Mathematics 3310.01
Homework 4
Due September 28, 2018

Please remember that if your submission is longer than one page, you must use a stapler or paper clip.

1. Let G be the group U_{19}. For each of the following subgroups, write down the cosets of the subgroup.
   (a) $\langle 7 \rangle$.
   (b) $\langle 12 \rangle$.
   (c) $\langle 5 \rangle$.

2. Now that you computed the cosets in the previous problem, write the multiplication tables for
   (a) $U_{19}/\langle 7 \rangle$.
   (b) $U_{19}/\langle 12 \rangle$.
   (c) $U_{19}/\langle 5 \rangle$.

3. The group U_{16} has 8 elements. Let $H_1 = \langle 15 \rangle$ and let $H_2 = \langle 9 \rangle$. Both H_1 and H_2 have 2 elements, so U_{16}/H_1 and U_{16}/H_2 both consist of 4 cosets. Write the multiplication tables for those two quotient groups.

4. If $m \in \mathbf{Z}$ and $m \geq 2$, and α is any integer, the function $L_\alpha : \mathbf{Z}/m\mathbf{Z} \to \mathbf{Z}/m\mathbf{Z}$ defined by $L_\alpha(k) = \alpha k$ is a group homomorphism (where the group operation on $\mathbf{Z}/m\mathbf{Z}$ is addition). You do not need to verify that L_α is a group homomorphism. Instead, compute $\ker(L_\alpha)$ if
   (a) $m = 10$ and $\alpha = 6$.
   (b) $m = 11$ and $\alpha = 9$.
   (c) $m = 12$ and $\alpha = 8$.

5. Suppose that G_1, G_2, and G_3 are groups, and $f_1 : G_1 \to G_2$ and $f_2 : G_2 \to G_3$ are both group homomorphisms. Show that $f_2 \circ f_1 : G_1 \to G_3$ is also a group homomorphism. As usual, $f_2 \circ f_1(g_1)$ is defined to be $f_2(f_1(g_1))$ for every $g_1 \in G_1$.