Rob Gross
Homework 8
Mathematics 2216.01
Due September 21, 2022

1. Use Euclid's algorithm to find integers x and y so that $ax + by = \gcd(a, b)$ if
   (1) $a = 86$ and $b = 16$.
   (2) $a = 21$ and $b = 91$.
   (3) $a = -72$ and $b = 312$.

2. Suppose that n is a positive integer. Prove using induction and integration by parts that
$$\int_0^1 (1 - x^2)^n \, dx = \frac{2^{2n}(n!)^2}{(2n+1)!}.$$
HINT: The case $n = 1$ is easy (but you need to do it anyway). The rest is not trivial. For the induction, write $(1 - x^2)^{k+1} = (1 - x^2)^k(1 - x^2)$ and expand. For integration by parts, set $u = x$ and $dv = -x(1 - x^2)^k \, dx$.