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Homework 21
Mathematics 2216.01
Due November 11, 2022

1. Suppose that $f : X \to Y$, $g : Y \to Z$, and $g \circ f : X \to Z$ is a bijection. On the examination, we saw that under these circumstances, f is an injection and g is a surjection.

(a) Give explicit functions $f : \mathbf{Z} \to \mathbf{Z}$ and $g : \mathbf{Z} \to \mathbf{Z}$ so that $g \circ f$ is a bijection and f is *not* a bijection.

(b) Give explicit functions $f : \mathbf{Z} \to \mathbf{Z}$ and $g : \mathbf{Z} \to \mathbf{Z}$ so that $g \circ f$ is a bijection and g is *not* a bijection.

2. Let $A = \{x \in \mathbf{R} : 0 < x < 1\}$. Define a function $f : A \to \mathbf{R}$ with the formula
$$f(x) = \frac{2x - 1}{2x(1 - x)}.$$

Prove that f is a bijection by constructing an inverse function $g : \mathbf{R} \to A$. To check that you have the right answer, check a few numerical cases. For example, you could check

- $f(g(-1.1)) = -1.1$.
- $f(g(0)) = 0$.
- $f(g(3.7)) = 3.7$.
- $g(f(0.1)) = 0.1$.
- $g(f(0.9)) = 0.9$.

You do not need to show me any of this work, only your function g.