Rob Gross
Homework 4
Mathematics 2216.01
Due September 12, 2022

1. As usual, define the Fibonacci numbers with

$$F_1 = 1$$
$$F_2 = 1$$
$$F_n = F_{n-1} + F_{n-2}, \quad n \geq 3$$

Prove that $F_n < 1.9^n$.

2. Let n be a positive integer. Prove using induction that

$$\lim_{x \to 0^+} x(\log x)^n = 0.$$

Hint: Apply l'Hôpital's rule, but make sure that you do it correctly.
NOTE: The notation $\log x$ means the natural logarithm of x. The notation $\lim_{x \to 0^+}$ means that x tends to 0 and is positive. The inequality $x > 0$ is required because $\log x$ is only defined for positive x.

3. Let n be a positive integer. Prove using induction that $\dfrac{(2n)!}{2^n n!}$ is always an integer.