Rob Gross
Homework 7
Mathematics 2216.01
Due September 19, 2022

1. Let a be any positive real number, and let k be a positive integer. Prove that
$$\lim_{x\to\infty} \frac{(\log x)^k}{x^a} = 0.$$
Here, as usual $\log x$ refers to the natural logarithm of x.

2. Suppose that $f(x)$ and $g(x)$ are functions with derivatives of all orders. For simplicity in what follows, write f and g rather than $f(x)$ and $g(x)$. The product rule is $(fg)' = f'g + fg'$.

Write $f^{(n)}$ for the nth derivative of f. In other words,
$$f^{(n)} = \frac{d^n f}{dx^n}.$$
We also define $f^{(0)} = f$.

Prove by induction that
$$(fg)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(k)} g^{(n-k)}$$
The case $n = 1$ is the product rule, so you do not need to verify that the formula is true when $n = 1$.

3. Use the formula in the previous problem to compute the fourth derivative of $e^{2x}\sin(3x)$. You might want to check your answer using a computer algebra system.