MATH1007
Homework 9
Due Friday, November 18

When submitting homework, please remember the following:

• Show all work leading to each solution.

• *You must use a staple* (not paper clip) if your answers are longer than a single page.

• Do not submit crossed-out or sloppy work.

• Do not submit ripped or torn pages.

• Be sure to submit your own work.

1. On Monday morning, the price of a share of a particular stock is $6. Suppose that the price increases by the same percentage amount on Monday, Tuesday, Wednesday, Thursday, and Friday, and at the end of the day on Friday, the price of the stock is $10. What was the percentage increase each day of the week?

2. Suppose in a geometric sequence, $P_5 = 7$ and $P_{10} = 6$. What is P_3? Compute the answer to 4 decimal places.

3. Compute the Banzhaf power of each player in the following voting systems:
 (a) $[7 : 4, 3, 2, 1]$.
 (b) $[8 : 4, 3, 2, 1]$.
 (c) $[9 : 4, 3, 2, 1]$.

4. Compute the Banzhaf power of each player in the voting system $[6 : 5, 1, 1, 1, 1, 1]$. HINT: Compute the power of P_1, and use that to compute the power of the other 5 players.

5. There are 100 members of the United States Senate. How many ways are there to pick a committee consisting of 3 of them?

6. There are 20 female Senators and 80 male Senators. How many ways are there to pick a committee consisting of
 (a) 3 women?
 (b) 3 men?
 (c) 2 women and 1 man?
 (d) 2 men and 1 woman?
What is the relationship between your answers to this problem and to the previous problem?

7. The Venusian War Council consists of 3 little green men, and 3 giant rabbits. In order to declare war on their enemies, at least 2 of the little green men must vote yes, and at least 2 of the giant rabbits must vote yes.
 (a) How many winning coalitions are there?
 (b) What is the Banzhaf power of each of the 6 members of the Venusian War Council?
 (c) Explain why it is *not* possible to describe the Venusian War council as a weighted voting system.

8. The Martian War Council consists of 3 little green women, and a giant squid. In order to declare war on their enemies, at least 2 of the little green women must vote yes, and the giant squid must also vote yes.

(a) What is the Banzhaf power of each of the 4 members of the Martian War Council?

(b) Is it possible to describe the Martian War Council using a weighted voting system? If so, find the weights; if not, explain why it is not possible.

9. To try to find an election system which avoids some of the problems associated with plurality voting, someone suggests the following complicated system:

(i) If there is a Condorcet winner (that is, if there is a winner of all head-to-head match-ups), then that person is the winner.

(ii) If there is no Condorcet winner, then use Borda count to determine the winner.

Here is a preference schedule:

Number of voters

	7	6	2
First choice	P	D	W
Second choice	D	W	P
Third choice	W	P	D

(a) Who is the winner using the system outlined above?

(b) Suppose that candidate W (a losing candidate) withdraws from the election. Who wins the resulting election between P and D?

(c) Comment on this situation.

10. A weighted voting system with 3 players has exactly 3 winning coalitions: $\{P_1, P_2, P_3\}$, $\{P_1, P_2\}$, and $\{P_1, P_3\}$.

(a) Find the critical players in each winning coalition.

(b) Find the Banzhaf power distribution of this system.

(c) Find the Shapley–Shubik distribution of this system.