

Rob Gross
Homework 28
Mathematics 2216.01
Due December 5, 2022

1. Suppose that $f(x), g(x) \in \mathbf{C}[x]$ are monic polynomials, with $\deg(f) = \deg(g) = n \geq 1$. Suppose also that

$$f(1) = g(1) \quad f(2) = g(2) \quad \cdots \quad f(n) = g(n).$$

Show that $f(x) = g(x)$. HINT: Let $h(x) = f(x) - g(x)$. What is the degree of h ? What are its roots?

2. Factor $x^4 + 1$ into irreducible factors in $\mathbf{C}[x]$.
3. Factor $x^4 + 1$ into irreducible factors in $\mathbf{R}[x]$.
4. Factor $x^4 + 1$ into irreducible factors in $\mathbf{Q}[x]$.
5. Factor $x^4 + 1$ into irreducible factors in $\mathbf{Z}/2\mathbf{Z}[x]$.
6. Factor $x^4 + 1$ into irreducible factors in $\mathbf{Z}/3\mathbf{Z}[x]$.