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Homework 9
Mathematics 2216.01
Due September 23, 2022

1. Suppose that a and b are positive integers, and d is the greatest common divisor of a and b. Show that $\frac{a}{d}$ and $\frac{b}{d}$ are relatively prime.

2. The Gamma function is defined by the formula

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$$

for $x \geq 1$. This is an improper integral, and you may assume that the integral converges if $x \geq 1$. Prove that $\Gamma(1) = 1$.

3. Use integration by parts, along with a limit from a previous homework, to prove that $\Gamma(n+1) = n\Gamma(n)$ if n is a positive integer.

4. Prove using induction that if n is a positive integer, then $\Gamma(n) = (n-1)!$.