MATH1007

Homework 8

Due Friday, November 11

When submitting homework, please remember the following:

- Show all work leading to each solution.
- *You must use a staple* (not paper clip) if your answers are longer than a single page.
- Do not submit crossed-out or sloppy work.
- Do not submit ripped or torn pages.
- Be sure to submit your own work.

As mentioned in class, sometimes Borda count uses point values other than 1 for last place, 2 for next-to-last, and so on. The first few problems explore various other point counts.

1. Baseball's Cy Young Award uses a variant of Borda count in which the point are awarded using a 7-4-3-2-1 scheme. In other words, first place is worth 7 points, second place is worth 4 points, third place is worth 3 points, fourth place is worth 2 points, and fifth place is worth 1 point.

Construct a preference schedule with 5 candidates so that one candidate wins when using the standard Borda count, with 5-4-3-2-1 awards, and a different candidate wins when using the modified 7-4-3-2-1 scheme.

2. Another modification of the Borda count awards 0 points for last place, 1 point for next-to-last, and so on. In an election with 3 candidates, this modification gives 2 points for first-place, 1 point for second place, and 0 points for last place. Show that in an election with 3 candidates, the modified 2-1-0 Borda count will always rank the candidates in the same order as the standard 3-2-1 Borda count.

3. Yet another modification of the Borda count awards 1 point for *first* place, 2 points for *second* place, and so on. In this version, the winner is the candidate with the *fewest* points.

Show that in an election with 3 candidates, awarding the points using the 1-2-3 modification, and ranking the candidates from fewest to most points, yields the same election result as the standard Borda count (ranking the candidates in the usual way, from most to fewest points).

4. Suppose that in an election among 3 candidates, we award 5 points for first place, 3 points for second place, and 1 point for third place. Does this method produce the same result as the standard Borda count method? If this method yields the same ranking of candidates as the standard Borda count, explain why. If it is not equivalent, give a preference schedule which demonstrates that this method can give a different result.

5. In class, we saw that Borda count can violate the majority criterion.

(a) As a warm up, recapitulate the example given in class with 3 candidates, in which one candidate has a majority of first-place votes, and a different candidate wins the Borda count.

(b) When you look at your answer to part (a), it seems as if the problem is that first-place is not worth enough points. Let's try a variation of Borda count in which first place is worth 20 points, second place is worth 2 points, and third place is worth 1 point. Is it now possible for a candidate to have a majority of first-place votes and not win the Borda count? If you think that it is possible, give a preference table that shows a candidate with the majority of first-place votes does not win the Borda count. If you think that it is not possible, give a convincing argument in support of your position.

6. For many years, elections in Louisiana have been conducted using what is called an "open" primary system, also sometimes called "plurality with runoff" or "jungle primary." If someone gets a majority, that candidate wins. Otherwise, the top two candidates compete in a runoff election. We will assume that no voters change their preferences between the two elections.

(a) Give an example to show that plurality with runoff can produce a different outcome from plurality with elimination.

(b) Give an example to show that plurality with runoff violates the monotonicity condition.

(c) Give an example to show that a Condorcet winner does not necessarily win using plurality with runoff.

7. Here is a preference schedule:

	Number of voters			
	6	**5**	**4**	**2**
First choice	G	M	D	S
Second choice	M	G	S	D
Third choice	D	D	M	G
Fourth choice	S	S	G	M

(a) Compute the winner of an election using instant runoff voting.

(b) Now suppose that in the last column, G switches place with D, giving this new preference schedule:

	Number of voters			
	6	**5**	**4**	**2**
First choice	G	M	D	S
Second choice	M	G	S	G
Third choice	D	D	M	D
Fourth choice	S	S	G	M

Who wins using instant runoff voting?

(c) Comment on this situation.