Rob Gross
Homework 15
Mathematics 2216.01
Due October 21, 2022

1. Given sets A and B define the *symmetric difference*

$$A \triangle B = (A \cup B) \setminus (A \cap B).$$

(*a*) Prove that $A \triangle B = (A \setminus B) \cup (B \setminus A)$.
(*b*) Prove that $A \triangle B = B \triangle A$.
(*c*) Find a set X with the property $X \triangle A = A$ for every set A.
(*d*) Prove that

$$(A \triangle B) \triangle C = A \triangle (B \triangle C).$$

2. If n is any nonnegative integer, write $g_n = 2^{2^n} + 1$. Prove using induction that

$$g_0 g_1 g_2 \cdots g_{n-1} = g_n - 2.$$

The numbers g_n were first studied by the mathematician Pierre de Fermat, who conjectured that they are always prime. If your calculator is sufficiently good, you can verify that g_5 in fact is not prime. When you consider that $g_5 = 4294967297$, it's hard to blame Fermat for being unable to notice that it is not prime.