

MATH1007

Homework 10

Due Friday, December 2

When submitting homework, please remember the following:

- Show all work leading to each solution.
- *You must use a staple* (not paper clip) if your answers are longer than a single page.
- Do not submit crossed-out or sloppy work.
- Do not submit ripped or torn pages.
- Be sure to submit your own work.

1. Compute the Shapley–Shubik power of each player in the following voting systems:

(a) [7 : 4, 3, 2, 1].

(b) [8 : 4, 3, 2, 1].

(c) [9 : 4, 3, 2, 1].

2. Compute the Shapley–Shubik power of each player in the voting system [6 : 5, 1, 1, 1, 1, 1].

3. Here is a preference schedule which only contains the rankings of candidate A. There are, of course, also candidates B, C, D, and E in the race.

	Number of voters		
	7	6	2
First choice	A	*	*
Second choice	*	*	*
Third choice	*	*	*
Fourth choice	*	*	*
Fifth choice	*	A	A

Is it possible for A to win using standard Borda count? If so, then fill in the preference schedule so that A wins. If not, explain why it is not possible for A to win.

4. Suppose that the United Nations Security Council expanded to include Germany as a sixth permanent member. In this scenario, there are 6 permanent members and 10 rotating members. A winning coalition must contain at least 12 countries, and include all 6 of the permanent members.

- (a) Can this voting system be modelled as a weighted voting system? If so, what are the weights for the permanent and for the rotating members? If not, explain why it is not possible to find such weights.
- (b) What is the Banzhaf power of Germany?
- (c) What is the Shapley–Shubik power of Germany?

5. The government of Freedonia has a king (K), a prime minister (P), and three royal advisors (A_1 , A_2 , and A_3). The rules of the government are:

- Every winning coalition must contain either the king or the prime minister.

- The king and the prime minister together can combine to form a winning coalition, with or without any royal advisors.
 - The king and 2 or 3 royal advisors is a winning coalition.
 - The prime minister and all 3 royal advisors is a winning coalition.
- (a) Can this government be modelled by using a weighted voting system? If so, what are the weights for the king, the prime minister, and for a royal advisor? If not, explain why it is not possible to find such weights.
- (b) Compute the Banzhaf power of the king, the prime minister, and a royal advisor.
- (c) Compute the Shapley–Shubik power of the king, the prime minister, and a royal advisor. We know that there are $5!$ sequential coalitions to list, but if you are clever, you can solve the problem by listing fewer than half that many sequential coalitions.
6. Suppose that 5 men (α , β , γ , δ , and ε) and 5 women (A, B, C, D, and E) rank the members of the opposite gender as follows:

	A	B	C	D	E
α	1,4	3,2	2,4	4,2	5,1
β	3,2	2,3	4,2	5,1	1,5
γ	1,3	4,1	5,1	3,3	2,4
δ	5,1	1,4	2,5	4,5	3,2
ε	2,5	4,5	1,3	3,4	5,3

This chart means that α has the ranking A, C, B, D, E, and C has the ranking γ , β , ε , α , δ , for example.

- (a) Use the Gale–Shubik matching algorithm with the men choosing to produce a stable assignment.
- (b) Use the Gale–Shubik matching algorithm with the women choosing to produce a stable assignment.
7. Enormous State University has been given a total of 200 faculty positions to allocate to various departments, based on their enrollment figures. The departments and their enrollments are:

Department	A	B	C	D	E
Enrollment	1646	762	2081	1066	6945

- (a) Find the standard divisor, and find each department’s standard quota.
- (b) Allocate the new faculty positions to the 5 departments using Hamilton’s method.
- (c) Allocate the new faculty positions to the 5 departments using Jefferson’s method.
- (d) Allocate the new faculty positions to the 5 departments using Webster’s method.
- (e) Allocate the new faculty positions to the 5 departments using Huntington–Hill’s method.