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Homework 5
Mathematics 2216.01
Due September 14, 2022

1. Suppose that a is a real number larger than 1, and k is a positive integer. Prove using induction and l'Hôpital's rule that

$$\lim_{x \to \infty} \frac{x^k}{a^x} = 0.$$

2. Using trial and error, find the smallest positive integer N so that $5N^2 < 2^N$, and then prove by induction that if $n \geq N$, then $5n^2 < 2^n$.

3. Let n be a positive integer. Prove that

$$\binom{3n}{n}$$

is a multiple of 3. NOTE: You do not need to use induction.