MATH4410
Homework 2
Due February 19, 2021
Your Name Here

Your answers must be in the form of a typed PDF file, and must be e-mailed to me by 5PM EST on February 19. Please name your file hw02-lastname-firstname.pdf. For example, my solution file is hw02-gross-robert.pdf.

I will try to acknowledge receipt of each e-mail.

1. Sometimes a differential equation can be solved by a clever change of variables. That is the theme of the first few problems.
   - (a) Solve the first-order linear differential equation $y' = y + t$ by finding an integrating factor, as we have done many times by now.
   - (b) You can also solve this differential equation with a change of variables. Substitute $y + t = z$, and eliminate y from the differential equation. The resulting differential equation is separable. Solve by separation of variables, and then remove z from the result and express y as a function of t.

2. The differential equation

$$y' - 2ty = ty^2$$

is not linear.
   - (a) Show that this differential equation is separable, and solve by separation of variables.
   - (b) This equation can also be solved with the substitution $z = y^{-1}$. Make that substitution, solve the resulting linear differential equation for z, and then remove z from the solution and express y as a function of t.

3. Make the substitution $y = tz$ to solve each of these differential equations:

   - (a) $y' = \dfrac{t+y}{t-y}$.
   - (b) $y' = \dfrac{y^2}{ty+t^2}$.
   - (c) $y' = \dfrac{t^2+ty+y^2}{t^2}$.
   - (d) $y' = \dfrac{y+te^{-2y/t}}{t}$.

In most cases, you will only be able to find an equation relating y and t.

4. Each of the differential equations in the previous problem has the form $y' = f(t, y)$, where $f(kt, ky) = f(t, y)$ for any non-zero real number k. Show that every differential equation of this form becomes separable after making the substitution $y = tz$.

5. Suppose that $y = \phi(t)$ solves the differential equation

$$y' + (\cos t)y = e^{-\sin t}.$$

Suppose as well that k is any integer. Show that $\phi(k\pi) - \phi(0) = k\pi$.

6. In class, we solved the first-order linear differential equation

$$2y' + ty = 2$$

with an initial condition $y(0) = 1$. This is also in the text: section 2.1, example 5. Change the initial condition to be $y(0) = A$ for any real number A, repeat the process of solving the differential equation, and evaluate $\lim_{t\to\infty} y(t)$. Your answer might depend on A.