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Homework 25
Mathematics 2216.01
Due November 21, 2022

1. Suppose that D is an integral domain. (Remember that an integral domain is a commutative ring with identity with the property that if r and s are non-zero, then rs is non-zero.) Define a relation \sim on $D \times (D \setminus \{0\})$ with the formula $(a, b) \sim (c, d)$ if $ad = bc$. Prove that the relation \sim is reflexive, symmetric, and transitive.
2. Now define a relation \sim on $\mathbf{Z}/24\mathbf{Z} \times (\mathbf{Z}/24\mathbf{Z} \setminus \{0\})$ with the same formula: $(a, b) \sim (c, d)$ if $ad = bc$. Show that \sim is *not* transitive. (Note that $\mathbf{Z}/24\mathbf{Z}$ is not an integral domain, because $[3]_{24} \cdot [16]_{24} = [0]_{24}$.)