Rob Gross
Homework 26
Mathematics 2216.01
Due November 28, 2022

1. What is the remainder when 116^{100} is divided by 11?

2. Suppose that p is a prime. Show that the solutions to the congruence $x^2 \equiv 1 \pmod{p}$ are $x \equiv \pm 1 \pmod{p}$.

3. Find all solutions to the congruence $x^2 \equiv 1 \pmod{8}$. How many different solutions are there? Trial and error is acceptable.

4. For each field F and each pair $a(x), b(x) \in F[x]$, find $q(x), r(x) \in F[x]$ such that $a(x) = b(x)q(x) + r(x)$ and $\deg(r) < \deg(b)$ or $r = 0$.
   (a) $F = \mathbf{Q}$, $a(x) = x^6 + 3x^2 - 2$, $b(x) = x^3 + 3x^2 + 1$.
   (b) $F = \mathbf{Z}/13\mathbf{Z}$, $a(x) = x^4 + 3x + 2$, $b(x) = 2x^2 + x - 1$. Compute inverses by trial and error if you don't have the energy to use the Euclidean algorithm.
Typing this is hard, so you can submit a hand-written solution if you prefer.