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Homework 23
Mathematics 2216.01
Due November 16, 2022

1. Find the smallest positive integer x that satisfies both of these congruences:

$$x \equiv 11 \pmod{13}$$
$$x \equiv 12 \pmod{18}$$

2. Find the smallest positive integer x that satisfies these three congruences:

$$x \equiv 5 \pmod{13}$$
$$x \equiv 4 \pmod{14}$$
$$x \equiv 8 \pmod{15}$$

3. Recall that $\mu_{15} = \{z \in \mathbf{C} : z^{15} = 1\}$, and that there are 15 elements in μ_{15}. Define an equivalence relation on μ_{15} by setting $z \sim w$ if $o(z) = o(w)$. You do not need to prove that this is an equivalence relation.

List the equivalence classes of this relation. REMINDER: $o(z)$, the *order* of z, is defined to be the smallest positive integer k so that $z^k = 1$. If $z \in \mu_{15}$, we know that $o(z) \leq 15$. We proved quite a bit of useful information about the order.