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Homework 16
Mathematics 2216.01
Due October 24, 2022

1. Define $f : \mathbf{Z} \rightarrow \mathbf{Z}$ with the multipart definition

$$f(n) = \begin{cases} n + 6 & n \text{ is even} \\ 2n + 7 & n \text{ is odd} \end{cases}$$

Is this function surjective? Is it injective?

2. If n is any positive integer, remember that how we defined the set μ_n :

$$\mu_n = \{z \in \mathbf{C} : z^n = 1\}.$$

Let m and n be positive integers, and suppose that $m|n$. Prove that $\mu_m \subseteq \mu_n$.

3. If n is any nonnegative integer, write $g_n = 2^{2^n} + 1$. We proved that

$$g_0 g_1 g_2 \cdots g_{n-1} = g_n - 2.$$

Let m and k be any two unequal nonnegative integers. Prove that $\gcd(g_m, g_k) = 1$.