Rob Gross
Homework 19
Mathematics 2216.01
Due October 31, 2022

1. Define a function $f : \mathbf{Q} \to \mathbf{Q}$:

$$f(x) = \begin{cases} \dfrac{x}{2x-1} & x \neq \dfrac{1}{2} \\ \dfrac{1}{2} & x = \dfrac{1}{2} \end{cases}$$

   (a) Is this function injective?
   (b) Is this function surjective?

2. Suppose that $f : X \to Y$ is a function, and $U \subseteq X$.
   (a) Show that $U \subseteq f^{-1}(f(U))$.
   (b) Find an explicit example of a function $f : X \to Y$ and a subset $U \subseteq X$ so that $U \neq f^{-1}(f(U))$.
   (c) Show that if f is an injection, then $U = f^{-1}(f(U))$.
   (d) Show that if f is not an injection, you can always find a particular subset $U \subseteq X$ so that $U \neq f^{-1}(f(U))$.