MATH1007
Homework 2
Answers

1. What is the sum of all numbers from 1 to 100 which are *not* multiples of 3? In other words, what is the sum of $1+2+4+5+7+8+\cdots+98+100$?

Answer: One way to do this problem is to add up $1+2+\cdots+100$ and then subtract $3+6+\cdots+99$. The first sum is $(1+100)100/2 = 5050$, and the second sum is $(3+99)33/2 = 1683$, and the difference is 3367.

Another way to do the problem is to think of it as the sum of two arithmetic sequences: $1+4+7+\cdots+100 = 1717$, and $2+5+\cdots+98 = (2+98)33/2 = 1650$. The sum is 3367, as before.

2. Nirvana Corporation manufactures widgets, and currently has 311 in stock. Nirvana plans to manufacture and store 14 widgets per week for the next 75 weeks. The storage cost is \$0.07/widget/week. This means that at the end of week 1, there are 325 widgets in stock, and the storage cost for that week is $325 \cdot 0.07$. What is the cost of storing all of these widgets for 75 weeks?

Answer: There are 325 widgets in stock at the end of the first week, and there are $311 + 14 \cdot 75 = 1361$ in stock at the end of 75 weeks. The storage cost is $(325+1361)(75)(0.07)/2 = \$4425.75$

3. A population decreases according to the formula $P_n = 0.8P_{n-1}$ with $P_0 = 3456$. What is the smallest value of n for which $P_n < 200$?

Answer: The formula is $P_n = 0.8^n P_0 = 0.8^n \cdot 3456$. If we set $P_n = 200$, we get $200 = 0.8^n \cdot 3456$, or $0.8^n \approx 0.0579$. One way to solve this is to use logarithms, but trial and error also works. You get $n \approx 12.8$. You can compute $P_{12} \approx 237.4945$, and $P_{13} \approx 189.9956$, so the answer to the question is 13.

4. Suppose that $P_0 = 13.4$ and $P_1 = 14.3$.
(*a*) If this is a sequence with linear growth, compute P_5.
(*b*) If this is a sequence with exponential growth, compute P_5.

Answer: (*a*) The formula is $P_n = P_0 + 0.9n = 13.4 + 0.9n$, and so $P_5 = 17.9$.
(*b*) The formula is $P_n = 1.0672^n(13.4)$, and so $P_5 \approx 18.5465$.

5. Suppose that you own a stock worth \$655.
(*a*) Suppose that the stock price increases 10% daily for 5 days. What is the stock price at the end of those 5 days?
(*b*) Suppose instead that the stock price starts at \$655 and increases 5% daily for 10 days. What is the stock price at the end of those 10 days?

Answer: (*a*) The stock price is $655 \cdot 1.10^5 \approx \$1054.88$. (*b*) The stock price is $655 \cdot 1.05^{10} \approx \$1066.93$.

6. According to Wikipedia, the population of England was 7,754,875 people on January 1, 1801 and 8,762,178 people on January 1, 1811.
(*a*) Assume that the population grew linearly. What was the first year in which the population was larger than 10 million on January 1?
(*b*) Assume that the population grew exponentially. What was the first year in which the population was larger than 10 million on January 1?

Answer: (*a*) We have $P_0 = 7754875$, $P_{10} = 8762178$, and $P_10 = P_0 + 10d$. That tells us that $d = (8762178 - 7754875)/10 = 100730.3$. The formula now is $P_n = 7754875 + 100730.3n$. Set $P_n = 10000000$ and solve for n, and we get $n \approx 22.2885$. Therefore, it takes 23 years for the population to be larger than 10 million on January 1, and so this would happen first in 1824.

(*b*) Now the formula is $P_n = R^n P_0$. We set $P_0 = 7754875$ and $P_{10} = 8762178$ as before, and get $R^{10} \approx 1.1299$, and therefore $R \approx 1.0123$. Now, we solve $10000000 = 7754875(1.0123^n)$, giving $1.0123^n \approx 1.2895$, and $n \approx 20.8203$. Therefore, it takes 21 years before the population is larger than 10 million on January 1, so this would happen first in 1822.

7. Suppose that you deposit \$750 in a bank account paying 5.34% annual interest, compounded annually. How many months must pass before the account will have more than \$900?

Answer: We solve $900 = 750(1.0534)^n$, and get $1.0534^n = 1.2$. Using logarithms or trial and error, we get $n \approx 3.5046$. Therefore it takes 4 years or 48 months before there is more than \$900 in the account.

8. Suppose that you deposit \$750 in a bank account paying 5.34% annual interest, compounded monthly. How many months must pass before the account will have more than \$900?

Answer: The monthly rate is 5.34/12, and so our formula is $900 = 750(1.0045)^n$. That gives $1.0045^n = 1.2$, or $n \approx 41.06$. The balance surpasses 900 in 42 months.

9. Suppose that you deposit \$800 in a bank account paying compound interest, compounded monthly, at some unknown rate. Suppose that in 20 months, the account has \$900. What is the APR for the account? What is the APY?

Answer: We have $900 = 800R^{20}$. That means $R^{20} = 1.125$, or $R \approx 1.0059$. But this gives the monthly rate of interest. The APR is $(1.0059 - 1) \cdot 12 \approx 0.0709$, or 7.09%. The APY is computed by first calculating $1.0059^{12} \approx 1.0732$, and then subtracting 1 to give 7.32%.