MATH4426
Robert Gross
Homework 1
Due September 8, 2023

Please remember that your answers to these problems must be typed and stapled, and are due at the start of class.

1. How many different arrangements are there of the letters CONTENTMENT?

2. How many ways can 8 people named A, B, C, D, a, b, c, and d line up if:
(a) there are no restrictions and all possibilities are allowed?
(b) A and a are adjacent? (Either can be to the left of the other.)
(c) no 2 people with upper-case names can be next to each other and no two people with lower-case names can be next to each other?
(d) all 4 of A, B, C, and D must be adjacent? For example, a possible line-up could be $cACDBadb$.
(e) all 4 of A, B, C, and D must be adjacent, and all 4 of a, b, c, and d must be adjacent? For example, a possible line-up could be $CDABcbda$.
(f) A and a must be adjacent, B and b must be adjacent, C and c must be adjacent, and D and d must be adjacent? For example, a possible line-up could be $BbdDaACc$.
NOTE: Each part of the problem is independent of the other parts.

3. Suppose that a 3-digit number is any whole number from 0 to 999.
(a) How many 3-digit numbers are there with 3 unequal digits?
(b) How many 3-digit numbers are there with 3 unequal digits in decreasing order?
(c) How many 3-digit numbers are there with 3 unequal digits in increasing order?

4. Suppose instead that a 3-digit number is any whole number from 100 to 999.
(a) How many 3-digit numbers are there with 3 unequal digits?
(b) How many 3-digit numbers are there with 3 unequal digits in decreasing order?
(c) How many 3-digit numbers are there with 3 unequal digits in increasing order?

5. A committee consists of 5 Republicans, 4 Democrats, and 7 independent voters.
(a) How many ways are there to pick a sub-committee of 6 voters?
(b) How many ways are there to pick a sub-committee of 6 voters without including any independents?
(c) How many ways are there to pick a sub-committee with 3 Republicans and 3 Democrats?
(d) How many ways are there to pick a sub-committee with 2 Republicans, 2 Democrats, and 2 independents?
(e) How many ways are there to pick a sub-committee of 6 voters with at least 1 Republican and 1 Democrat?

6. How many solutions are there to the equation

$$a + b + c + d + c = 42$$

if
(a) a, b, c, d, and e are nonnegative integers?

(b) a, b, c, d, and e are positive integers?

(c) a, b, c, d, and e are positive even integers?

7. Suppose that n, m, and r are non-negative integers.

(a) Prove that

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}.$$

Hint: Consider a group of n men and m women. How many different ways are there to choose r of them?

(b) Show that

$$\binom{2n}{n} = \sum_{k=0}^{n}\binom{n}{k}^2.$$