Mathematics 3310.01
Homework 8
Due November 2, 2018

Please remember that if your submission is longer than one page, you must use a stapler or paper clip.

1. Suppose that F is a field, and $f(x) \in F[x]$ is a polynomial of degree $n > 1$. Suppose that $f(x)$ is *not* irreducible. Show that one of the irreducible factors of f must have degree no greater than $\frac{n}{2}$.

2. Decompose $\frac{1}{(x^2+x+1)(x^2+2x+1)}$ into partial fractions with powers of irreducible polynomials in the denominators.

3. Suppose that K and L are fields, both characteristic 0, and $K \subset L$. (For example, you can think of $\mathbf{Q} \subset \mathbf{R}$.) Suppose that $f(x) \in K[x]$. You can also think of $f(x) \in L[x]$ because K is a subset of L. Suppose that when $f(x)$ is factored into irreducible factors in $K[x]$, it has no multiple factors. Show that when $f(x)$ is factored into irreducible factors in $L[x]$, it has no multiple factors.

4. Suppose that $f(x), g(x) \in \mathbf{Z}[x]$. Suppose that $f(x)g(x)$ is primitive. Prove that $f(x)$ and $g(x)$ are both primitive.

5. Suppose that $f(x) \in \mathbf{Z}[x]$ is monic. Suppose that $f(x) = g(x)h(x)$, where $g(x), h(x) \in \mathbf{Q}[x]$ and $g(x)$ and $h(x)$ are both monic. Prove that $g(x), h(x) \in \mathbf{Z}[x]$.