, $\rho_{\pi y}$, $\rho_{\pi v}$, ρ_{yr}, $\rho_{y\pi}$, ρ_{yy}, ρ_{yv}, ρ_{vv}, σ_τ, σ_r, σ_π, σ_y, $\sigma_{\pi\tau}$, $\sigma_{y\pi}$, $\sigma_{y\tau}$, σ_{vr}, $\sigma_{v\pi}$, σ_{vy}, and $\sigma_{v\tau}$ determining the autoregressive behavior, volatility, and comovement between the inflation gap, output gap, and risk variable v_t in (3)-(5), the coefficients Λ^r, Λ^π, Λ^y, Λ^τ describing time-variation in the prices of risk in (9) and (11), and the coefficients σ_4, σ_8, σ_{16} governing the volatility of the bond-yield measurement errors in (25).

4 Results

Table 1 displays the maximum likelihood estimates of the parameters just listed, together with their standard errors, computed using a bootstrappng method outlined by Efron and Tibshirani (1993, Ch.6), according to which the model, with its parameters fixed at their estimated values, is used to generate 1,000 samples of artificial data on the same eight variables found in the actual US data. These artificial series then get used to re-estimate the 31 parameters 1,000 times, and the standard errors reported in table 1 correspond to the standard deviations of the parameter estimates taken over all of the 1,000 replications. Conveniently and by construction, therefore, this bootstrapping procedure accounts for the finite-sample properties of the maximum likelihood estimates as well as the constraints, discussed above, that are imposed during the estimation.

Most notable in the table are the estimated parameters from the Taylor-type interest rate rule for monetary policy shown in (2). The estimate of $\rho_r = 0.59$ implies a considerable amount of interest rate smoothing by the central bank, a finding that is consistent with many other studies that estimate Taylor rules in various ways. The point estimates of $\rho_\pi = 0.12$ and $\rho_y = 0.16$ measure monetary policy responses to changes in inflation and the output gap that are roughly balanced, though somewhat stronger for output than prices; in addition, the large standard error attached to ρ_π makes the maximum likelihood estimate of this parameter statistically insignificant.

Both of these policy response coefficients are considerably smaller than estimates reported in studies that use macroeconomic variables alone. Clarida, Gali, and Gerter's (2000) single-equation instrumental variables estimates are $\rho_\pi = 0.83$ and $\rho_y = 0.27$ for their "Pre-Volcker" sample running from 1960 through mid-1979 and $\rho_\pi = 2.15$ and $\rho_y = 0.93$ for their "Volcker-Greenspan" sample running from mid-1979 through 1996. Similarly, Lubik and Schorfheide's (2004) Bayesian estimates of a multi-equation dynamic, stochastic, general equilibrium New Keynesian model are $\rho_\pi = 0.77$ and $\rho_y = 0.17$ for the same pre-Volcker sample used by Clarida, Gali, and Gertler and $\rho_\pi = 2.19$ and $\rho_y = 0.30$ for a "post-1982" sample running from 1982 through 1997. In New Keynesian models, the forward-looking "IS curve" is a log-linearized Euler equation, obtained under the assumption that the representative household has additively-time separable preferences over consumption of the constant relative risk aversion form. Here, the no-arbitrage condition (13), with the more flexible specification for the nominal asset pricing kernel given by (8)-(11), takes the place of the New Keynesian IS curve and the parameters of the modified Taylor rule (2) are identified, in part, by the timing assumptions, made above, that monetary policy shocks affect the output gap and inflation with a one-quarter lag. Thus, the comparison between the estimated coefficients of the Taylor rule obtained here and those reported in previous studies speaks directly to the practical importance of the issues examined from a variety of different angles by Sims and Zha (2006), Ang, Dong, and Piazzesi (2007), Atkeson and Kehoe (2008), Cochrane (2011), Backus, Chernov, and Zin (2013), and Joslin, Le, and Singleton (2013), each of which shows in one way or another how the identification of the parameters of interest rate rules for monetary policy is complicated by the similarities between the Taylor (1993) rule, which links the nominal interest rate to output and inflation, and the Euler equation, which in models without investment does much the same thing. Changes in the specification of one of these equations, therefore, can easily change the estimated values of coefficients in the other, implying vastly different behavior on the part of consumers, the central bank, or both. Ang, Dong, and Piazzesi (2007), in particular, estimate values for Taylor rule coefficients in an affine term structure model that, like those obtained here, are much smaller than those from Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), and other previous studies that focus solely on macroeconomic variables.

Of course, (2) differs from standard Taylor-rule specifications by including the risk variable v_t among those to which the Federal Reserve can respond by adjusting the short-term nominal interest rate. As noted above, whereas the smoothing coefficient ρ_r is constrained during estimation to lie between zero and one and the response coefficients ρ_π and ρ_y are constrained to be nonnegative, ρ_v is allowed to take either sign, so that the central bank can respond to higher bond risk premia by either lowering the short-term rate, presumably to offset some of the upward pressure on long-term rates, or raising the long-term rate. In fact, the positive and statistically significant estimate of $\rho_v = 0.08$ is consistent with Goodfriend (1993) and McCallum's (2005) earlier analyses, which suggest that the Fed has historically interpreted upward movements in long-term bond yields, for a given setting of the short-term rate, as evidence of rising inflationary expectations, which it has then tried to tamp down by raising the short-term rate.

Most of the other parameter estimates reported in table 1 are like those from a more conventional vector autoregression: they determine the volatility and dynamics of the model's observable and unobservable variables in a way that is easier to summarize by plotting impulse response functions and tabulating forecast error variance decompositions than by trying to interpret each coefficient individually. Hence, figures 1 through 5 plot impulse responses to each of the model's five shocks, and tables 2 through 4 report on the variance decompositions. In the graphs, the output gap is shown as a percentage deviation from its steady state, while the inflation and interest rates are all expressed in annualized, percentage-point terms.

Thus, the left-hand column of figure 1 shows how a one-standard deviation monetary policy shock ε_r raises the short-term nominal interest rate by about 65 basis points on impact; the short rate then converges back to its initial value over the following six quarters. The output gap falls and, after a brief but very small increase that resembles the "price puzzle" that frequently appears in more conventional vector autogressive models of monetary policy shocks and their effects, inflation declines persistently. The risk variable v_t rises in response to the monetary policy shock, so that the long-term interest rates shown in the figure's middle column rise by more than the average of expected future short rates. The right-hand column of the figure confirms, therefore, that the rise in v_t is mirrored by a rise in risk premia built into all five of the longer-term bond rates.

Figure 2 then displays impulse responses to a one-standard deviation shock to v_t, which as shown in the right-hand column, gives rise to increases in all bond risk premia. The output gap and inflation both fall in response to this shock, consistent with the "practitioner view" described by Rudebusch, Sack, and Swanson (2007) that higher long-term interest rates, reflecting larger bond risk premia, work to slow aggregate economic activity in the same way that more traditional aggregate demand shocks do. As noted above, the positive estimate of ρ_v in the policy rule (2) causes monetary policy to tighten when bond risk premia rise.

Figure 3 plots the impulse responses to shocks to the inflation target τ_t. With the persistence parameter ρ_τ in (1) fixed at 0.999, this is the model's most persistent shock, and the simultaneous and roughly equal upward movements in interest rates on bonds of all maturities shown in the figure's middle column indicate that this shock plays the role of the "level factor" that appears in more traditional, affine models of the term structure without macroeconomic variables. The figure's left-hand column shows how actual inflation rises gradually to meet the new, higher target that results from this shock. The output gap increases, presumably because of the implied monetary expansion.

In figure 4, the orthogonal shock ε_π to inflation has small effects on all of the model's variables: its effect are mainly on inflation itself although, consistent with the interpretation of this as a "cost-push" shock, the disturbance works as well to decrease the output gap. In figure 5, meanwhile, the shock ε_y to the output gap itself has effects that might be expected from a non-monetary shock to aggregate demand: it increases both the output gap and inflation and causes interest rates to rise. The risk variable v_t declines following this shock, however, so that bond risk premia fall. Taken together, all these impulse responses are indicative of important multi-directional effects running between monetary policy, bond risk premia, output, and inflation.

Table 2 decomposes the k-quarter-ahead forecast error variance in the output gap, inflation, the short-term interest rate, and all bond risk premia into components attributable to each of the model's five fundamental shocks. Since (1) makes the inflation target evolve as an exogenous process, unrelated to any of the models other shocks or variables, 100 percent of its forecast error variance is by assumption allocated to the shock ε_τ; hence, it is excluded from the table. In addition, the restrictions, mentioned above, allowing for only contemporaneous effects running from the model's other variables to the risk variable v_t, coupled with the assumption that all movements in risk premia are driven by fluctuations in the single variable v_t, get reflected in the table's lower panel, which shows that the fraction of the volatility in all bond risk premia attributable to each of the five shocks is invariant to the forecast horizon considered: no matter which shock hits, the persistence in v_t and all risk premia is governed by the single parameter ρ_{vv} in (5), estimated at 0.88 in table 1.

The various panels of table 2 show that the monetary policy shock ε_r accounts for sizable components of the variation in the output gap, the short-term nominal interest rate, and all bond risk premia. According to the estimated model, in fact, nearly one-quarter of all historical movements in bond risk premia are related to monetary policy shocks. The shock ε_v, meanwhile, accounts exogenously for about 45 percent of all movements in bond risk premia. Meanwhile, the "practitioner view" referred to by Rudebusch, Sack, and Swanson

(2007) is still reflected, but less strongly so, in the variance decompositions: exogenous shocks to bond risk premia account for between 2.5 and 4 percent of the forecast error variance in the output gap and between 1.5 and 2.5 percent of the forecast error variance in inflation at horizons between 3 and 5 years. On the other hand, stronger effects run from the shock ε_y, which, as noted above, acts in the model like a non-monetary aggregate demand disturbance, to bond risk premia: accounting for more than 28 percent of their variance, this shock is even more important than monetary policy in driving movements in risk premia. Finally, table 2 confirms that long-run movements in inflation and the short-term interest rate mainly reflect movements in the inflation target τ_t.

Tables 3 and 4 break down, in a similar manner, the forecast error variance in longer-term bond yields into components attributable to the five fundamental shocks and, in the cases of the one, two, and four-year bonds, to the measurement errors added to the empirical model to facilitate maximum likelihood estimation. Reassuringly, those tables reveal that measurement errors are quite small, soaking up only slightly more than 3 percent of the one-quarter-ahead variance in the one-year rate, less than 2 percent of the one-quarter-ahead variance in the two-year rate, and less than one percent of the one-quarter-ahead variance in the four-year rate. Consistent with the association, made through the impulse response analysis, of the model's inflation target with the level factor in more traditional affine models, shocks to the inflation target are shown in tables 3 and 4 to account for the largest movements in interest rates up and down the yield curve. The monetary policy shock also plays an important role in affecting bond rates, particularly at shorter horizons and for the bonds with shorter terms to maturity. The shock ε_v to bond risk premia, meanwhile, also appears as a key factor in driving sizable movements, especially in the two through four year bond rates, over horizons extending out one to two years.

Returning to table 1, it is also of interest to make note of the estimated parameters from the matrix Λ in (11), governing how movements in the variable v_t translate into changes in the prices of risk attached to the model's fundamental shocks. While Cochrane and Piazzesi

(2008) find that the single, observable factor that they associate with time-variation in bond risk premia works to change the pricing of their model's level factor – which, as already noted, seems to resemble most closely the inflation target in the model used here – table 1 shows that the estimate of Λ_τ is small and statistically insignificant. Instead, time variation appears most important in the prices of risk attached to the monetary policy shock ε_r and the output shock ε_y. Again, the impulse response analysis makes both of these shocks look like traditional, monetary and non-monetary aggregate demand disturbances. These results join with others from above, therefore, to suggest that these shocks feed through financial markets and the economy as a whole through multiple channels, most of which are simply not present in existing theoretical models.

Finally, figure 6 plots full-sample estimates of the two model's unobservable state variables, the inflation target τ_t and the risk variable v_t, obtained using the Kalman smoothing algorithm that is also described by Hamilton (1994, Ch.13). After remaining stable at an annualized rate of about one percent through the mid-1960s, the inflation target rises to a peak above 10.5 percent in the third quarter of 1981. Comparing the top and bottom panels of the left-hand column shows how the inflation target remains elevated through the end of 1984, even as actual inflation declines. Hence, the estimated model attributes the persistence of high bond yields into the early to mid-1980s in large part to continued high expected inflation during that period, indicative of credibility problems associated with the Federal Reserve's fight against inflation. The inflation target begins its long-run trend downward in 1985 and stabilizes back at a rate of one percent in last five or six years of the sample.

The two panels on the right-hand side of figure 6, meanwhile, exhibit evidence of shifting cyclical patterns in bond risk premia, with the estimated risk premium in the five-year bond rate appearing as strongly countercyclical in the 1960s, 1970s, and 1980s, approximately acyclical during the 1990s, and strongly procyclical since 2000. The model can account for these shifting correlations since, as shown in figures 1-5, different shocks give rise to different patterns of comovement between the output gap and bond risk premia, with monetary policy shocks, shocks to the risk variable v_t itself, and shocks to output pushing these variables in opposite directions and shocks to both inflation and the inflation target moving them in the same direction.

Campbell, Sunderam, and Viceira (2013) focus on similarly shifting patterns of nominal and real correlations evident in data on nominal and real bond yields and stock returns over the same time periods, suggesting that the preponderance of supply-side shocks hitting the economy during the 1970s and 1980s may explain the positive comovement between bond and stock returns during those decades and the prevalence of demand-side shocks may explain the negative comovement across bond and stock returns in more recent years. Compared to Campbell, Sunderman, and Viceira's, the empirical analysis here excludes data on stock prices and inflation-indexed bond yields but includes data on output itself; moreover, the analysis here uses restrictions on the empirical model to identify shocks with specific, structural interpretations. It is of interest to note, therefore, that the results here seem to point to aggregate demand shocks as drivers of countercyclical bond risk premia both during the inflationary period of the 1960s and 1970s and the disinflationary episode of the 1980s and to ongoing uncertainty regarding the Federal Reserve's willingness or ability to stabilize long-run inflation as a source of procyclical bond risk premia since 2000. In any case, more detailed structural modeling, both theoretical and empirical, is clearly needed to better understand and reconcile these findings.

5 Conclusion

The Federal Reserve's recent policies of large scale asset purchases, more popularly known as "quantitative easing," rely on the widely-held view that monetary policy actions can influence the risk premia built into long-term bond rates and that changes in bond risk premia can then have impacts, working through aggregate demand channels, on output and inflation as well. As Rudebusch, Sack, and Swanson (2007) explain, however, little evidence has been compiled to support this "practitioner view," even in data covering more normal periods when Federal Reserve policy has not been constrained by the zero lower bound on the short-term rate of interest.

Using an affine model of the term structure with observable and unobservable macroeconomic factors, the empirical analysis here looks for – and finds – such evidence. Monetary policy shocks, identified using restrictions borrowed from the literature that works with more conventional, macroeconomic vector autoregressions but imposed here, instead, on the driving processes for the macroeconomic state variables in a term structure model, do appear to influence bond risk premia, with monetary policy tightenings working to increase those premia and, consistent with the goals of quantitative easing, monetary policy easings working to decrease them. In addition, purely exogenous shocks to bond risk premia, identified by restricting the determinants of those risk premia in a manner that is inspired by the work of Bauer (2011), Cochrane and Piazzesi (2006, 2008), and Dewacheter, Iania, and Lyrio (forthcoming), do appear in the estimated model to work like aggregate demand disturbances, with higher risk premia associated with slower output growth and inflation and, again consistent with the intended workings of quantitative easing, lower risk premia associated with faster output growth and inflation.

The estimated model, however, also allows for and provides evidence of other channels through which monetary policy, bond risk premia, and the macroeconomy interact. The extended version of the Taylor (1993) rule, for example, that is included in the estimated model indicates that, historically, the Federal Reserve has moved to raise the short-term interest rate, not only in response to shocks that increase output and inflation, but when bond risk premia rise, in a manner that is consistent with Goodfriend (1993) and McCallum's (2005) earlier analyses. In addition, different structural disturbances identified by the model move output, inflation, and bond risk premia in a variety of directions, helping to account for the shifting correlations between these variables seen in the data.

Thus, monetary policy affects bond risk premia and the economy; bond risk premia affect monetary policy and the economy, and the economy affects monetary policy and bond risk premia. Standard, textbook New Keynesian models like Woodford (2003) and Gali's (2008) do not even begin to consider the channels through which all of these connections are made; and even the most ambitious extensions thus far, such as Andres, Lopez-Salido, and Nelson's (2004), account only for a small subset. Much more research along these lines is needed, to fully understand how the workings of monetary policy and financial markets have and will continue to interact to shape the performance of the American economy.

6 Appendix

6.1 Derivation of the Bond Pricing Equations

To derive the bond pricing equations (14)-(17), start by setting $n = 0$ and substitute (8) and $p^0_{t+1} = 0$ (the price of at $t+1$ of a claim to a dollar at $t+1$ equals one dollar, hence zero after taking logs) into the right-hand side of (13) to obtain

$$\exp(p^1_t) = E_t[\exp(m_{t+1})] = \exp(-r_t).$$

Hence, consistency between (7) and (12) requires that

$$\bar{A}_1 + \bar{B}_1' X_t = -\delta' X_t$$

or, equivalently, that (14) and (15) hold.

Next, for an arbitrary value of $n = 1, 2, 3, \ldots$, substitute (7), (8), and (12) into the right-hand side of (13) to obtain

$$\begin{aligned}
\exp(p_t^{n+1}) &= E_t\left[\exp\left(-\delta' X_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)\exp\left(\bar{A}_n + \bar{B}_n' X_{t+1}\right)\right] \\
&= \exp\left(-\delta' X_t - \frac{1}{2}\lambda_t'\lambda_t + \bar{A}_n\right) E_t\left[\exp\left(-\lambda_t'\varepsilon_{t+1} + \bar{B}_n' X_{t+1}\right)\right] \\
&= \exp\left(-\delta' X_t - \frac{1}{2}\lambda_t'\lambda_t + \bar{A}_n\right) E_t\left\{\exp\left[-\lambda_t'\varepsilon_{t+1} + \bar{B}_n'\left(\mu + PX_t + \Sigma\varepsilon_{t+1}\right)\right]\right\} \\
&= \exp\left[\bar{A}_n + \bar{B}_n'\mu + \left(\bar{B}_n' P - \delta'\right) X_t - \frac{1}{2}\lambda_t'\lambda_t\right] E_t\left\{\exp\left[-(\lambda_t' - \bar{B}_n'\Sigma)\varepsilon_{t+1}\right]\right\} \\
&= \exp\left[\bar{A}_n + \bar{B}_n'\mu + \left(\bar{B}_n' P - \delta'\right) X_t - \frac{1}{2}\lambda_t'\lambda_t\right] \\
&\quad \times \exp\left[\frac{1}{2}\lambda_t'\lambda_t - \bar{B}_n'\Sigma\left(\lambda + \Lambda X_t\right) + \frac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n\right] \\
&= \exp\left[\bar{A}_n + \bar{B}_n'\left(\mu - \Sigma\lambda\right) + \frac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n + \left(\bar{B}_n' P - \bar{B}_n'\Sigma\Lambda - \delta'\right) X_t\right],
\end{aligned}$$

where the second equality simply moves objects that are known at t outside of the conditional expectation, the third equality uses the law of motion (6) for the state variable, the fourth equality again moves objects that are known outside the expectation, the fifth equality uses the normality of ε_{t+1} to compute the expectation of the exponential function involving the vector of shocks, and the sixth equality simplifies the resulting expression. Matching coefficients after (12) is substituted into the left-hand side of this equation then implies that the difference equations (16) and (17) must hold as well.

6.2 Matching Average Long-Term Bond Yields

To see how the five elements of the vector λ in (9) and (10) can be calibrated to match the average yields on one through five-year discount bonds, multiply both sides of (17) by $\bar{\mu} = (1 - P)^{-1}\mu$, which by (6) keeps track of the steady-state values of the variables in the state vector X_t, and add the results to (16), after replacing μ with $(I - P)\bar{\mu}$, to obtain

$$\bar{A}_{n+1} + \bar{B}_{n+1}'\bar{\mu} = \bar{A}_n + \bar{B}_n'\bar{\mu} - \bar{B}_n'\Sigma\lambda \ + \frac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n - \bar{B}_n'\Sigma\Lambda\bar{\mu} - \delta'\bar{\mu}.$$

From (18), each term of the form $\bar{A}_n + \bar{B}_n'\bar{\mu}$ that enters into this difference equation equals $-ny^n$, where y^n denotes the steady-state yield on an n-period discount bond. Hence, the difference equation can be written more compactly as

$$(n+1)y^{n+1} = ny^n + z_n + \bar{B}_n'\Sigma\lambda,$$

where

$$z_n = \bar{B}_n'\Sigma\Lambda\bar{\mu} + \delta'\bar{\mu} - \frac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n.$$

Solving this difference equation forward starting from $y_1 = r$, where r is the steady-state value of the short-term nominal interest rate, yields

$$y^n = \frac{1}{n}\left(r + \sum_{j=1}^{n-1} z_j\right) + \left(\frac{1}{n}\sum_{j=1}^{n-1} \bar{B}_n'\Sigma\right)\lambda$$

for all $n = 2, 3, 4, \ldots$.

The average yield on an n-period bond can be used to measure y^n, the average values of the macroeconomic variables can be used to measure r and the elements of $\bar{\mu}$, and the estimated values of the parameters governing the model's dynamics can be used to compute z_n and $\bar{B}_n'\Sigma$ for all $n = 2, 3, 4, \ldots$. Since observations on yields at five longer maturities are used in the estimation, five versions of this last equation, with $n = 4$, $n = 8$, $n = 12$, $n = 16$, and $n = 20$ can be stacked into a vector, and the 5×5 matrix formed from the partial sums of the $\bar{B}_n'\Sigma$ terms on the right-hand side inverted, so as to solve uniquely for the elements of the 5×1 vector λ that both accurately demean the data and allow the model to match the average slope of the yield curve.

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Table 1. Maximum Likelihood Estimates and Standard Errors

Parameter	Estimate	Standard Error
ρ_r	0.5861	0.0431
ρ_π	0.1150	0.0785
ρ_y	0.1649	0.0296
ρ_v	0.0800	0.0154
$\rho_{\pi r}$	0.1119	0.1397
$\rho_{\pi\pi}$	0.8494	0.0472
$\rho_{\pi y}$	0.0099	0.0255
$\rho_{\pi v}$	-0.0154	0.0171
ρ_{yr}	-0.9778	0.4403
$\rho_{y\pi}$	0.0158	0.1576
ρ_{yy}	1.0267	0.0821
ρ_{yv}	0.0349	0.0499
ρ_{vv}	0.8767	0.0377
σ_τ	0.0012	0.0001
σ_r	0.0014	0.0001
σ_π	0.0027	0.0001
σ_y	0.0079	0.0004
$\sigma_{\pi\tau}$	-0.4955	0.1856
$\sigma_{y\pi}$	-0.2457	0.2154
$\sigma_{y\tau}$	1.3996	0.6080
σ_{vr}	5.3628	1.7549
$\sigma_{v\pi}$	-0.5328	0.6886
σ_{vy}	-1.0008	0.2722
$\sigma_{v\tau}$	1.1539	1.3095
Λ_r	-7.5343	3.1940
Λ_π	-6.2755	6.2920
Λ_y	-42.8948	5.4782
Λ_τ	3.9606	3.2547
σ_4	0.0003	0.0000
σ_8	0.0002	0.0000
σ_{16}	0.0001	0.0000

Table 2. Forecast Error Variance Decompositions

Output Gap

Quarters Ahead k	Monetary Policy ε_r	Risk Premium ε_v	Inflation Target ε_τ	Inflation ε_π	Output ε_y
1	0.0	0.0	4.1	0.7	95.2
4	5.9	0.1	3.9	1.0	89.2
8	13.0	1.0	3.2	1.4	81.3
12	16.2	2.5	2.9	1.7	76.8
20	17.5	3.9	2.7	1.9	73.9
∞	17.5	4.1	2.7	1.9	73.7

Inflation

Quarters Ahead k	Monetary Policy ε_r	Risk Premium ε_v	Inflation Target ε_τ	Inflation ε_π	Output ε_y
1	0.0	0.0	4.6	95.4	0.0
4	0.0	0.4	9.7	87.2	2.6
8	0.3	1.0	17.9	71.2	9.6
12	1.4	1.5	24.9	57.5	14.6
20	3.0	2.3	35.7	42.8	16.4
∞	0.3	0.3	94.2	3.7	1.5

Short-Term Interest Rate

Quarters Ahead k	Monetary Policy ε_r	Risk Premium ε_v	Inflation Target ε_τ	Inflation ε_π	Output ε_y
1	58.5	2.4	37.4	0.0	1.7
4	32.0	6.2	54.5	0.0	7.3
8	19.2	5.5	64.4	0.1	10.9
12	15.2	4.2	69.6	0.1	10.9
20	11.5	3.1	76.5	0.2	8.7
∞	0.7	0.2	98.6	0.0	0.5

Bond Risk Premia

Quarters Ahead k	Monetary Policy ε_r	Risk Premium ε_v	Inflation Target ε_τ	Inflation ε_π	Output ε_y
1	24.8	45.4	0.8	0.9	28.1
4	24.8	45.4	0.8	0.9	28.1
8	24.8	45.4	0.8	0.9	28.1
12	24.8	45.4	0.8	0.9	28.1
20	24.8	45.4	0.8	0.9	28.1
∞	24.8	45.4	0.8	0.9	28.1

Table 3. Forecast Error Variance Decompositions

One-Year Bond Rate

Quarters Ahead k	Monetary Policy ε_r	Risk Premium ε_v	Inflation Target ε_τ	Inflation ε_π	Output ε_y	Measurement Errors ε_4	ε_8	ε_{12}
1	32.0	12.5	50.3	0.0	2.2	3.1	0.0	0.0
4	16.2	13.3	63.7	0.0	5.9	1.0	0.0	0.0
8	9.7	10.0	71.9	0.1	7.7	0.6	0.0	0.0
12	7.8	7.6	76.7	0.2	7.3	0.4	0.0	0.0
20	5.9	5.5	82.6	0.3	5.5	0.3	0.0	0.0
∞	0.3	0.3	99.0	0.0	0.3	0.0	0.0	0.0

Two-Year Bond Rate

Quarters Ahead k	Monetary Policy ε_r	Risk Premium ε_v	Inflation Target ε_τ	Inflation ε_π	Output ε_y	Measurement Errors ε_4	ε_8	ε_{12}
1	16.5	17.7	61.8	0.1	2.0	0.0	1.8	0.0
4	7.8	15.2	72.3	0.2	4.0	0.0	0.5	0.0
8	4.7	10.8	79.3	0.3	4.6	0.0	0.3	0.0
12	3.7	8.1	83.4	0.4	4.1	0.0	0.2	0.0
20	2.7	5.7	88.1	0.3	3.0	0.0	0.2	0.0
∞	0.1	0.3	99.4	0.0	0.2	0.0	0.0	0.0

Three-Year Bond Rate

Quarters Ahead k	Monetary Policy ε_r	Risk Premium ε_v	Inflation Target ε_τ	Inflation ε_π	Output ε_y	Measurement Errors ε_4	ε_8	ε_{12}
1	9.7	16.1	72.1	0.3	1.7	0.0	0.0	0.0
4	4.3	12.6	79.9	0.4	2.8	0.0	0.0	0.0
8	2.5	8.6	85.5	0.5	3.0	0.0	0.0	0.0
12	2.0	6.4	88.7	0.5	2.5	0.0	0.0	0.0
20	1.4	4.3	92.2	0.4	1.7	0.0	0.0	0.0
∞	0.1	0.2	99.6	0.0	0.1	0.0	0.0	0.0

Table 4. Forecast Error Variance Decompositions

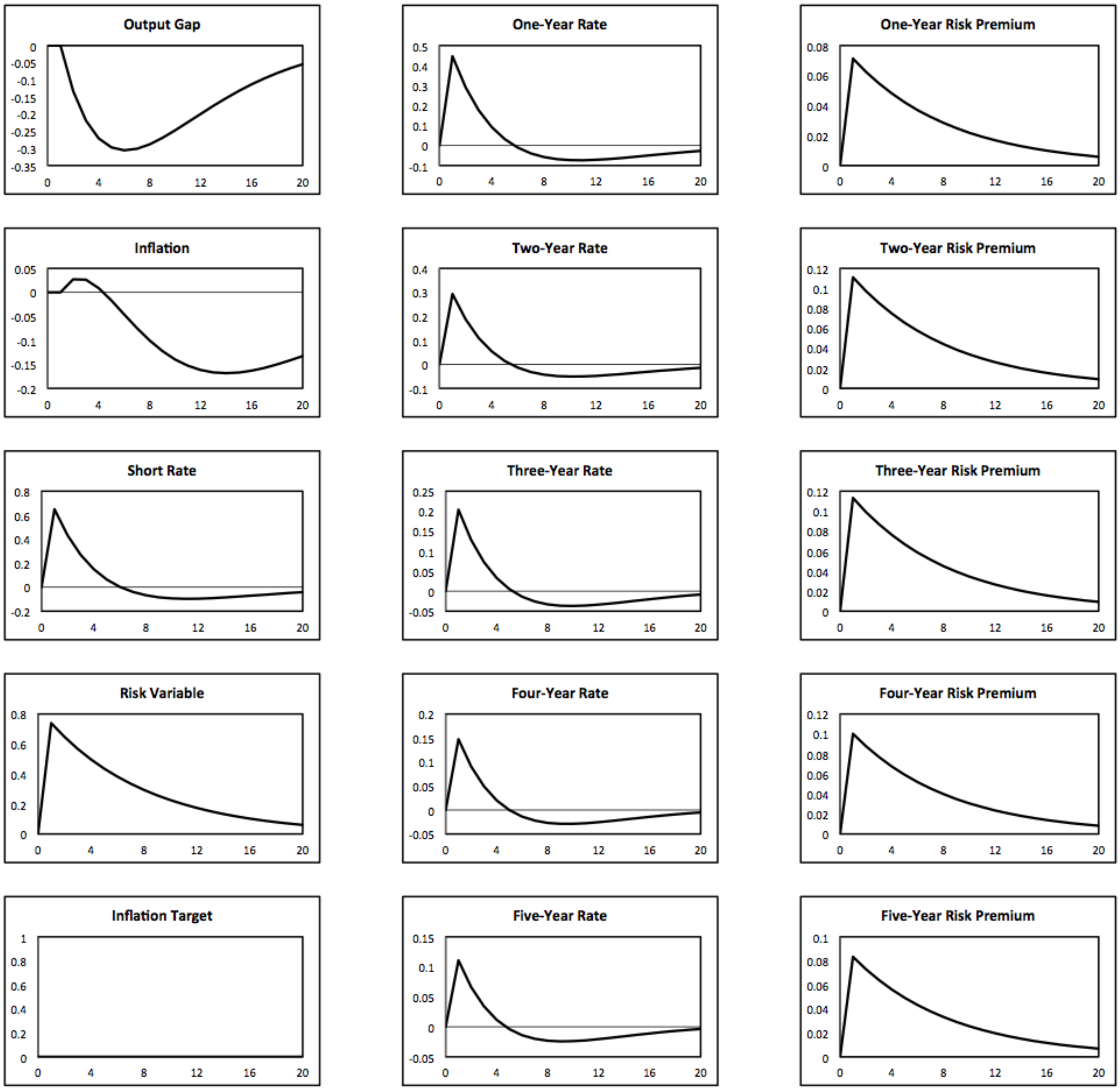
Four-Year Bond Rate

Quarters Ahead k	Monetary Policy ε_r	Risk Premium ε_v	Inflation Target ε_τ	Inflation ε_π	Output ε_y	Measurement Errors ε_4	ε_8	ε_{12}
1	5.9	12.1	79.1	0.4	1.5	0.0	0.0	0.9
4	2.5	9.1	85.6	0.4	2.1	0.0	0.0	0.3
8	1.4	6.0	89.9	0.5	2.0	0.0	0.0	0.1
12	1.1	4.4	92.3	0.4	1.7	0.0	0.0	0.1
20	0.8	2.9	94.8	0.3	1.1	0.0	0.0	0.1
∞	0.0	0.1	99.8	0.0	0.1	0.0	0.0	0.0

Five-Year Bond Rate

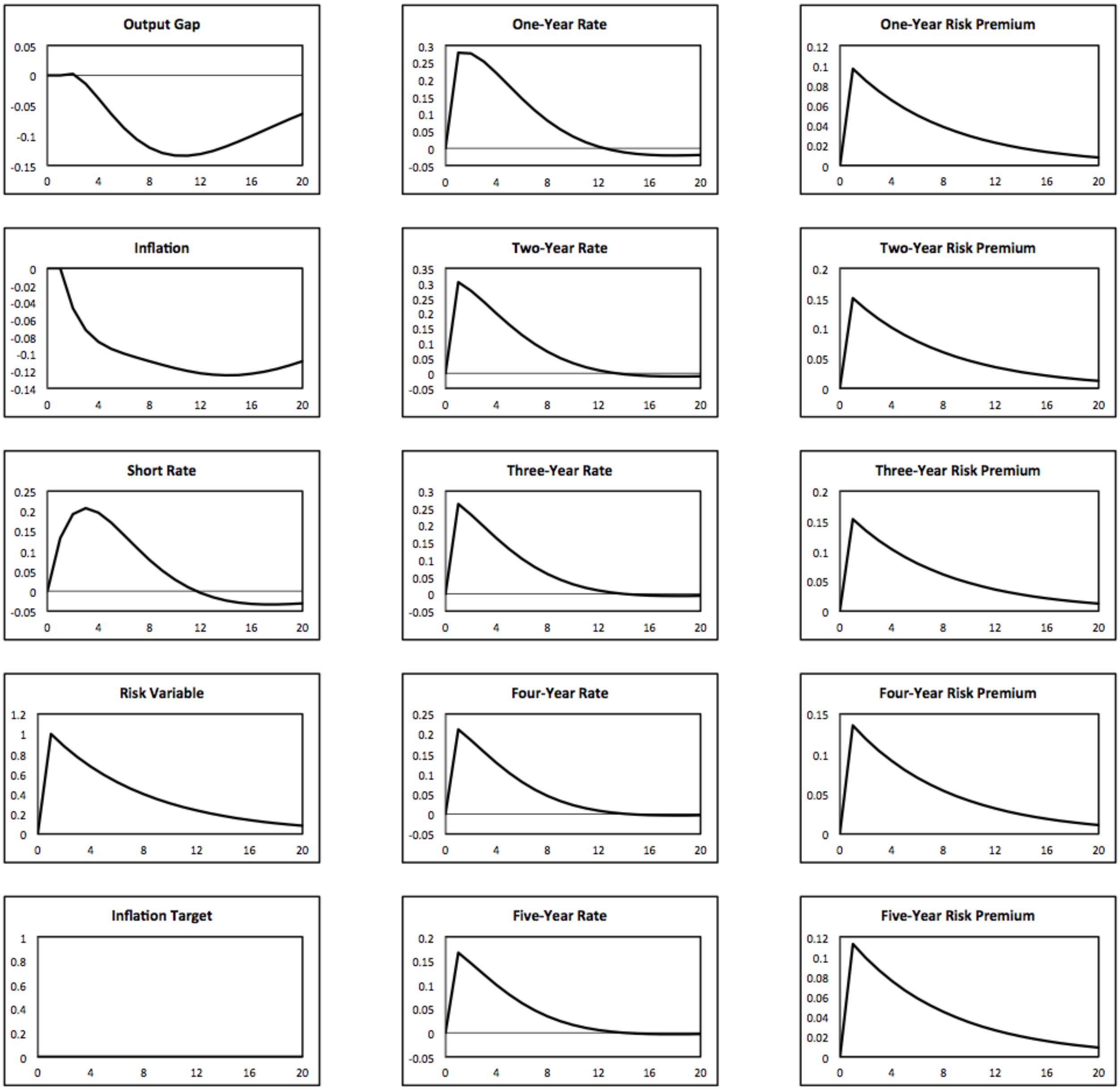
Quarters Ahead k	Monetary Policy ε_r	Risk Premium ε_v	Inflation Target ε_τ	Inflation ε_π	Output ε_y	Measurement Errors ε_4	ε_8	ε_{12}
1	3.8	8.7	85.8	0.4	1.4	0.0	0.0	0.0
4	1.5	6.2	90.2	0.4	1.7	0.0	0.0	0.0
8	0.9	4.0	93.2	0.4	1.5	0.0	0.0	0.0
12	0.7	2.9	94.9	0.4	1.2	0.0	0.0	0.0
20	0.5	1.8	96.6	0.3	0.8	0.0	0.0	0.0
∞	0.0	0.1	99.9	0.0	0.0	0.0	0.0	0.0

Figure 1. Impulse Responses to a Monetary Policy Shock



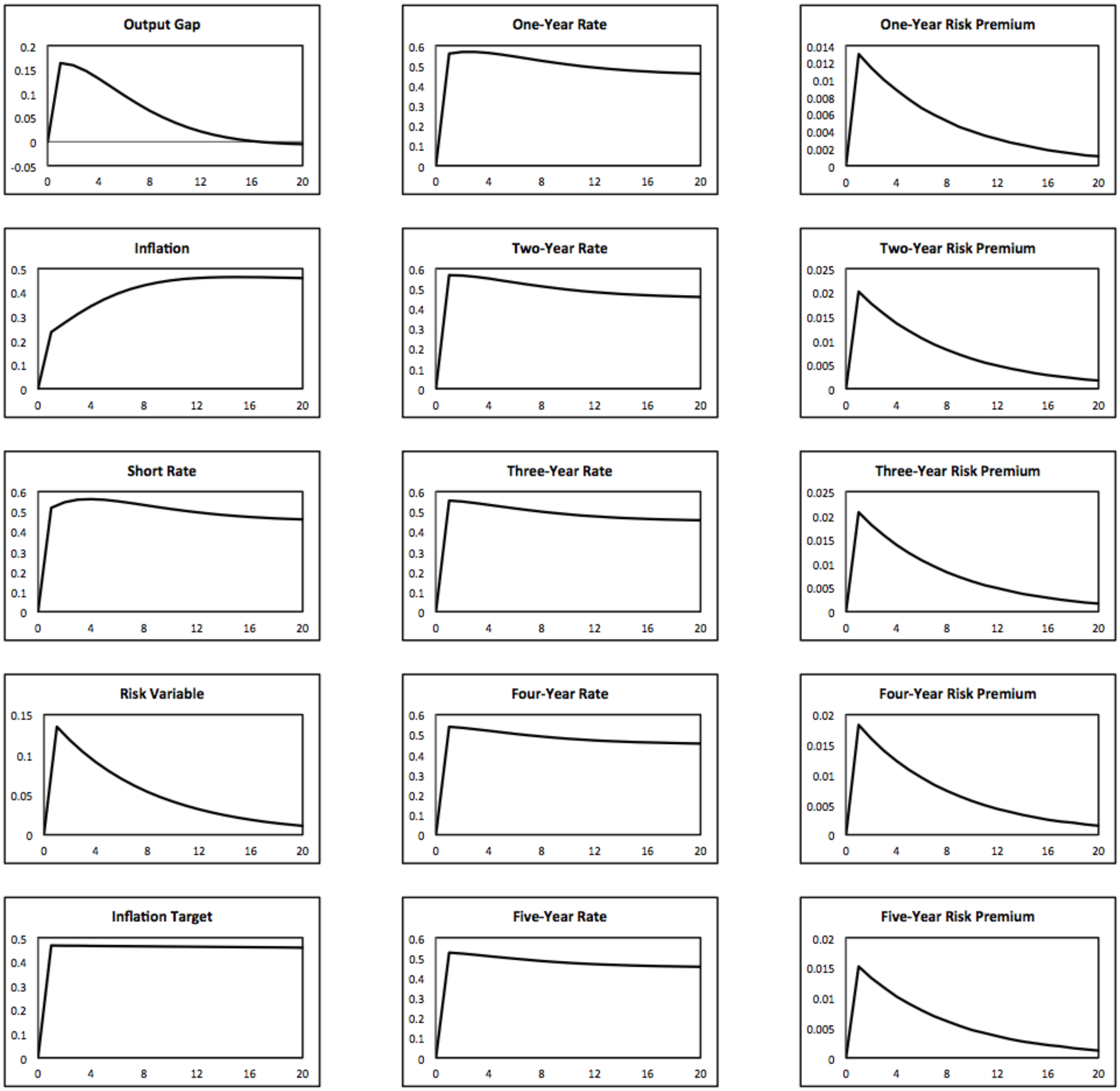
Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation monetary policy shock ε_r. The inflation and interest rates are in annualized terms.

Figure 2. Impulse Responses to a Risk Premium Shock



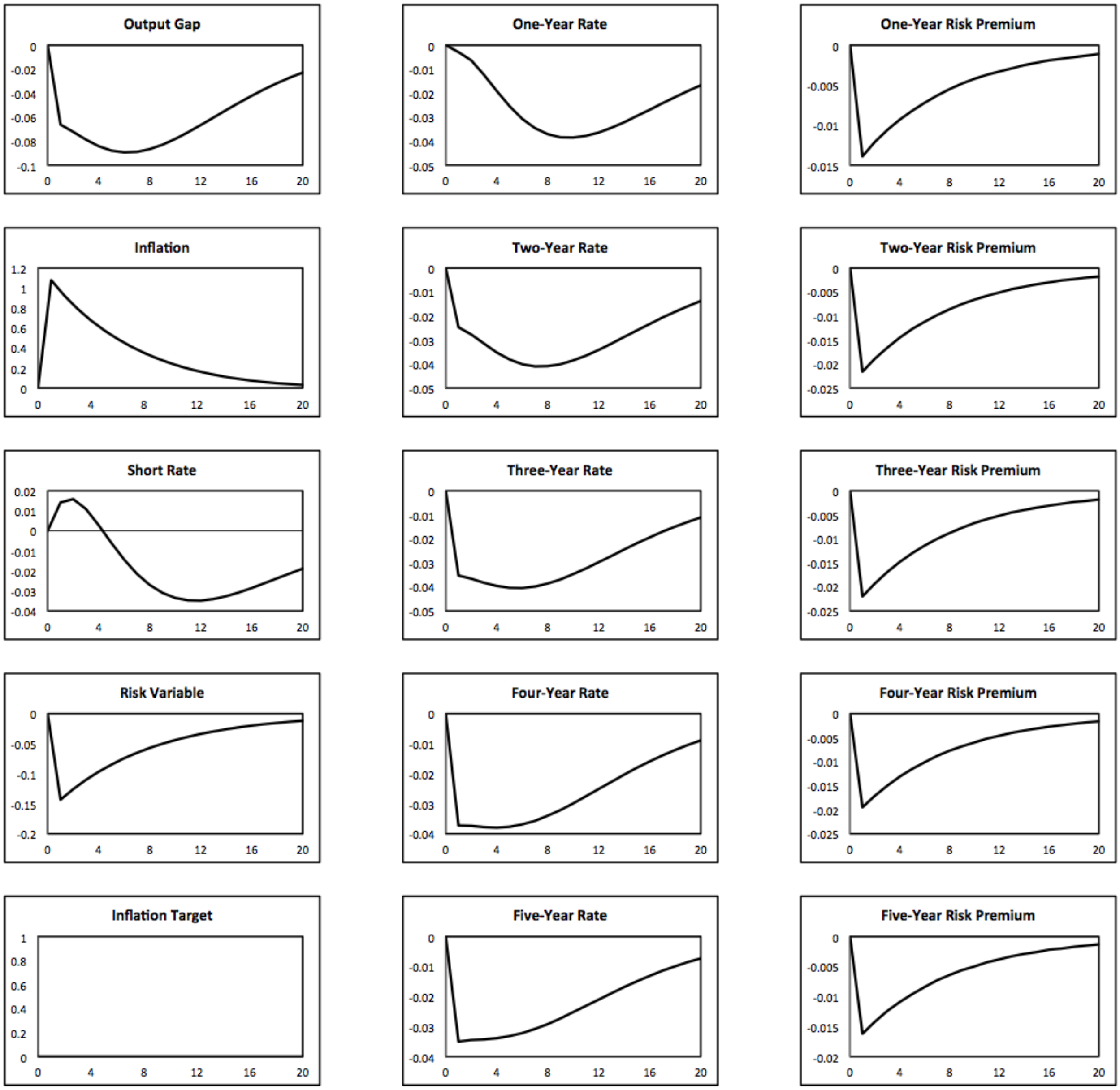
Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation risk premium shock ε_v. The inflation and interest rates are in annualized terms.

Figure 3. Impulse Responses to an Inflation Target Shock



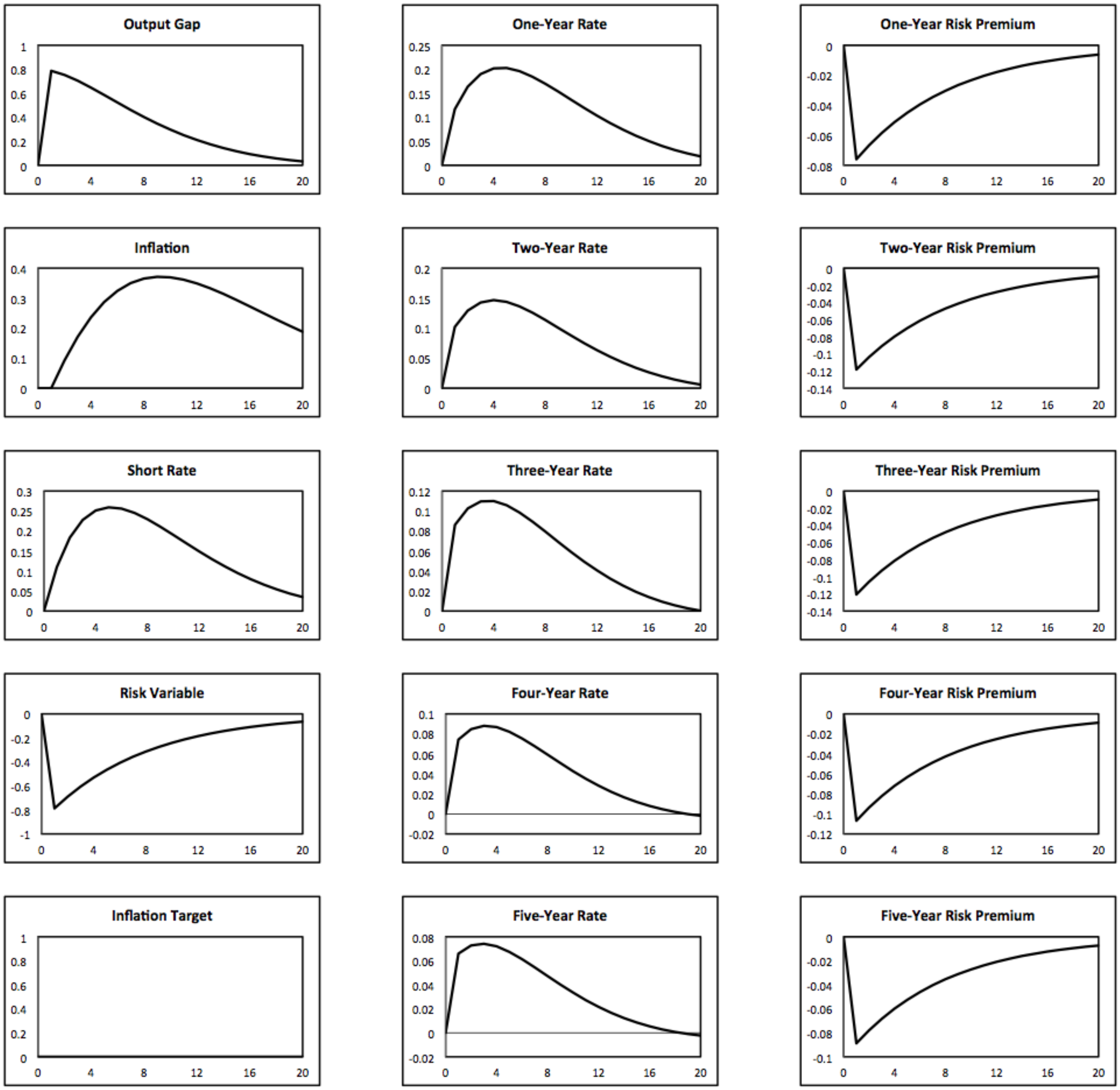
Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation inflation target shock ε_τ. The inflation and interest rates are in annualized terms.

Figure 4. Impulse Responses to an Inflation Shock



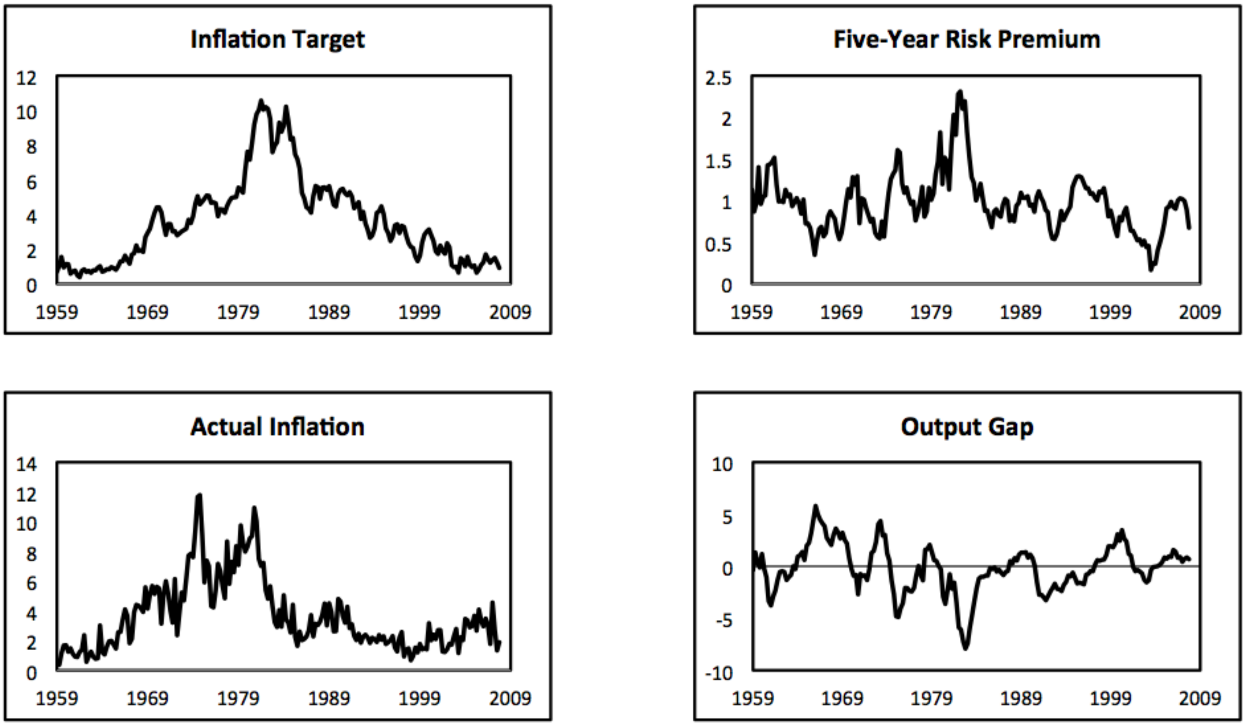
Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation inflation shock ε_π. The inflation and interest rates are in annualized terms.

Figure 5. Impulse Responses to an Output Shock



Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation output shock ε_y. The inflation and interest rates are in annualized terms.

Figure 6. Smoothed Estimates of the Inflation Target and the Five-Year Bond Risk Premium



The left-hand column compares the smoothed (full-sample) series for the estimated inflation target from the model to the actual series for the inflation rate, when both measures are expressed in annualized terms. The right-hand column compares the smoothed series for the estimated risk premium in the five-year bond yield to the actual series for the output gap, when the risk premium is expressed in annualized terms and the output gap in percentage points.