

MATH1180
Homework 7
Due Monday, April 3, 2017

Please (re)read Chapter 4 and sections 5.1–4 of the Baglivo textbook.

Please submit solutions to the following problems. Use 4 decimal places of accuracy for probabilities unless a problem says otherwise.

When submitting homework, please remember the following:

- Show all work leading to each solution.
- *Staple* all sheets together. A paper clip is not acceptable.
- Do not submit crossed-out or sloppy work.
- Do not submit ripped or torn pages.
- Be sure to submit your own work.

PROBLEM 1 (BM). *Osteoporosis* is a condition in which the bones become brittle due to loss of minerals. To diagnose osteoporosis, an elaborate apparatus measures bone mineral density (BMD).

BMD is usually reported in standardized form, where the standardization is based on a population of healthy young adults. The World Health Organization (WHO) criterion for osteoporosis is a BMD that is 2.5 or more standard deviations *below* the mean for healthy young adults. Assume that BMD measurements for young healthy adults are normally distributed.

- (a) Using the standard established by the WHO, what percentage of healthy young adults have osteoporosis?

Osteopenia is low bone mineral density, defined by the WHO as a BMD that is between 1 and 2.5 standard deviations *below* the mean for healthy young adults.

- (b) Using this standard, what percentage of healthy young adults have osteopenia?
(c) Using the standards established by the WHO, what percentage of healthy young adults have neither osteoporosis nor osteopenia?

PROBLEM 2 (DSB, BM). High blood cholesterol increases your risk of heart disease and stroke. Individuals with serum cholesterol levels between 200 milligrams per deciliter (mg/dl) and 240 mg/dl are classified as having “borderline-high” cholesterol; individuals with serum cholesterol levels greater than 240 mg/dl are classified as having “high” cholesterol.

- (a) For women aged 35–44, serum cholesterol levels follow a normal distribution with mean 205 mg/dl and standard deviation 45 mg/dl. Let X be the serum cholesterol level of a randomly chosen woman in this age range. Find $P(X \leq 200)$, $P(200 < X \leq 240)$, and $P(X > 240)$.
(b) For men aged 35–44, serum cholesterol levels follow a normal distribution with mean 210 mg/dl and standard deviation 49 mg/dl. Let X be the serum cholesterol level of a randomly chosen man in this age range. Find $P(X \leq 200)$, $P(200 < X \leq 240)$, and $P(X > 240)$.

PROBLEM 3 (PG). A recent report from the National Center for Health Statistics (NCHS) states that the distribution of weights for men in the United States aged 35 to 45 is well-approximated by a normal distribution with mean 176.4 pounds and standard deviation 31.3 pounds.

- (a) Find the probability that the weight of a randomly chosen man from this population is
- (i) Less than 130 pounds,
 - (ii) Greater than 210 pounds,
 - (iii) Between 130 and 210 pounds.

- (b) If 5 men are randomly chosen from this population, find the probability that the weights of at least 2 of these men are outside the interval $[130, 210]$.

PROBLEM 4 (PG). Continuing with the setting of the previous problem, let \bar{X} be the sample mean of a simple random sample of 16 men chosen from this population.

- (a) Find the interval containing the central 80% of the \bar{X} distribution.
(b) Repeat part (a) using the sample size of 64 instead of 16.
(c) The length of an interval is the difference between its upper endpoint and its lower endpoint. Compute and compare the lengths of the intervals from parts (a) and (b).

PROBLEM 5 (AF). The Psychomotor Development Index (PDI) is a scale of infant development. Assume that PDI scores follow a normal distribution with mean 100 and standard deviation 15.

- (a) Find the probability that a randomly chosen infant's PDI score is between 98 and 107.
(b) Find the interval containing the central 90% of the PDI scores distribution.

PROBLEM 6 (AF). Continuing with the previous problem, let \bar{X} be the sample mean of PDI scores for a simple random sample of 25 infants.

- (a) Find the standard error of the sample mean.
(b) Find the probability that the sample mean lies between 98 and 107.
(c) Find the interval containing the central 90% of the \bar{X} distribution.

PROBLEM 7 (AF). For the population of people who suffer occasionally from migraine headaches, suppose that $p = 0.55$ is the proportion who get some relief from their symptoms by taking ibuprofen.

Let X be the number of individuals who get some relief by taking ibuprofen in a simple random sample of 84 individuals chosen from this population, and let $\hat{p} = \frac{X}{84}$ be the sample proportion.

- (a) Find the standard error of the sample proportion.
(b) Find the probability that the sample proportion is greater than 0.65.
(c) Find the interval containing the central 70% of the \hat{p} distribution.

PROBLEM 8 (PG). Tetanus is a potentially life-threatening infection of the nervous system caused by the bacteria *Clostridium tetani*. Recent government reports tell us that, on average, $\lambda = 4.2$ cases of tetanus are reported per month in the United States.

Let X be the number of cases of tetanus reported in a 3-year (36-month) period, and let $\hat{\lambda} = \frac{X}{36}$ be the sample monthly mean rate.

- (a) Find the standard error of the sample monthly mean rate. Explain what the standard error describes.
(b) Find the probability that the sample monthly mean rate is at most 4.9 cases per month.
(c) Find the interval containing the central 95% of the $\hat{\lambda}$ distribution.

PROBLEM 9 (AF). The Department of Public Health at the University of Western Australia conducted a survey in which they randomly sampled general practitioners (GPs) in Australia. One question asked whether the GP had ever studied alternative therapy, such as acupuncture, hypnosis, homeopathy, and yoga. Of 304 respondents, 142 said yes.

Assume the respondents are a simple random sample of GPs in Australia.

- (a) Find the sample proportion of GPs responding yes.
(b) Find the estimated standard error of the sample proportion.
(c) Construct and interpret a 99% confidence interval for the population proportion.

(d) From your answer to part (c), do you believe that fewer than 60% of GPs in Australia have studied alternative therapy? Why?

PROBLEM 10. Public health officials interested in determining the monthly mean rate of suicides in western Massachusetts gathered information over a 5-year period. The following table gives the frequency distribution of the number of cases reported each month during this time:

Number of cases, x :	0	1	2	3	4	5	6	7	<i>Total:</i>
Number of months:	2	10	11	15	12	3	3	4	60

(There were no cases reported in each of two months, 1 case reported in each of ten months, 2 cases reported in each of eleven months, and so forth.)

- Find the total number of suicides reported over the 5-year period and the sample monthly mean rate of suicides.
- Find the estimated standard error of the sample monthly mean rate.
- Construct and interpret a 90% confidence interval for the population monthly mean rate.

PROBLEM 11 (PG). One of the goals of the Edinburgh Artery Study was to investigate the risk factors of peripheral arterial disease among persons 55 to 74 years of age. As part of this study, researchers collected LDL (or bad) cholesterol levels for simple random samples from four separate sub-populations of subjects:

- patients with intermittent claudication or interruptions in movement,
- those with major asymptomatic disease,
- those with minor asymptomatic disease, and
- those with no evidence of disease at all.

The following table gives summary information: sample size (n), sample mean LDL cholesterol level in mmol/liter (\bar{x}), sample standard deviation of LDL levels in mmol/liter (s), estimated standard error of the sample mean (se), and a 99% confidence interval for the mean level of LDL cholesterol in the population from which the sample was drawn.

	n	\bar{x}	s	se	99%CI
1. Intermittent Claudication	73	6.22	1.62	0.190	[5.731, 6.709]
2. Major Asymptomatic Disease	105	5.81	1.43	0.140	[5.449, 6.171]
3. Minor Asymptomatic Disease	240	5.77	1.24	????	????
4. No Disease	1080	5.47	1.31	????	????

- Fill in the missing information. Please use 3 decimal places of accuracy in your table.
- If confidence intervals for two population means do not overlap, then we have fairly strong evidence that the two population means are different.

Based on this criterion, do you think that any pairs of means are different? (You will need to consider $\binom{4}{2} = 6$ pairs of means: means for the first and second populations, means for the first and third populations, and so forth.)

PROBLEM 12 (AF). A simple random sample of 50 records yields a 95% confidence interval of 21.5 to 23.0 years for the mean age at first marriage of women in a certain county. Explain what is wrong with each of the following interpretations of this interval.

- If simple random samples of 50 records were repeatedly selected, then 95% of the time the sample mean age at first marriage for women would be between 21.5 and 23.0 years.

- (b) Ninety-five percent of the ages at first marriage for women in the county are between 21.5 and 23.0 years.
- (c) We can be 95% confident that \bar{x} is between 21.5 and 23.0 years.
- (d) If we repeatedly sampled the entire population, then 95% of the time the population mean would be between 21.5 and 23.0 years.

PROBLEM 13 (AF). A tax assessor wants to estimate the mean property tax bill for all homeowners in Madison, Wisconsin, to within 100 dollars. A survey ten years ago produced a sample mean of 2500 dollars, with a sample standard deviation of 800 dollars.

- (a) Using the sample standard deviation from the ten year old survey as your best guess for the standard deviation, find the sample size needed to estimate the mean property tax bill to within 100 dollars with 95% confidence.
- (b) A sample standard deviation of 1250 dollars is more realistic than the one from the ten year old survey. Repeat part (a) using 1250 dollars as your best guess for the sample standard deviation.

PROBLEM 14 (DS). Glaucoma is a disease of the optic nerve. Suppose that the proportion of adults in a certain population who have glaucoma is unknown.

- (a) Find the sample size needed to estimate this proportion to within 0.04 with 90% confidence.
- (b) Find the sample size needed to estimate this proportion to within 0.02 with 90% confidence.

PROBLEM 15 (DS). Cryptosporidiosis is a painful diarrheal disease caused by the microscopic parasite *Cryptosporidium*. Occasionally, oocysts (the dormant form of the parasites causing this disease) are detected in public drinking water supplies. A recent study found that the sample mean rate of oocysts was 7.6 per fifty-liter drum of water.

- (a) Using the results from the recent study as your best guess for the mean rate per fifty-liter drum, find the number of fifty-liter drums needed to estimate the current mean rate to within 0.5 parasites per drum with 99% confidence.
- (b) Using twice the results from the recent study as your best guess for the mean rate per fifty-liter drum (that is, using 15.2 instead of 7.6), find the number of fifty-liter drums needed to estimate the current mean rate to within 0.5 parasites per drum with 99% confidence.