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Homework 6
Mathematics 2216.01
Due September 16, 2022

1. There are times when it is useful to define F_k for $k \leq 0$. The standard definition is that $F_0 = 0$ and if $k < 0$, then $F_k = (-1)^{k+1}F_{-k}$. For example, $F_{-1} = (-1)^0 F_1 = 1$, $F_{-2} = (-1)^{-1}F_2 = -1$, $F_{-3} = (-1)^{-2}F_3 = 2$, and $F_{-4} = (-1)^{-3}F_4 = -3$.

It is easy to see that $F_1 = F_0 + F_{-1}$, $F_0 = F_{-1} + F_{-2}$, and $F_{-1} = F_{-2} + F_{-3}$. Show that if k is a negative integer, and $k < -1$, then $F_k = F_{k-1} + F_{k-2}$.

2. There is a formula that allows us to compute Fibonacci numbers more easily. That is the point of this problem.

The two solutions of the equation $x^2 - x - 1 = 0$ are $\alpha = \frac{1+\sqrt{5}}{2}$ and $\beta = \frac{1-\sqrt{5}}{2}$. It is sometimes helpful to use the formulas $\alpha + \beta = 1$, $\alpha\beta = -1$, and $\alpha - \beta = \sqrt{5}$. You may use all of those without verification.

Prove using induction that

$$F_k = \frac{\alpha^k - \beta^k}{\sqrt{5}}$$

if $k \geq 1$.