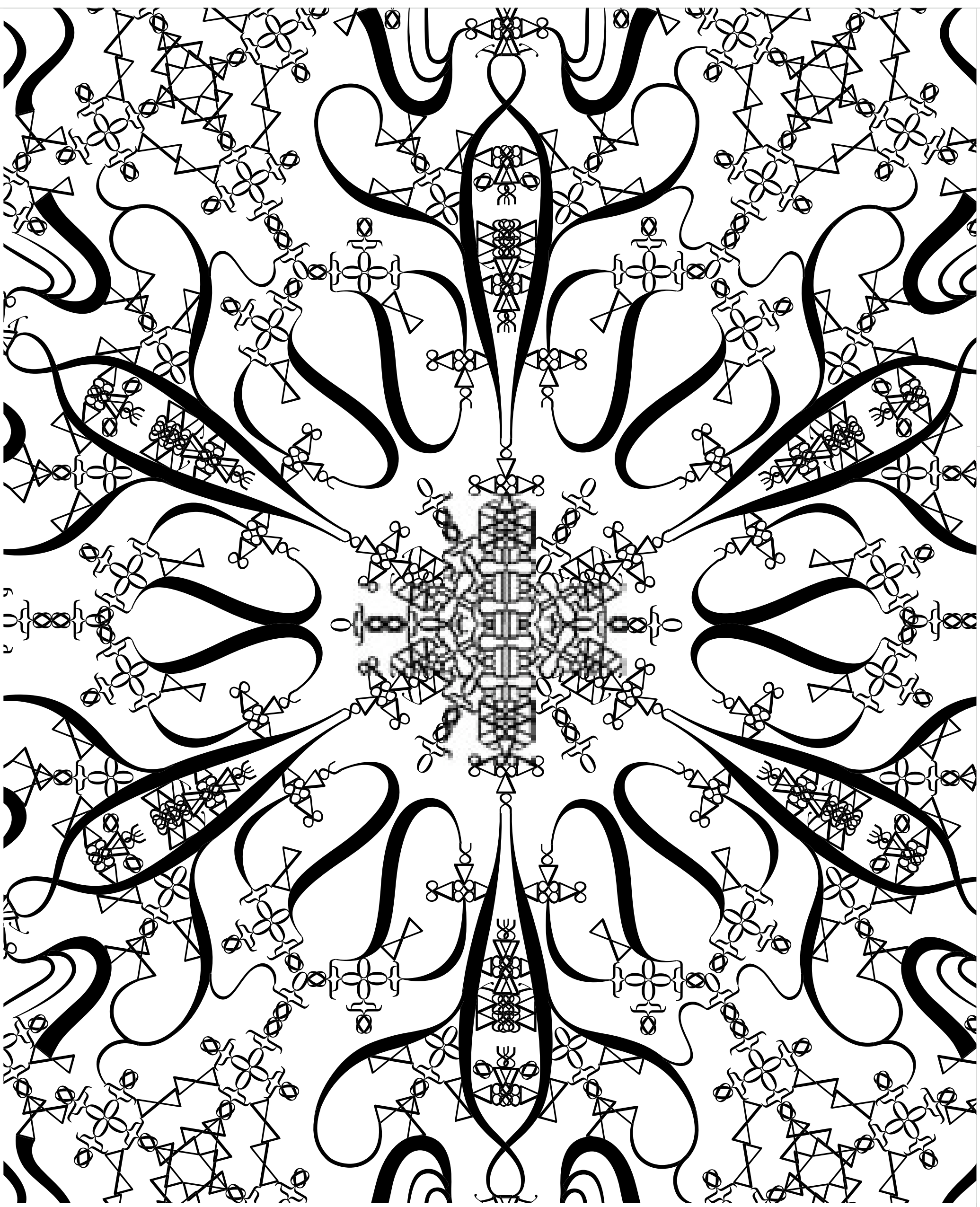
GUIDE TO GLOBAL SCIENCE CULTURE

# {REVIEWS}

# A FESTIVAL OF LIKENESS

Everything from butterflies, to algebra, to the universe is based on symmetry —so how come we know so little about it? By Jordan Ellenberg



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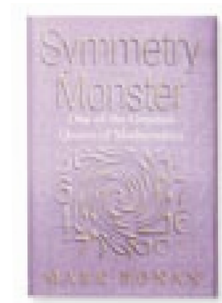
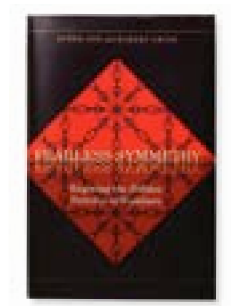


***Symmetry and the Monster***
By Mark Ronan
(Oxford University Press)



***Fearless Symmetry***
By Avner Ash and Robert Gross
(Princeton University Press)

Compared to other key concepts of contemporary mathematics and physics—infinity, uncertainty, undecidability, relativity—the notion of symmetry might seem a bit pedestrian. Things look the same as their reflection in the mirror—big whoop!

But symmetry conditions our understanding of the universe more completely than any of these other ideas. It would not be far off to say that our basic understanding of what the universe is depends, fundamentally, on the symmetries we believe it possesses. In the Newtonian universe, the symmetries were pretty simple—essentially, physics didn't change if you stood on a moving boat and everything worked perfectly. Except that it didn't actually describe the universe we live in. All the famous heart-worrying features of relativity—the contraction of objects moving at high speed, the constancy of the speed of light—are consequences of the fact that the universe obeys a different set of symmetries than the ones the Newtonian physicists imagined. These laws involve more complicated transformations, called the Lorentz symmetries, in which space and time can't be separated.

But here's where it starts getting tricky. We don't know—even now, with all the increased orders-of-magnitude in our powers of observation—what the symmetries of the universe truly are. We are still trying to understand what they could be. When string theorists contemplate their proposed 11-dimensional universe, they aren't (yet) asking whether the theory matches the real world—they're asking whether there even exists a theory to be tested. Which is where mathematicians come in. We get the fun job of working out what kinds of symmetries are mathematically possible, without the pressure of having to show that our constructions actually conform to anything in the universe.

Two new books, Mark Ronan's *Symmetry and the Monster* and Avner Ash and Robert Gross's *Fearless Symmetry*, treat different aspects of this attempt to discover and classify all possible symmetries. Ronan's topic is the classification of the finite simple groups and the development of the so-called "Monster group." Ash and Gross take on the whole grand subject of Galois representations and reciprocity laws. If the words in the preceding two sentences mean nothing to you, then you've begun to understand the terrific challenges these authors are taking on. Mathematics has proceeded so far and so quickly that a survey of contemporary study means catching up on a century or two of homework before being able to understand what someone proved last Tuesday.

That said, it's not so hard to describe what mathematicians mean by symmetry. A symmetry is no more or less than a way of transforming a mathematical object which is reversible. For instance, if our object is the set $\{0,1\}$, a perfectly good symmetry is the transformation T which exchanges 0 and 1. It's easy to reverse this transformation—just switch the two numbers back to their original positions! Similarly, there is a symmetry of the sphere (i.e. the surface of the Earth) which transforms each point into its antipode, the point directly opposite on the globe. Call this symmetry A. The fundamental insight that animates both these books is that it is the symmetries themselves, not the objects on which the symmetries operate, that are the true entities of interest. The antipodal symmetry is the antipodal symmetry whether it is transforming the surface of the Earth, the Moon or a tennis ball. What's more, the symmetry A and the symmetry T are themselves quite similar: when you execute them twice, the overall effect is the same as doing nothing at all. So though the surface of the Earth and the set $\{0,1\}$ look nothing alike, they share this very profound property.

*Symmetry and the Monster* tells the story of a decades-long project in which dozens of mathematicians joined forces against a problem of extraordinary difficulty known as "the classification of finite simple groups." The problem is to produce a list of fundamental systems of symmetries out of which the symmetries of every conceivable object, from planet, to set, to tennis ball, could be built. Ronan cleverly likens the project to the construction of a periodic table of elements, out of which the whole chemical smorgasbord of the material universe can be constructed by basic combination.

"The Monster" is the largest and still most mysterious of the "elements of symmetry" that mathematicians know as "simple groups." Perhaps the most striking aspect of the story is that, before it was shown to exist, nearly all of the Monster's identifying features were already known. To see how this could be, suppose that scientists were searching for llamas. And suppose that all of Peru has been mapped out and all llamas therein located, except for one very small area where the presence of a llama could be neither ascertained nor ruled out. And finally, suppose that tiny region were shaped exactly like a giant llama. Understandably you'd be pretty sure there was a llama there. You could even figure out the alleged llama's size, shape and position—all without actually observing the llama. This was precisely the status of the Monster until Robert Griess announced its existence in 1982, providing a satisfying denouement to Ronan's story. But not a conclusion—that comes with the development of "Monstrous Moonshine," a still poorly-understood connection between the Monster, number theory and physics.

Ash and Gross's *Fearless Symmetry* asks that its readers be fearless indeed—in fewer than 300 pages it delivers a tour through basic number theory, Galois theory and the rudiments of arithmetic geometry. This amount of material would be suitable for a one- or two-semester college course, but the authors manage to make it all accessible with an affable prose style and a healthy willingness to pause for a philosophical aside. The symmetries discussed by Ash and Gross are not the general kind that characters in *Symmetry and the Monster* labor to classify. They are symmetries attached to very special objects coming from number theory, sometimes called "motives." It is the interaction between modern ideas about symmetry and the classical apparatus of number theory—a subject in vogue since the Greeks—that forms the spine of *Fearless Symmetry*. Even a reader who knows the end of the story will delight at the relatively straight path the authors walk from the definition of sets and functions in the first chapter to the achievements of Andrew Wiles in the last. Wiles proved Fermat's Last Theorem in 1995, by showing that if the theorem were false, there would be a geometric object whose symmetries were so strange as to be impossible—such an object would be a llama in a part of Peru in which llamas had been positively ruled out. To get a less grievously vague sense of what Wiles did, one has to work a bit more—and for those who are willing, Ash and Gross's book will be an excellent companion. (Full disclosure: Ash and Gross work in my research area; one of my theorems is described in their book.)

Both of these books succeed in bringing to the fore an aspect of mathematics that some popularizers miss—that math is not a science of monuments, but a living tradition as vibrant as physics or ethics or law, one in which new monuments pop up weekly and old ones are retrofitted for purposes inconceivable to their creators. It's happening as we speak. And readers of these two books will know, at least in part, where it's happening now, and even (maybe) where it's going to happen next. ∞

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## 05.6 OOH, OOH, THAT SMELL

Though the scent of another woman on your man would usually spark an argument, female mice are attracted to males bearing the lingering odor of sexually primed females. The attraction is so strong that female mice will reject an otherwise-yummy and parasite-free male in favor of an infected one if he also carries the smell of a female in heat. Mate selection seems to involve oxytocin: females deficient of the hormone had impaired odor differentiation.