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Homework 2  
Mathematics 2216.01  
Due September 7, 2022

1. Prove that  $\log_{10} 11$  is irrational.
2. Prove using induction that

$$\sum_{k=0}^n x^k = \frac{x^{n+1} - 1}{x - 1}$$

for  $n \geq 0$ .

3. Define two sequences of numbers  $a_n$  and  $b_n$  recursively with the formulas  $a_0 = 2$ ,  $b_0 = 1$ , and

$$\begin{aligned} a_{n+1} &= 2a_n + 3b_n \\ b_{n+1} &= a_n + 2b_n \end{aligned}$$

Show using induction that

$$\left(\frac{a_n}{b_n}\right)^2 - 3 = \frac{1}{b_n^2}.$$