MATH1007
Homework 5
Answers

1. Suppose that in an arithmetic sequence, $P_3 = 7$ and $P_{11} = 41$.
(a) What is P_{15}?
(b) What is $P_3 + \cdots + P_{11}$?

Answer: Start by computing the common difference d. We know that $P_{11} = P_3 + 8d$, and solving tells us that $d = 4.25$. Therefore, $P_{15} = P_{11} + 4d = 58$. The formula for the sum of an arithmetic sequence tells us that $P_3 + \cdots + P_{11} = \frac{P_3 + P_{11}}{2}(9) = 216$. Notice that there are 9 terms in the sum, not 8.

2. Suppose that in a geometric sequence, $P_3 = 7$ and $P_{11} = 41$.
(a) What is P_{12}?
(b) What is $P_3 + \cdots + P_{11}$?

Answer: Start by computing the common ratio R. We know that $P_{11} = P_3 R^8$, and solving gives $R \approx 1.2473$. Therefore $P_{12} = 1.2473 P_{11} \approx 51.1381$. The formula for the sum of a geometric sequence tells us that $P_3 + \cdots + P_{11} = \frac{P_{12} - P_3}{R - 1} \approx 178.5011$.

3. A quiz with 4 questions was administered to a class, and scored on a scale of 0 through 4. Here are the results:

Grade	0	1	2	3	4
Number of students	13	18	19	25	22

(a) What is the mean grade?
(b) What is the median grade?
(c) What are the first quartile (Q_1) and third quartile (Q_3)?
(d) What is the 90th percentile?

Answer: Notice that there are $N = 97$ students in the class
(a) The mean is $\mu = \frac{13 \cdot 0 + 18 \cdot 1 + 19 \cdot 2 + 25 \cdot 3 + 22 \cdot 4}{13 + 18 + 19 + 25 + 22} \approx 2.2577$. ($b$) The median M is the 50th percentile. If there are 97 students, then the median is the 49th one, counting from the smallest to the largest score, and that student's score is 2. (c) The first quartile Q_1 is the 25th percentile. Our definition makes that the 25th student, with a score of 1. The third quartile Q_3 is the 75th percentile, and our definition makes that the 73rd student, with a score of 3. (d) The 90th percentile is the 88th student, with a score of 4.

4. You are purchasing a home, and need to borrow $135000. You have two options:
(a) Bank A offers a 30-year loan with 5.5% APR and monthly payments.
(b) Bank B offers a 30-year loan with 5.3% APR, monthly payments, and a 4% fee that can be added to the initial loan.

Which bank offers a better deal?

Answer: In each case, we apply the usual formula. For the first one, we have $P = 135000$, $n = 360$, $r = 0.055/12$, and $R = 1 + r$. We get a monthly payment of approximately

$766.52. For the second one, we have $P = 140400$, $n = 360$, $r = 0.053/12$, and $R = 1 + r$. We get a monthly payment of approximately $779.65. Bank A offers a better deal.

5. I have a $350,000 mortgage with a 6.75% APR, compounded monthly, and a 25-year term.

(a) What is my monthly payment?

(b) After I have made 15 payments, what is the outstanding principal on the loan? The *outstanding principal* is the amount of principal still owed to the bank.

Answer: (a) We apply the usual formula, with $P = 350000$, $r = 0.0675/12$, $R = 1 + r$, and $n = 300$, and get $M = 2418.19$.

(b) We keep r, R, and M as above, set $n = 300 - 15 = 285$, and solve for P. We get $P = 342986.36$, and that is the outstanding principal.

6. Suppose that a set of examination scores is normally distributed, with mean $\mu = 81.4$ and standard deviation $\sigma = 2.6$.

(a) What are Q_1 and Q_3?

(b) Find scores A and B so that 95% of the examination scores are between A and B.

Answer: (a) The formulas are $Q_1 = \mu - 0.675\sigma = 79.645$ and $Q_3 = \mu + 0.675\sigma = 83.155$.

(b) We compute $\mu - 2\sigma = 76.2$ and $\mu + 2\sigma = 86.6$. Those are the values of A and B.

7. Suppose that the mean (average) of 6 numbers is 11.2, and that the smallest of the 6 numbers is 5.1. Let x be the *maximum* of the 6 numbers.

(a) What is the smallest possible value for x?

(b) What is the largest possible value for x?

Answer: (a) This needs some thought. The smallest possible value for x occurs when all 5 of the remaining numbers are the same. We have $\frac{5.1+5x}{6} = 11.2$, which gives $x = 12.42$. The smallest possible value for x occurs when the 6 numbers are 5.1, 12.42, 12.42, 12.42, 12.42, and 12.42.

(b) The largest possible value for x occurs when 5 of the numbers are equal to 5.1. Then we get $\frac{5 \cdot 5.1+x}{6} = 11.2$, and solving gives $x = 41.7$. The largest possible value for x occurs when the 6 numbers are 5.1, 5.1, 5.1, 5.1, 5.1, and 41.7.